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OPTIMAL CONTROL AND COST-EFFECTIVENESS ANALYSIS OF TAENIASIS AND CYSTICERCOSIS IN HUMANS, PIGS AND CATTLE

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Abstract. Taeniasis and cysticercosis are neglected food-borne diseases that pose challenge to food safety, human health and livelihood of rural livestock farmers. In this paper, an optimal control problem for the dynamics and control of taeniasis and cysticercosis in humans, pigs and cattle with its cost-effectiveness analysis is presented and analysed to determine the optimal and cost-effective strategy for disease control. A combination of two or more time dependent controls involving vaccination of pigs and cattle, meat inspection, environmental hygiene and sanitation, and the treatment of humans who are infected with taeniasis is carried out to study their impacts on disease control. The Pontryagin's maximum principle is adopted to find the necessary conditions for existence of the optimal controls. The Runge Kutta order four forward-backward sweep method is implemented to solve the optimal control problem. The incremental cost-effectiveness ratio (ICER) is applied to determine the most cost-effective strategy for disease control. The optimal control results indicate that the strategy which focus on the combination of all interventions or that exclude vaccination of pigs and cattle is the most effective optimal control strategy in disease control. However, cost-effectiveness analysis results show that a strategy which excludes vaccination of pigs and cattle is the most cost-effective strategy for disease control. Based on these results, we recommend that interventions which focus on meat inspection, treatment of humans who are infected with taeniasis

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and improvement in hygiene and sanitation should be considered to control the transmission of taeniasis and cysticercosis in humans, pigs and cattle at a minimal cost.

Keywords: taeniasis; cysticercosis; optimal control; Hamiltonian function; cost-effectiveness analysis.

2010 AMS Subject Classification: 35F21.

1. INTRODUCTION

Taeniasis refers to the dwelling of adult tapeworms in the human's small intestine caused by consumption of raw or undercooked beef or pork infected with *Taenia sagnata* or *Taenia solium* tapeworm larval cysts respectively [1]. Cysticercosis is a tissue infection with the larval cysts of tapeworms due to consumption of taenia eggs from the contaminated environment [2, 3]. Humans who are infected with taeniasis contaminate the soil, fodder, pastures or water sources when they defecate in the fields [4]. Cattle become infected by cysticercosis when they feed on the contaminated environment while pigs acquire cysticercosis through direct consumption of human feaces or indirectly when they ingest taenia eggs from the contaminated environment [5]. Cysticercosis in humans is a result of consuming $T.$ *solium* eggs from the contaminated environment via contaminated water, fruits and vegetables, or by putting contaminated fingers in the mouth [6]. Once the $T.$ *solium* eggs are consumed, they hatch in the small intestine, and develop into larvae which penetrate the intestinal wall and migrate to various parts of the body such as eyes, muscles, skin and the central nervous system through the blood circulatory system, where they form larval cysts [7]. When the cysts reach and infect the human brain, they cause neurocysticercosis which is the most severe form of tapeworm infection of the central nervous system and is the major cause of epilepsy worldwide [1]. $T.$ *solium* and $T.$ *sagnata* parasites have global distribution, being more endemic in many developing countries of Latin America, Africa and Asia [5]. These parasites affect people's health and the economy of rural livestock farmers by reducing the market value of pigs and cattle [7, 8].

Current interventions against human taeniasis and cysticercosis include the use of prescribed medication of praziquantel, tribendimidine, nitazoxanide, albendazole or niclosamide [9, 7, 5]. The treatment of neurocysticercosis may involve prolonged doses of albendazole and/or praziquantel with supporting therapy such as anti-epileptic drugs, corticosteroids, and possibly surgery for some cases [7]. Control strategies in pigs and cattle involve the use of vaccines

such as S3Pvac & TSOL18 for pigs and TSA-18 & TSA-9 for cattle [10, 11], and anthelmintic treatment with flubendazole, fenbendazole, oxfendazole, praziquantel and nitazoxanide [7, 8].

Mathematical modeling plays an important role in analysing the transmission dynamics of infectious diseases and helps in designing appropriate disease control measures. There are only few mathematical models that have been formulated and analysed to study the transmission dynamics and control of taeniasis and cysticercosis in humans and pigs only. Winskill et al. [8] formulated a mathematical model to assess the impact of various control strategies against *T. solium* taeniasis and cysticercosis. Six interventions that were applied either singularly or in combination are pig vaccination, mass drug administration for pigs; improved animal husbandry, improved sanitation, meat inspection, and treatment of humans who are infected with taeniasis. The results show that chemotherapeutic intervention focusing on humans or pigs is highly effective for disease control when applied singly, with annual chemotherapy of humans and pigs. Kyvsgaard et al. [12] formulated and analysed a model for simulating transmission and control of humans taeniasis and porcine cysticercosis due to *T. solium*. Three groups of interventions that were considered are the use of latrines, meat inspection, and cooking habits; rapid detection and treatment of human carriers or pig vaccination; and treatment of either infected humans or pigs. The results show that mass-treatment results in a short term reduction in disease prevalence, whereas interventions focusing on interruption of the parasite's life cycle lead to long-term reduction in disease prevalence. José et al. [13] formulated a mathematical model for control of life cycle of *T. solium* parasite through chemotherapy. The results show that chemotherapeutic interventions against pig cysticercosis or human taeniasis reduces the mean intensity of human taeniasis, pig cysticercosis and human cysticercosis. Sánchez-Torres et al. [14] formulated a mathematical model to asses the control of porcine cysticercosis and human taeniasis using pig vaccination and human chemotherapy. The results indicate that pig vaccination influences the transmission dynamics among vaccinees and other hosts, and when the protective efficacy and/or the coverage rate is less than 100%, a combination of pig vaccination with chemotherapeutic treatment against human taeniasis, leads to elimination of the infection in both pigs and humans.

Recently, the theory of optimal control and cost effectiveness analysis have gained popularity due to their power to analyse various mathematical models for determining effective control strategies. To the best of our knowledge, very little has been done in applying the theory of optimal control to analyse mathematical models that describe the dynamics of food-borne disease. Currently, there is no study that has applied the optimal control theory and cost-effectiveness analysis to assess the best strategy for controlling the transmission of taeniasis and cysticercosis in humans, pigs and cattle. In this paper, we formulate and analyse the optimal control model for taeniasis and cysticercosis and apply the cost-effectivess analysis method to determine the most cost-effective strategy.

The rest of this article is arranged as follows: In Section 2, we derive and analyse a mathematical model for the transmission dynamics of taeniasis and cysticercosis in humans, pigs and cattle with some control measures. In Section 3, we present and analyse the optimal control problem for taeniasis and cysticercosis. Section 4 is devoted to numerical simulations of the optimal control problem with cost effectives analysis when two or more control measures are implemented. The conclusion is presented in Section 5.

2. Model Formulation

A mathematical model for the transmission dynamics of taeniasis and cyticercosis with control strategies is formulated basing on the basic model in Mwasunda et al. [15]. We divide the human population into S_H, I_{HT} and I_{HC} classes that represent susceptible humans, humans infected with taeniasis and humans infected with cysticercosis respectively. Pig and cattle populations are divided into susceptible, vaccinated, infected and recovered classes represented by S_P, V_P, I_P, R_P and S_C, V_C, I_C, R_C respectively. Classes P_I and B_I denote pork and beef infected with larval cysts respectively, while E_T is the number of taenia eggs in the environment.

We assume a costant recruitment rate Λ_H for susceptible humans through birth. Susceptible humans are assumed to consume raw or insufficiently cooked infected pork or beef at rates α_P and α_b respectively. Following the consumption of raw or insufficiently cooked infected pork or beef, the probability of susceptible humans getting infected with taeniasis is β_T. Susceptible humans can get infected with cysticercosis at a rate θ when they consume *T. solium* eggs from the contaminated water, fruits and vegetables or by putting contaminated fingers in their mouth

[6]. Humans infected with taeniasis and cysticercosis recover due to treatment at rates χ and χ_c, respectively and move to the susceptible class. All humans are assumed to die naturally at a rate μ_h, except humans infected with cysticercosis who have an additional disease induced mortality rate μ_d. Taenia eggs in the environment develop and grow at a rate ν due to open defecation by humans who are infected with taeniasis and diminish at a rate μ_e due to natural death.

