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AN OPTIMAL CONTROL PROBLEM OF MALARIA MODEL WITH SEASONALITY EFFECT USING REAL DATA

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Abstract: In this study, we present a mathematical model of malaria transmission with a seasonality effect to describe the dynamics of the infection. In the absent seasonality effect, we prove the local stability of the malaria-free equilibrium point. The parameters of the model are fitted to the cumulative number of malaria cases of Papua province, Indonesia for the year 2018 and parameterized using the least-squares technique. The sensitivity analysis of the model to changes in the parameters is explored. Further, the malaria model with the seasonality effect via a periodic mosquito birth rate is investigated numerically. Finally, we formulate an optimal control problem with a control function and obtain the optimal control characterization. The optimal control problem is solved numerically, and the results comprised of a controls system for different strategies.

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1. INTRODUCTION

Malaria is a contagious disease that is very dominant in the tropics and sub-tropics. This disease is caused by the protozoan parasite of the genus plasmodium which is transmitted by *Anopheles* mosquito bites. Malaria is still one problem of public health because it affects high morbidity and mortality. If malaria patient is untreated, the patient can progress rapidly to convulsions, coma, and death. Nowadays, malaria is found in almost all parts of the world. In 2019, almost half of the world's population is at high risk of contracting malaria. WHO recorded the incident malaria in 2019 around 229 million cases with a death toll of around 409,000 cases. The highest risk of transmission occurs in sub-Saharan Africa. Even the regions in Southeast Asia, the Eastern Mediterranean, the Western Pacific and America are also at risk [1].

Indonesia is one of the contributing countries for malaria sufferers in the world. High-endemic malaria districts/cities are still concentrated in eastern Indonesia, including Papua Province, West Papua Province, and East Nusa Tenggara Province. The total number of malaria cases in Indonesia in 2019 was 250,644. About 86% occurred in Papua (216,380 cases). Then, followed by East Nusa Tenggara with 12,909 cases and West Papua with 7,079 cases. Meanwhile, 300 districts/cities (58%) have eliminated malaria in Indonesia [2].

Malaria as one of the diseases transmitted by mosquitoes is strongly influenced by climatic conditions (temperature, rainfall, and humidity). Specifically, rainfall and temperature affect mosquito reproduction and development and parasitic survival in mosquitoes [3, 4]. Indonesia as a tropical country, of course, breeding mosquitoes influenced by two seasons namely the rainy season and the dry season. Thus, it is very important to be included in seasonal factors of malaria transmission in Indonesia.

The mathematical model approach has been used by the researcher to capture the complexities of the phenomenon on the transmission dynamics of mosquito-borne diseases, such as malaria. Numerous mathematical models have been constructed to assess the dynamics of malaria transmission in population [5-10]. In recent years, research investigating the climate factors on the malaria transmission model has been reported [11-15]. For instance, the authors in [11] have developed a malaria model involving the impact of climate variables on population of *Anopheles arabiensis*, and the effect of rainfall and temperature on mosquito breeding. An age-structured mathematical model to assess the impact of temperature and rainfall variability on the dynamics of malaria transmission in a population was studied in [12]. A deterministic malaria model to examine the impact of temperature and rainfall on malaria epidemics over Limpopo province, South Africa can be found in [13]. The authors in [14] have explored the *Plasmodium vivax* malaria transmission model associated with climate-dependent and climate-independent parameters, which are estimated by using previous articles and actual data. Recent study on developing a malaria model that take into account temperature- and rainfall-dependent parameters has been studied in [15].

In this work, we extend the malaria model in [10, 16] by incorporating the seasonal factors based on the real data of malaria cases in Indonesia. We estimate the parameter model using the cumulative monthly number of the malaria cases in Papua province, Indonesia in 2018. To examine the impact of the optimal level of intervention in the form of insecticides, prevention, and treatment effort, we carry out the malaria model to be the optimal control problem.

2. MALARIA MODEL TRANSMISSION

In this section, we describe a host-vector model for malaria transmission. The host population is divided into two subpopulations, namely, the susceptible subpopulation S_h and the infectious subpopulation I_h. Moreover, the vector population is also divided into the susceptible subpopulation S_V and the infectious subpopulation I_v. The total human and mosquito populations are denoted by N_h and N_v where $N_h = S_h + I_h$ and $N_v = S_v + I_v$, respectively.

The system of differential equations describing the host-vector model is as following

$$(1)\begin{cases}\frac{dS_h}{dt} = \Lambda_h - p_1\,\beta\,I_v\frac{S_h}{N_h} + \gamma_h\,I_h - \mu_h\,S_h,\\ \frac{dI_h}{dt} = p_1\,\beta\,I_v\frac{S_h}{N_h} - (\mu_h + \gamma_h + \delta_h)\,I_h,\\ \frac{dS_v}{dt} = \mu_v\,N_v - p_2\,\beta\,S_v\frac{I_h}{N_h} - \mu_v\,S_v,\\ \frac{dI_v}{dt} = p_2\,\beta\,S_v\frac{I_h}{N_h} - \mu_v\,I_v.\end{cases}$$

Description of parameters of model (1) could be seen in Table 1.

TABLE 1. Description of parameter for model (1)

Parameter	Description
Λ_h	Recruitment rate of human
p_1	Transmission probability from I_v to S_h
β	Contact rate of mosquito and human
γ_h	Recovery rate of infectious human
μ_h	Per capita natural death rate of human
δ_h	Disease induced death rate of human
μ_v	Per capita birth/death rate of mosquito
p_2	Transmission probability from I_h to S_v

In the malaria model (1) all of the parameters are positive and constant. Moreover, all of the state variables are non-negative. It can also be confirmed that the solution of the malaria model with non-negative initial conditions will remain non-negative for $t \geq 0$.

