ARFIMA MODEL APPLIED TO MALAYSIAN STOCK MARKET

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Abstract. In this paper we study Sukuk on the Islamic financial market of Malaysia, since they present favorable conditions for investment. The aim of our work is to concretely study the characteristic of long memory for our series; to study the impact of maturity on the long memory for the different studied series. We use government indexes of several maturities while comparing them with their counterparts in each maturity, over the periods from 2007 to 2017, while basing on the rate of return, risk measures that we calculate using Garch and EGarch Models; we have confirmed that more the maturity is higher more the volatility is higher, for the conventional bonds and conversely for the Islamic bonds. We confirmed also the presence of the long memory for conventional bonds of short and medium maturity while we captured it for Sukuk of long maturity using ARFIMA model to establish the relationships between maturity and long memory of our bonds.

Keywords: Sukuk; Islamic financial market of Malaysia; bonds; Shairah; maturity; rate of return; risk; ARFIMA; Garch; EGarch models.

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1. INTRODUCTION

Nowadays, most analysts consider that the key of the Islamic system resides in Islamic credits, named "Sukuk". The equivalent of bonds in classical finance, this is actually a sharia compliant investment certificate, which prohibits the concept of "money for money", and which would offer more stability than traditional investments experiencing significant growth across the world and has become a competitor of the conventional financial system. If in common language Sukuk are commonly called "Islamic bonds", it should be kept in mind that these are not classic obligations whose coupon is a remuneration of the debt, Indeed, with the Sukuk the coupon is linked to the performance of the underlying asset. Sukuk are yield products that can behave similar to an action or an obligation, more exactly investment Sukuk are tradable hybrid financial securities, whose remuneration and, where applicable, the principal, are indexed to the performance of one or more underlying assets held directly by the issuer. This issuer can be corporate (company) or sovereign (state). Defined as such, Sukuk security holders are exposed to various types of asset-related risks: market risk (yield), credit risk (default risk), operational risk, liquidity risk (illiquidity premium), risk of the underlying, etc. As other conventional bonds, Sukuk may also promote default risk. Sukuk however are claiming to be safer than conventional bonds as they theoretically transfer ownership of the underlying assets to the holders, who in turn will earn a return on holding that asset. This is regarded as protection for the Sukuk holders in case of default. Even if the issuer defaults or goes bankrupt, investors should be in a good position to recover much of their contributions. Therefore, providing asset security or corporate guarantees (referred as Special Vehicle in Sukuk contracts) to investors is vital in Sukuk structures [3]. In this paper, we study the Islamic financial Market of Malaysia comparing it to the conventional financial market, in terms of long memory of the volatility of each serie for both conventionnal and islamic bonds, also in term of risk and return. The slow hyperbolic decay of the autocorrelation function characterizing many economic and financial time series has been the subject of much research for over two decades. Several explanations and models have been provided in an attempt to explain this phenomenon, also known as long memory phenomenon. Among the most relevant explanations offered so far, three main mechanisms, which can act separately or jointly, are of particular interest: first (i) the aggregation

approach suggested by Granger and Joyeux (1980) [7, 8]who showed than the time series resulting from the aggregation of micro-variables with short memory can lead to long memory, then (ii) the presence of rare regime changes that reproduce the long-term non-stationarity of economic activity and financial systems.

Our aim in this article is to provide a model relating the long memory of the realized volatility of asset returns, which is an important characteristic of series temporal financial (Taylor 1986, Ding, Granger, and Engle 1993, Dacoragna, Müller, Nagler, Olsen, and Pictet 1993, for pioneer work), to the heterogeneous behavior of agents economic. Based on the fact that, in the market, participants form heterogeneous expectations about the future level of volatility and that they act as agents with limited rationality (Simon 1957, Simon 1991), we offer an explanation of the long memory phenomenon which is based on the market aggregation of heterogeneous investor opinions, revealed through the process of setting market prices.

In order to justify these different sources of heterogeneity, but still keep a parsimonious representation, we propose a model that establishes a link between the behavior investors and a small number of parameters of heterogeneity that justify the individual tendency of these investors to manage their portfolio on the basis of information they receive or, conversely, on the basis of a self-referential approach which leads them to focus primarily on their past expectations and past observations of market volatility.

This approach allowed us to capture the long memory of the volatilities of our different studied series on the different maturities while basing on the parameter d of ARFIMA model, to position well in the financial market and to make the right decisions thereafter. The remainder of this paper is organized as follows. we will present a brief literature review. Section 2 describes the data and their statistical properties. Section 3 outlines the empirical methodology, reports and discusses the empirical results, we conclude our research and summarize our findings in section 4.

A number of researchers wanted to follow the behavior of conventional and islamic bonds in front of the long memory, risk and volatility, these results contradicted or applied to specific markets such as those of the Golf regions and European countries. Therefore, our research question is " what are between the two categories of bonds those that are more profitable and

those that are more risky on the different studied maturities in Malaysian market? and what are between the two kind of time series those that present the long memory on the different studied maturities? First, we have specified Garch and Egarch models that allow us to model the conditional volatility of the indices returns of our sample, using family GARCH models and to model the influence of the bearish and bullish movements on the financial behavior series using Egarch Models. after we have applied ARFIMA model to our data to detect the long memory of the obligations that validated it.

Independence assumption of the time series in most cases, is only an approximation of the true series correlation structure. Important correlations for small delays can sometimes be detected and short-memory processes, such as ARMA, can be sufficient to model dependency structure series.

However, there are many examples of data where the correlations taken only these are small, but extremely high in sum. The periodogram of these series show a peak in the spectrum at zero frequency. So equivalent, in the time domain, the autocorrelations of these series decrease very slowly. Such a phenomenon can be the reflection of several possibilities, in particular: non-stationarity and stationarity with long-term dependency. We focus here only on the second explanation through a study of long memory processes.

Long-term memory has also been the subject of analyzes of the traffic density in the networks at high speed, but also in geophysics, astronomy and economics.

In finance several studies have been done,[13], verified the efficiency of the estimating methods of long memory process based on the linear relationship between the Hurst exponent (H) and the fractional differencing parameter (d), which are the two approaches used to identify the long memory process. By using the Monte Carlo simulation and empirical examinations, they showed that there is a distinct linear relationship between the Hurst exponent and the fractional differencing parameter.