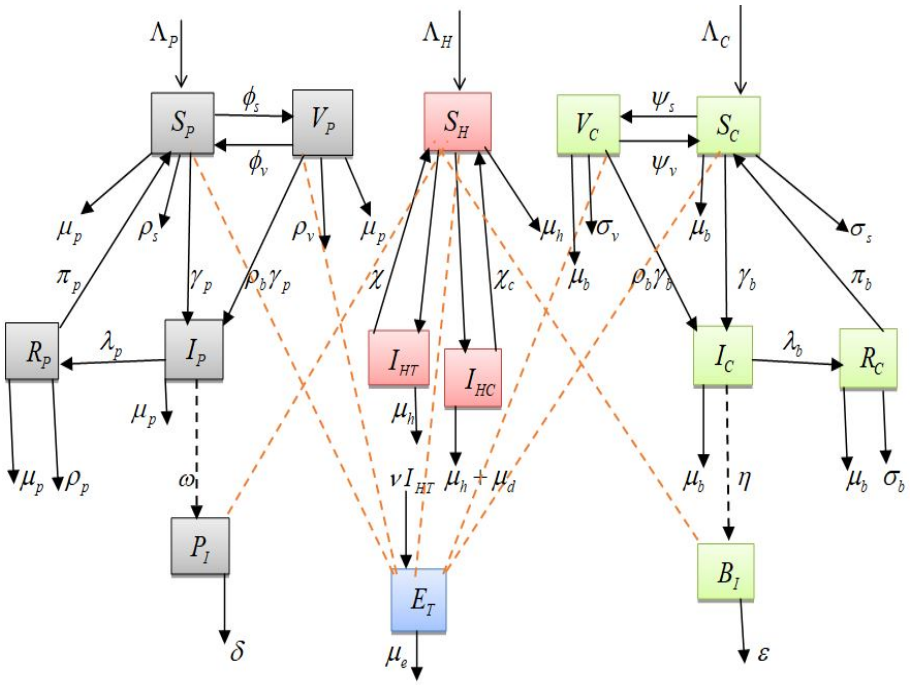

FIGURE 1. Model Schematic Diagram

Susceptible pigs and cattle are recruited at per capta rates Λ_P and Λ_c respectively due to birth. Parameters γ_p and γ_b are the rates at which pigs and cattle acquire cysticercosis from the contaminated environment respectively, and ρ_b is the vaccine efficacy for protecting vaccinated cattle and pigs against infection. The parameters λ_p and ω are the rates at which infected pigs are treated and slaughtered for consumption respectively while λ_b and η are the rates at which infected cattle are treated and slaughtered for consumption respectively. Both recovered pigs and cattle loose their immunity at rates π_p and π_b respectively and move to susceptible classes. Susceptible pigs and cattle are vaccinated at rates ϕ_s and ψ_s respectively and slaughtered for consumption at rates ρ_s and σ_s respectively. The parameters ϕ_v and ψ_v are the rates at which the vaccines wane in vaccinated pigs and cattle respectively. Moreover, vaccinated pigs and cattle

are slaughtered for consumption at rates ρ_v and σ_v respectively while recovered pigs and cattle are slaughtered at the rates ρ_p and and σ_b respectively. All pigs and cattle suffer natural death at rates μ_p and μ_b respectively. The parameters δ and ε are the proportions of infected pork and beef which are not consumed by susceptible humans.

In formulating the mathematical model, we consider the free range farming system for both pigs and cattle populations, and we do not consider immigration. We assume that humans can be infected by either taeniasis or cysticercosis; the number of taenia eggs consumed by humans, pigs and cattle has negligible effect on the total number of eggs in the environment and infected humans, pigs and cattle cannot recover naturally without treatment. We further assume that pigs and cattle do not suffer disease induced mortality, they become carriers for their life and the rates at which susceptible humans consume infected raw or undercooked pork or beef depend on the amount of infected pork or beef that is present. Humans, pigs and cattle contact rates with taenia eggs in the environment are assumed to be density dependent. We also assume that infected humans, pigs and cattle cannot recover from infections without treatment and the vaccines which are provided to susceptible pigs and cattle are not 100% effective, hence they wane after sometime. The model parameters are summarized in TABLE 1. FIGURE 1 describes the interaction of human, pigs and cattle in the presence of taeniasis and cysticercosis.

TABLE 1. Description of Model Parameters

Parameter	Description
Λ_H	Per capita recruitment rate of human population
Λ_C	Per capita recruitment rate of cattle population
Λ_P	Per capita recruitment rate of pig population
μ_h	Per capita natural death rate of humans
α_p	Rate of eating raw/undercooked infected pork
λ_p	Recovery rate of infected pigs from cysticercosis
π_p	Immunity waning rate for recovered pigs
λ_b	Recovery rate of infected cattle from cysticercosis

Parameter	Description
π_b	Immunity waning rate for recovered cattle
χ	Recovery rate of infected humans from taeniasis
χ_c	Recovery rate of infected humans from cysticercosis
α_b	Rate of eating raw/undercooked infected beef
γ_p	*T. solium* eggs to pig transmission rate
γ_b	*T. saginata* eggs to susceptible cattle transmission rate
ρ_b	Vaccine efficacy to protect cattle and pigs against infection
ω	Slaughter rate of infected pigs
θ	*T. solium* eggs to human cysticercosis transmission rate
η	Slaughter rate of infected cattle
δ	Proportion of unconsumed infected pork by humans
β_T	Probability of humans getting taeniasis
ν	Defecation rate by humans with taeniasis
μ_e	Per capita death rate of taenia sagnata eggs
ρ_s	Harvesting rate of susceptible pigs
ρ_v	Harvesting rate of vaccinated pigs
ρ_p	Harvesting rate of recovered pigs
ϕ_s	Rate at which susceptible pigs are vaccinated
ϕ_v	Waning rate of vaccine in pigs
ψ_s	Rate at which susceptible cattle are vaccinated
ψ_v	Waning rate of vaccine in cattle
μ_b	Per capita natural death rate of cattle
μ_p	Per capita natural death rate of pigs
μ_d	Human cysticercosis disease induced death rate
ε	Proportion of unconsumed infected beef by humans

Following the description above, the mathematical model for the transmission dynamics and control of taeniasis and cysticercosis in human, pig and cattle populations is governed by the following system of differential equations:

$$
\begin{aligned}
\frac{dS_H}{dt} &= \Lambda_H + \chi I_{HT} + \chi_c I_{HC} - \beta_T(\alpha_p P_I + \alpha_b B_I)S_H - \theta S_H E_T - \mu_h S_H,\\
\frac{dI_{HT}}{dt} &= \beta_T(\alpha_p P_I + \alpha_b B_I)S_H - (\chi + \mu_h)I_{HT},\\
\frac{dI_{HC}}{dt} &= \theta S_H E_T - (\mu_h + \mu_d + \chi_c)I_{HC},\\
\frac{dS_P}{dt} &= \Lambda_P + \pi_p R_P + \phi_v V_P - \gamma_p S_P E_T - (\rho_s + \mu_p + \phi_s)S_P,\\
\frac{dV_P}{dt} &= \phi_s S_P - \rho_b \gamma_p V_P E_T - (\rho_v + \mu_p + \phi_v)V_P,\\
\frac{dI_P}{dt} &= \gamma_p S_P E_T + \rho_b \gamma_p V_P E_T - (\omega + \lambda_p + \mu_p)I_P,\\
\frac{dR_P}{dt} &= \lambda_p I_P - (\rho_p + \pi_p + \mu_p)R_P,\\
\frac{dP_I}{dt} &= \omega I_P - (\delta + \alpha_P)P_I,\\
\frac{dS_C}{dt} &= \Lambda_C + \pi_b R_C + \psi_v V_C - \gamma_b S_C E_T - (\sigma_s + \mu_b + \psi_s)S_C,\\
\frac{dV_C}{dt} &= \psi_s S_C - \rho_b \gamma_b V_C E_T - (\sigma_v + \mu_b + \psi_v)V_C,\\
\frac{dI_C}{dt} &= \gamma_b S_C E_T + \rho_b \gamma_b V_C E_T - (\eta + \lambda_b + \mu_b)I_C,\\
\frac{dR_C}{dt} &= \lambda_b I_C - (\sigma_b + \pi_b + \mu_b)R_C,\\
\frac{dB_I}{dt} &= \eta I_C - (\varepsilon + \alpha_b)B_I,\\
\frac{dE_T}{dt} &= \nu I_{HT} - \mu_e E_T,
\end{aligned}
\tag{1}
$$

with initial conditions:

$$S_H(0) > 0; I_{HT}(0) \geq 0; I_{HC}(0) \geq 0; S_P(0) > 0; V_P(0) > 0; I_P(0) \geq 0; R_P(0) \geq 0;$$

$$P_I(0) \geq 0; S_C(0) > 0; V_C(0) > 0; I_C(0) \geq 0; R_C(0) \geq 0; B_I(0) \geq 0 \text{ and } E_T(0) \geq 0.$$

2.1. Model Basic Properties. For the model system (1) to be epidemiologically meaningful, we need to show that all its state variables are non-negative and bounded.

2.1.1. *Positivity of Model Solutions.* From the first equation of the model system (1), we have:

$$\begin{aligned}
\frac{dS_H}{dt} &= \Lambda_H + \chi I_{HT} + \chi_c I_{HC} - \beta_T(\alpha_p P_I + \alpha_b B_I)S_H - \theta S_H E_T - \mu_h S_H,\\
\frac{dS_H}{dt} &\geq -\Big(\beta_T(\alpha_p P_I + \alpha_b B_I) + \theta E_T + \mu_h\Big)S_H,\\
\frac{dS_H}{S_H} &\geq -\Big(\beta_T(\alpha_p P_I + \alpha_b B_I) + \theta E_T + \mu_h\Big)dt,\\
S_H(t) &\geq S_H(0)e^{\int_0^t -\Big(\beta_T(\alpha_p P_I + \alpha_b B_I) + \theta E_T + \mu_h\Big)ds} \geq 0, \forall t \geq 0.
\end{aligned}$$

In similar manner, it can be shown that:

$$I_{HT}(t) \geq 0; I_{HC}(t) \geq 0; S_P(t) \geq 0; V_P(t) \geq 0; I_P(t) \geq 0; R_P(t) \geq 0; P_I(t) \geq 0;$$

$$S_C(t) \geq 0; V_C(t) \geq 0; I_C(t) \geq 0; R_C(t) \geq 0; B_I(t) \geq 0; E_T(t) \geq 0, \forall t \geq 0.$$

2.1.2. *Invariant Region.* To find the biologically feasible region, we let

$$N_H = S_H + I_{HT} + I_{HC},\ N_P = S_P + V_P + I_P + R_P, \text{ and } N_C = S_C + V_C + I_C + R_C \tag{2}$$

to be total population for humans, pigs and cattle respectively. Adding all the equations of the model system (1) for human population, we obtain:

$$\begin{aligned}
\frac{dN_H}{dt} &= \Lambda_H - \mu_h N_H - \mu_d I_{HC},\\
\implies \frac{dN_H}{dt} &+ \mu_h H \leq \Lambda_H.
\end{aligned} \tag{3}$$

Integrating throughout and assuming that the initial total human population $N_H(0)$ cannot exceed Λ_H/μ_h, then it can be shown that the total human population $N_H \to \Lambda_H/\mu_h$ as $t \to \infty$. Also, using the same procedure the total pig population $N_P \to \Lambda_P/(\rho_s + \mu_b)$ and cattle population $N_C \to \Lambda_C/(\sigma_s + \mu_b)$ as $t \to \infty$.

