Available online at http://scik.org
Commun. Math. Biol. Neurosci. 2021, 2021:68
https://doi.org/10.28919/cmbn/5859
ISSN: 2052-2541

DISCRETE MATHEMATICAL MODELING AND OPTIMAL CONTROL FOR BAYOUD DISEASE OF DATE PALM

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Abstract. In this work, we propose a discrete mathematical model that describes the Bayoud, the disease caused by the Fusarium oxysporum fungi [24], which spreads by contact between palm roots and the Fusarium oxysporum soil fungi, the susceptible palm (S), the infected (I), the recovered palm (R), the Fusarium oxysporum fungi (C). We also focus on the importance of treatments to find the optimal strategies to minimize the number of infected palms and also the number of Fusarium oxysporum fungi population and maximize the number of the recovered palm under treatment. We propose three optimal control strategies for this disease in order to slow down the growth of Fusarium oxysporum, the agent of Bayoud's disease. The result of this optimal system is solved numerically by Matlab. Consequently, the obtained results confirm the performance of our optimization strategy.

Keywords: discrete mathematical model; bayoud; fusarium oxysporum; optimal strategies.

2010 AMS Subject Classification: 39A05, 39A14, 39A12, 39A45, 39A60, 93C35, 93C55, 93C55.

1. Introduction

The date palm (Phoenix dactylifera L.) is one of the oldest cultivated plant species, best adapted to the difficult climatic conditions of the Saharan and pre-Saharan regions. It produces fruits rich in nutrients, provides a multitude of secondary products, and is subsequently a key

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Received April 14, 2021

source of income necessary for seal farmers and oasis inhabitants. Dates are also delicious with sweet taste that provides us good satiety feel that gives an extensive variety of essential nutrients with many potential health benefits[14] . Date palm has 36 chromosomes (n=18; 2n=36)[10], domesticated date palm fruits have quite variable shapes and range in size from 18 to 110x8 to 32 mm with the weight of an individual fruit varying from 2 to 60 g; they are significantly larger than in other Phoenix species[1].

The date palm also has other uses: the trone is used as wood for carpentry and framing; the palms are used for terraces, as firewood, fences and their leaflets in basketry.

The economy of the oases in some countries revolves around the exploitation of palm groves. This activity contributes an important agricultural income for several million farmers worldwide as date production is increasing every year[25].

Despite all these advantages, the date palm remains the cultivated species that has suffered from several problems in several oases in North Africa (e.g. Tata and Tafilalet in Morocco, Mzab and Gourara in Algeria, Jérid in Tunisia) and in the USA (e.g Louisiana, California, Florida and Nevada). These two regions have ranked among the top in the world in the production of dates[26].

This situation calls upon all stakeholders for this industry in North Africa and the USA to bolden their actions and to implement all the adequate means to improve the productivity of date palms, to ensure its cultural integration in the oasis system, and consequently to create the conditions for a sustainable development of the oases [28] .

1.1. Fusarium oxysporum. One of the problems this sector has been facing for over 100 years is Fusarium oxysporum f. sp. albedinis. The genus Fusarium takes its name from the Latin fuses because its spores are spindle-shaped and it proliferates in the soil and on organic matter. Like all soil fungi, it is characterized by its ability to adapt to a wide variety of environmental conditions due to its capacity to change form and function [15].The soil fungus, Fusarium oxysporum, causes a rapid dieback of date palms.In the event of an infestation, losses can reach up to 50 of production[4]. Fusarium oxysporum (Fo) is a soilborne fungus and is very common worldwide [8]. Most Fo strains are not patho- genic and some are known to be beneficial to plants [3].

This vascular fusarium, commonly known as Bayoud, particularly affects the best date-producing palm varieties; several million trees have been destroyed two−thirds of date palm in North Africa, especially in Morocco (more than 10 million trees) and in the western and central parts of Algeria in 20th centuries [17]. Fusarium oxysporum f.sp. was also reported as a potential danger to date production in California. It is reported that Fusarium spp., which negatively affected social and economic ecosystems in these regions.

