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CHARACTER OF IMAGES DEVELOPMENT ON GAUSSIAN COPULA MODEL USING DISTRIBUTION OF CUMULATIVE DISTRIBUTION FUNCTION

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Abstract. This paper is a preliminary study that aims to build the character of color image object through the Gaussian copula model. The development of the Gaussian copula model is carried out with the distribution of Cumulative Distribution Function (CDF) of the pixels as input. In contrast to other studies that use image pixel values. The random variables used are the CDF value at the point of differentiation determined by the Kulback-Leibler Divergence (KLD). The case discussed in this paper is the image modeling of red apples and green apples. The model is built on each Red, Green, and Blue (RGB) image with three distinguishing points. The result obtained is a Gaussian copula model with different parameters for red apples and green apples. The character of joint distribution is also different. The joint distribution value of green apples is greater than that of red apples in R and G images. The difference in the models obtained shows that the character of this image is different. Future research will use the acquired models to identify new images, whether they are recognized as red and green apples or not. Further development in the field of healthy vision. This model can be used to identify fruit images in an application.

Keywords: image; pixel; cumulative distribution function; Gaussian copula; Kulback-Leibler Divergence.

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1. INTRODUCTION

Health vision technology is developing rapidly, a machine that can recognize food images and extract the nutritional content of food. This kind of application is necessary for people who are on a diet to measure the number of calories consumed. A person simply takes a photo of the food and the machine will extract the calorie and nutrient content of the food. The basis of this application is the recognition of image characters through a model built from an image database [1]. Through this model, we no longer need to store a large image memory, but simply by storing the parameters of the image model.

Research related to image recognition was carried out by [2] and [3] using K-Nearest Neighbour Support Vector Machine (KNN-SVM), and [4][5] using Gaussian processes. In a Gaussian process, the conditions for a normal distribution cannot always be satisfied. The Gaussian copula is a more flexible method with a more general distribution. Research by [6], [7], [8], [9], [10], and [11] modeled the image object using the Gaussian copula model. This research generally uses pixel values as model input. In [12] states that the pixel approach is indeed easier to analyze, but it is not robust against extreme values. In this paper, we use the pixel distribution approach as input to the Gaussian copula model. The modeling does not use the pixel value itself but uses the cumulative distribution value of the pixel. The character of the image object can be represented by the distribution of pixels. The use of pixels in an image will be different from other images. Gaussian copula model parameters will determine the character of the image.

We apply this Gaussian copula model to the image data of red apples and green apples, each of which has 100 normalized data. The image is cropped with the center position and the edges touch the object of the image. Images are extracted in RGB images. The independent variable of the Gaussian copula is the value of the empirical CDF at the pixel point that can distinguish the character of a red apple and a green apple using KLD. This study aims to obtain the characteristics of the image by using the Gaussian copula model. This characteristic is indicated by distribution of empirical CDF in point of differentiation. In the case of the image of a red apple and a green apple, we will discuss whether the two images have the same image characteristics or not. In further research, this Gaussian copula model can be used to identify new images whether they can be recognized as red and green apple images or not. The structure of the paper compiled in section 2 is a method in the formation of the Gaussian copula model. In section 3, the obtained model results and their interpretation are discussed. The last is the conclusion of section 4.

2. METHOD

We have image data of red and green apples measuring $p \times p$ pixels of N each. The images are extracted in an RGB matrix of $p \times p$ each with elements being pixel intensity values. The matrix is then converted into a row vector measuring $1 \times p^2$ so that with N data, then three $N \times p^2$ matrices will be produced. The formation of the Gaussian Copula model is carried out by the following algorithm:

Input :

$F_u(x)$ pixel empirical CDF value

N lots of image

T evaluation pixel points

t point of differentiation

Process :

Step 1. For i = 1 : N

Determine the empirical CDF using Equation 1.

Evaluation of empirical CDF value at pixel point T.

end

Step 2. Determine t point of differentiation using Equation 2 and 3.