Considering the last equation of the model system (1) for taenia eggs in the environment, we have:

$$\frac{dE_T}{dt} = \nu I_{HT} - \mu_e E_T. \tag{4}$$

Since the total human population $N_H \to \Lambda_H/\mu_h$, it can be concluded that $I_{HT} \leq \Lambda_H/\mu_h$. Thus we have:

$$\begin{aligned} \frac{dE_T}{dt} &\leq \nu\Lambda_H/\mu_h - \mu_e E_T, \\ \implies \frac{dE_T}{dt} &+ \mu_e E_T \leq \nu\Lambda_H/\mu_h. \end{aligned} \tag{5}$$

Applying the same procedure as for equation (3), it can be shown that the number of taenia eggs in the environment $E_T \to \nu\Lambda_H/\mu_h\mu_e$ as $t \to \infty$. Similarly, considering the equations for infected pork and beef in the mode system (1), it can be shown that $P_I \to \omega\Lambda_P/\Big((\alpha_b+\delta)(\rho_s+\mu_b)\Big)$ and $B_I \to \eta\Lambda_C/\Big((\varepsilon+\alpha_b)(\sigma_s+\mu_b)\Big)$ as $t \to \infty$. Therefore, this indicates that the biological feasible region:

$$\begin{aligned} \Omega = \Big\{ & (S_H, I_{HT}, I_{HC}, S_P, V_P, I_P, R_P, P_I, S_C, V_C, I_C, R_C, B_I, E_T) \in \mathbb{R}_+^{14} : \\ & N_H \leq \Lambda_H/\mu_h; N_P \leq \Lambda_P/(\rho_s+\mu_b);\ N_C \leq \Lambda_C/\Big((\sigma_s+\mu_b)\Big); E_T \leq \nu\Lambda_H/\mu_h\mu_e; \\ & P_I \leq \omega\Lambda_P/\Big((\alpha_b+\delta)(\rho_s+\mu_b)\Big); B_I \leq \eta\Lambda_C/\Big((\varepsilon+\alpha_b)(\sigma_s+\mu_b)\Big) \Big\} \end{aligned} \tag{6}$$

is positive invariant and hence the model system (1) is epidemiologically and mathematically well posed. Therefore, it is sufficient to consider the model system (1) for analysis.

2.1.3. *The Disease Free Equilibrium and Effective Reproduction Number.* When there are no diseases in human, pig and cattle populations, then the disease free equilibrium E^0 is given by:

$$E^0 = \left(\frac{\Lambda_H}{\mu_h}, 0, 0, \frac{d_0\Lambda_P}{M_0}, \frac{\phi_s\Lambda_P}{M_0}, 0, 0, 0, \frac{c_0\Lambda_C}{K_0}, \frac{\psi_s\Lambda_C}{K_0}, 0, 0, 0, 0\right), \tag{7}$$

where

$$c_0 = (\sigma_v + \psi_v + \mu_b)\ ,\ K_0 = c_0(\sigma_s+\mu_b) + \psi_s(\sigma_v+\mu_b),$$

$$d_0 = (\rho_v + \phi_v + \mu_p)\ ,\ M_0 = d_0(\rho_s+\mu_p) + \phi_s(\rho_v+\mu_p).$$

The effective reproduction number R_e is the expected number of new infections that may arise as a consequence of introducing one infected individual in a susceptible population when some interventions are implemented to control the spread of the disease [16]. The controls will be effective if $R_e < 1$ and not effective if $R_e > 1$. In this paper, we adopt the next generation matrix method to compute the effective reproduction number R_e [17]. Let $\mathscr{F}_i$ and $\mathscr{V}_i$ be new infections

and transfer terms in compartment i respectively, given by:

$$\mathscr{F}_i = \begin{pmatrix} \beta_T(\alpha_p P_I + \alpha_b B_I)S_H \\ \theta S_H E_T \\ \gamma_p S_P E_T + \rho_b \gamma_p V_P E_T \\ 0 \\ \gamma_b S_C E_T + \rho_b \gamma_c V_C E_T \\ 0 \\ 0 \end{pmatrix}, \ \mathscr{V}_i = \begin{pmatrix} (\chi + \mu_h) I_{HT} \\ (\mu_h + \mu_d + \chi_c) I_{HC} \\ (\omega + \lambda_p + \mu_p) I_P \\ -\omega I_P + (\delta + \alpha_p) P_I \\ (\eta + \lambda_b + \mu_b) I_C \\ -\eta I_C + (\varepsilon + \alpha_b) B_I \\ -\nu I_{HT} + \mu_e E_T \end{pmatrix}. \tag{8}$$

The effective reproduction number R_e is given by:

$$R_e = \rho\left(FV^{-1}\right), \tag{9}$$

where F and V are Jacobian matrices evaluated at disease free equilibrium that are given by:

$$F = \frac{\partial \mathscr{F}_i}{\partial x_j}\left(E^0\right), \ V = \frac{\partial \mathscr{V}_i}{\partial x_j}\left(E^0\right). \tag{10}$$

Using the definitions in (10), we have:

$$F = \begin{pmatrix} 0 & 0 & 0 & \frac{\beta_T \alpha_p \psi}{\mu_h} & 0 & \frac{\beta_T \alpha_b \psi}{\mu_h} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\theta \psi}{\mu_h} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{(d_0 + \rho_b \phi_s)\gamma_p \Lambda_P}{M_0} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{(c_0 + \rho_b \psi_s)\gamma_p \Lambda_C}{K_0} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ and}$$

$$V = \begin{pmatrix} (\chi+\mu_h) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (\mu_h+\mu_d+\chi_c) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (\omega+\lambda_p+\mu_p) & 0 & 0 & 0 & 0 \\ 0 & 0 & -\omega & (\delta+\alpha_p) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (\eta+\lambda_b+\mu_b) & 0 & 0 \\ 0 & 0 & 0 & 0 & -\eta & (\varepsilon+\alpha_b) & 0 \\ -\nu & 0 & 0 & 0 & 0 & 0 & \mu_e \end{pmatrix}.$$

From equation (9), the effective reproduction number R_e is given by:

$$R_e = \sqrt{R_{e1}+R_{e2}}, \tag{11}$$

where

$$R_{e1} = \frac{\beta_T \nu \omega \alpha_p \gamma_p \Lambda_H \Lambda_P (d_o + \rho_b \phi_s)}{M_0 \mu_h \mu_e (\chi+\mu_h)(\delta+\alpha_p)(\omega+\lambda_p+\mu_p)} \text{ and } R_{e2} = \frac{\beta_T \nu \eta \alpha_b \gamma_b \Lambda_H \Lambda_C (c_o + \rho_b \psi_s)}{K_0 \mu_h \mu_e (\chi+\mu_h)(\varepsilon+\alpha_b)(\eta+\lambda_b+\mu_b)}.$$

R_{e1} and R_{e2} are the partial reproduction numbers due to interaction of humans with pigs and with cattle respectively.

When there are no interventions ($\phi_s = \phi_v = \rho_v = \rho_b = \lambda_p = \lambda_b = \pi_p = \pi_b = \chi = \chi_c = \psi_s = \psi_v = \sigma_v = \sigma_b = 0$), then the effective reproduction number R_e reduces to the basic reproduction number R_0 given by:

$$R_0 = \sqrt{R_{HP}+R_{HC}}, \tag{12}$$

where

$$R_{HP} = \frac{\beta_T \nu \alpha_p \gamma_p \omega \Lambda_H \Lambda_P}{\mu_h^2 \mu_e (\omega+\mu_p)(\alpha_p+\delta)(\mu_p+\rho_s)} \text{ and } R_{HC} = \frac{\beta_T \nu \alpha_b \gamma_b \eta \Lambda_H \Lambda_C}{\mu_h^2 \mu_e (\eta+\mu_b)(\alpha_b+\varepsilon)(\mu_b+\sigma_s)}.$$

FIGURE 2 shows the comparison between effective reproduction number (R_e) and basic reproduction number (R_0) when some parameters are varied using parameter values in TABLE 2. We aim at assessing whether implementation of control measures have significant impact in reducing or eliminating taeniasis and cysticercosis in humans, pig and cattle.