One of the first typical external symptoms of a Bayoud attack is a unilateral drying and bleaching of one or more palms (leaflets and rachis) in the middle crown of the palm. This symptom is at the origin of the name of this disease, Bayoud derives from the Arabic word "abyed" which means white [23], and from the special form of Fusarium oxysporum which is responsible for it, albedinis, from the Latin albus (white).

The same symptoms then appear on neighboring palms, and the attack then spreads to the whole palm, which rapidly withers.

But the symptoms are not always so typical and it has already happened to confuse an attack of Bayoud with drying out due to water stress. A thorough examination of diseased plants is therefore necessary to locate and identify the parasite.

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But the symptoms are not always so typical and it has already happened to confuse an attack of Bayoud with drying out due to water stress. A thorough examination of diseased plants is therefore necessary to locate and identify the parasite [6].

2. Formulation of the Mathematical Model

2.1. Description of the model.

The SIR model is the basic one used for modelling epidemics. Kermack and McKendrick created the model in 1927[16] in which ney considered a fixed population with only three compartments, asceptible (S), infected (I) and recovered (R).

Mathematically the spatial spread of palms infected with the fungus Fusarium oxysporum in a Date Palm Oasis can be first modeled using a system of $SIRS-C$ with four compartments.

- **The compartment** S : The number of palm trees belonging to the area that are not sick but likely to become sick (e.g. old, infected, poorly watered, rocky soil, ...).
- **The compartment** I : The number of palm trees infected with bayoude.
- **The compartment** R : The number of palm trees that have already had the disease and are now immune to bayoud.
- **The compartment** C : The growth rate of the fungus Fusaruim oxyporum.

The following diagram shows the flow directions of the palms between the compartments.

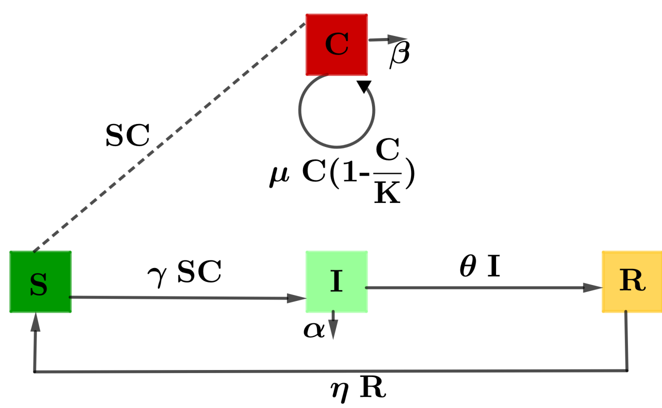

FIGURE 1. SIRS-C model.

γ	Rate of fungus encounters with palms.
β	Rate of fungus mortality .
μ_s	Rate of susceptible palm natural mortality .
α	Rate of palm tree deaths due to infection.
θ	Rate of palm healing.
η	Rate of cured palm that becomes susceptible due to loss of immunity.

TABLE 1. Definitions of parameters used in model 2.1

2.2. Model equations. The susceptibles palm S_i becomes infected at rate γ when they come in contact with the Fusarium oxysporum . That is, the change in population is equal to $-\gamma S_k C_k$. In addition, individuals from the recovered group become susceptible again at a certain rate " η " to give ηR_k, Thus, we have:

$$S_{k+1} = S_k - \gamma S_k C_k - \mu_s S_k + \eta R_k.$$

The infected palm begins with adding what was just removed from the susceptible population, $\gamma S_k C_k$ and then a reduction in two ways i.e. palm can either recover or they are killed by the virus. They recovered from the virus at rate " θ" and are killed at rate " α ". Thus, we have:

$$I_{k+1} = I_k + \gamma S_k C_k - \theta I_k - \alpha I_k.$$

The recovered palm R_i is increased by those that recovered from the virus and reduced by the number of ppalm that join the susceptible group at rate "λ". This can be expressed as:

$$R_{k+1} = R_k - \eta R_k + \theta I_k.$$

The fact that the Fusarium oxysporum fungi population is increasing exponentially is not biologically satisfactory, because even if a population arrives in an environment containing all the necessary resources, which is the case for invasive species, a population cannot increase exponentially to infinity [12][19]. Self-regulation phenomena will therefore take place [9]. These phenomena are taken into account in Verhulst's model (1838), also known as the logistic growth model. This model, in discrete time, is written:

$$C_{k+1} = C_k + \mu C_k \left(1 - \frac{C_k}{K}\right) - \beta C_k,$$

with μ the growth rate and K the carrying capacity of the environment.