Step 3. Determine the joint distribution using Equation 5 and 6.

Output :

Λ covariance matrix of Copula Gaussian.

Red apple and green apple image size N, we write X_u with $u = 1,2, \ldots, N$. We can create a pixel CDF for each image denoted by F_u. The CDF function used in this study is the empirical CDF [13], given in Equation 1.

$$F_u(x) = \frac{number\ of\ pixel\ in\ the\ image\ u \leq x}{total\ number\ of\ pixel\ in\ the\ image\ u} \tag{1}$$

The point of differentiation in this case is the pixel point that has the potential to distinguish the character of the image. The point of differentiation is determined by symmetrical KLD using the Probability Distribution Function (PDF) values at T pixel points. The Beta distribution is used with the PDF function in Equation 2.

$$f(x; \alpha, \beta) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1} \tag{2}$$

with $B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt$, and (α, β) is the beta distribution parameter [14].

The KLD score measures how much one opportunity distribution differs from another. The KLD between the two distributions Q and P is expressed by the formula $D_{KL}(P||Q) = \sum_x P(x) \log\left(\frac{P(x)}{Q(x)}\right)$ which is equivalent to $D_{KL}(P||Q) = -\sum_x P(x) \log\left(\frac{Q(x)}{P(x)}\right)$. Note that $D_{KL}(P||Q) \neq D_{KL}(Q||P)$. The KLD between the P and Q distributions is expressed by $D_{KL}(Q||P) = \sum_x Q(x) \log\left(\frac{Q(x)}{P(x)}\right)$ so that a symmetrical KLD is used [15].

$$KLD_{sym} = D_{KL}(P||Q) + D_{KL}(Q||P) \tag{3}$$

Three different points are taken with the highest KLD score.

At the point of differentiation, we determine the empirical CDF distribution using CDF Beta with the equation given in Equation 4.

$$F(x; \alpha, \beta) = \frac{1}{B(\alpha,\beta)} \int_0^x t^{\alpha-1}(1-t)^{\beta-1}\, dt \tag{4}$$

with three points of differentiation for each RGB image, nine Beta CDF will be generated for each red apple and green apple. The CDF of the Gaussian copula is given in Equation 5.

$$C_{\mathbf{\Lambda}}(\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_n) = \Phi_{\mathbf{\Lambda}}\left(\Phi^{(-1)}(\boldsymbol{u}_1), \Phi^{(-1)}(\boldsymbol{u}_2), \ldots, \Phi^{(-1)}(\boldsymbol{u}_n)\right) \tag{5}$$

with $\Phi_{\mathbf{\Lambda}}$ is a normal multivariate CDF with a mean of $\mathbf{0}$ and a covariance matrix $\mathbf{\Lambda}$. Next Φ is a standard normal CDF with $\boldsymbol{u}_i = F_i(\boldsymbol{x}_i)$, $i = 1,2, \ldots, n$. $F_i(\boldsymbol{x}_i)$ is the cumulative marginal distribution, and $\Phi^{(-1)}$ is invers of normal standard [16], [17], [18]. Parameter estimation is carried out using the maximum likelihood method. Due to Equation 5, the joint distribution Gaussian copula can be expressed as Equation 6.

$$f(\boldsymbol{u}_1, \boldsymbol{u}_2, \dots, \boldsymbol{u}_n) = c_{\boldsymbol{\Lambda}}(\boldsymbol{u}_1, \boldsymbol{u}_2, \dots, \boldsymbol{u}_n) \prod_{i=1}^{n} f_i(\boldsymbol{u}_i) \tag{6}$$

With $c_{\boldsymbol{\Lambda}}$ is a PDF of Gaussian copula and $f_i(\boldsymbol{u}_i)$ is marginal PDF of random variable.

2. Results and Discussion

The data used in this experiment is data of red and green apples measuring 50×50 pixels of 100 each. The images are extracted in an RGB matrix of 50×50 each then converted into a row vector measuring 1×2500. There are three 100×2500 matrices will be produced. In this matrix, the pixel empirical CDF is constructed for each image using Equation 1.