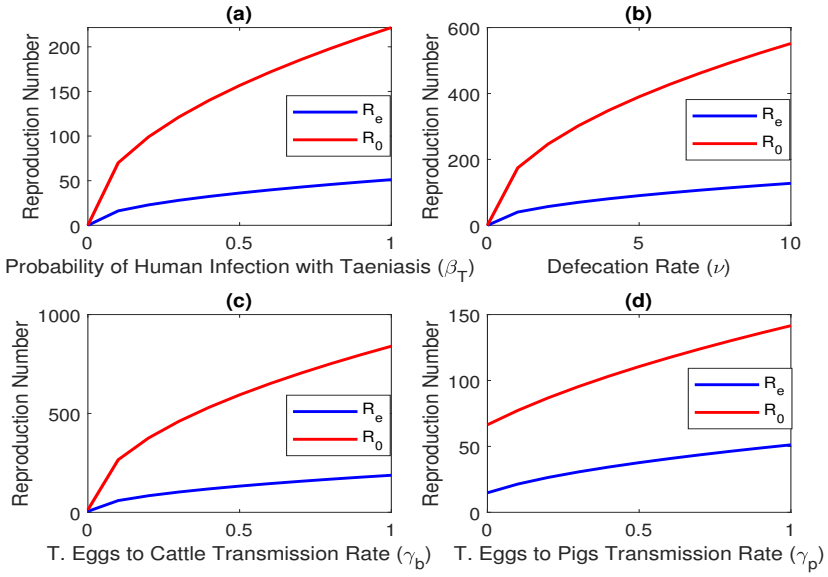

FIGURE 2. Variations in R_0 and R_e with respect to β_T, ν, γ_p and γ_b

It can be observed from FIGURE 2 (a)&(b) that both the basic reproduction number (R_0) and effective reproduction number (R_e) increase in proportion to probability of human infection with taeniasis β_T and human defecation rate ν respectively, having zero value when $\beta_T = 0$ and $\nu = 0$. Similarly, in FIGURE 2 (c)&(d) both R_0 and R_e increases with increase in the rate of taenia eggs to pigs transmission (γ_p) and the rate of taenia eggs to cattle transmission (γ_b) respectively, having different values throughout. In all cases, the effective reproduction number (R_e) is less than the basic reproduction number (R_0), implying that there is a possibility of reducing the prevalence of taeniasis and cysticercosis in humans, pigs and cattle when some interventions are implemented.

3. THE OPTIMAL CONTROL MODEL

In this section we administer time dependent control variables to see if they can eradicate or contain the diseases. The choice of control variables is based on the sensitivity analysis results in Mwasunda et al. [15] which shows that interventions which focus on reducing the probability of human infection with taeniasis and defecation rate by humans with taeniasis are effective for controlling the transmission of taeniasis and cysticercosis in humans, pigs and cattle. To optimally control taeniasis and cysticercosis, we incorporate five time dependent control variables

that are $u_1(t)$ which measures the effect of meat inspection in reducing the possibility of human infection with taeniasis, $u_2(t)$ and $u_3(t)$ which represent the control efforts due to vaccination of pigs and cattle respectively, $u_4(t)$ which measures treatment efforts of humans who are infected with taeniasis and $u_5(t)$ which measures the impact of improved hygiene and sanitation to reduce the rate of transfer of infections from contaminated environment to pigs, cattle and humans. Thus, incorporating these control variables in model system (1) we obtain:

$$
\begin{aligned}
&\frac{dS_H}{dt} = \Lambda_H + u_4 I_{HT} + \chi_c I_{HC} - \beta_T(1-u_1)(\alpha_p P_I + \alpha_b B_I)S_H \\
&-\theta(1-u_5)S_H E_T - \mu_h S_H, \\
&\frac{dI_{HT}}{dt} = \beta_T(1-u_1)(\alpha_p P_I + \alpha_b B_I)S_H - (u_4+\mu_h)I_{HT}, \\
&\frac{dI_{HC}}{dt} = \theta(1-u_5)S_H E_T - (\mu_h + \mu_d + \chi_c)I_{HC}, \\
&\frac{dS_P}{dt} = \Lambda_P + \pi_p R_P + \phi_v V_P - \gamma_p(1-u_5)S_P E_T - (\rho_s + \mu_p + u_2)S_P, \\
&\frac{dV_P}{dt} = u_2 S_P - \rho_b\gamma_p(1-u_5)V_P E_T - (\rho_v + \mu_p + \phi_v)V_P, \\
&\frac{dI_P}{dt} = \gamma_p(1-u_5)S_P E_T + \rho_b\gamma_p(1-u_5)V_P E_T - (\omega + \lambda_p + \mu_p)I_P, \\
&\frac{dR_P}{dt} = \lambda_p I_P - (\rho_p + \pi_p + \mu_p)R_P, \\
&\frac{dP_I}{dt} = \omega I_P - (\delta + u_1 + \alpha_P)P_I, \\
&\frac{dS_C}{dt} = \Lambda_C + \pi_b R_C + \psi_v V_C - \gamma_b(1-u_5)S_C E_T - (\sigma_s + \mu_b + u_3)S_C, \\
&\frac{dV_C}{dt} = u_3 S_C - \rho_b\gamma_b(1-u_5)V_C E_T - (\sigma_v + \mu_b + \psi_v)V_C, \\
&\frac{dI_C}{dt} = \gamma_b(1-u_5)S_C E_T + \rho_b\gamma_b(1-u_5)V_C E_T - (\eta + \lambda_b + \mu_b)I_C, \\
&\frac{dR_C}{dt} = \lambda_b I_C - (\sigma_b + \pi_b + \mu_b)R_C, \\
&\frac{dB_I}{dt} = \eta I_C - (\varepsilon + u_1 + \alpha_b)B_I, \\
&\frac{dE_T}{dt} = \nu(1-u_5)I_{HT} - \mu_e E_T,
\end{aligned}
\tag{13}
$$

We aim at minimizing the number of infected humans, infected pigs and cattle, and the cost of administering the controls. The objective function that minimizes infected humans, pigs, cattle and the cost for administering the controls is given as:

$$
J = \int_0^{T_f} \Big(C_1 I_{HT} + C_2 I_{HC} + C_3 I_P + C_4 I_C + C_5 u_2 S_P + C_6 u_3 S_C + \frac{1}{2}\sum_{i=1}^{i=5} A_i u_i^2 \Big) dt \tag{14}
$$

subject to system of differential equations (13), where C_1 and C_2 are the constants for minimizing humans who are infected with taeniasis and cysticercosis respectively, C_3 and C_4 are the weight constants for infected pigs and cattle that are slaughtered for consumption respectively whereas the terms u_2S_P and u_3S_C aim at minimizing the number of vaccines used for pigs and cattle with weight constants C_5 and C_6 respectively [18]. The coefficients A_1, A_2, A_3, A_4, A_5 are relative cost weight for each individual control measure that are used to transform the integral into cost expended over a period of T_f years which is the time period for applying the control strategy [19]. Control variables u_i^2 aim at minimizing the rate of implementing controls and their associated costs. The square term of the controls reflects the non-linearity nature for the cost of controls, and the half-term minimizes the effect of applying the controls [20]. In this paper, the weight constants C_i are chosen to be $C_i = (0.8, 1.2, 25, 40.5, 0.0075, 0.05)$ for $i = 1,2,3,4,5,6$ while the relative cost weight for each individual control measure $A_i = (500, 50, 450, 100, 3000)$ for $i = 1,2,3,4,5$. The initial values are set to be 2800, 2500, 550, 250, 140, 220, 100, 52, 340, 130, 250, 90, 83 and 100 for $S_H, I_{HT}, I_{HC}, S_P, V_P, I_P, R_P, P_I, S_C, V_C, I_C, R_C, B_I$ and E_T compartments respectively.

Therefore, we seek to find the optimal controls $u_1^*, u_2^*, u_3^*, u_4^*$ and u_5^* such that:

$$J(u_1^*, u_2^*, u_3^*, u_4^*, u_5^*) = \min_U J(u_1, u_2, u_3, u_4, u_5), \tag{15}$$

where $U = \{u : u \text{ is measurable and } 0 \leq u_i(t) \leq 1 \text{ for } t \in [0, T_f]\}$ is the control set.

3.1. Characterization of the Optimal Control Problem. We apply Pontryagin's maximum principle [21, 22] which provides the necessary conditions that an optimal control problem must satisfy. This principle converts the system of differential equations (13) and equation (14) into minimization problem point-wise Hamiltonian ($\mathscr{H}$), with respect to control variables $(u_1, u_2, u_3, u_4, u_5)$.

If we defined a Lagrangian $\mathscr{L}$ for the control problem by:

$$\mathscr{L} = C_1 I_{HT} + C_2 I_{HC} + C_3 I_P + C_4 I_C + C_5 u_2 S_P + C_6 u_3 S_C + \frac{1}{2} \sum_{i=1}^{i=5} A_i u_i^2, \tag{16}$$

then the Hamiltonian function $\mathscr{H}$ for the control problem is given as:

$$\mathscr{H} = \mathscr{L} + \sum_{i=1}^{i=14} \lambda_i g_i, \tag{17}$$

where g_i are the right hand sides of the model system (13) and λ_i are the adjoint variables associated with the states $S_H, I_{HT}, I_{HC}, S_P, V_P, I_P, R_P, P_I, S_C, V_C, I_C, R_C, B_I$ and E_T.