The feasible region of model (1) is given by $\Omega = \Omega_v \times \Omega_h \subset \mathbb{R}_+^4$, where

$$\Omega_h = \left\{\left(S_h(t), I_h(t)\right) \in \mathbb{R}_+^2 : N_h(t) \leq \frac{\Lambda_h}{\mu_h}\right\} \text{ and}$$

$$\Omega_v = \left\{\left(S_v(t), I_v(t)\right) \in \mathbb{R}_+^2 : S_v + I_v = N_v\right\}.$$

Here, parameter N_v is constant because the mosquito population is constant. The model (1) is well-posed in the region Ω due to all the solutions of model (1) with initial conditions in the region remains in the region for all $t \geq 0$.

3. STABILITY ANALYSIS

This section examines the equilibria of model (1) and analyze their local stability. Model (1) has two equilibria, namely the disease-free equilibrium and the endemic equilibrium. The disease-free equilibrium is a condition when there is no spread of malaria in the population. In model (1), the disease-free equilibrium (DFE) is

$$E^0 = (S_h^0, I_h^0, S_v^0, I_v^0) = \left(\frac{\Lambda_h}{\mu_h}, 0, N_v, 0\right). \tag{2}$$

Next, we determine the basic reproduction number R_0 which represents the expected average number of new malaria infections due to contact between infected mosquitoes and susceptible humans as well as infected humans and susceptible mosquitoes [17, 18]. In this study, the Next Generation Matrix (NGM) method developed by [19] is used to obtain R_0. The basic reproduction number R_0 of the model (1) is given by

$$R_0 = \sqrt{\frac{p_1\, p_2\, \beta^2\, \mu_h\, N_v}{\Lambda_h\, \mu_v\, (\mu_h + \gamma_h + \delta_h)}}. \tag{3}$$

Endemic equilibrium is a condition when malaria has been endemic in the population. The system of equation (1) can be solved using the strength of infection during steady-state conditions (κ_1^* dan κ_2^*), where

$$\kappa_1^* = \beta \frac{I_v^*}{N_h^*} \text{ dan } \kappa_2^* = \beta \frac{I_h^*}{N_h^*}. \tag{4}$$

By making the right-hand side of model (1) zero and noticing that $\kappa_1 = \kappa_1^*$ dand $\kappa_2 = \kappa_2^*$ at equilibrium conditions, it is obtained

$$\begin{cases} S_h^* = \dfrac{\Lambda_h + \gamma_h\, I_h^*}{p_1\, \kappa_1^* + \mu_h}, \\ I_h^* = \dfrac{p_1\, \kappa_1^*\, \Lambda_h}{p_1\, \kappa_1^*\, (\mu_h + \delta_h) + \mu_h\, (\mu_h + \gamma_h + \delta_h)}, \\ S_v^* = \dfrac{\mu_v\, N_v}{p_2\, \kappa_2^* + \mu_v}, \\ I_v^* = \dfrac{p_2\, \kappa_2^*\, N_v}{p_2\, \kappa_2^* + \mu_v}. \end{cases} \tag{5}$$

By using equation (5) to obtain κ_1^* and κ_2^* of equation (4), it shows that the endemic equilibrium of model (1) satisfying

$$\begin{cases} \kappa_1^* = \dfrac{\beta\, p_2\, \kappa_2^*\, N_v\, \mu_h}{\Lambda_h\, (p_2\, \kappa_2^* + \mu_v)}, \\ \kappa_2^* = \dfrac{\Lambda_h\, \mu_v\, (\mu_h + \gamma_h + \delta_h)(R_0^2 - 1)}{p_1\, p_2\, \beta\, N_v\, (\mu_h + \delta_h) + p_2\, \Lambda_h\, (\mu_h + \gamma_h + \delta_h)}. \end{cases} \tag{6}$$

Based on equation (5), $S_h^*, I_h^*, S_v^*, I_v^* > 0$ whenever $\kappa_1^*, \kappa_2^* > 0$. Next on the equation (6), $\kappa_1^* > 0$ whenever $\kappa_2^* > 0$ and also $\kappa_2^* > 0$ if $R_0 > 1$. Hence, the endemic equilibrium E^* exists when $R_0 > 1$.

Next, we analyze the local stability of DFE. The Jacobian matrix of model (1) at the DFE in equation (2) is as follows.

$$J_E^0 = \begin{pmatrix} -\mu_h & \gamma_h & 0 & -p_1\beta \\ 0 & -(\mu_h + \gamma_h + \delta_h) & 0 & p_1\beta \\ 0 & -\frac{p_2\beta N_v \mu_h}{\Lambda_h} & -\mu_v & 0 \\ 0 & \frac{p_2\beta N_v \mu_h}{\Lambda_h} & 0 & -\mu_v \end{pmatrix}. \tag{7}$$

The local stability of the DFE can be found through the eigenvalues of matrix J_E^0. The characteristic equation of matrix J_E^0 is

$$(\lambda + \mu_h)(\lambda + \mu_v)(\lambda^2 + a_1\lambda + a_2) = 0,$$

where $a_1 = \mu_v + \mu_h + \gamma_h + \delta_h$ and $a_2 = \mu_v(\mu_h + \gamma_h + \delta_h)(1 - R_0^2)$.