3.1 Pixel empirical CDF results.

One apple image yields an empirical CDF function. This function describes the distribution of pixel usage. The results obtained are 100 empirical CDFs for each RGB image. The empirical CDF results of the red apple pixels are given in Figure 1.

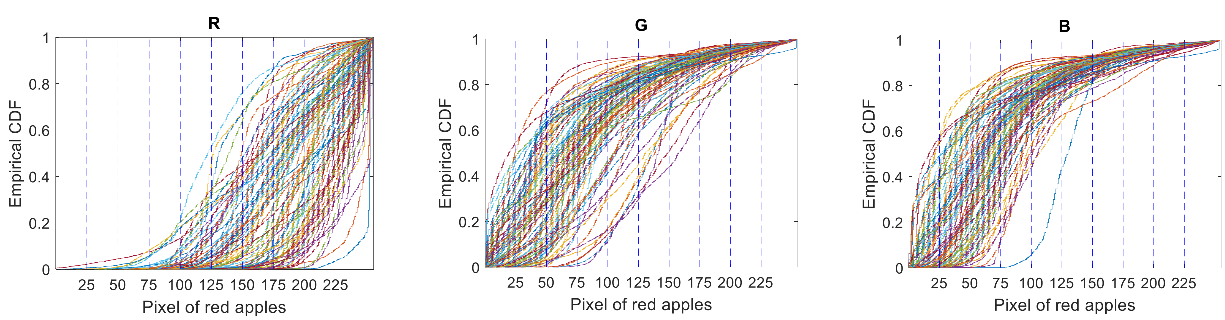

Figure 1. Red apple empirical CDF for Red, Green and Blue images

In Figure 1, we can see that in the R image, the empirical CDF function of the initially formed pixels is low. Next, at pixels from 100 to 255, the function increases. That is, the R image is used more in the pixel intensity interval [100,255]. In contrast to the G and B images, these two images instantly increase in pixels from 1 to 150. At pixels greater than 150, the empirical CDF function is relatively flat. It means that G and B images are used more in the pixel intensity interval [1,150]. In this empirical CDF function, it can be seen that the use of the R image on red apples is mostly used for large pixels, meaning that the red color on red apples is sharper than other images. Figure 2 is the pixel empirical CDF results for green apples.

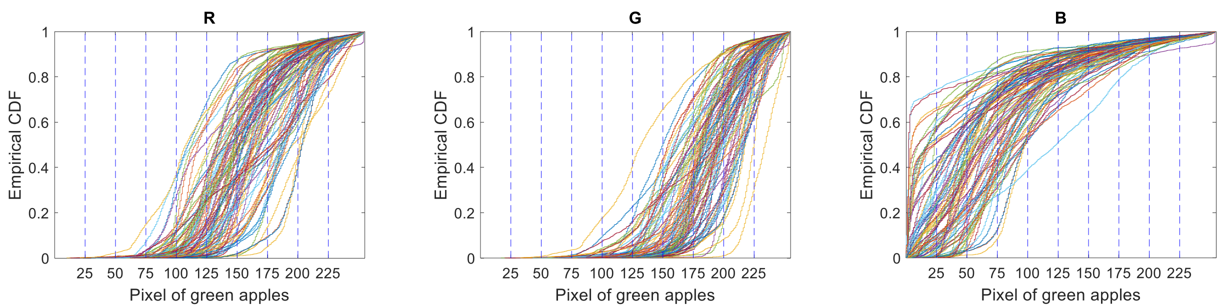

Figure 2. Empirical CDF pixel of green apples for Red, Green and Blue image.