Using the Pontryagin's maximum principle [23, 24], there exist adjoint variables that satisfy:

$$\frac{d\lambda_i}{dt} = -\frac{\partial \mathscr{H}}{\partial i} \tag{18}$$

with transversality conditions:

$$\lambda_i(T_f) = 0. \tag{19}$$

Therefore, the adjoint system is given as:

$$\begin{aligned}
\frac{d\lambda_1}{dt} &= \beta_T(1-u_1)(\lambda_1-\lambda_2)(\alpha_p P_I + \alpha_b B_I) + \theta(1-u_5)(\lambda_1-\lambda_3)E_T + \mu_h\lambda_1,\\
\frac{d\lambda_2}{dt} &= (u_4+\mu_h)\lambda_2 - C_1 - u_4\lambda_1 - \nu(1-u_5)\lambda_{14},\\
\frac{d\lambda_3}{dt} &= (\mu_h+\mu_d+\chi_c)\lambda_3 - C_2 - \chi_c\lambda_1,\\
\frac{d\lambda_4}{dt} &= \gamma_p(1-u_5)(\lambda_4-\lambda_6)E_T + (\rho_s+\mu_p+u_2)\lambda_4 - u_2\lambda_5 - u_2C_5,\\
\frac{d\lambda_5}{dt} &= \rho_b\gamma_p(1-u_5)(\lambda_5-\lambda_6)E_T + (\rho_v+\mu_p+\phi_v)\lambda_5 - \psi_v\lambda_4,\\
\frac{d\lambda_6}{dt} &= (\omega+\lambda_p+\mu_p)\lambda_6 - \lambda_p\lambda_7 - \omega\lambda_8 - C_3,\\
\frac{d\lambda_7}{dt} &= (\rho_p+\pi_p+\mu_p)\lambda_7 - \pi_p\lambda_4,\\
\frac{d\lambda_8}{dt} &= \beta_T(1-u_1)(\lambda_1-\lambda_2)\alpha_p S_H + (\delta+u_1+\alpha_P)\lambda_8,\\
\frac{d\lambda_9}{dt} &= \gamma_b(1-u_5)(\lambda_9-\lambda_{11})E_T + (\sigma_s+\mu_b+u_3)\lambda_9 - u_3\lambda_{10} - u_3C_6,\\
\frac{d\lambda_{10}}{dt} &= \rho_b\gamma_b(1-u_5)(\lambda_{10}-\lambda_{11})E_T + (\sigma_v+\mu_b+\psi_v)\lambda_{10} - \psi_v\lambda_9,\\
\frac{d\lambda_{11}}{dt} &= (\eta+\lambda_b+\mu_b)\lambda_{11} - \lambda_b\lambda_{12} - \eta\lambda_{13} - C_4,\\
\frac{d\lambda_{12}}{dt} &= (\sigma_b+\pi_b+\mu_b)\lambda_{12} - \pi_b\lambda_9,\\
\frac{d\lambda_{13}}{dt} &= \beta_T(1-u_1)(\lambda_1-\lambda_2)\alpha_b S_H + (\varepsilon+u_1+\alpha_b)\lambda_{13},\\
\frac{d\lambda_{14}}{dt} &= \theta(1-u_5)(\lambda_1-\lambda_3)S_H + \gamma_p(1-u_5)(\lambda_4-\lambda_6)S_P + \rho_b\gamma_p(1-u_5)(\lambda_5-\lambda_6)V_P\\
&\quad + \gamma_b(1-u_5)(\lambda_9-\lambda_{11})S_C + \rho_b\gamma_b(1-u_5)(\lambda_{10}-\lambda_{11})V_C + \mu_e\lambda_{14}.
\end{aligned} \tag{20}$$

To obtain the optimality conditions, we differentiate the Hamiltonian function (17) with respect to the control variables and solve it when the derivative is zero, that is:

$$
\begin{aligned}
&\frac{\partial \mathscr{H}}{\partial u_1} = A_1 u_1 - (\lambda_2 - \lambda_1)\beta_T(\alpha_p P_I + \alpha_b B_I)S_H - \lambda_8 P_I - \lambda_{13} B_I = 0,\\
&\frac{\partial \mathscr{H}}{\partial u_2} = A_2 u_2 + C_5 S_P - (\lambda_4 - \lambda_5)S_P = 0,\\
&\frac{\partial \mathscr{H}}{\partial u_3} = A_3 u_3 + C_6 S_C - (\lambda_9 - \lambda_{10})S_C = 0,\\
&\frac{\partial \mathscr{H}}{\partial u_4} = A_4 u_4 - (\lambda_2 - \lambda_1) I_{HT} = 0,\\
&\frac{\partial \mathscr{H}}{\partial u_5} = A_5 u_5 - (\lambda_3 - \lambda_1)\theta S_H E_T + (\lambda_4 - \lambda_6)\gamma_p S_P E_T + (\lambda_5 - \lambda_6)\rho_b \gamma_p V_P E_T\\
&\quad + (\lambda_9 - \lambda_{11})\gamma_b S_C E_T + (\lambda_{10} - \lambda_{11})\rho_b \gamma_b V_C E_T - \lambda_{14}\nu I_{HT} = 0.
\end{aligned}
\tag{21}
$$

Since the characterization of the optimal control problem holds on the interior of the control set U, thus we have:

$$
\begin{aligned}
u_1^* &= max\Big\{0, min\Big(1, \frac{(\lambda_2 - \lambda_1)\beta_T(\alpha_p P_I + \alpha_b B_I)S_H + \lambda_8 P_I + \lambda_{13} B_I}{A_1}\Big)\Big\},\\
u_2^* &= max\Big\{0, min\Big(1, \frac{(\lambda_4 - \lambda_5 - C_5)S_P}{A_2}\Big)\Big\},\\
u_3^* &= max\Big\{0, min\Big(1, \frac{(\lambda_9 - \lambda_{10} - C_6)S_C}{A_3}\Big)\Big\},\\
u_4^* &= max\Big\{0, min\Big(1, \frac{(\lambda_2 - \lambda_1) I_{HT}}{A_4}\Big)\Big\},\\
u_5^* &= max\Big\{0, min\Big(1, \frac{\mathscr{F}_{var}}{A_5}\Big)\Big\}.
\end{aligned}
\tag{22}
$$

where

$$
\begin{aligned}
\mathscr{F}_{var} &= (\lambda_3 - \lambda_1)\theta S_H E_T + (\lambda_6 - \lambda_4)\gamma_p S_P E_T + (\lambda_6 - \lambda_5)\rho_b \gamma_p V_P E_T + (\lambda_{11} - \lambda_9)\gamma_b S_C E_T\\
&\quad + (\lambda_{11} - \lambda_{10})\rho_b \gamma_b V_C E_T + \lambda_{14}\nu I_{HT}.
\end{aligned}
$$

4. Numerical Results and Cost-Effectiveness Analysis

In this section, we present numerical simulations for the optimal control to assess the impact of various interventions on the control of taeniasis and cysticercosis. Also, cost-ffectiveness analysis is carried out to determine the most cost-effective strategy for controlling the diseases.

4.1. Numerical Results for the Optimal Control Problem. To obtain numerical solutions for the model, the forward-backward sweep method for the model system (13) and the adjoint system (20) are implemented in Matlab using parameter values in TABLE 2. The method begins by solving the model system (13) forward in time using Runge Kutta method of the fourth order relying on the supplied initial values of the controls. Then, the backward fourth order Runge Kutta method uses the obtained values of the state variables and initial values of controls to solve the adjoint equations (20) with given final condition (19). The controls $u_1(t), u_2(t), u_3(t), u_4(t), u_5(t)$ are then updated and used to solve the state and adjoint systems. Some parameter values have been assumed since only little has been done on this area and the diseases are common in rural areas where free range farming system for pigs and cattle is dominant, there is inadequate or no meat inspection and the treatment of these diseases is not readily available [25].

TABLE 2. Model Parameter Values (unit: yr^{-1})

Parameter	Value	Source	Parameter	Value	Parameter	Value	Parameter	Value
Λ_H	2247	[26]	Λ_C	750	η	0.235	π_p	0.213
μ_p	0.996	[8]	Λ_P	1450	θ	0.00523	ψ_v	0.115
μ_d	0.0925	[27]	ε	0.225	α_p	0.012	ψ_s	0.213
μ_h	0.0141	[27]	δ	0.358	ρ_b	0.1968	ϕ_v	0.413
ω	0.332	[12]	β_T	0.093	γ_b	0.00625	ϕ_s	0.317
μ_e	10.42	[27]	ν	0.150	λ_b	0.125	ρ_p	0.105
π_b	0.213	-	λ_p	0.125	α_b	0.023	ρ_v	0.075
μ_b	0.33	[27]	σ_s	0.213	χ_c	0.192	ρ_s	0.252
γ_p	0.01	[12]	σ_v	0.183	χ	0.225	σ_b	0.153

Since the implementation of only one control may not be effective in disease control, we illustrate the impact of combining atleast two controls as follows: Strategy 1: Meat inspection ($u_1(t)$), pigs and cattle vaccination ($u_2(t)$ and $u_3(t)$ respectively), treatment of humans who are infected with taeniasis ($u_4(t)$) and improved hygiene and sanitation ($u_5(t)$), Strategy 2: Meat inspection ($u_1(t)$), pigs and cattle vaccination ($u_2(t)$ and $u_3(t)$ respectively) and treatment of

humans who are infected with taeniasis ($u_4(t)$), Strategy 3: Meat inspection ($u_1(t)$), treatment of humans who are infected with taeniasis ($u_4(t)$) and improved hygiene and sanitation ($u_5(t)$), Strategy 4: Meat inspection ($u_1(t)$) with pigs and cattle vaccination ($u_2(t)$ and $u_3(t)$ respectively), Strategy 5: Pigs and cattle vaccination ($u_2(t)$ and $u_3(t)$ respectively), and treatment of humans who are infected with taeniasis ($u_4(t)$), Strategy 6: Meat inspection ($u_1(t)$) and improved hygiene and sanitation ($u_5(t)$), Strategy 7: Meat inspection ($u_1(t)$) and treatment of humans who are infected with taeniasis ($u_4(t)$), Strategy 8: Treatment of humans who are infected with taeniasis ($u_4(t)$) and improved hygiene and sanitation ($u_5(t)$), and Strategy 9: Pigs and cattle vaccination ($u_2(t)$ and $u_3(t)$ respectively).