[15], used fractional integration techniques to examine the stochastic behaviour of high and low stock prices in Europe and then to test for the possible existence of long-run linkages between them by looking at the range, i.e., the difference between the two logged series.

[16], analyzed the long memory effects of earthquake time series and volcano volcanic eruptions. By using a class of auto-regressive techniques, they studied the seismograms of a set of earthquake and volcanic eruptions. The autoregressive model characterizes the key variables that have a long-range dependence with a slowly decaying auto-covariance function. Sabatier, showed that this geometric distribution is only a particular distribution case and that many other distributions (an infinity) are in fact possible. From the networks obtained, a class of partial differential equations (heat equation with a spatially variable coefficient) is then deduced. This class of equations is thus another tool for power law type long memory behaviour modelling, that solves the drawback inherent in fractional heat equations that was proposed to model anomalous diffusion phenomena.

[17], investigated a bivariate pure-jump model of stock prices with long memory in volatility, using a marked log-Gaussian Cox process. We show that, due to the non-synchronicity of transactions, the ordinary least squares estimator of the slope in a contemporaneous regression of returns on returns converges to different targets depending on the sampling frequency. Therefore, they proposed a transaction level estimator that makes full use of data in the complete continuous-time record, and showed that the estimator of the slope had slow convergence with rate determined by the memory parameter in volatility.

[18], used a variety of fractional GARCH models to describe typical volatility characteristics like long memory, volatility clustering, asymmetry and thick tail. The autoregressive conditional heteroscedasticity in the mean model (ARCH-M) and peaks-over-threshold model of extreme value theory (EVT-POT) are taken into account to develop a hybrid time-varying long memory GARCH-M-EVT model for calculation of static and dynamic VaR. Empirical results of this study showed that the WTI crude oil has a significantly long memory feature. All the fractional integration GARCH models can describe the long memory appropriately and the FIAPARCH model is the best in regression and out of sample one-step-ahead VaR forecasting. Back-testing results showed that the FIAPARCH-M-EVT model is superior to other GARCH-type models which only consider oil price fluctuation characteristics partially and traditional methods including Variance-Covariance and Monte Carlo in price risk measurement.

[19], calculated, Hurst exponent wish is used to examine the efficiency of Hong Kong REITs market in China, and time-varying Hurst exponent is used to explore the dynamic changes of its efficiency. The empirical results of the Hong Kong REITs market show that the Hong Kong REITs market has not yet reached weak efficiency, and it is basically in a state of inefficiency from November 25 2005 to October 10, 2018, with only three times approximate to weak-form efficiency, but the duration is very short. Furthermore, compared with the Hong Kong stock market and the real estate market.

[20], suggested that long memory and regime switching can be effectively distinguished, if the cause of the confusion between them is properly controlled for to distinguish between them in modelling stock return volatility. They firstly model long memory and regime switching in volatility via the Long-Memory GARCH(LMGARCH) and Markov Regime-Switching GARCH (MRS-GARCH) models, respectively. A theoretical cause of the confusion between those processes is proposed with simulation evidence. Adopting the ideas of existing studies, an MRS-LMGARCH framework is further developed to control for this cause.

[21], evaluated the adaptive pattern of long memory in the volatility of intra-day bitcoin returns. It also tests the impact of the trading volume on time-varying long memory. they had confirmed long memory in the volatility of intra-day bitcoin returns is not an all-or-nothing phenomenon; it is adaptive to change in time and creation of events and, therefore, adheres to the proposition of the adaptive market hypothesis.

[22], discussed macroeconomic models, which take into account effects of power-law fading memory. The power-law long memory was described by using the mathematical tool of fractional calculus that included the fractional derivatives and integrals of non-integer orders. They obtained solutions of the fractional differential equations of these macroeconomic models. Examples of dependence of macroeconomic dynamics on the memory effects was suggested. Asymptotic behaviors of the solutions, which characterize the rate of technological growth with memory, was described. They formulated principles of economic dynamics with one-parametric and multi-parametric memory.

[23], proposed a simple test on structural change in long-range dependent time series. It was based on the idea that the statistic test of the standard CUSUM test retains its asymptotic

distribution if it is applied to fractionally differenced data. they proved in this study that their approach was asymptotically valid, if the memory was estimated consistently under the null hypothesis. Therefore, the well-known CUSUM test can be used on the differenced data without any further modification. In a simulation study, they compared the test with a CUSUM test on structural change that was specifically constructed for long-memory time series and showed that this approach performs reasonably well

[24], transfered the recently introduced fast fractional differencing that utilizes fast Fourier transforms (FFT) to long memory variance models and showed that this approach offers immense computation speedups. They demonstrated how calculation times of parameter estimations benefit from this new approach without changing the estimation procedure. A more precise depiction of long memory behavior becomes feasible.

[25], performed predictions of foreign exchange rates series by taking into account their long-term memory property. To this end, this paper proposes the use of ARFIMA processes in order to make predictions of three exchange rate series: $/Canadian$, $/FrenchFrancand$/ltalian Lira

While [26] tested for the presence of long memory in daily stock returns and their squares using a robust semiparametric procedure of Lobato and Robinson. Spurious results can be produced by nonstationarity and aggregation. he addressed these problems by analyzing subperiods of returns and using individual stocks. The test results showed no evidence of long memory in the returns. By contrast, there is strong evidence in the squared returns.

[27]applied the modified rescaled range test to the return series of 1,952 common stocks. The results indicated that long memory is not a widespread characteristic of these stocks. But logit models of the event of a test rejection reveal that rejections are linked to firms with large risk-adjusted average returns. The maximal moment of a return distribution is also found to influence the event of a rejection, but not in a way suggestive of moment-condition failure. Evidence suggestive of survivorship bias is also uncovered. They concluded that there is some evidence consistent with persistent long memory in the returns of a small proportion of stocks.