infection model by the following system of difference equations:

$$
\begin{cases}
S_{k+1} = S_k - \gamma S_k C_k - \mu_s S_k + \eta R_k, \\
I_{k+1} = I_k + \gamma S_k C_k - \theta I_k - \alpha I_k, \\
R_{k+1} = R_k - \eta R_k + \theta I_k \\
C_{k+1} = C_k + \mu C_k \left(1 - \frac{C_k}{K}\right) - \beta C_k.
\end{cases} \tag{2.1}
$$

2.3. Numerical simulation without control. In this section, we shall solve numerically the optimal control problem for our $SIRS-C$ model. Here, we obtain the optimality system from the state and adjoint equations. The proposed optimal control strategy is obtained by solving the optimal system which consists of four difference equations and boundary conditions. The optimality system can be solved by using an iterative method. Using an initial guess for the control variables, $u_{1,k}$ and $u_{2,k}$, the state variables, SIR, and C, are solved forward and the adjoint variables λ_i for $i = 1$, are solved backwards at time steps $k = 0$ and $k = N$. If the new values of the state and adjoint variables differ from the previous values, the new values are used to update $u_{1,k}$and $u_{2,k}$ and the process is repeated until the system converges. The numerical solution of model (1) is executed using Matlab, the table below shows the parameters used in the simulations and the parameters are chosen arbitrarily. 2. We begin by presenting the solution evolution of our model (1) without controls that are represented in Figure 7.

We firstly consider the numerical simulation of the SIRS model when $\gamma = 0.01, \beta = 0.00001$, $\alpha = 0.00003, \mu = 0.09, \theta = 0.00001$ and $\eta = 0.01$. In our simulation, we also assume that population size is constant with natural mortality rate of individuals is equal to the birth rate.

Parameter	Values
γ	0.01
β	0.00001
α	0.00003
θ	0.00001
μ	0.09
η	0.01

The figure 2 represent the evolution of the four populations (susceptible, influenced, recovered, Fusarium oxysporum fungi) in the uncontrolled situation. Following this figure, we can clearly shows that when the level of the Fusarium oxysporum fungi population increases the level of

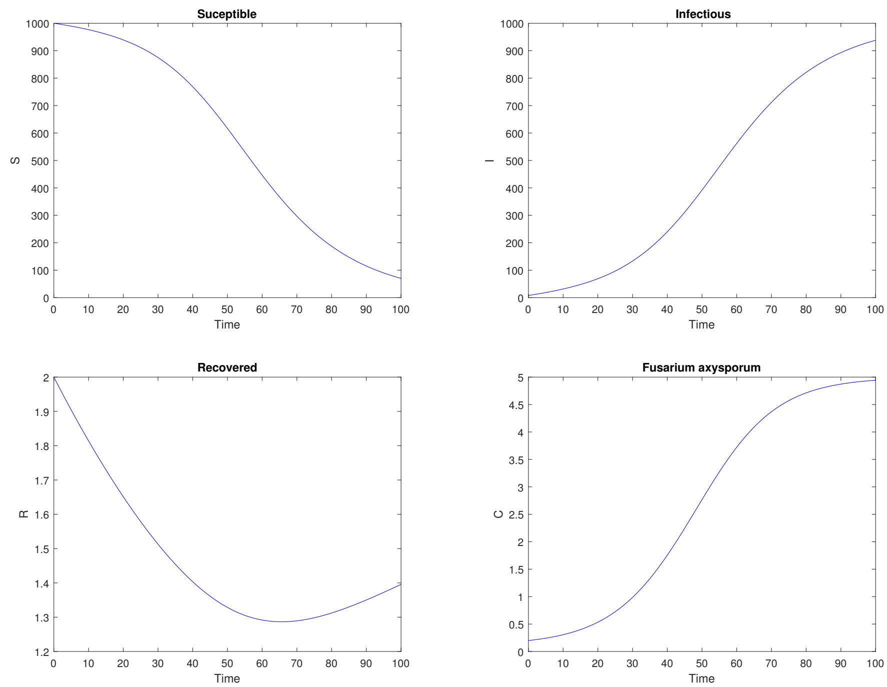

FIGURE 2. SIRS-C model without control.

infected population increases too, and for the recovered it decreases. So, we can deduce that importance of controlling these populations.