4.1.1. *When all Controls are Implemented.* When all controls are implemented, all infected populations and taenia eggs in the environment decrease with time while humans with taeniasis decrease to zero in 3 years. Infected pigs, cattle and taenia eggs diminish to zero in 3, 5.5 and 1.5 years respectively. Only a small portion of humans who are infected with cysticercosis remains at the final time.

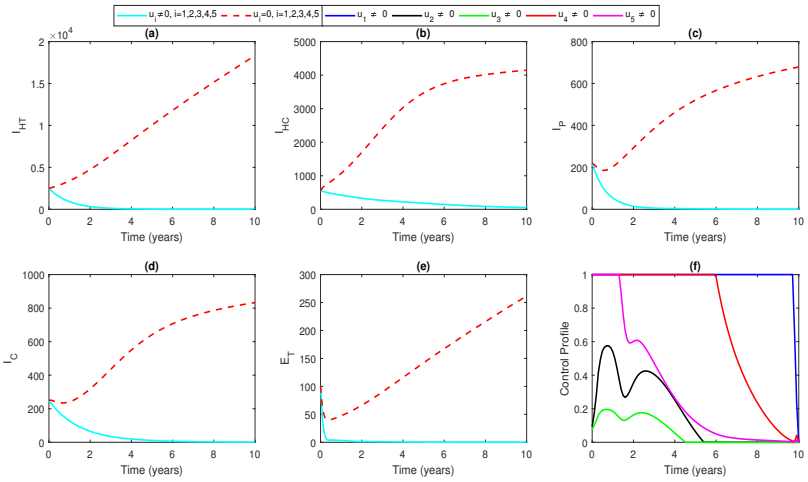

FIGURE 3. Impact of Applying All Controls on Infected Humans, Cattle, Pigs and Taenia Eggs

The control profiles for this strategy in FIGURE 3 (f) show that the control variables $u_1(t), u_4(t)$ and $u_5(t)$ are at their peak for the first 9.75, 6.5 and 1.5 years respectively and finally decline to zero. The control profiles for $u_2(t)$ and $u_3(t)$ variables have up and down movement and then decline to zero in the 5.5^{th} and 4.5^{th} years respectively.

4.1.2. *Meat Inspection, Pigs and Cattle Vaccination, and Treatment of Humans Infected with Taeniasis.* In this strategy, we consider a combination of meat inspection $(u_1(t))$, vaccination of pigs and cattle ($u_2(t)$ and $u_3(t)$ respectively) and treatment of humans with taeniasis $(u_4(t))$. It can be observed in FIGURE 4 that, cysticercosis in pigs and cattle can be eradicated after 5 and 7 years respectively while human taeniasis and taenia eggs in the environment decreases to zero after 3 years. However, a small proportion of humans infected with cysticercosis remains at the final time. The control profiles in FIGURE 4 (f) show that the control variables $u_1(t)$, $u_2(t), u_3(t)$ and $u_4(t)$ are at their peak for the first 9.5, 3, 1.75 and 6 years respectively. Thereafter $u_1(t)$ and $u_4(t)$ decline to zero whereas $u_2(t)$ and $u_3(t)$ reduces to zero in the 5^{th} and 5.5^{th} years respectively.

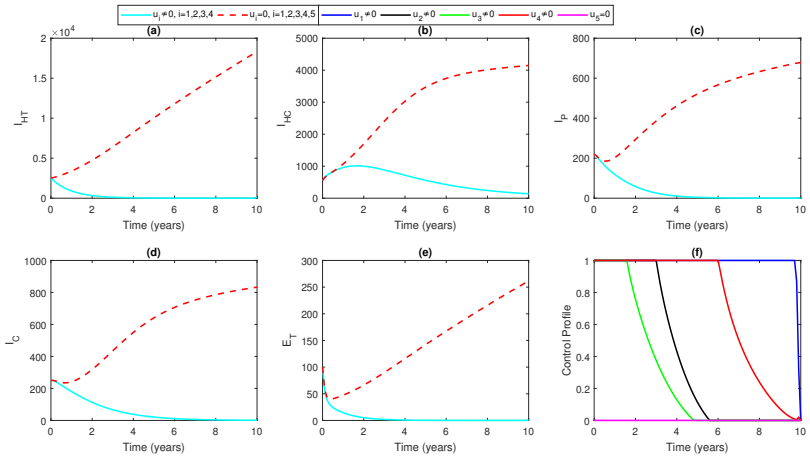

FIGURE 4. Impact of Meat Inspection, Pigs and Cattle Vaccination, and Treatment of Humans Infected with Taeniasis on Infected Humans, Cattle, Pigs and Taenia Eggs

4.1.3. *Meat Inspection, Improved Hygiene and Sanitation, and Treatment of Humans with Taeniasis.* Here, a combination of meat inspection $(u_1(t))$, improved hygiene and sanitation $(u_5(t))$ and treatment of humans with taeniasis $(u_4(t))$ is considered . It can be seen in FIGURE 5 that cysticercosis in pigs and cattle can be eliminated after 4 and 6 years respectively whereas human taeniasis and taenia eggs reduces to zero after 4 and 2 years respectively. Only a small proportion of humans remains infected with cysticercosis at the final time. The control profiles in FIGURE 5 (f) show that the control variables $u_1(t)$ and $u_4(t)$ are at their peak for the first

9.5 and 6.5 years respectively and then decline to zero in the final time. On the other hand, the control profile for $u_5(t)$ variable has up and down movement in the first 1.75 years and then declines to zero in the final time.

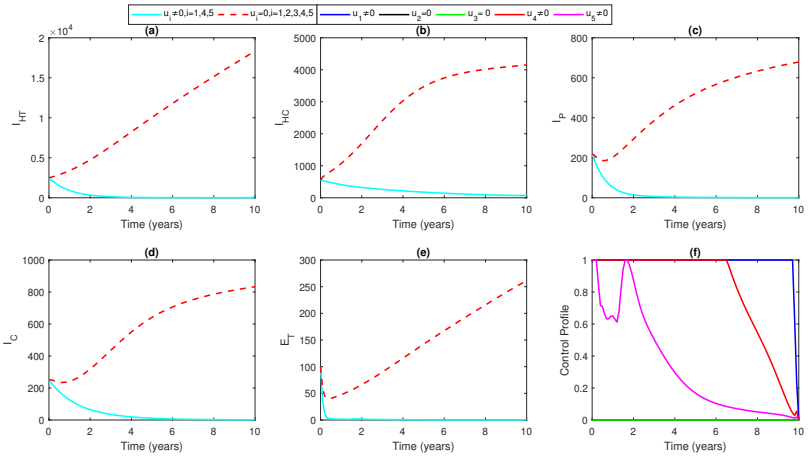

FIGURE 5. Impact of Meat Inspection, Improved Hygiene and Sanitation, and Treatment of Humans with Taeniasis on Infected Humans, Cattle, Pigs and Taenia Eggs

4.1.4. *Meat Inspection and Vaccination of Pigs and Cattle.* In this strategy a combination of meat inspection $(u_1(t))$ and vaccination of pig and cattle populations ($u_2(t)$ and $u_3(t)$ respectively) is implemented. The results in FIGURE 6 show that inially infected pigs decrease with time in the first year and then maintained around 170 pigs throughout. On the other hand, infected cattle approach to zero after the first 2 years while humans infected with taeniasis and taenia eggs decrease and remain intact. A different trend is observed for humans infected with cysticercosis where a small proportion declines for the first five years but flourish later. This is the case when hygiene and sanitation is not improved. The control profiles in FIGURE 6 (f) show that the control variable $u_1(t), u_2(t)$ and $u_3(t)$ are at their peak for the first 9.8, 9.75 and 9.25 years respectively and then decline finally to zero.

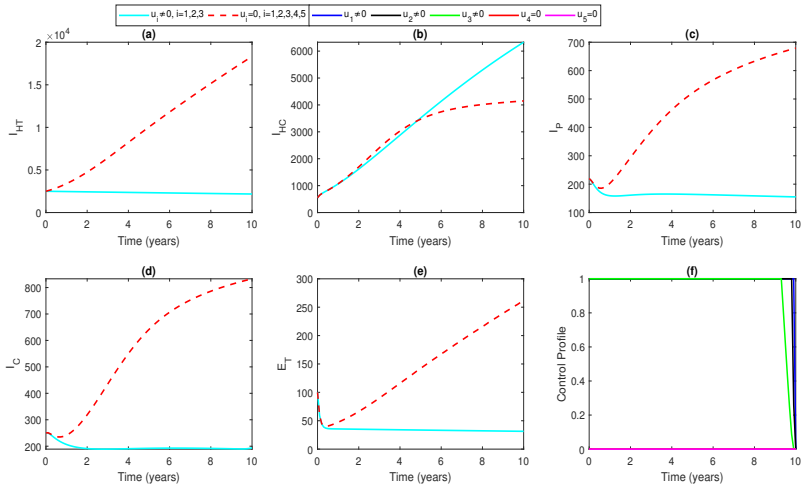

FIGURE 6. Impact of Meat Inspection and Vaccination of Pigs and Cattle on Infected Humans, Cattle, Pigs and Taenia Eggs