2. Presentation of the Model

In order to compare the Islamic financial Market with the conventional financial market of malaysia, we used a database containing (132 observations), this data is monthly from 2007 to

2017, and is recuperated from the international base of THOMSON REUTERS. We use govern-ment indices of several maturities while comparing them with their counterparts (conventionnel bonds) in each maturity, over the periods from 2007 to 2017 as the table 1 shows, while applying ARFIMA model to the rate of return in order to capture the long memory of our studied series, causal links using Granger causality test and some risk measures using Garch and EGarch Models to study conditional volatilities. In this section we will present a set of models and tools that allowed us to answer the problem, ie models that aim to capture the long memory of our data.

2.1. Yield. After collecting the indices historical data of our sample, we calculated logarith-mic daily yields, in order to mitigate heteroscedasticity and keep the additivity of financial series. The daily yields R_t are calculated by the following formula:

$$R_t = \log(\frac{p_t}{p_{t-1}}).$$

where P_t is the index closing price at the date t. To compare indices returns series of Sukuk with their conventional counterparts we implemented the Student test (t-test) which allows to test the following hypotheses:

$$\begin{cases} H_0: \text{ the average returns of the two indices are equal}, \\ \qquad\qquad against \\ H_1: \text{ the average returns of the two indices are different}. \end{cases}$$

2.2. Volatility measures. Volatility, is an indispensable statistical tool in the financial market that makes it possible to measure the risk of the yields indices variability. Indeed volatility is a measure to study the dispersion of financial assets returns around their average return. In our study we used two types of volatilities, namely; historical volatility also referred to as unconditional volatility and conditional volatility based on GARCH models.

2.2.1. *Unconditional volatility.* First, we calculated the historical (unconditional) volatility using the unbiased standard deviation of the return changes rate, it is estimated ex-post on the basis of past returns. The estimated standard deviation of daily returns can be used as a tool to measure volatility, as it measures the dispersion of returns on the average by the following

formula,

$$\sigma = \frac{\sqrt{\sum_{i=t}^{T}(R_t - \bar{R})}}{T - 1}$$

If the standard deviation is large, it means that there is a high probability of capturing a high return in the positive or negative direction, wich means that the volatility of returns is high [28].

2.3. conditional volatility. GARCH-Model

This part will be devoted to the modeling of the indices returns conditional volatility of our sample, using family GARCH models, which were introduced by Bollerslev (1986). Indeed, these models allowed us to make an autoregressive representation of returns series conditional variance of our indices. In this context, the conditional variance of each returns series is estimated by the square of the p past error terms and the delayed conditional variances. Thus, the conditional variance is calculated by the following formula:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}$$

where β_j represents the autoregressive coefficient that gives an explanation of the impact of past volatilities on current volatility; ie d. impact of shocks on performance. Asymmetry is a central property to study financial series, but ARCH and GARCH type models do not take this hypothesis into account, which leads us to use the EGARCH model.

EGARCH-Model

GARCH models are characterized by squared residuals, which hides the impact of bearish or bullish movements, EGARCH models introduced by Nelson aim to overcome this insufficiency, they make it possible to model the influence of the bearish and bullish movements on the financial behavior series. This model may be represented as follows:

$$ln(h_t^2) = \alpha_0 + \sum_{i=1}^{q} (\alpha_i Z_{t-i} + \gamma[|Z_{t-i}| - E|Z_{t-i}|] + \sum_{j=1}^{p} \beta_j ln(h_{t-j}^2)$$

where $Z_{t-j} = \frac{\varepsilon_{t-j}}{\sigma_{t-j}}$, ε_t represents the standardized error, based on the empirical observation that financial assets volatility tends to increase after peaks in negative returns, unlike peaks of positive returns. This observation has been taken into account in the EGARCH model, because it takes into consideration the signs of past residues in calculating the conditional variance. Contrary to GARCH model, the EGARCH model assumes no assumptions about the parameters to

ensure the positivity of the conditional variance. The presence of negative and positive Having a negative coefficient means a negative yield shock that generates higher volatility than a positive shock. Indeed, the main difference between GARCH and EGARCH model lies in the fact that the latter takes into account bad news wich have different impacts compared to good news on volatility.

2.4. ARFIMA/ARIMA.

Autoregressive Fractionally Integrated Moving Average(ARFIMA) Model. The autoregressive fractionally integrated moving average (ARFIMA) model is a linear model introduced by Granger and Joyeux and is widely used to fit and forecast time series. The ARFIMA (p, d, q) model is represented as follows

$$\Psi(B)(1-B)^d X_t = \Theta(B)e_t,$$

where B is the back shift operator such that $BX_t = X_{t-1}$; e_t is a white noise process; $-0.5 \prec d \prec 0.5$ $\Psi(B) = 1 - \Psi_1 B - \ldots\ldots - \Psi_p B^p$; $\Theta(B) = 1 - \Theta_1 B - \ldots\ldots - \Theta_q B^q$; p is the number of the autoregressive terms; d is the number of the fractional differences; and q is the number of the lagged forecast errors in the prediction equation. We obtain the following equation by the binomial expansion.

$$(1-B)^d = \sum \frac{d!}{k!(d-k)!}(-1)^k B^k$$
$$= 1 - dB + \frac{d(d-1)B^2}{2!} - \frac{d(d-1)(d-2)}{B}^3 3! + \ldots$$

where

$$\sum \frac{d!}{k!(d-k)!}(-1)^k B^k$$
$$= \frac{\Gamma(d+1)}{\Gamma(k-1)\Gamma(d-k+1)}$$

3. NUMERICAL SIMULATIONS

As a first step, we have downloaded monthly data for Sukuk indices of different maturities, and their conventional consorts. Then, we calculated the geometric returns of each index. The table summarizes the main statistical properties of each index and its conventional counterpart.

