Available online at http://scik.org
Commun. Math. Biol. Neurosci. 2021, 2021:94
https://doi.org/10.28919/cmbn/6848
ISSN: 2052-2541

ON THE IMPLEMENTATION OF A VARIATIONAL ITERATION METHOD FOR A SEIQR COVID-19 EPIDEMIC MODEL

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Abstract. We consider a SEIQR epidemic model which describes the spread of COVID-19. The SEIQR epidemic model is built by introducing an isolation compartment in the SEIR model. We implement the variational iteration method (VIM) to find the approximate solution for the SEIQR model. We first implement the VIM by applying restricted variations for both linear and nonlinear terms in the correction functionals and find the Lagrange multiplier for the VIM. The comparison between the solution obtained by such VIM and the solution obtained by the fourth-order Runge-Kutta method shows that the VIM is accurate only for relatively small time domains. For larger time domains, the VIM solution is inaccurate and unrealistic. Then we improve the previous VIM by reducing the restricted variations and show that the improved VIM is more accurate than the previous VIM for larger time domains.

Keywords: COVID-19; SEIQR epidemic model; variational iteration method; Lagrange multiplier.

2010 AMS Subject Classification: 34A34; 47J30; 92D30.

1. Introduction

In March 2020, the World Health Organization (WHO) announced the coronavirus disease 2019 (COVID-19) outbreak as a global pandemic. This is because the COVID-19 case, which

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Received October 03, 2021

was first identified in Wuhan – China, in December 2019, has already spread to almost all countries in the world [1]. The COVID-19 pandemic has had a devastating impact on almost all aspects of human life, including health, economic and social. Therefore, researchers, scientists and policymakers are struggling with how to control this pandemic to guarantee the lives of many people [2, 3].

To understand the spread of COVID-19, many researchers have developed mathematical epidemic models. There are two types of epidemic models, namely phenomenological models and mechanistic compartment models. Phenomenological epidemic models predict the epidemic curves based on the observed data, without assuming any biological/physical mechanisms involved in the transmission dynamics of disease [4]. The applications of phenomenological models for describing the spread of COVID-19 can be found for example in [5, 6, 7, 8, 9]. On the contrary, the mechanistic compartment epidemic models assume clear biological mechanisms, which represent the dynamics of disease transmission [10]. These models describe the movement of sub-populations through a sequence of compartments that correspond to health states. By considering that the COVID-19 epidemic consists of three health states, namely the susceptible state (S), infectious state (I) and recovered state (R), authors in [11, 13, 12] have applied a SIR epidemic model. In the case of COVID-19, there exists an exposed state (E), i.e. class of individuals that have been exposed to the disease but are not yet infectious. Considering such exposed state, the SIR model is then modified into an SEIR model, see [14, 15, 16]. Furthermore, the spread of COVID-19 is mainly due to close contact with infected symptomatic cases or via respiratory droplets. Many countries have implemented quarantine measures to prevent the spread of COVID-19. Since the quarantined individuals (Q) play a significant role in controlling the spread of COVID-19, Zeb et al. [17] have extended the SEIR epidemic model into the SEIQR epidemic model.

Compartmental epidemic models are generally written as system of nonlinear differential equations, where their exact solution are difficult to be determined. In recent years, many scholars have developed various techniques to find approximate solutions for nonlinear differential equations. One of the approximate analytical techniques is the variational iteration method (VIM) which was firstly suggested by He [18, 19]. VIM is based on the correction functionals and the implementation of restricted variations. One of the main advantages of this method is that the resulting approximated solution is represented as a continuous function. VIM has been applied to find a solution for both nonlinear ordinary and partial differential equations, see for example [20, 21, 22, 23, 24, 25, 26]. The key point of VIM implementation relies on a Lagrange multiplier, which is optimally determined by the variational theory. In many applications, this is performed by introducing restricted variations in the correction functionals to get the Lagrange multipliers in simple forms.

In this paper we aim to apply VIM to determine an analytical approximated solution for the SEIQR COVID-19 epidemic model proposed by Zeb et al. [17]. We first apply VIM by assuming restricted variations for both linear and nonlinear terms in the correction functionals, except for the differential operator. Then we improve such VIM by reducing the number of restricted variations. The results of the two methods (VIM and its improvement) will be evaluated by comparing those results with the solution obtained by the fourth-order Runge-Kutta method.

This paper is organized as follows. We describe the SEIQR COVID-19 epidemic model in Section 2. Section 3 presents the implementation of VIM to determine the solution of the SEIQR epidemic model. In Section 4, we provide results and discussion. The conclusion is given in Section 5.