Further, we obtain the eigenvalues $\lambda_1 = -\mu_h$, $\lambda_2 = -\mu_v$, and the remainder are the roots of the following equation:

$$\lambda^2 + a_1\,\lambda + a_2 = 0. \tag{8}$$

Based on the Routh-Hurwitz criteria, equation (8) has roots whose real part is negative if $a_1, a_2 > 0$. It is clear that $a_1 > 0$ because all parameters are assumed to be positive. Moreover, $a_2 > 0$ when $R_0^2 < 1 \Leftrightarrow R_0 < 1$. Thus the DFE E^0 is local asymptotically stable when $R_0 < 1$ and unstable when $R_0 > 1$.

4. PARAMETER ESTIMATION OF MALARIA MODEL

Next, we estimate the parameters of model (1) based on data on malaria infective in Papua province in 2018. The data for individuals infected with malaria discussed here are data from people who have been confirmed in the laboratory and tested positive for malaria. Data obtained from Papua Health Office from January-December 2018 [20]. We used cumulative data on malaria cases per month from January to December 2018.

In this study, we used the least squares method to estimate parameters of model (1) except for parameters μ_h obtained from demographic conditions in Papua. Natural human mortality rate μ_h is obtained from the inverse of the average life expectancy in Papua. The average life expectancy

in Papua is 65.36 years [21], then $\mu_h = \frac{1}{65.36}$ per year. For parameter Λ_h, the human recruitment rate is calculated as follows. The total population of Papua province in 2018, which is equal to 2,264,615 [20]. Hence, we have $\frac{\Lambda_h}{\mu_h} =$ 2,264,615, which is the total human population without disease, so that $\Lambda_h =$ 34,648.3323 per year. The remainder of the parameters of model (1) are estimated using the least squares method [22].

Based on the least squares method, the results of parameter estimation of model (1) are given in Table 2. The comparison of the solution of model (1) and data on malaria patients is given in Figure 1. Using the parameter values stated in Table 2, the basic reproduction number in Papua is $R_0 \approx 1.3141$ which mean the malaria disease will persist in the province.

Table 2. Estimated parameter values. Time unit is day.

Parameter	Value	Source
$\boldsymbol{\Lambda_h}$	94.9269	Estimated
$\boldsymbol{\gamma_h}$	0.0972	Fitted
$\boldsymbol{\mu_h}$	$\frac{1}{65.36 \times 365}$	Estimated
$\boldsymbol{\delta_h}$	0.0244	Fitted
$\boldsymbol{\beta}$	0.6910	Fitted
$\boldsymbol{\mu_v}$	0.0591	Fitted
$\boldsymbol{p_1}$	0.5841	Fitted
$\boldsymbol{p_2}$	0.9981	Fitted

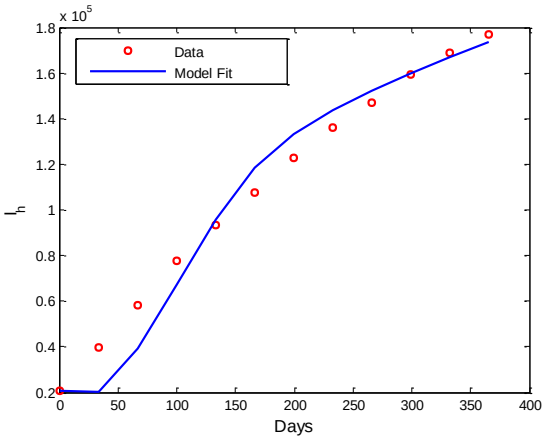


Figure 1. Comparison of dynamics $I_h(t)$ of model (1) and real data

5. SENSITIVITY ANALYSIS

In this section, we carry out the local sensitivity in order to investigate the model's parameters that are sensitive or significant (positive or negative) to the basic reproduction number of system (1) by calculating their sensitivity indices. To achieve this, we apply the following elasticity formula

$$\Gamma_{\Psi}^{R_0} = \frac{\partial R_0}{\partial \Psi} \times \frac{\Psi}{R_0}, \tag{9}$$

to equation (3) as defined in [23], where Ψ represents each parameter of interest. Applying equation (9) to (3) we obtain

$$\begin{cases} \Gamma_{\Lambda_h}^{R_0} = -\frac{1}{2}, \quad \Gamma_{\gamma_h}^{R_0} = -\frac{\gamma_h}{2(\gamma_h+\delta_h+\mu_h)}, \quad \Gamma_{\mu_h}^{R_0} = \frac{\gamma_h+\delta_h}{2(\gamma_h+\delta_h+\mu_h)}, \\ \Gamma_{\delta_h}^{R_0} = -\frac{\delta_h}{2(\gamma_h+\delta_h+\mu_h)}, \ \Gamma_{\beta}^{R_0} = 1, \ \Gamma_{\mu_v}^{R_0} = -\frac{1}{2}, \quad \Gamma_{\rho_1}^{R_0} = \frac{1}{2}, \quad \Gamma_{\rho_2}^{R_0} = \frac{1}{2}. \end{cases} \tag{10}$$

Next, we evaluate (10) by substituting the parameter values from Table 2 and obtain the sensitivity indices with respect to each parameter as summarised in Table 3.

TABLE 3. Sensitivity indices

Parameter	Sensitivity index
Λ_h	-0.5000
p_1	+0.5000
β	+1.000
γ_h	-0.3995
μ_h	+0.4998
δ_h	-0.10029
μ_v	-0.5000
p_2	+0.5000

From Table 3, we observe that the parameters Λ_h and μ_v have a great negative impact on R_0, while the parameters β, p_1 and p_2 have a strong positive impact on R_0. Further, parameter β is the most sensitive parameter to R_0 with a positive sign and 100% index value. An increase in β will lead to an increase in the malaria infections. Also, parameters Λ_h and μ_v are less significant with negative signs and a decrease (increase) by 50% will decrease the value of R_0 respectively.