	TRBBG13	TRBSG13	TRBBG31	TRBSG31	TRBBG37	TRBSG37	TRBBG7P	TRBSG7P
Mean	0,00286	0,00282	0,00248	0,00221	0,00321	0,00320	0,00327	0,00314
Median	0,00268	0,00298	0,00249	0,00270	0,00350	0,00389	0,00414	0,00405
Maximum	0,01418	0,02783	0,00591	0,02405	0,03019	0,02238	0,06098	0,03094
Minimum	-0,01329	-0,04814	-0,00368	-0,10205	-0,02745	-0,09699	-0,07340	-0,05068
Std. Dev.	0,00288	0,00686	0,00103	0,01038	0,00617	0,01042	0,01614	0,00963
Skewness	-0,65086	-2,65536	-0,98713	-7,55134	-0,30500	-6,81995	-1,09361	-2,17274
Kurtosis	11,16985	25,68200	12,54152	77,05809	9,12382	65,22641	9,27711	12,31427
Jarque-Bera	384,98	3052,56	534,03	32133,89	213,04	22827,23	248,55	594,22
Probability	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000
Sum	0,38636	0,38086	0,33516	0,29833	0,43317	0,43243	0,44142	0,42417
Sum Sq. Dev.	0,00111	0,00631	0,00014	0,01445	0,00510	0,01456	0,03490	0,01243

FIGURE 1. *Statistical properties of different maturities indices*

The table presents the main statistical properties of the Sukuk indices series returns of different maturities, and their conventional counterparts (Mean, Median, Maximum, Minimum, Standard Deviation, Skewness, Kurtosis, and Bera Jar Test). The table shows that the averages indices returns in our sample are all positive, as well as the average yields of conventional bond indices are slightly higher than those of sukuk indices for all maturities.

Looking at the standard deviation we can easily notice that the standard deviation of the returns series Sukuk is well above the standard deviation of their conventional counterpart except for the maturity more than 7 years. The SKEWNESS coefficients of the indices are all negative, which means that the returns distributions series indices are asymmetric, bearish movements are stronger than bullish movements. Examining the KURTOSIS coefficients we notice that the returns series distributions of all indices have long distribution tails, since the coefficient is greater than 3, which means that the tails are thicker on the ends than those of the normal distribution. This result allowed us to assume the presence of a nonlinear (dynamic) distribution, because in general, volatility depends on the past. The Jarque-Bera test in the table rejects the normality assumption of all indices returns series distributions of our sample. These statistical

properties confirm the classic characteristics of the financial series, including negative asymmetry and thick distribution tails. In following graphs figures we represent returns series of the stock indices Sukuk of different maturities and their conventional counterparts.

3.1. Return of each maturity: In the figures below we represent Sukuk returns indices series of different maturities and their conventional counterparts.
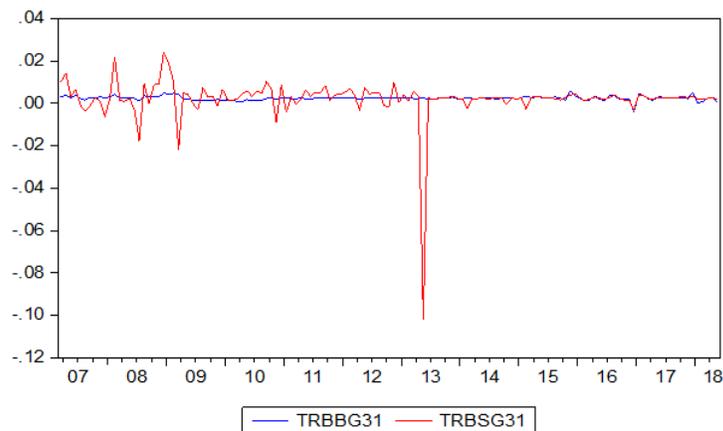
FIGURE 2. *Return of Islamic Bonds(Sukuk) and conventional Bonds for the maturity 3M-1Y*

the yield serie of conventional bonds for the short maturity ranging from 3 months to one year decreases slowly which confirms the presence of a long memory, on the other hand the yield series of sukuk decreases rapidly justifying the absence of the long memory. This is due to their liquidity on the market compared to their conventional bonds of the same maturity.
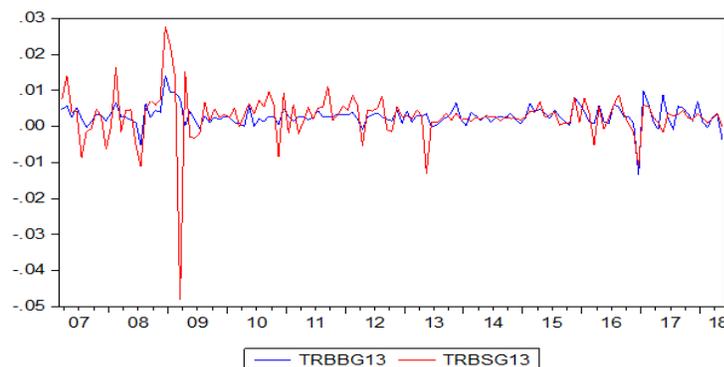
FIGURE 3. *Return of Islamic Bonds(Sukuk) and conventional Bonds for the maturity 1Y-3Y*

the same for the average maturity ranging from one year to 3 years, the yield serie of the conventional bonds presents a long memory unlike its counterpart which invalidates the presence of the long memory.
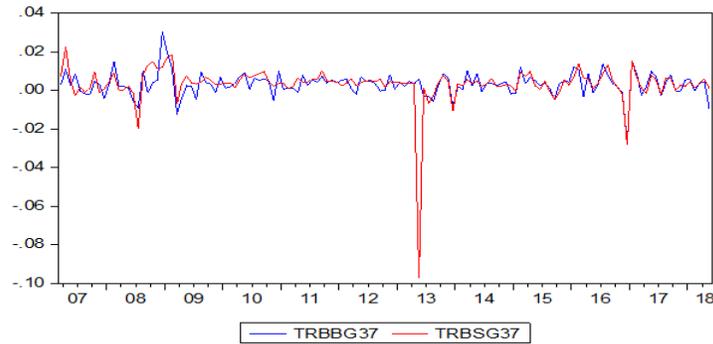
FIGURE 4. *Return of Islamic Bonds(Sukuk) and conventional Bonds for the maturity 3Y-7Y*