2. COVID-19 EPIDEMIC MODEL WITH ISOLATION

Recently, Zeb et al. [17] have proposed an SEIQR epidemic model to describe the spread of COVID-19. The model consists of five compartments, namely Susceptible population ($\hat{S}(t)$), Exposed population $\hat{E}(t)$, Infected population $\hat{I}(t)$, Isolated population $\hat{Q}(t)$, and Recovered population $\hat{R}(t)$, which are represented by the following system of first order differential equations.

TABLE 1. Description of parameters.

Parameter	Description
A	Recruitment rate
m	Death rate (both naturally and due to disease)
b	Infection rate
p	Rate at which exposed population moves to infected population
g	Rate at which exposed population moves to isolated population
σ	Rate at which infection population moves to isolated population
θ	Rate at which isolated individuals recovered

$$\begin{aligned} \frac{d\hat{S}(t)}{dt} &= A - m\hat{S}(t) - b\hat{S}(t)\left(\hat{E}(t)+\hat{I}(t)\right) \\ \frac{d\hat{E}(t)}{dt} &= b\hat{S}(t)\left(\hat{E}(t)+\hat{I}(t)\right) - (p+m+g)\hat{E}(t) \\ \frac{d\hat{I}(t)}{dt} &= p\hat{E}(t) - (\sigma+m)\hat{I}(t) \\ \frac{d\hat{Q}(t)}{dt} &= g\hat{E}(t) + \sigma\hat{I}(t) - (\theta+m)\hat{Q}(t) \\ \frac{d\hat{R}(t)}{dt} &= \theta\hat{Q}(t) - m\hat{R}(t). \end{aligned} \tag{1}$$

All parameters in system (1) are positive and described in Table 1.

If we define the total population as $N = \hat{S}+\hat{E}+\hat{I}+\hat{Q}+\hat{R}$, then from system (1) we obtain that

$$\frac{dN(t)}{dt} = A - mN(t), \tag{2}$$

where its solution is given by $N(t) = A/m + (N(0) - A/m)\exp(-mt)$. It is clear that $N(t) \to A/m$ as $t \to \infty$. Therefore, we can rescale system (1) by introducing transformation $N = A/m, \beta = bN, S = \hat{S}/N, E = \hat{E}/N, I = \hat{I}/N, Q = \hat{Q}/N$ and $R = \hat{R}/N$ to get the following normalized system

(3)
$$\begin{aligned}
\frac{dS(t)}{dt} &= m - mS(t) - \beta S(t)\left(E(t)+I(t)\right)\\
\frac{dE(t)}{dt} &= \beta S(t)\left(E(t)+I(t)\right) - (p+m+g)\,E(t)\\
\frac{dI(t)}{dt} &= pE(t) - (\sigma+m)\,I(t)\\
\frac{dQ(t)}{dt} &= gE(t) + \sigma I(t) - (\theta+m)\,Q(t),\\
\frac{dR(t)}{dt} &= \theta Q(t) - mR(t).
\end{aligned}$$

In this paper we focus on solving the system (3) with the initial conditions

(4)
$$S(0) = S_0 \geq 0, S(0) = S_0 \geq 0, S(0) = S_0 \geq 0, S(0) = S_0 \geq 0.$$

3. Variational Iteration Method

For a general nonlinear differential equation

(5)
$$Lu + N(u) = f(t),$$

where L and N are respectively a linear and nonlinear operators, and $f(t)$ is a nonhomogeneous term, He [18, 19] proposed a correction functional based on the variational method as follows

(6)
$$u_{n+1}(t) = u_n(t) + \int_{t_0}^{t} \lambda\left[Lu_n(s) + N\tilde{u_n}(s) - f(s)\right] ds.$$