6. MALARIA MODEL WITH SEASONALITY EFFECT

In this section, we develop the malaria transmission model given in equation (1) considering climatic factors. Climatic factors that are considered in this study are limited to temperature and rainfall factors. The temperature factor can determine the speed of growth and development of mosquitoes and the resistance of adult mosquitoes. This of course affects the rate of transmission of malaria from mosquitoes to humans and vice versa. Meanwhile, the rainfall factor also affects the development of mosquitoes and the malaria epidemic. From this, we included the effect of rainfall on the reproductive patterns of the mosquitoes so that seasonal effects in the modeling of malaria transmission were considered, allowing the total number of mosquitoes to vary periodically over time. Referring to [24, 25], we included this seasonal effect in model (1) by modifying the mosquito birth rate to periodic as given in the following system of differential equations.

$$(11)\begin{cases} \dfrac{dS_h}{dt} = \Lambda_h - p_1\,\beta\,I_v\dfrac{S_h}{N_h} + \gamma_h\,I_h - \mu_h\,S_h, \\ \dfrac{dI_h}{dt} = p_1\,\beta\,I_v\,\dfrac{S_h}{N_h} - (\mu_h + \gamma_h + \delta_h)\,I_h, \\ \dfrac{dS_v}{dt} = \mu_v\left(1 + \alpha\cos\left(\dfrac{2\,\pi\,t}{365} + \varphi\right)\right)N_v - p_2\,\beta\,S_v\dfrac{I_h}{N_h} - \mu_v\,S_v, \\ \dfrac{dI_v}{dt} = p_2\,\beta\,S_v\dfrac{I_h}{N_h} - \mu_v\,I_v. \end{cases}$$

where α is the amplitude of the seasonal variation $(0 \le \alpha < 1)$, and φ is the periodic phase of the birth of mosquito. Figure 2 gives some simulation results of model (1) and model (10). The simulation is carried out using the Runge-Kutta method. The parameter values used refer to the parameter values given in Table 2 and it is also assumed that $\alpha = 0.3$ and $\varphi = 0$. From Figure 2(a) - (d), it can be seen that the model without seasonal effects (solid blue curves) for the susceptible humans and the susceptible mosquitoes tends to decline, whereas the infected humans tend to rise and then fall, and the infected mosquitoes tend to increase. Meanwhile, the model with seasonal effects (periodic) tends to fluctuate depending on the current season phase.

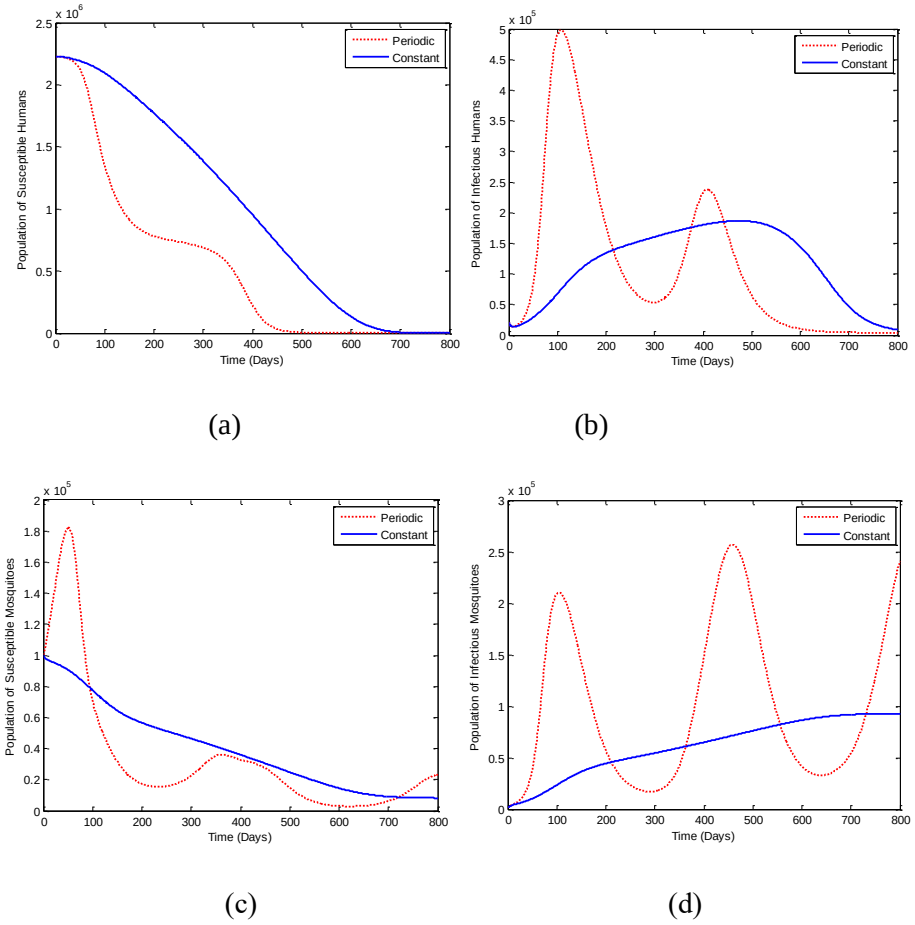


FIGURE 2. Seasonal effects on the malaria model

7. OPTIMAL CONTROL PROBLEM

In this study, an optimal control problem will be conducted for the malaria model with seasonal effects based on the sensitivity analysis. These results show that the strategy control that reduces the contact rate of mosquitoes and humans will be useful to reduce malaria prevalence in the community. Meanwhile, strategies to increase mosquito mortality and cure infected humans will provide benefits in order to reduce the incidence of malaria. Therefore, three control variables are applied to model (11), namely the malaria prevention control u_1, malaria treatment control u_2, and insecticide control u_3. The efforts of the malaria prevention can be in the form of using anti-mosquito lotion and the use of insecticide-treated mosquito net, while the efforts of the malaria treatment include the provision of anti-malaria drugs. Next, insecticide efforts include spraying