The characteristics of the empirical CDF produced are different from the empirical CDF of red apples. The use of dominant pixels in the R image is [100,200]. The variance of pixel usage is also smaller than that of red apples. In the G image, the empirical CDF increases in the pixel intensity interval [125,225]. The empirical CDF for B images on green apples is almost the same as for red apples. This difference in pixel distribution is interesting to study in image analysis. There are no other studies that have done modeling using this pixel distribution value.

The next is the result of the pixel distribution function at 9-pixel points to determine which point can best distinguish the characteristics of red apples and green apples. Determination of PDF Beta using Equation 2 and KLD using Equation 3. Table 1 presents the results obtained. The maximum KLD values for R images are at pixels 175, 200, 225, G images at pixels 100, 125, 150, and B images at pixels 175, 200, 225. These points have the potential to distinguish the characteristics of red and green apples. Table 2 shows the results of CDF distribution at the point of differentiation for each image.

Table 1. Kullback-Leibler Divergence for RGB image

Pixels	Kullback Leibler Divergence		
	R	G	B
25	412.802	1.343	110.769
50	108.973	1.657	107.903
75	39.410	2.022	59.109
100	203.719	**7.569**	65.730
125	567.118	**5.026**	6.029
150	756.451	**5.071**	89.342
175	**3101.267**	5.012	**112.166**
200	**5581.858**	4.120	**178.227**
225	**15762.994**	1.223	**283.475**

Table 2. The results of fitting distribution for CDF pixel

Image	Pixel	Fitting distribution	
		Red apples	Green apples
R	175	Beta(0.570,1.458)	Beta(2.840,1.628)
	200	Beta(1.083,1.228)	Beta(8.527,1.861)
	225	Beta(2.343,1.088)	Beta(49,669,3.106)
G	100	Beta(4.086,2.242)	Beta(0.486,54.712)
	125	Beta(8.580,2.723)	Beta(0.519,16.799)
	150	Beta(18,635,3.529)	Beta(0.590,4.711)
B	175	Beta(131.134,10.145)	Beta(114.613,9.891)
	200	Beta(224.271,10.285)	Beta(278.849,14.190)
	225	Beta(491.927,11.908)	Beta(696.783,18.804)

The fitting distribution produces a Beta distribution with two parameters, with the CDF function as in Equation 4. Table 2 shows that the CDF function formed in red apples and green apples is different. Figure 3 is a Beta CDF plot for an RGB image.