conversely, at short and medium maturity, the yield serie begins to decrease rapidly, which this time confirms the absence of the long memory for bonds ranging from 3 years to 7 years confirming the presence of the long memory for its counterpart.
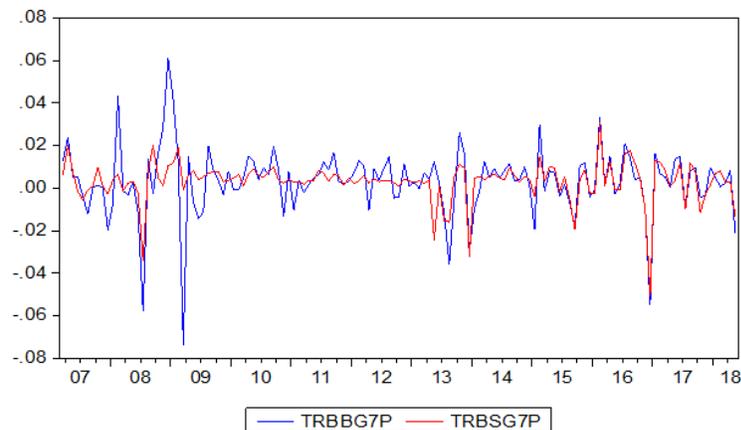
FIGURE 5. *Return of Islamic Bonds(Sukuk) and conventional Bonds for the maturity more than 7Y*

The graph confirms the presence of the long memory for the long-maturity yield serie, inversely to its counterpart; this is due to the liquidity of conventional long-maturity bonds on the financial market. We can conclude that more the maturity is higher more the conventional

bonds become more and more liquids confirming the absence of the long memory and inversly for their contreparts.

3.2. Conditional volatility of different maturities indices. After studying the econometric characteristics of each indices returns serie. We will use heteroscedasticity tests to detect the importance of applying ARCH and GARCH models.

index	Ljung-Box-Q(1)		Ljung-Box-Q(5)	
trbbg13	0,5172	0 ,4720	13,2846	0,0209
trbbg31	3,0037	0,0831	10,1373	0,0714
trbbg37	0,5085	0,4758	11,6949	0,0392
trbbg7p	1,3729	0,2413	15,0896	0,0100
trbbgll	1,1833	0,2767	16,2734	0,0061
trbsg13	0,3735	0,5411	6,3337	0,2751
trbsg31	0,0466	0,8290	0,7872	0,9778
trbsg37	0,6491	0,4205	1,8896	0,8642
trbsg7p	0,8792	0,3484	3,0629	0,6903
trbsgll	0,8577	0,3544	9,0998	0,1051

TABLE 1. *Ljung-Box (Q(1),Q(5))test for different indices notation*

index	Ljung-Box-Q(10)		Ljung-Box-Q(15)		Ljung-Box-Q(20)	
trbbg13	19,6985	0,0322	28,5457	0,0184	31,9511	0,0438
trbbg31	14,3316	0,1584	30,7876	0,0094	37,3213	0,0107
trbbg37	13,2215	0,2115	18,6731	0,2289	20,3001	0,4393
trbbg7p	20,2866	0,0267	32,4026	0,0057	36,3536	0,0140
trbbgll	18,9894	0,0404	28,3588	0,0194	30,9419	0,0560
trbsg13	10,4621	0,4009	15,4895	0,4168	28,7712	0,0924
trbsg31	2,9855	0,9818	3,6925	0,9986	4,1546	0,9999
trbsg37	3,9517	0,9495	4,4147	0,9960	4,7405	0,9998
trbsg7p	7,4498	0,6824	11,4968	0,7166	13,4508	0,8572
trbsgll	11,4588	0,3229	18,5428	0,2352	21,1481	0,3885

TABLE 2. *Ljung-Box(Q(10),Q(15),Q(20)) test for different indices notation*

The Ljung-Box-Q (1979) test that we applied on all the indices of our sample allowed us to notice that generally the correlations are significant for all returns, since most of the p-values associated with the LJUNG - BOX statistics are all less than 5%. Unlike the Sukuk, conventional bond yields are not significantly correlated, because the p-values of LJUNG - BOX test are all higher than 5%. We therefore generally conclude the rejection of the homoscedasticity hypothesis of all returns indices series. Indeed, there is an ARCH effect for all indices returns of our sample.

These results led us to model conditional volatility by performing the famous GARCH model (1.1). The minimization of the AKAIKE criteria has proposed us to use the Gaussian distribution compared to the Student distribution and the GED distribution.

index	C	α	β	$\alpha + \beta$
TRBBG31	0,000000	0,402691	0,774953	1,177644
	0,638338	0,0000***	0,000***	non
TRBSG31	0,000067	-0,013758	0,555319	0,541561
	0,463976	0,000***	0,362086	
TRBBG13	0,000001	0,136877	0,829709	0,966585
	0,184644	0,09874*	0,000***	
TRBSG13	0,000004	1,085351	0,318031	1,403383
	0,0009***	0,000***	0,0008***	non
TRBBG37	0,000007	0,076649	0,740382	0,817031
	0,392800	0,339900	0,003***	
TRBSG37	0,000060	-0,014298	0,577899	0,563600
	0,639582	0,450645	0,524643	
TRBBG7P	0,000057	1,179700	0,165976	1,345676
	0,0000***	0,000***	0,0004***	
TRBSG7P	0,000032	1,645580	0,038486	1,684065
	0,000***	0,000***	0,356927	
TRBBGLL	0,000017	1,568798	-0,003616	1,565182
	0,000***	0,000***	0,918081	
TRBSGLL	0,0000130	0,1241610	0,6004392	0,724600
	0,1881284	0,2228604	0,03486**	

TABLE 3. *Garch(1,1) with * significance at the 10% threshold, ** significance at the 5% threshold,*

*** *significance at the 1% level*

3.3. GARCH models applied to the different indices of all studied maturities. After reading the table, we find that most coefficients of the GARCH model (1,1) are positive and are significant at the 1% threshold for all the indices of our sample. This finding allowed us to conclude that the GARCH model (1,1) successfully managed to model the behavior of the returns indices of our sample. We can note as easily that α coefficients values of the model GARCH (1,1) equation are all lower than the β coefficients values. This means that the market brings significant corrections to future conditional volatility and also informs that conditional variance is certainly impacted by past variance, which implies that past information shocks have a significant impact on current returns.

The first parameter C means the threshold of the minimum conditional variance, we notice that it is almost negligible (close to 0) for all the indices of our sample.