and fogging to reduce mosquitoes. The malaria model (11) by incorporated the control variables is given as follows

$$(12)\begin{cases} \dfrac{dS_h}{dt} = \Lambda_h - (1-u_1)\,p_1\,\beta\,I_v\,\dfrac{S_h}{N_h} + \gamma_h\,I_h - \mu_h\,S_h + \theta\,u_2\,I_h, \\ \dfrac{dI_h}{dt} = (1-u_1)\,p_1\,\beta\,I_v\,\dfrac{S_h}{N_h} - (\mu_h + \gamma_h + \delta_h)\,I_h - \theta\,u_2\,I_h, \\ \dfrac{dS_v}{dt} = \mu_v\left(1 + \alpha\cos\left(\dfrac{2\,\pi\,t}{365} + \varphi\right)\right)N_v - (1-u_1)\,p_2\,\beta\,S_v\,\dfrac{I_h}{N_h} - \mu_v\,S_v - \alpha_v u_3 \qquad S_v, \\ \dfrac{dI_v}{dt} = (1-u_1)\,p_2\,\beta\,S_v\,\dfrac{I_h}{N_h} - \mu_v\,I_v - \alpha_v\,u_3 I_v. \end{cases}$$

The parameters θ and α_v represent the malaria cure rate due to treatment and the mosquito mortality rate due to insecticide, respectively.

The objective function of model (12) that minimized the infected human and infected mosquito populations on the time interval $[0,\ t_f]$ is formulated as

$$(13)\qquad \min J = \int_0^{t_f} b_1\,I_h + b_2\,I_v + \frac{1}{2}\,c_1\,u_1^2 + \frac{1}{2}\,c_2\,u_2^2 + \frac{1}{2}\,c_3\,u_3^2\,dt,$$

where $u_1, u_2, u_3 \in [0,1]$ and b_1, b_2, c_1, c_2, dan c_3 respectively state weighting constants for infected human, infected mosquito, costs for malaria prevention, malaria treatment, and insecticide, respectively. The terms $c_1u_1^2$, $c_2u_2^2$, and $c_3u_3^2$ were described as measures of control costs for malaria prevention, malaria treatment, and insecticide, respectively.

The optimal control strategy problem in model (12) is solved by applying the Pontryagin Maximum Principle [26]. The first step in analyzing the optimal control strategy problem is to form the Hamiltonian (H) function as follows

$$(14)\qquad H = b_1\,I_h + b_2\,I_v + \frac{c_1}{2}\,u_1^2 + \frac{c_2}{2}\,u_2^2 + \frac{c_3}{2}\,u_3^2 + \sum_{i=1}^{4}\lambda_i\,f_i\,,$$

where f_i is the right-hand side of model (12) which represents the i-th variable, while λ_i represents the co-state variable with $i = 1, 2, 3, 4$. The existence of the optimal controls u_1, u_2, and u_3 are established by using the convexity of the objective function (13) with respect to the controls and the boundedness and Lipschitz property of the state system (12) [27].

The optimal controls u_1, u_2, and u_3 are obtained as stationary conditions $\frac{\partial H}{\partial u} = 0$, where $u = (u_1, u_2, u_3)$. Hence, we have the following the optimal controls of system (12).

$$(15)\quad \begin{cases} u_1^* = \min\left(1, \max\left(0, \frac{(\lambda_2-\lambda_1)\,\beta\, p_1\, S_h\, I_v + (\lambda_4-\lambda_3)\,\beta\, p_2\, S_v\, I_h}{c_1\, N_h}\right)\right), \\ u_2^* = \min\left(1, \max\left(0, \frac{(\lambda_2-\lambda_1)\,\theta\, I_h}{c_2}\right)\right), \\ u_3^* = \min\left(1, \max\left(0, \frac{(I_v\,\lambda_4 + S_v\,\lambda_3)\,\alpha_v}{c_3}\right)\right). \end{cases}$$

Differentiating the Hamiltonian function with respect to the state variables provide the co-state variables corresponding to the system given as follows

$$(16)\quad \begin{cases} \dot{\lambda_1} = -\frac{\partial H}{\partial S_h} = (\lambda_2 - \lambda_1)\left(\frac{(1-u_1)\,p_1\,\beta\, I_v\, S_h}{N_h^2} - \frac{(1-u_1)\,p_1\,\beta\, I_v}{N_h}\right) + \lambda_1\,\mu_h \\ \qquad\qquad + (\lambda_4 - \lambda_3)\left(\frac{(1-u_1)\,p_2\,\beta\, S_v\, I_h}{N_h^2}\right), \\ \dot{\lambda_2} = -\frac{\partial H}{\partial I_h} = -b_1 + (\lambda_2 - \lambda_1)\left(\frac{(1-u_1)\,p_1\,\beta\, I_v\, S_h}{N_h^2} + \gamma_h + \theta\, u_2\right) + \lambda_2(\mu_h + \delta_h) \\ \qquad\qquad + (\lambda_4 - \lambda_3)\left(\frac{(1-u_1)\,p_2\,\beta\, S_v\, I_h}{N_h^2} - \frac{(1-u_1)\,p_2\,\beta\, S_v}{N_h}\right), \\ \dot{\lambda_3} = -\frac{\partial H}{\partial S_v} = (\lambda_3 - \lambda_4)\left(\frac{(1-u_1)\,p_2\,\beta\, I_h}{N_h}\right) + \\ \qquad\qquad \lambda_3\left(-\mu_v\left(1 + \alpha\cos\left(\frac{2\,\pi\, t}{365} + \varphi\right)\right) + \mu_v + \alpha_v\, u_3\right), \\ \dot{\lambda_4} = -\frac{\partial H}{\partial I_v} = -b_2 + (\lambda_1 - \lambda_2)\left(\frac{(1-u_1)\,p_1\,\beta\, S_h}{N_h}\right) + \lambda_3\left(-\mu_v\left(1 + \alpha\cos\left(\frac{2\,\pi\, t}{365} + \varphi\right)\right)\right) \\ \qquad\qquad + \lambda_4\,(\mu_v + \alpha_v\, u_3). \end{cases}$$

with transversality condition $\lambda_i(t_f) = 0$, for $i = 1, 2, 3, 4$.