3. The Optimal Control Problem Approach

In the absence of the available effective treatment for Bayoud's disease caused by the fungus Fusarium oxysporum, infected palms are not controlled, they can serve the disease, and the fungus can spread. According to the previous simulations, it can be seen that without the introduction of an external strategy, the entire palm population is doomed to disappear.

So adopted the following mathematical model we introduce our controls into system (1) as the control measures to fight the spread of the fungus Fusarium oxysporum; we extend our system by including three kinds of controls v and w. The first control v is the selection of cultivars and clones resistant to Bayoud and of good date quality, and also the use of fungicides with systemic or endotherapeutic action. the second straegy is applying the control w on the Fusarium oxysporum fungi population for stop its propagation.

$$
\begin{cases}
S_{k+1} = S_k - \gamma S_k C_k - \mu_s S_k + \eta R_k, \\
I_{k+1} = I_k + \gamma S_k C_k - \theta I_k - \varepsilon_1 v_k I_k - \alpha I_k, \\
R_{k+1} = R_k - \eta R_k + \theta I_k + \varepsilon_1 v_k I_k, \\
C_{k+1} = C_k + \mu C_k \left(1 - \frac{C_k}{K}\right) - \varepsilon_2 w_k C_k.
\end{cases} \tag{3.1}
$$

with

$$
\varepsilon_i = \begin{cases} 1 \\ 0 \end{cases} \quad \text{for } i = 1,2
$$

for exemple $\varepsilon_1 = 1$ and $\varepsilon_2 = 0$ it means that we apply only a single control,$\varepsilon_1 = 1$ and $\varepsilon_2 = 1$ it means that we apply two controls.

and we consider the objective functional :

$$
J(v_k, w_k) = \sum_{k=0}^{N} (a_k I_k - b_k R_k + c_k C_k) + \sum_{k=0}^{N-1} \left(A^k \frac{v_k^2}{2} + B^k \frac{w_k^2}{2} \right). \tag{3.2}
$$

where the parameters $a_k > 0$, $b_k > 0$ and $c_k > 0$ are the cost coefficients; they are selected to weigh the relative importance of $S_k, I_k, R_k^t, C_k^t, v_k$, and w_k at time $k.N$ is the final time.

In other words, we seek the optimal controls v_k, and w_k such that

$$
J(v_k^*, w_k^*) = \min_{(v_k, w_k) \in U_{ad}} J(v_k, w_k). \tag{3.3}
$$

where U_{ad} ad is the set of allowable controls defined by :

$$
U_{ad} = \{(v_k, w_k) \quad 0 \leq v_k \leq 1, \quad 0 \leq w_k \leq 1 \quad k = 0, 1 \ldots N-1\} \tag{3.4}
$$

The sufficient condition for the existence of optimal controls (v_k, w_k) for problems (3.1) and (3.2) comes from the following theorem:

Theorem 3.1. *There exists an optimal control (v_k^*, w_k^*) such that*

$$
J(v_k^*, w_k^*) = \min_{(v_k, w_k) \in U_{ad}} J(v_k, w_k).
$$

subjet to the control system (3.1)

Proof. Since the coefficients of the state equations are bounded and there are a finite number of time steps $S\,I$, R, C are uniformly bounded for all $(\mathrm{v_k})$ in the control set U_{ad}; thus $J\,(v_k, w_k)$ is bounded for all $(v_k, w_k) \in U_{ad}$.
Since $J\,(v_k, w_k)$ is bounded,

$$\inf_{(v_k, w_k) \in U_{ad}} J\,(v_k, w_k)\,.$$

is finite, and there exists a sequence $(\mathrm{v_k^n}, \mathrm{w_k^n}) \in U_{ad}$ such that

$$\lim_{n \to +\infty} (\mathrm{v_k^n}, \mathrm{v_k^n}) = \inf_{(v_k, w_k) \in U_{ad}} J\,(v_k, w_k)$$