The second parameter α is a coefficient that captures the impact of shocks on volatility, it is calculated by summing the residuals squares. Since we could not deduce results in a general way for all the studied maturities so we will go into the details and make interpretations for each government indices studied with its counterpart, on different maturities. From the table we can notice that the impact of financial shocks on conventional bond indices is greater than that of Sukuk indices for all maturities except the maturity more than 7 years. Similarly, the impact of delayed conditional volatilities is greater for conventional bonds, compared to that of the Sukuk on almost maturities except for the indices of maturity more than 7 years and for global indices.

The third parameter β aims to measure the persistence of the volatility, it is obtained by the sum of the delayed variances. Indeed, it gives an idea of the contribution of past volatilities in current volatility. We can easily notice in the table that for all maturities the β coefficient of the Sukuk indices is lower than its conventional counterpart. This means that the persistence of the conventional bond indices volatility is stronger than that of the Islamic counterparts.

The sum of the parameters $\alpha + \beta$ for all returns indices is lower than 1 for all the Sukuk indices, which implies the stationarity of the conditional volatility of these indices. For conventional bond indices, this sum is greater than 1 for all maturities except for maturity between 1 year and 3 years. Indeed, the calculation of the $\alpha + \beta$ coefficients gives an important information on the extent of the conditional volatility persistence.

4. GRAPHICAL REPRESENTATIONS OF THE CONDITIONAL VOLATILITY ESTIMATED BY THE GARCH MODEL (1.1)

In this subsection we have applied the GARCH model (1,1) for all returns indices series of our sample, then we graphically represent the conditional volatility of our series as shown in the figures below.
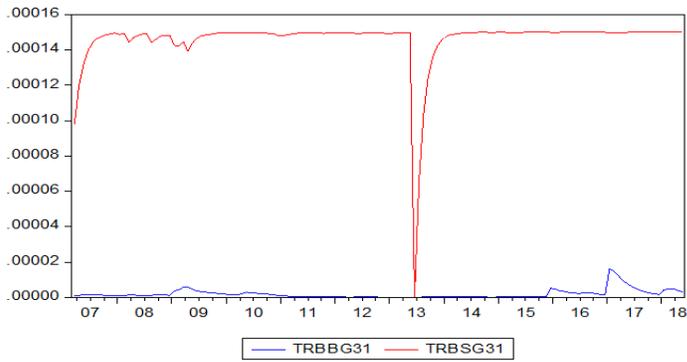
FIGURE 6. *Volatility of Islamic Bonds(Sukuk) and conventional Bonds for the maturity 3M-1Y*
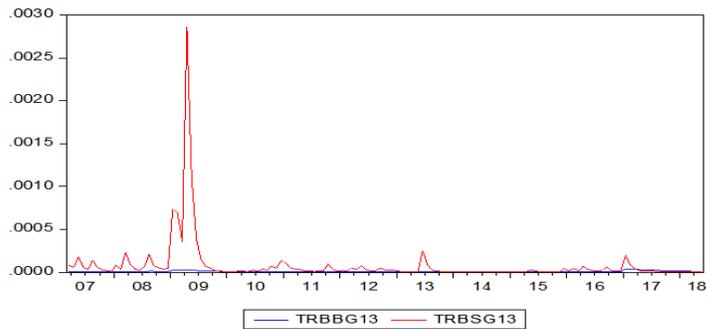
FIGURE 7. *Garch Volatility of Islamic Bonds(Sukuk) and conventional Bonds for the maturity 1Y-3Y*
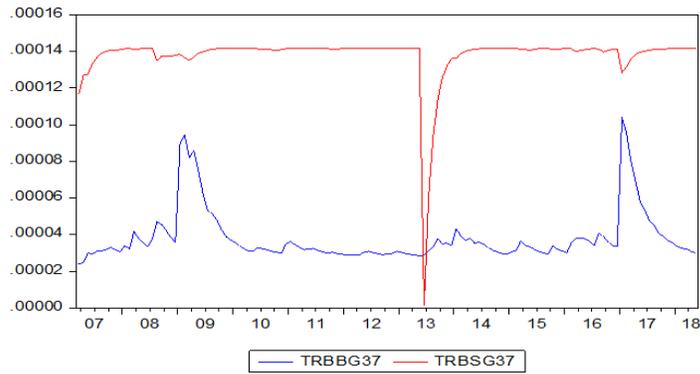
FIGURE 8. *Garch Volatility of Islamic Bonds(Sukuk) and conventional Bonds for the maturity 3Y-7Y*
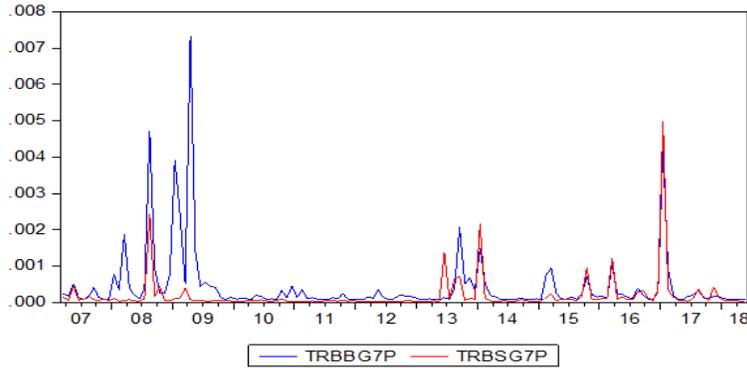
FIGURE 9. *Garch Volatility of Islamic Bonds(Sukuk) and conventional Bonds for the maturity more than 7 years*

Given the figures above, which represent the dynamics of the returns indices volatility , we usually notice that the peaks of volatility do not occur on the same dates, which indicates that the degree of impact of the financial shocks on the Sukuk indices is totally different to that of conventional bonds. It is also clear from the graphs that all Sukuk indices are significantly more volatile than their conventional counterparts, this remark allows us to conclude that the Sukuk indices are riskier than their counterparts. These peaks had generally been due to the crises that the history had and have marked their impact on the Malaysian Financial Market explained by the peaks of 2008 due to the financial crisis of the US market; peaks of 2013 as a consequence of the Asian crisis; or oil crisis; 2017 peaks triggered by the crisis in China. We can even note that as maturity increases, the volatility of conventional bonds increases inversely for sukuk indices.