The solutions of S_h, I_h, S_v, and I_v from the optimal control $u^* = (u_1^*, u_2^*, u_3^*)$ will be solved numerically.

8. NUMERICAL RESULTS

To examine the effectiveness of the optimal control implementation strategy, numerical simulations were carried out. In this study, simulations were conducted by using the forward–

backward scheme [28] to solve the numerical solutions of model (12). A comparison of the dynamics of the malaria transmission without the control variables in model (11) and with the control variables in model (12) will be demonstrated to investigate the behavior of both models.

The parameter values used for the numerical simulation refer to the parameter values given in Table 2. The initial values used in this simulation refer to the malaria incidence data in January 2018 in Papua with $S_h(0) = 2{,}223{,}198$ and $I_h(0) = 20{,}711$ with five people dying from malaria. Furthermore, it is assumed that the initial value of the mosquito population is $S_v(0) = 100{,}000$ and $I_v(0) = 1{,}000$. The time horizon for the simulation is 200 days. Weighting constants for infected human populations, infected mosquitoes, costs for prevention, malaria treatment and insecticides are $b_1 = b_2 = 1$, $c_1 = 1$, $c_2 = 10$, and $c_3 = 3$, respectively. It is also assumed that the parameter values $\theta = 0.0035$, $\alpha_v = 0.1$, $\alpha = 0.5$ and $\varphi = 0$. Next, we investigate the following four control scenarios as follows

1. Combination of malaria prevention (u_1) and malaria treatment (u_2).
2. Combination of malaria prevention (u_1) and insecticide (u_3).
3. Combination of malaria treatment (u_2) and insecticide (u_3).
4. Combination of malaria prevention (u_1), malaria treatment (u_2), and insecticide (u_3).

8.1 FIRST SCENARIO

In the first scenario, the combination of malaria prevention u_1 and malaria treatment u_2 is applied, meanwhile, the insecticide was not used ($u_3 = 0$). The optimal control profiles for u_1 and u_2 are presented in Figure 3. Malaria prevention should be given intensively for 141 days, whereas malaria treatment should be done intensively for 32 days.

The dynamics of infected humans and infected mosquitoes are presented in Figure 4(a)-(b). From Figure 4, it can be seen that malaria prevention and malaria treatment controls provide a significant reduction in infected human and infected mosquito populations compared to having no controls.

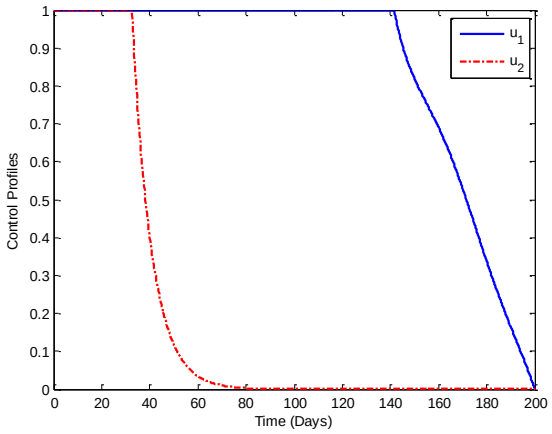
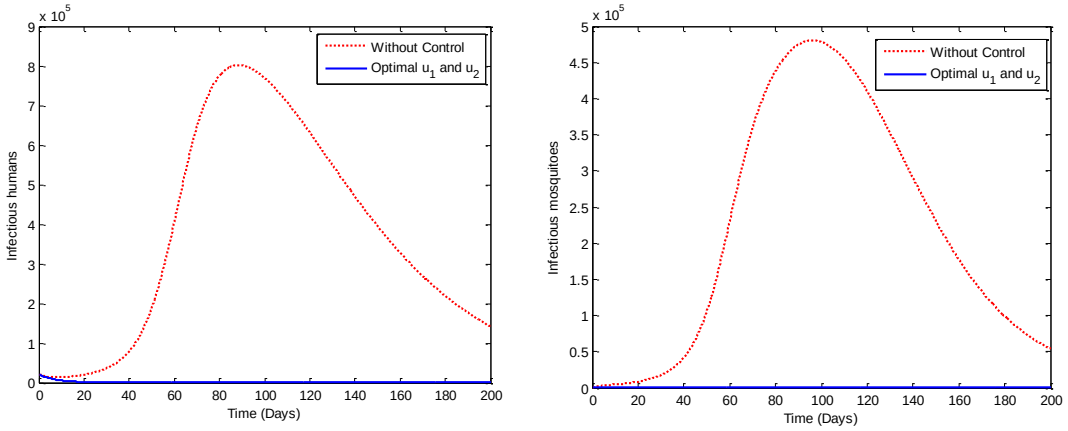


FIGURE 3. Optimal control profiles of u_1^* and u_2^*.



(a) (b)

FIGURE 4. The dynamics of I_h (a) and I_v (b) without and with controls u_1^* and u_2^*.