and corresponding sequences of states $S^n \to S$, $I^n \to I$, $R^n \to R$, $C^n \to C$
Since there is a finite number of uniformly bounded sequences, there exist $(\mathrm{v_k^*}, \mathrm{w_k^*}) \in U_{ad}$ and $(S^*,\, I^*,\, R^*,\, C^*) \in \mathbb{R}^{t_{\mathrm{end}}+1}$ such that, on a subsequence,

$$(\mathrm{v_k}, \mathrm{w_k}) \to (, \mathrm{v_k^*}, \mathrm{w_k^*}).$$

Finally, due to the finite dimensional structure of system (3.1) and the objective function $J(\mathrm{v_k}, \mathrm{w_k})$,
$(\mathrm{v_k^*}, \mathrm{w_k^*})$ is an optimal control with corresponding states $(S^*,\, I^*,\, R^*,\, C^*)$. Therefore

$$\min_{(\mathrm{v_k}, \mathrm{w_k}) \in U_{ad}} J(\mathrm{v_k}, \mathrm{w_k}).$$

□

In order to derive the necessary condition for optimal control, the pontryagins maximum principle in discrete time. This principle converts into a problem of minimizing a Hamiltonian H at time step k defined by

$$H_k = L\,(S_k, I_k, R_k, C_k) + \sum_{i=1}^{4} \lambda_{i,k+1} f_{i,k+1}\,, \tag{3.5}$$

where $f_{i,k+1}$ is the right side of the system of difference equations (3.1) of the i^{th} state variable at time step $k+1$

$$L(S_k,I_k,R_k,C_k)=a_kI_k-b_kR_k+c_kC_k+A^k\frac{v_k^2}{2}+B^k\frac{w_k^2}{2}. \tag{3.6}$$

Theorem 3.2. *Given an optimal control* $\left(v_k^*,w_k^*\right)$ *and the solutions* S_k^*,I_k^*,R_k^* *and* C_k^* *of the corresponding state system (3.1), there exist adjoint functions* $\lambda_{1,k},\lambda_{2,k},\lambda_{3,k}$, *and* $\lambda_{4,k}$ *satisfying*

$$\lambda_{1,k}=\lambda_{1,k+1}\left[1-\gamma C_k-\mu_s\right]+\lambda_{2,k+1}\gamma C_k,$$

$$\lambda_{2,k}=a_k+\lambda_{2,k+1}\left[1-\theta-\varepsilon_1v_k-\alpha\right]+\lambda_{3,k+1}\left[\theta+\varepsilon_1v_k\right],$$

$$\lambda_{3,k}=-b_k+\eta\lambda_{1,k+1}+(1-\eta)\lambda_{3,k+1},$$

$$\lambda_{4,k}=c_k-\lambda_{1,k+1}\gamma S_k+\lambda_{2,k+1}\gamma S_k+\lambda_{4,k+1}\left[1+\mu-\frac{2C_k}{K}-\varepsilon_2w_k\right].$$

With the transversability conditions at time $N,\lambda_{1,N}=\lambda_{5,N}=0,\lambda_{2,N}=B_N,\lambda_{3,N}=A_N$, *and* $\lambda_{4,N}=C_N$

Furthermore, for $k=0,1,2\ldots N-1$ *the optimal controls* v_k^*, *and* w_k^* *are given by*

$$v_k^*=\min\left\{\max\left\{\frac{1}{A^k}(\lambda_{2,k+1}-\lambda_{3,k+1})\varepsilon_1I_k^*,1\right\},0\right\}. \tag{3.7}$$

$$w_k^*=\min\left\{\max\left\{\frac{1}{B^k}\lambda_{4,k+1}\varepsilon_2C_k^*,1\right\},0\right\}. \tag{3.8}$$

Proof. The Hamiltonian at time step k is given by

$$\begin{aligned}H_k=L(S_k,I_k,R_k,C_k)&+\lambda_{1,k+1}\left(S_k-\gamma S_kC_k-\mu_sS_k+\eta R_k\right)\\&+\lambda_{2,k+1}\left(I_k+\gamma S_kC_k-\theta I_k-\varepsilon_1v_kI_k\right)\\&+\lambda_{3,k+1}\left(R_k-\eta R_k+\theta I_k+\varepsilon_1v_kI_k\right)\\&+\lambda_{4,k+1}\left(C_k+\mu C_k\left(1-\frac{C_k}{K}\right)-\varepsilon_2w_kC_k\right)\end{aligned}$$