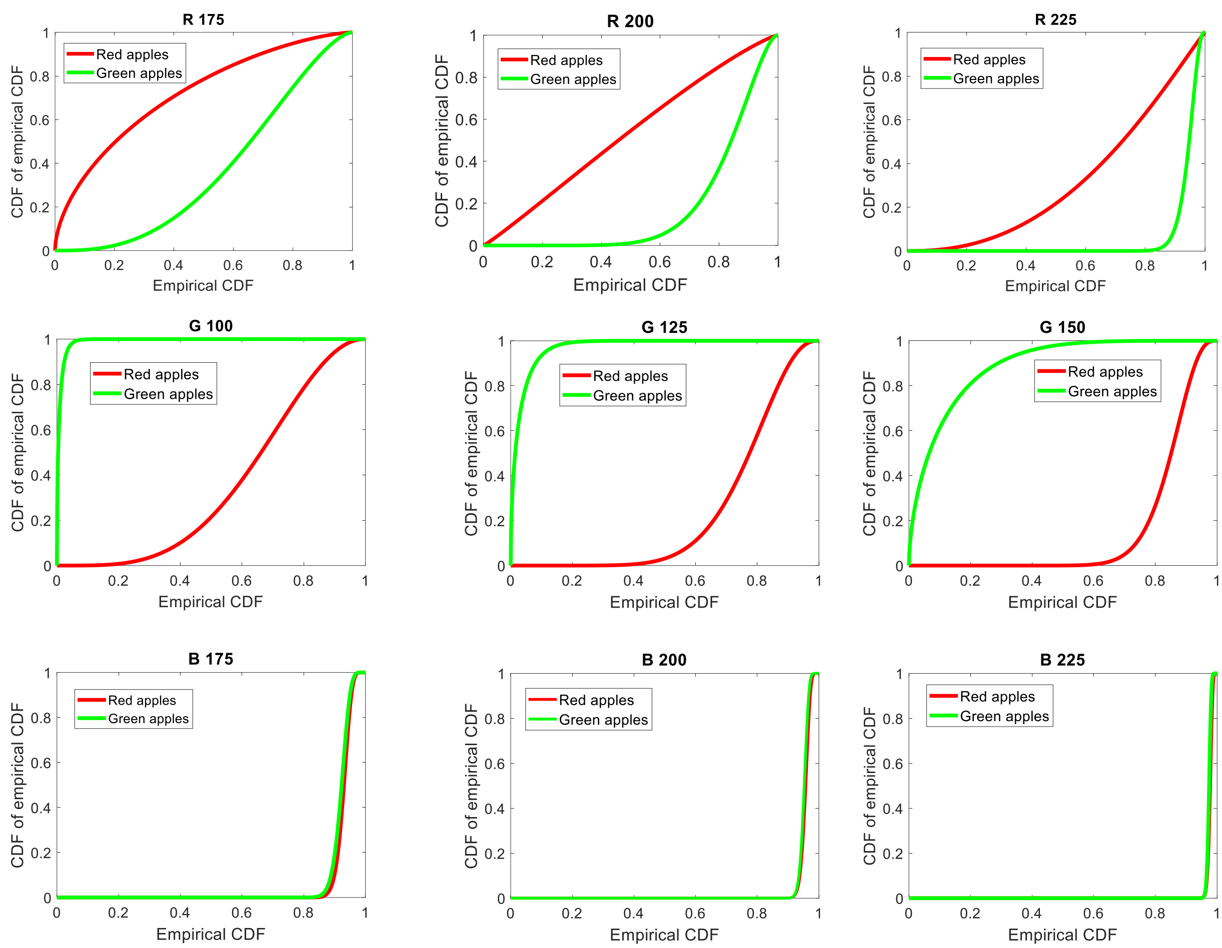

Figure 3. Distribution of CDF pixel for RGB image.

The CDF Beta function in image R and image G looks far apart, meaning that the distribution of pixel usage is different for red apples and green apples. In the R image, the CDF value of the empirical CDF of red apples is higher than that of green apples. At point R225 the intensity of the red apple pixels increases at the empirical CDF of 0.2, while at the green apple it increases at the empirical CDF of 0.8. We can also see in Figure 1 and Figure 2 points R175, R200, and R225. The empirical distribution of the CDF of red apples is different from that of green apples. In the G image, the CDF from the empirical CDF is different at points G100, G125, and G150. The

CDF values of the empirical CDFs are relatively the same in the B image.

The CDF value at the point of differentiation will be a random variable in the copula Gaussian model. We model each RGB image to produce 6 copula Gaussian models. Each Gaussian copula model consists of 3 random variables with marginal distributions in Table 2 and Figure 3. The three marginal distributions will be expressed in the Gaussian copula distribution function as a joint distribution. By using Equation 5, the results of the model parameters are as seen in Table 3. The copula Gaussian model results from different parameters for red apples and green apples. Parameters in the form of a covariance matrix characterize the image characteristics of each image. The covariance matrix element shows the relationship of CDF pixels between the three points of differentiation.