In order to deepen our modeling, we have incorporated the phenomenon of asymmetry for the volatility behavior. The EGARCH models introduced by Nelson (1990) are used to analyze the asymmetry of conditional volatility. The table below presents the different results estimations of the EGARCH model (1,1).

index	C	α	γ	β
TRBBG31	-11,1003	1,1542	-0,4974	0,2599
	0,0000***	0,0000***	0,0007***	0,1528
TRBSG31	-0,5363	-0,2510	0,4196	0,9337
	0,0000***	0,0000***	0,0000***	0,0000***
TRBBG13	-10,3823	1,0045	-0,4323	0,1794
	0,0000***	0,0000***	0,0081***	0,2778
TRBSG13	-0,7170	0,2159	0,3979	0,9498
	0,0000***	0,0000***	0,0000***	0,0000***
TRBBG37	-15,9505	0,3648	-0,0485	-0,5308
	0,0001***	0,1631	0,6839	0,1938
TRBSG37	-9,1325	-0,3442	-0,0894	0,0099
	0,3208	0,1671	0,6142	0,9921
TRBBG7P	-2,0183	0,4662	-0,1181	0,7942
	0,0343**	0,0065***	0,1493	0,0000***
TRBSG7P	-7,9706	0,9991	-0,3214	0,2281
	0,0000***	0,0000***	0,0428**	0,0608*
TRBBGLL	-8,1133	1,4165	-0,1353	0,2929
	0,0000***	0,0000***	0,3233	0,0022***
TRBSGLL	-3,4496	0,2048	0,0384	0,6673
	0,1767	0,3747	0,8091	0,0082***

TABLE 4. *EGarch(1,1) with * significance at the 10% threshold, ** significance at the 5% threshold, *** significance at the 1% level*

From the table, we notice that all the coefficients of the model EGARCH (1,1) are significant, which implies the presence of the asymmetry effect for all the indices of our sample. By examining the parameters α that measure the impact of financial shocks on performance, we find that the degree of impact of the financial shocks on the Sukuk indices is lower than that of their conventional counterparts except for the maturity more than 7 years. Concerning the γ coefficients, we notice that they exist on the studied maturities those which are significant; others that are insignificant (positive or negative) at 1%, it is a sign of the presence of the lever effect,

which implies that the indices of our sample are sensitive to bad news and good news. This remark allows us to conclude that financial shocks have an impact on the indices volatility in our sample that have posted coefficients that are significant. The β coefficients are used to calculate the magnitude of the impact of past volatility on current volatility, it means the presence of volatility groups (volatility cluster), because most β coefficients are positive and significant for all Sukuk indices and their conventional counterparts. We also note that all studied series are stationary, as all the β coefficients are less than 1.

In order to visualize conditional volatility behavior obtained by models EGARCH (1,1) estimation, we have represented the dynamics of each returns Sukuk index volatility and their conventional consorts as the following figures indicate.
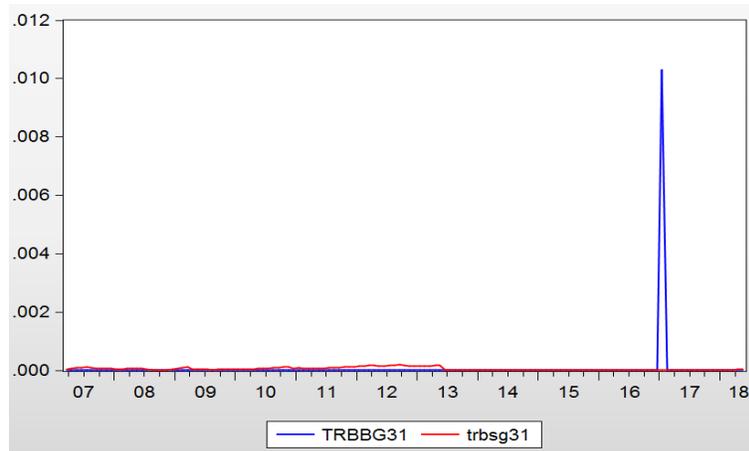
FIGURE 10. *Egarch Volatility of Islamic Bonds(Sukuk) and conventional Bonds for the maturity 3M-1Y*
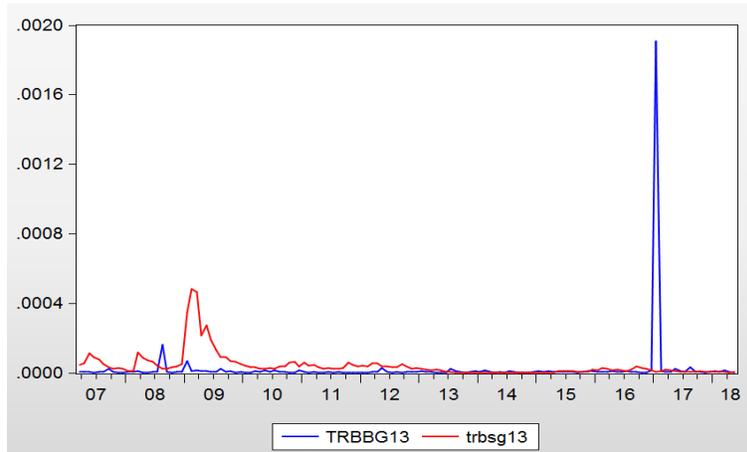
FIGURE 11. *Egarch Volatility of Islamic Bonds(Sukuk) and conventional Bonds for the maturity 1Y-3Y*
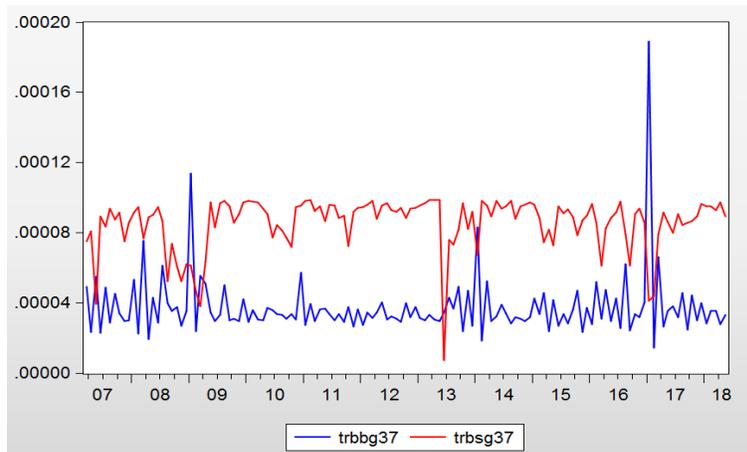
FIGURE 12. *Egarch Volatility of Islamic Bonds(Sukuk) and conventional Bonds for the maturity 3Y-7Y*
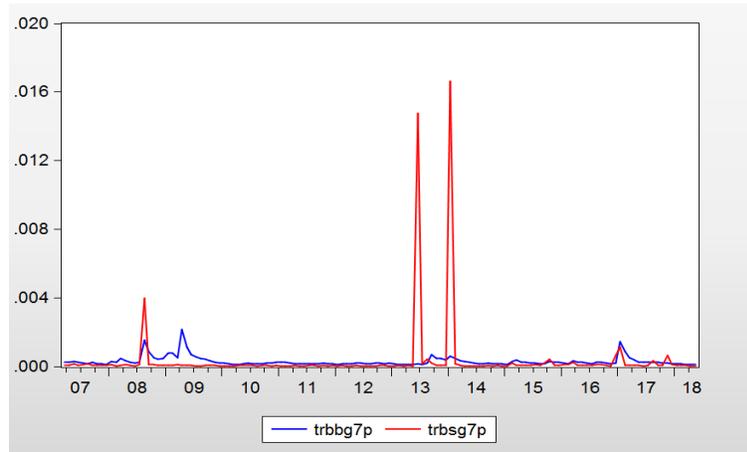
FIGURE 13. *Egarch Volatility of Islamic Bonds(Sukuk) and conventional Bonds for the maturity more than 7 years*