8.2 SECOND SCENARIO

In the second scenario, the combination of malaria prevention u_1 and insecticide u_3 is employed, while malaria treatment was not used ($u_2 = 0$). The optimal control profiles for u_1 and u_3 are displayed in Figure 5. Based on Figure 5, it can be seen that malaria prevention is implemented intensively for 57 days, whereas insecticide should be kept intensively for 36 days.

The dynamics of infected humans and infected mosquitoes are presented in Figure 6(a)-(b). From Figure 6, it can be seen that malaria prevention and insecticide controls provide a significant reduction in infected human and infected mosquito populations compared to those without controls.

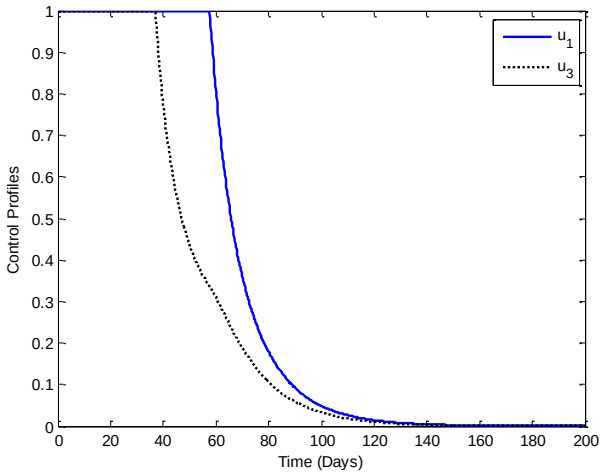
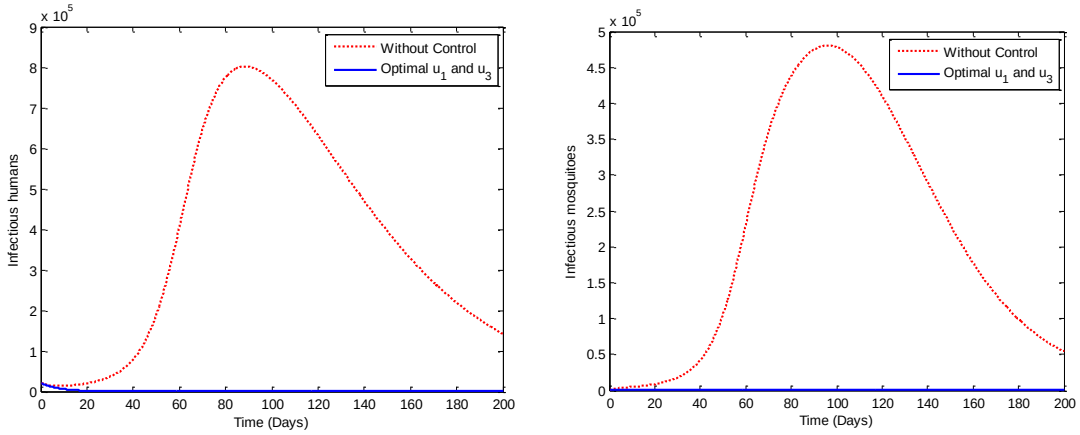


FIGURE 5. Optimal control profiles of u_1^* and u_3^*.



(a) (b)

FIGURE 6. The dynamics of I_h (a) and I_v (b) without and with controls u_1^* and u_3^*.

8.3 THIRD SCENARIO

In the third scenario, the combination of malaria treatment u_2 and insecticide u_3 are utilized, while malaria prevention was not used ($u_1 = 0$). The optimal control profiles for u_2 and u_3 are summarized in Figure 7. The malaria treatment should be adopted intensively for 47 days, whereas insecticide should be done intensively for 65 days.

The dynamics of infected humans and infected mosquitoes are set out in Figure 8(a)-(b). From this figure, it is apparent that malaria treatment and insecticide controls provide a significant reduction in infected human and infected mosquito populations compared to having no controls.

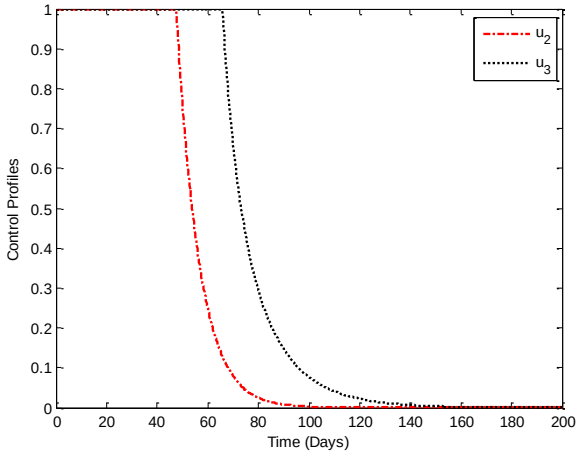
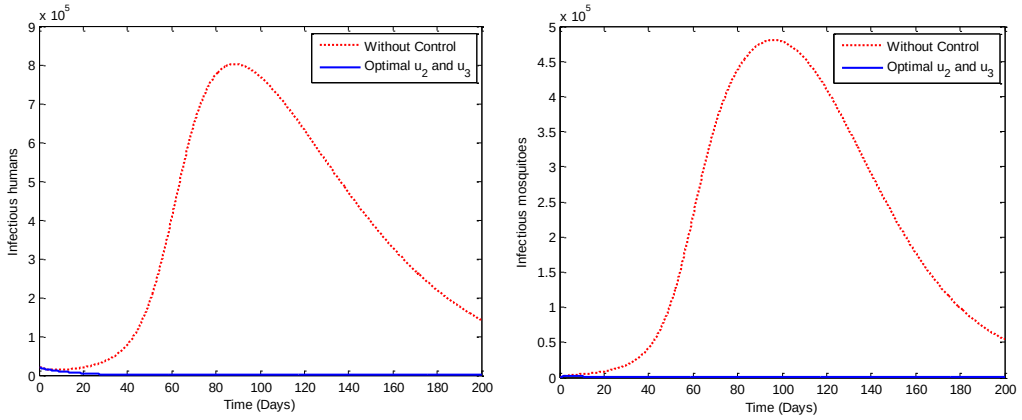


FIGURE 7. Control profiles of u_2^* and u_3^*



(a) (b)

FIGURE 8. The dynamics of I_h (a) and I_v (b) without and with controls u_2^* and u_3^*.