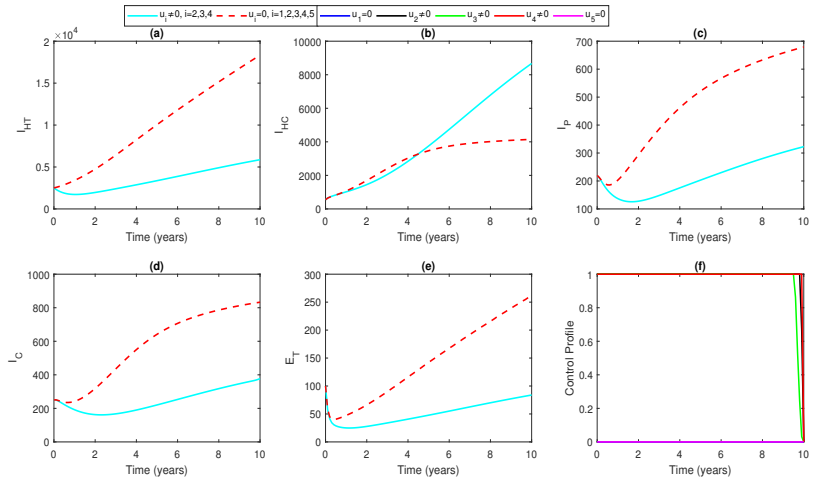

FIGURE 7. Impact of Vaccination of Pigs and Cattle, Treatment of Humans with Taeniasis on Infected Humans, Cattle, Pigs and Taenia Eggs

4.1.5. *Vaccination of Pigs and Cattle, and Treatment of Humans Infected with Taeniasis.*

In this strategy, the optimal control model with control variables that involve a combination of pigs and cattle vaccination ($u_2(t)$ and $u_3(t)$ respectively) and treatment of humans with taeniasis ($u_4(t)$) is simulated. The results in FIGURE 7 indicate that cases of human taeniasis, taenia eggs, and porcine and bovine cysticercosis reduce after implementing this strategy, however there is no possibility to control these infections. On the other hand, there is a small decline in cases of humans infected with cysticercosis in the first five years where thereafter humans with cysticercosis exceed the case when no control is applied. The control profiles in FIGURE 7 (f) show that the control variables $u_2(t)$, $u_3(t)$ and $u_4(t)$ are at their peak for the first 9.75, 9.5 and 9.8 years respectively and then eventually reduces rapidly to zero in the final time.

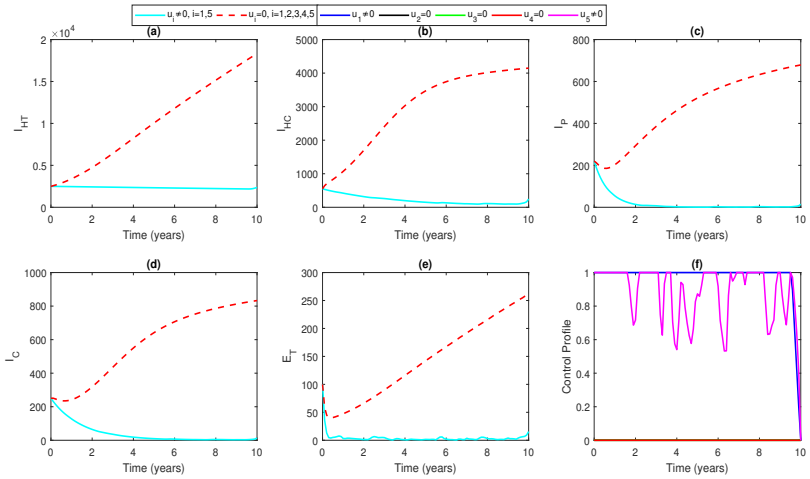

FIGURE 8. Impact of Meat Inspection and Improved Hygiene and Sanitation on Infected Humans, Cattle, Pigs and Taenia Eggs

4.1.6. *Meat Inspection, and Improved Hygiene and Sanitation.* In this strategy a combination of meat inspection ($u_1(t)$) with improved hygiene and sanitation ($u_5(t)$) is implemented. It can be observed that the trend has changed whereby infected pig and cattle populations decline with time, reaching zero in 4 and 6 years respectively while taenia eggs reduce quickly in the first 0.5 years and thereafter there are small fluctuations throughout the time. On the other hand, there is reduction in number of humans with taeniasis and humans with cysticercosis, however human infections cannot be eliminated for the whole time of implementing the controls. The control profiles in FIGURE 8 (f) show that the control variable $u_5(t)$ is at its peak for the first 1.75 years followed by up and down movements until the 9.5 year when it drops down quickly

to zero in the final time. On the other hand, the control profiles $u_1(t)$ is at its peak for the first 9.5 years and finally declines to zero in the final time.

4.1.7. *Meat Inspection and Treatment of Humans with Taeniasis.* This strategy aims at assessing the impact of implementing a combination of meat inspection $(u_1(t))$ with treatment of humans who are infected with taeniasis $(u_4(t))$. It can be seen in FIGURE 9 that humans taeniasis, porcine cyticercosis, bovine cysticercosis, and taenia eggs in the environment can be elimnated within 4, 6, 8 and 4 years respectively. Also, humans who are infected with cysticercosis decline with time, however a small proportion remains in the final time. The control profiles in FIGURE 9 (f) indicated that the control variables $u_1(t)$ and $u_4(t)$ are at their peak for the first 9.5 and 6.25 years respectively and finally decline to zero in the final time.

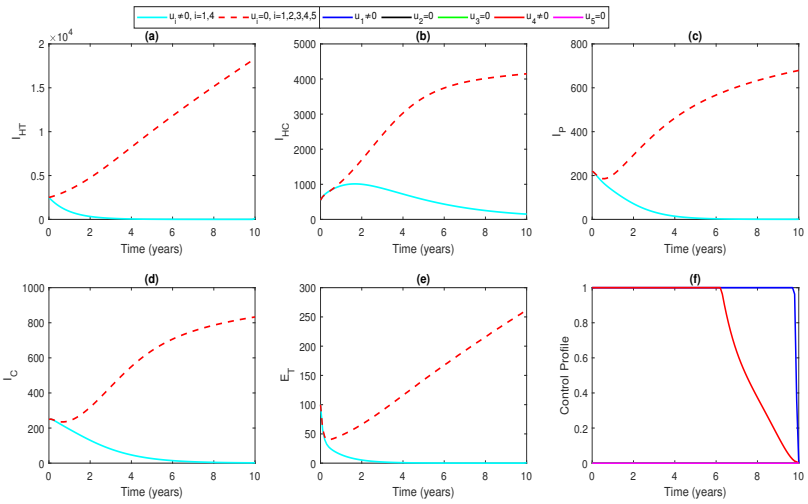

FIGURE 9. Impact of Meat Inspection and Treatment of Humans with Taeniasis on Infected Humans, Cattle, Pigs and Taenia Eggs

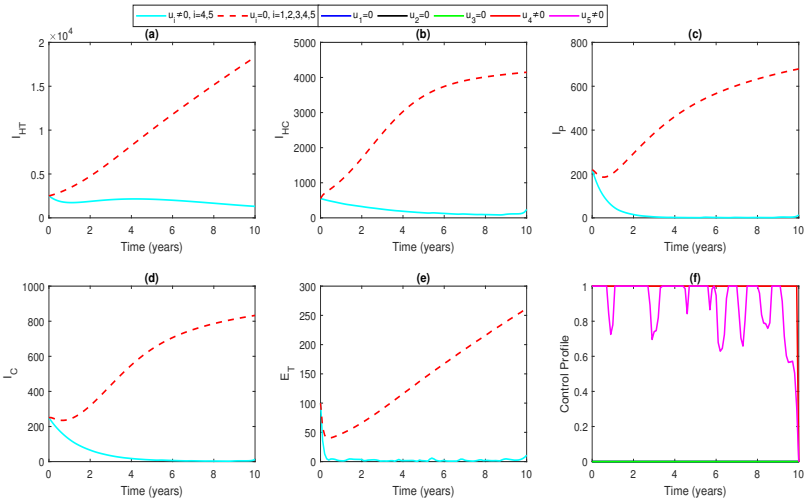

FIGURE 10. Impact of Improved Hygiene and Sanitation with Treatment of Humans having Taeniasis on Infected Humans, Cattle, Pigs and Taenia Eggs

4.1.8. *Improved Hygiene and Sanitation & Treatment of Humans with Taeniasis.* This strategy gives similar results as strategy 6 whereby porcine and bovine cysticercosis can be eliminated within 4 and 6 years respectively whereas taenia eggs in the environment reduce quickly in the first 0.5 years and thereafter small fluctuations can be seen throughout the time. On the other hand, humans with taeniasis and humans with cysticercosis have reduced significantly whereby a small portion of humans remain infected at the final time. The control profiles in FIGURE 10 (f) show that the control variable $u_5(t)$ is at its peak for the first year followed by up and down movements until the 9^{th} year where it starts to decline, reaching zero in the final time while $u_4(t)$ peaks for the first 9.9 years and finally declines to zero.

4.1.9. *Pigs and Cattle Vaccination.* Here, we assess the impact of implementing pigs and cattle vaccination on disease control in humans, pigs and cattle. It can be observed that the implementation of this strategy is ineffective since only a small portion of humans with taeniasis, infected pigs and cattle, and taenia eggs in the environment is reduced after its implementation of this strategy. Also there is no changes for cases of humans with cysticercosis in the first five years and thereafter number of cases are higher compared to the case when there is no any control. The control profiles in FIGURE 11 (f) show that the control variable $u_2(t)$ and $u_3(t)$

are always at their peak for the first 9.9 and 9.5 years respectively and quickly declines to zero in the final time.