For $k = 0, 1 \dots N-1$ the optimal controls u_k, v_k, w_k can be solved from the optimality condition,

$$\begin{aligned} \frac{\partial H_k}{\partial v_k} &= 0, \\ \frac{\partial H_k}{\partial w_k} &= 0, \end{aligned} \tag{3.9}$$

that are

$$\begin{aligned} \left.\frac{\partial H_k}{\partial v_k}\right|_{v_k=v_k^*} &= A^k v_k^* + \left(\lambda_{3,k+1} - \lambda_{2,k+1}\right) \varepsilon_1 I_k^* = 0, \\ \left.\frac{\partial H_k}{\partial w_k}\right|_{w_k=w_k^*} &= B^k w_k^* - \lambda_{4,k+1} \varepsilon_2 C_k^* = 0, \end{aligned}$$

we get :

$$\begin{aligned} v_k^* &= \frac{1}{A^k} \left(\lambda_{2,k+1} - \lambda_{3,k+1}\right) \varepsilon_1 I_k^*, \\ w_k^* &= \frac{1}{B^k} \lambda_{4,k+1} \varepsilon_2 C_k^*. \end{aligned} \tag{3.10}$$

□

4. Numerical Simulation with Control

In this part of the numerical simulation, we deal with three cases. In the first case we apply a single control, in the second case two controls. In the case of a single control, we start with the optimal control v that is applied for the infected population and its objective is to minimize this population. Next, we discuss the results obtained by applying control w that has as purpose the minimization of the Fusarium oxysporum fungi population. Then, we move on to show the importance of increasing the number of controls in the study.

4.1. Date palm resistant to bayoud and the genetic control. The first strategy to control Bayoud is to select disease-resistant material of good date quality. Use the resistance of certain palm varieties to Bayoud[7]. The search for more resistant palms is also the general direction taken to control vascular fusarium of the different cultivated palms. This method has the disadvantage of being long to implement, not allowing the conservation of certain sensitive varieties that are highly appreciated for their yield and the quality of their fruit[27].

As another solution, the world has rallied around a genetic fight against the disease[2][5]. A second strategy in the field has been to induce the date palm's defense reactions using salicylic acid (SA)[18] .

This led to a significant reduction in mortality rates of date palms inoculated with Foa. This result was obtained in correlation with a marked increase in the content of phenolic compounds, H2O2 and malondialdehyde on the one hand and phenylalanine ammonia-lyase and peroxidase activities on the other. In addition, we noted that activation of these components of date palm resistance by SA was greater after inoculation of the pathogen [20]. In addition, a localized necrosis, reminiscent of the necrosis formed during the establishment of the hypersensitive reaction (HR), was formed at the site of infection. It was positively correlated with the accumulation of H2O2 and malonyldialdehyde and with the establishment of resistance. Furthermore, analysis of phenolic compounds by HPLC, led to the identification of caffeoylshikimic acid isomers constitutively present in date palm roots. When these roots are triated by salycilic acid and noculated by Foa, new phenolic compounds identified as hydroxycinnamic acid derivatives with strong antifungal activity were induced. Their accumulation is thought to be responsible for the improvement in the resistance of date palm, obtained following treatment with SA. The histochemical revelation of phenolic compounds in the date palm root tissues revealed flavonoids in the cell walls of the vascular parenchyma of the xylem[17]. These flavonoids are only expressed in the roots which are both treated by the SA and inoculated by the pathogen, and would be at the origin of the cytological alterations observed in the Foa mycelium, in electronographs of date palm roots.

According to Figure 3, the control of the infected population leads to the following results: A decrease in the susceptible population that converges to 0. A significant increase in the infected population that exceeds the level reached in the situation without control. An increase, in an interval with a small amplitude, of the recovered, followed by their decrease, but they always remain higher than the number of recovered in the case without control. An increase in Fusarium oxysporum level.