8.4 FOURTH SCENARIO

In the fourth scenario, the combination of malaria prevention u_1, malaria treatment u_2, and insecticide u_3 is implemented. The optimal control profiles for u_1, u_2, and u_3 are depicted in Figure 9. Malaria prevention should be given intensively for 57 days, malaria treatment should be kept intensively for 32 days, whereas insecticide should be preserved intensively for 36 days.

The dynamics of infected humans and infected mosquitoes are shown in Figure 10(a)-(b). From this figure, we can see that malaria prevention, malaria treatment, and insecticide controls provide a significant reduction in infected humans and infected mosquitoes compared to the scenario without controls.

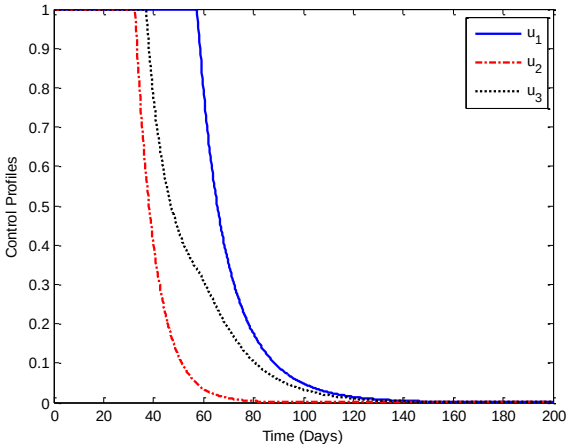
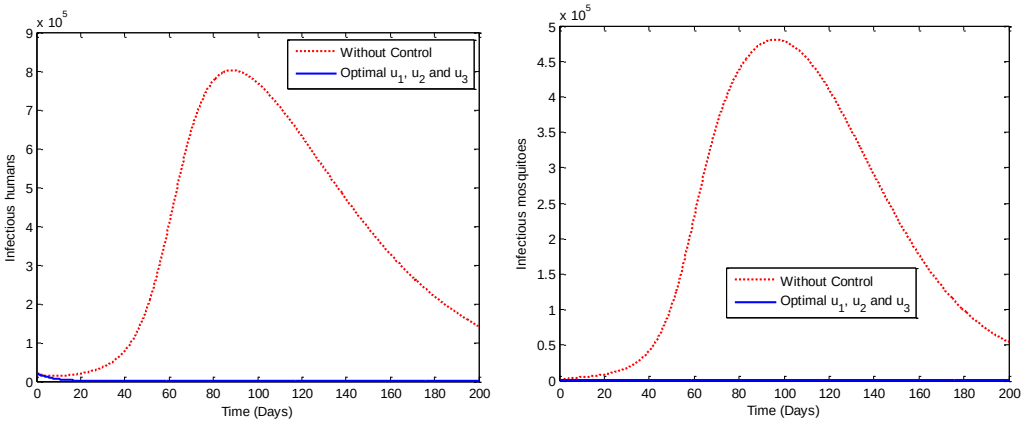


FIGURE 9. Optimal control profiles of u_1^*, u_2^*, and u_3^*.



(a) (b)

FIGURE 10. The dynamics of I_h (a) and I_v (b) without and with controls u_1^*, u_2^*, and u_3^*.

Table 4 presents a comparison of the cost functions required for various scenarios of applying the controls used.

TABLE 4. Cost function of every scenario

Scenario	Cost Function
Controls u_1 and u_2	182,740
Controls u_1 and u_3	176,670
Controls u_2 and u_3	299,210
Controls u_1, u_2, and u_3	172,130

From Table 4, it is shown that of the four simulated scenarios, the implementation of control in the form of prevention control u_1, treatment control u_2, and insecticide control u_3 which are utilized simultaneously have the minimum value of the cost function. Hence, it can be observed that scenario four is the best strategy to minimize the infected human and the infected mosquito populations, as well as the costs of the implementation for the controls.

9. CONCLUSION

In this work, we have analyzed the host-vector model with seasonality effect to depict the dynamics of malaria transmission. Initially, we briefly present the stability of the malaria model without the seasonality effect. The disease-free equilibrium is locally asymptotically stable if the basic reproduction number is less than one. Thus, the parameters of the model are estimated by using the malaria cases of Papua province, Indonesia for the year 2018. From the result of the estimation of the parameters, we obtain the basic reproduction number that is $R_0 \approx 1.3141$. This finding indicates that malaria infection persists in the province. Thus, we developed the malaria model by incorporating the seasonal effects to capture the climate factor in Indonesia. The simulation results suggest the seasonal effects very influential on the dynamics of infected humans and both susceptible and infected mosquitoes. Next, we present an optimal control problem with a control function and obtain the optimal control characterization. The optimal

control problem is solved numerically, and the results comprised of the control system for different strategies. Finally, we concluded that the best control strategy is the fourth scenario i.e., activating all control variables at the same time is more prominent to reduce malaria disease in the community.

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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