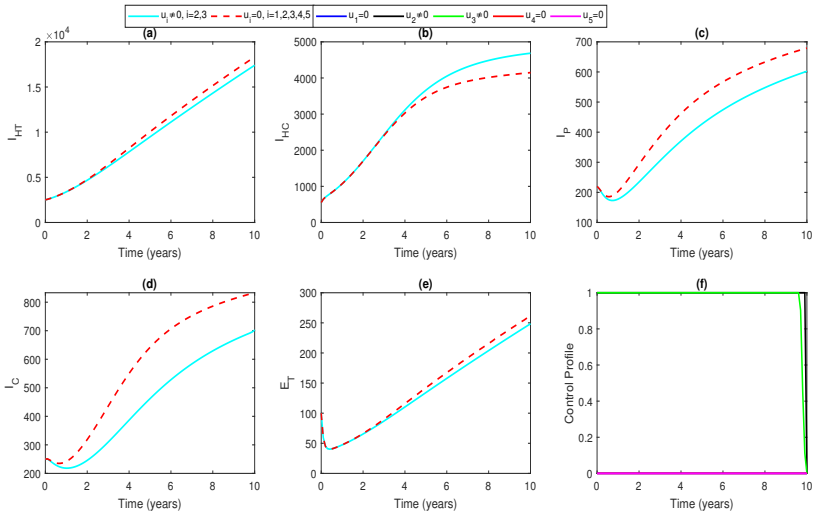

FIGURE 11. Impact of Pigs and Cattle Vaccination on Infected Humans, Cattle, Pigs and Taenia Eggs

Generally, it can be observed that the first and third strategies that involve implementation of all control measures or by excluding pigs and cattle vaccination are effective in controlling the transmission of taeniasis and cysticercosis in humans, pigs and cattle.

4.2. Cost-Effectiveness Analysis. In this section, we carryout a cost-effectiveness analysis to justify the costs associated with various control strategies that include meat inspection, vaccination of pigs and cattle, treatment of humans who have taeniasis, and improved hygiene and sanitation. The analysis is based on incremental cost-effectiveness ratio (ICER) as applied by Agusto and ELmojtaba [28, 29] and Omame et al. [30], which is given by:

$$\text{ICER(X)} = \frac{\text{Difference in infection averted costs in strategies X and Y}}{\text{Difference in total number of infection averted in strategies X and Y}}$$

To implement the ICER, the optimal control model is simulated using various intervention strategies. Then, using the simulation results, the control strategies are ranked in increasing order of effectiveness based on cases of infection averted. The ICER for various control strategies are given in TABLE 3.

TABLE 3. ICER in increasing order of total infection averted

Strategy	Total Cost ($)	Total Infection Averted	ICER
1	313,188.91	1,388,893.43	0.2255
3	312,002.18	1,388,898.50	-234.0690
2	442,817.08	1,390,133.11	105.9565
7	472,839.59	1,390,136.79	8158.2908
8	515,079.26	1,390,326.56	222.5835
5	2,349,121.73	1,391,602.56	1437.3374
6	569,467.05	1,392,116.77	-3460.9492
4	1,874,540.84	1,393,457.10	973.6959
9	4,045,734.84	1,394,988.27	1417.9967

By comparing two strategies with the highest ICER in TABLE 3, it can be observed that the ICER for strategy 7 is greater than the ICER for strategy 5. This shows that Strategy 7 is more expensive to implement compared to Strategy 5. Therefore, Strategy 7 can be excluded from the set of controls for implementation so as to preserve the available limited resources. Thus, Strategy 7 is removed and Strategy 5 is further compared with other Strategies. The procedure is repeated by computing the ICER and comparing the strategies until when two strategies remain for comparison purposes. Thus, the ICER for the remaining strategies are presented in TABLE 4.

TABLE 4. ICER in increasing order of total infection averted

Strategy	Total Cost ($)	Total Infection Averted	ICER
1	313,188.91	1,388,893.43	0.2255
3	312,002.18	1,388,898.50	-234.0690
2	442,817.08	1,390,133.11	105.9565
8	515,079.26	1,390,326.56	373.5445
5	2,349,121.73	1,391,602.56	1437.3374
6	569,467.05	1,392,116.77	-3460.9492
4	1,874,540.84	1,393,457.10	973.6959
9	4,045,734.84	1,394,988.27	1417.9967

It can be observed further from TABLE 4 that the ICER for Strategy 5 is greater than ICER for Strategy 9 which implies that Strategy 9 is less expensive to implement than Strategy 5. Thus, strategy 5 can be removed from the list of control options so that Strategy 9 can be compared with other strategies. The ICER for the remaining strategies are shown in TABLE 5.

TABLE 5. ICER in increasing order of total infection averted

Strategy	Total Cost ($)	Total Infection Averted	ICER
1	313,188.91	1,388,893.43	0.2255
3	312,002.18	1,388,898.50	-234.0690
2	442,817.08	1,390,133.11	105.9565
8	515,079.26	1,390,326.56	373.5445
6	569,467.05	1,392,116.77	30.3807
4	1,874,540.84	1,393,457.10	973.6959
9	4,045,734.84	1,394,988.27	1417.9967

Similarly, the ICER for Strategy 9 from TABLE 5 is greater than ICER for Strategy 4 which means that Strategy 4 is less costly than Strategy 9. Thus, we remove strategy 9 from the list of

intervention strategies so that Strategy 4 can be compared with the remaining strategies. Thus, the ICER for the remaining strategies are shown in TABLE 6.

TABLE 6. ICER in increasing order of total infection averted

Strategy	Total Cost ($)	Total Infection Averted	ICER
1	313,188.91	1,388,893.43	0.2255
3	312,002.18	1,388,898.50	-234.0690
2	442,817.08	1,390,133.11	105.9565
8	515,079.26	1,390,326.56	373.5445
6	569,467.05	1,392,116.77	30.3807
4	1,874,540.84	1,393,457.10	973.6959

Likewise, it can be seen from TABLE 6 that the ICER for Strategy 4 is greater than ICER for Strategy 8, implying that Strategy 4 is more expensive to implement than Strategy 8. Thus, Strategy 4 is excluded from the set of control strategies and Strategy 8 is compared with other strategies. Following the same procedure, the results are shown in TABLES 7, 8, 9 and 10.

TABLE 7. ICER in increasing order of total infection averted

Strategy	Total Cost ($)	Total Infection Averted	ICER
1	313,188.91	1,388,893.43	0.2255
3	312,002.18	1,388,898.50	-234.0690
2	442,817.08	1,390,133.11	105.9565
8	515,079.26	1,390,326.56	373.5445
6	569,467.05	1,392,116.77	30.3807

TABLE 8. ICER in increasing order of total infection averted

Strategy	Total Cost ($)	Total Infection Averted	ICER
1	313,188.91	1,388,893.43	0.2255
3	312,002.18	1,388,898.50	-234.0690
2	442,817.08	1,390,133.11	105.9565
6	569,467.05	1,392,116.77	63.8466

TABLE 9. ICER in increasing order of total infection averted

Strategy	Total Cost ($)	Total Infection Averted	ICER
1	313,188.91	1,388,893.43	0.2255
3	312,002.18	1,388,898.50	-234.0690
6	569,467.05	1,392,116.77	80.0010

TABLE 10. ICER in increasing order of total infection averted

Strategy	Total Cost ($)	Total Infection Averted	ICER
1	313,188.91	1,388,893.43	0.2255
3	312,002.18	1,388,898.50	-234.0690

TABLE 7 shows the cost of saving \$0.2255 for Strategy 3 over Strategy 3. In addition, TABLE 7 indicates that Strategy 3 (with negative ICER) strongly dominates Strategy 1. This means that Strategy 3 is less costly and more effective to implement compared to Strategy 1. Thus, these results indicate that the strategy that involves a combination of meat inspection, improved hygiene and sanitation, and treatment of humans who are infected with taeniasis saves more money and gives the best outcomes. Therefore, this strategy is the most cost-effective strategy in combating taeniasis and cysticercosis transmission in humans, pigs and cattle.

5. CONCLUSION

In this paper, a deterministic model is presented and analysed for studying the impact of implementing various time dependent controls on the dynamics and control of taeniasis and cysticercosis in humans, pigs and cattle. The controls that are administered include meat inspection, improved hygiene and sanitation, pigs' and cattle's vaccination, and treatment of humans who are infected with taeniasis. The optimal control theory has been employed for finding the necessary conditions for existence of the optimal controls and to determine the optimal strategy for controlling the diseases. The cost-effectiveness analysis also has been carried out through incremental cost effective ratio (ICER) to obtain the strategy that can be implemented at minimal cost with highest health outcomes. Simulation results for the optimal control problem suggest that strategies which involve implementation of all controls or that exclude vaccination of pigs and cattle perform well than other strategies in controlling the spread of taeniasis nd cysticercosis in humans, pigs and cattle. However, cost-effectiveness analysis shows that the strategy which excludes vaccination of pigs and cattle is the most cost-effective strategy for controlling the spread of taeniasis and cysticercosis in humans, pigs and cattle. Therefore, to control taeniasis and cysticercosis in humans, pigs and cattle at minimal cost. Therefore, we recommend that initiatives which focus on meat inspection, improvement in hygiene and sanitation, and treatment of humans who are infected with taeniasis should be encouraged.

AVAILABILITY OF DATA AND MATERIAL

Most of data used in this paper were found from different literature and some were assumed.

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AUTHORS' CONTRIBUTION

J.A. Mwasunda: Conceptualization, Model Formulation, Model Analysis and Drafting of the Manuscript; J.I. Irunde: Model Formulation and Supervision; D. Kajunguri: Supervision ; and D. Kuznetsov: Supervision.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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