4.2. Chemical control. Fungicides are pesticides that kill or inhibit the fungi responsible for certain diseases. On the other hand, not all fungal diseases can be controlled with a fungicide,

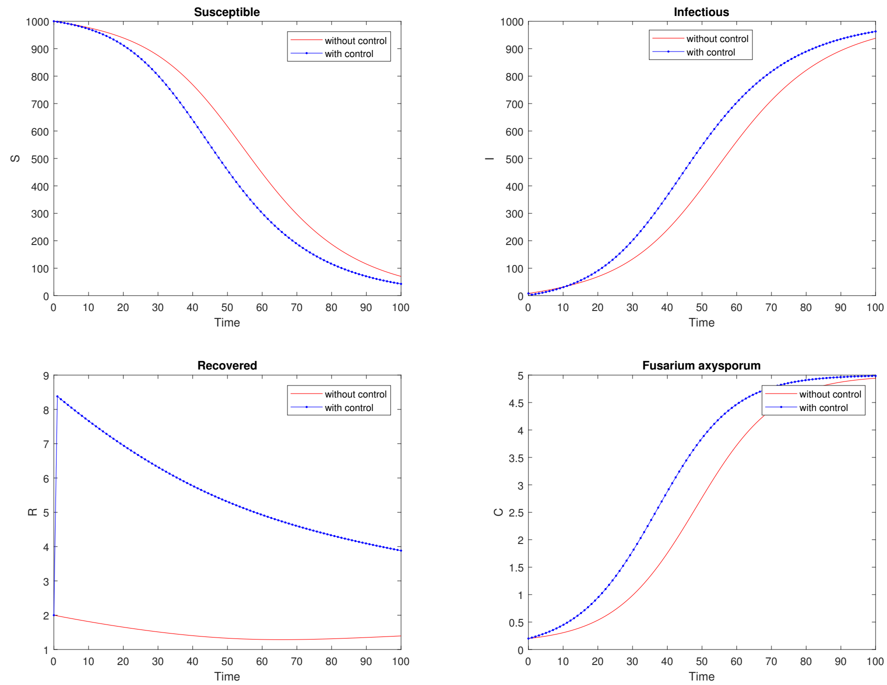

FIGURE 3. SIRS-C model with control v.

we can think of vascular diseases such as Fusarium. So the first strategy is the use of fungicides with systemic or endotherapeutic action.

This method is ruled out, as the practical possibilities for the use of systematic fungicides for tracheomycoses are very limited[21]. In addition, these products are not very stable in the soil and may favor the selection of resistant strains [22]. If used repeatedly over many years, these chemicals can harm the environment [29].

In the figure 4 , we illustrate the results obtained after applying the control on the Fusarium oxysporum fungi population. Here, the alarming result is that even if we apply this control for minimizing the Fusarium oxysporum fungi population over time, this last one still increasing. However, the number of susceptible is minimized, it decreases to zero. The number of infected becomes lower than that in the case without control, it does not exceed 200. The recovered increases and after a certain time it decreases so as not to exceed the number of recovered in the case without control.