Table 3. Parameter Gaussian copula for red and green apples

Image	Parameter copula	
	Red apples	Green apples
R	$\begin{pmatrix} 1 & 0.919 & 0.785 \\ 0.919 & 1 & 0.933 \\ 0.785 & 0.933 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0.911 & 0.662 \\ 0.911 & 1 & 0.816 \\ 0.662 & 0.816 & 1 \end{pmatrix}$
G	$\begin{pmatrix} 1 & 0.974 & 0.912 \\ 0.974 & 1 & 0.950 \\ 0.912 & 0.950 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0.878 & 0.669 \\ 0.878 & 1 & 0.860 \\ 0.669 & 0.860 & 1 \end{pmatrix}$
B	$\begin{pmatrix} 1 & 0.895 & 0.688 \\ 0.895 & 1 & 0.849 \\ 0.688 & 0.849 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0.837 & 0.575 \\ 0.837 & 1 & 0.836 \\ 0.575 & 0.836 & 1 \end{pmatrix}$

To simplify the discussion, we will create a PDF visualization of the Gaussian copula using the contour plot in Figure 3 and Figure 4. This two-dimensional contour plot is formed by assigning the value of one of the random variables. In the R and B images, the values for R200 and B200 are set to 0.7, and in the G image the values for G125 are set at 0.7.