We usually notice that the peaks of volatility do not occur on the same dates, which indicates that the impact degree of the financial shocks on the Sukuk indices is totally different to that of the conventional bonds. The graphs show also that all Sukuk indices significantly more volatile than their conventional counterparts, this observation allows us to conclude that Sukuk indices are riskier than their counterparts, which validates what was obtained using Garch volatility (1,1).

4.1. ARFIMA Model Results. We simulated The ARFIMA model using EViews 6.0, we have grouped the results in the table below

index	D
TRBBG31	0,1003
TRBSG31	-0,4363
TRBBG13	0,3823
TRBSG13	-0,3170
TRBBG37	-0,4505
TRBSG37	0,1325
TRBBG7P	-0,0183
TRBSG7P	0,1706

TABLE 5. Parameter D values of the ARFIMA model

This table show that conventional bonds of short-maturity ranging from 3 months to one year have a long memory $0 \leq d \leq 0.5$ while sukuk of the same maturity presents a short memory $-0.5 \leq d \leq 0$.

The same thing for conventional bonds of the medium maturity, they have a long memory process unlike sukuk ranging from one year to 3 years.

contrary to short and medium conventional bonds that present long memory process, conventional bonds of long maturity have a short memory process due to their liquidity in the financial market unlike islamic bonds of long maturity ranging from 3 years to 7 years.

Factors influencing the long memory:

We observed in the previous section thanks to the figures that the autocorrelation function of short and medium-maturity bonds price returns decrease was slow, so we speak of hyperbolic decay. This trait is characteristic of the presence of long memory in volatility time series, as opposed to a rapid decrease of exponential type, synonymous with short memory. In other words, it means that in the presence of long memory, the volatility of an asset's returns on day d can have an impact on future volatility, while in the presence of short memory, the impact most often disappears completely after no more than ten days.

Taking into account the phenomenon of long memory is a capital aspect of volatility modeling. Indeed, it will contribute to better modeling, and by therefore a better prediction of volatility in the long term. In addition, the more accurate the modeling over the long term, the more portfolio resource allocation will be efficient. The manager will be able to have a more precise idea of the medium or long risk term and thus manage its portfolio over the longer term. It will then save costs transactions that put a strain on the performance of its portfolio.

In conclusion, since an efficient market is a very liquid market (this is necessary for the arrival of information is immediately reflected in the share price), the more liquid a market, the smaller d (negative). Furthermore, since efficiency results from the interaction of a large number of heterogeneous agents, then we infer that the greater the heterogeneity large among agents plus d is small. In other words, the less long memory there is. In particular, in perfectly efficient markets there is no long memory at all (d = 0). Since long memory seems to be linked to the hererogeneity of agents, it is appropriate to consider how to model this heterogeneity.

5. Conclusion

In our view, taking long memory into account is a key point for a better assessment of equity risk and constitutes a two-part challenge. First, it is essential to better interpret this phenomenon on the financial markets, and this requires the development of a volatility model for our sukuk and bonds series of each maturity able of linking long memory to investor behavior. Second, it is worth considering how to integrate this characteristic of volatility series operationally, in particular in portfolio management and the consequences that this implies both in the context of theoretical than practical. In fact, it is essential, in this context, to know whether the classic theoretical portfolio of choice can still apply, or on the contrary, whether to consider other means of implementation.

After considering long memory from a theoretical and conceptual point of view we put it in a context of portfolio management for the practical side. In this area, the modeling of financial assets returns plays a central role in the process selection, first qualitative, then quantitative, of the portfolio constituents. In Indeed, the modeling makes it possible to apprehend in a rigorous way two essential elements what are the expected return and the level of risk. This imperative arises in fact in a way evident in the theory of the choice of optimal allocation, whether in discrete time or in continuous time.

To model volatility as closely as possible and thus obtain the portfolio closest to the optimal portfolio, two characteristics must imperatively be introduced:the dynamics of volatility on the one hand and long memory on the other. However, in portfolio management, taking long memory into account leads to a change in framework of the studied problem:from Markovian, it becomes non-Markovian. The consequence of the point from a practical point of view is that the management strategy must take into account all the past risk, unlike a classic approach.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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