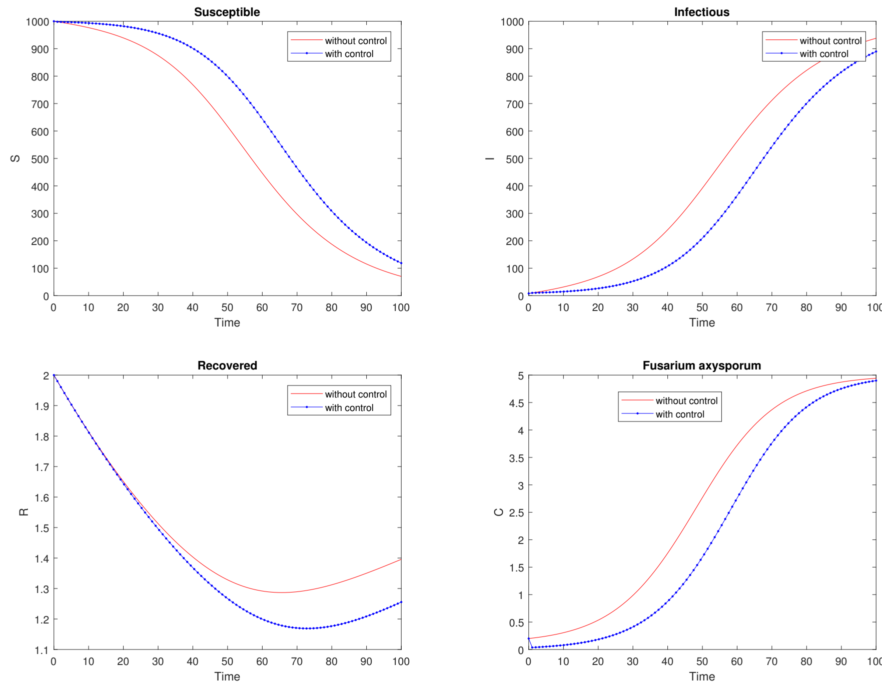

FIGURE 4. SIRS-C model with control w.

4.3. Date palm resistant to bayoud , the genetic control and chemical control. Now we move to the case of the two controls v and w. By figure 5, we show that, in this case, the susceptible population is minimized and converges to zero. The infected population is minimized too and it not exceeds the threshold of 200. The recovered population diminutes by staying lower than the number of recovered in the uncontrolled situation. The Fusarium oxysporum fungi population is minimized by attaining values less than in the uncontrolled situation.

We can deduce that the results obtained in this case are more convincing than those of the case of a single control w. Because, the control w alone leads to an increase in the number of the Fusarium oxysporum fungi population, however, when a second control is added, we can minimize the number of individuals of this population. And the same goes for the susceptible and the infected controls.

5. CONCLUSION

In this work, in spite of their inapplicability in areas infested with on a large scale, the results of our study show that combination of reproduction of species resistant to Bayoud's disease and genetic control remains the most effective way to fight the disease, and in case of introduction

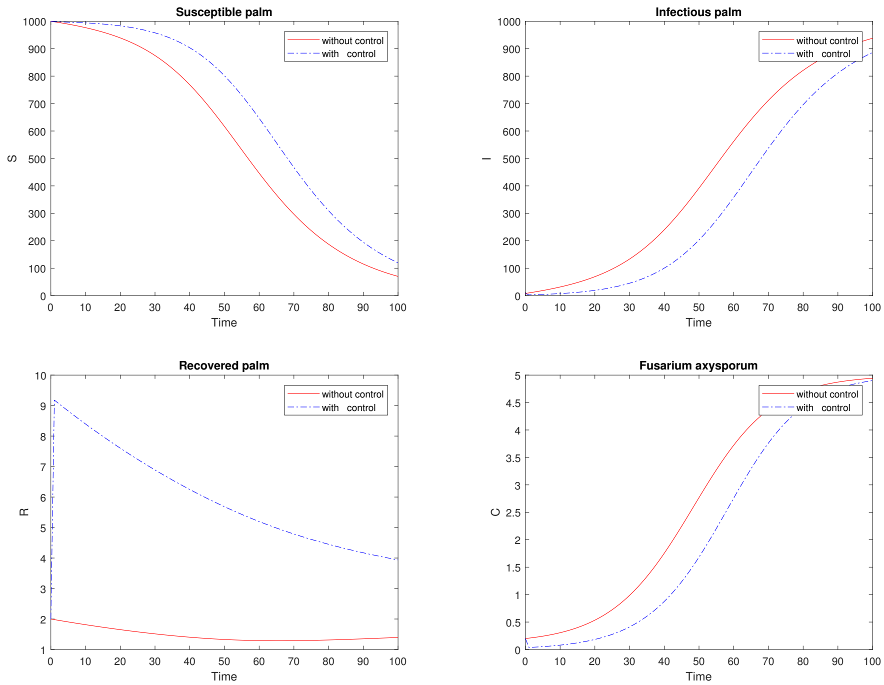

FIGURE 5. SIRS-C model with control v and w.

of bayoud disease in the news. Plantations, a timely spot treatment based on the chemical control, could eradicate the disease and prevent its progression in these areas. The technical management of palm orchards in the areas of expansion, including the practice of localized irrigation, does not allow the pathogen to be spread through water irrigation, and measures to prevent and control the spread of strict exclusion practiced by investors, are also factors that complement this control strategy. This could help protect the plantations against the effect of the devastating disease thus secure long-term investments.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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