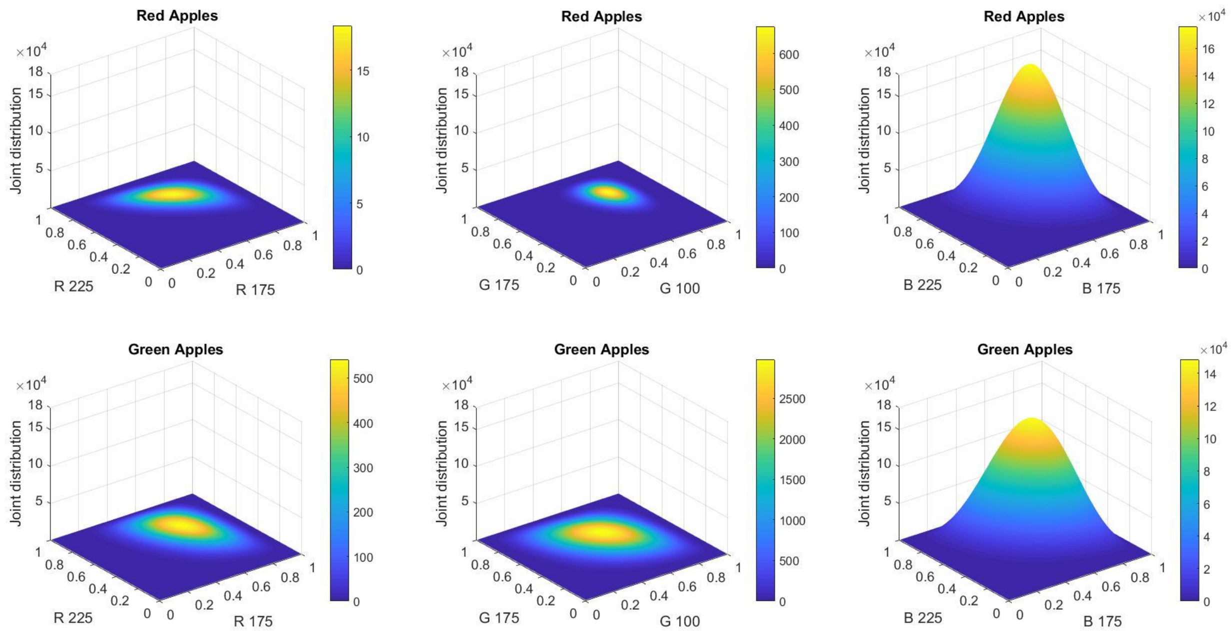

Figure 3. Surface plot of PDF copula Gaussian for RGB image

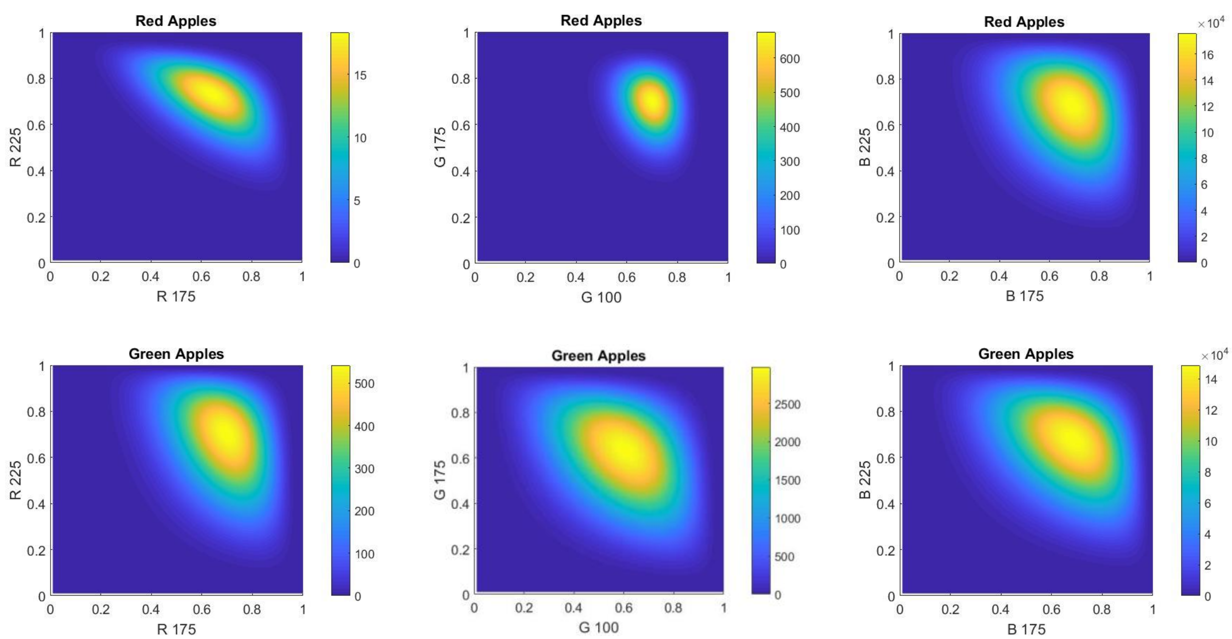

Figure 4. Contour plot of PDF copula Gaussian for RGB image

In the surface plot and contour plot, we can see that the character of the contour plot for red apples is different from that for green apples. The red apple has a copula Gaussian PDF value that reaches a maximum value of around 17 in the R image, around 650 in the G image, and around 17×10^4 in the B image. In the R image, the maximum area is at intervals of [0.5,0.7] for R175 and [0.6,0.8] for R225. This value is at intervals of [0.65,0.75] for G100 and [0.6,0.8] for G150 in the G image. In the B image, this value is at intervals of [0.55,0.8] for B175 and [0.5,0.85] for B225. In this red apple contour plot, the maximum area is quite small. In contrast to the green apples, which have a wide maximum area. In the green apples, the maximum value for the RGB image is around 500 in R image, 2800 in G image, and 17×10^4 in B image. In the R image, the maximum area is at the intervals of [0.55,0.8] for R175 and [0.5,0.8] for R225. This value is in the intervals [0.4,0.8] for G100 and [0.5,0.8] for G150 in the G image. In the B image, this value is in intervals of [0.5,0.8] for B175 and [0.5,0.8] for B225. The characters of red apples had a maximum area at smaller intervals than green apples. The maximum value obtained for red apples is also greater than for green apples. Determination of this maximum area is needed to classify new images. When we insert a new image into the model, we can determine whether it is an image of a red apple or a green apple.

3. CONCLUSION

We conclude that the use of the pixel distribution value as the input of the Gaussian copula model can represent the characteristics of an image. The model parameters Gaussian copula and contour of joint distribution describe the character of the image object. Different objects will generate not similar parameters according to their character. In the case of red apples and green apples, it was found that the characteristics of the apples image determined by the R image and the G image. Green apples joint distribution value is higher than red apple. The maximum area for green apples is quite large compared to that for red apples. The B image is relatively the same between red apples and green apples. We can do further research by identifying new images so that we can classify objects from those images.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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