Here λ is a general Lagrange multiplier which can be obtained optimally via the variational theory and $\tilde{u_n}$ is a restricted variation, namely $\delta\tilde{u_n} = 0$ with δ is a variational derivative. Following [20, 24], we apply the variational iteration method (6) by considering restricted variations for both linear and nonlinear parts except for the differential operator. The correction functionals for the system (3) with initial conditions (4) are then constructed as follows

$$
\begin{aligned}
S_{n+1}(t) =& S_n(t)+\int_0^t \lambda_1(s)\left[\frac{dS_n(s)}{ds}-m+m\tilde{S_n}(s)+\beta\tilde{S_n}(s)\left(\tilde{E_n}(s)+\tilde{I_n}(s)\right)\right]ds,\\
E_{n+1}(t) =& E_n(t)+\int_0^t \lambda_2(s)\left[\frac{dE_n(s)}{ds}-\beta\tilde{S_n}(s)\left(\tilde{E_n}(s)+\tilde{I_n}(s)\right)+(p+m+g)\tilde{E_n}(s)\right]ds,\\
I_{n+1}(t) =& I_n(t)+\int_0^t \lambda_3(s)\left[\frac{dI_n(s)}{ds}-p\tilde{E_n}(s)+(\sigma+m)\tilde{I_n}(s)\right]ds,\\
Q_{n+1}(t) =& Q_n(t)+\int_0^t \lambda_4(s)\left[\frac{dQ_n(s)}{ds}-g\tilde{E_n}(s)-\sigma\tilde{I_n}(s)+(\theta+m)\tilde{Q_n}(s)\right]ds\\
R_{n+1}(t) =& R_n(t)+\int_0^t \lambda_5(s)\left[\frac{dR_n(s)}{ds}-\theta\tilde{Q_n}(s)+m\tilde{R_n}(s)\right]ds.
\end{aligned} \tag{7}
$$

By taking variations of the system (7) and remembering that $\delta\tilde{S_n}=\delta\tilde{E_n}=\delta\tilde{I_n}=\delta\tilde{Q_n}=\delta\tilde{R_n}=0$, we get

$$
\begin{aligned}
\delta S_{n+1}(t) =& \delta S_n(t)+\delta\int_0^t \lambda_1(s)\left(\frac{dS_n(s)}{ds}\right)ds,\\
=& \delta\left[\left(1+\lambda_1(t)\right)S_n(t)\right]-\delta\int_0^t \lambda_1^{'}(s)S_n(s)\,ds,\\
\delta E_{n+1}(t) =& \delta E_n(t)+\delta\int_0^t \lambda_2(s)\left(\frac{dE_n(s)}{ds}\right)ds,\\
=& \delta\left[\left(1+\lambda_2(t)\right)E_n(t)\right]-\delta\int_0^t \lambda_2^{'}(s)E_n(s)\,ds,\\
\delta I_{n+1}(t) =& \delta I_n(t)+\delta\int_0^t \lambda_3(s)\left(\frac{dI_n(s)}{ds}\right)ds,\\
=& \delta\left[\left(1+\lambda_3(t)\right)I_n(t)\right]-\delta\int_0^t \lambda_3^{'}(s)I_n(s)\,ds,\\
\delta Q_{n+1}(t) =& \delta Q_n(t)+\delta\int_0^t \lambda_4(s)\left(\frac{dQ_n(s)}{ds}\right)ds\\
=& \delta\left[\left(1+\lambda_4(t)\right)Q_n(t)\right]-\delta\int_0^t \lambda_4^{'}(s)I_n(s)\,ds\\
\delta R_{n+1}(t) =& \delta R_n(t)+\delta\int_0^t \lambda_5(s)\left(\frac{dR_n(s)}{ds}\right)ds\\
=& \delta\left[\left(1+\lambda_5(t)\right)R_n(t)\right]-\delta\int_0^t \lambda_5^{'}(s)R_n(s)\,ds.
\end{aligned} \tag{8}
$$

System (8) gives the stationary conditions $1+\lambda_i(s)|_{s=t}=0$ and $\lambda_i^{'}(s)|_{s=t}=0,\ i=1,2,3,4,5;$ from which we obtain the Lagrange multipliers for the system (7)

$$\lambda_1(s)=-1,\ \lambda_2(s)=-1,\ \lambda_3(s)=-1,\ \lambda_4(s)=-1,\text{and } \lambda_5(s)=-1.$$

By identifying the Lagrange multipliers, the variational iteration method (VIM) for system (3) is now given by

$$
\begin{aligned}
S_{n+1}(t) =& S_n(t) - \int_0^t \left[\frac{dS_n(s)}{ds} - m + mS_n(s) + \beta S_n(s)\left(E_n(s) + I_n(s)\right) \right] ds, \\
E_{n+1}(t) =& E_n(t) - \int_0^t \left[\frac{dE_n(s)}{ds} - \beta S_n(s)\left(E_n(s) + I_n(s)\right) + (p+m+g)\, E_n(s) \right] ds, \\
I_{n+1}(t) =& I_n(t) - \int_0^t \left[\frac{dI_n(s)}{ds} - pE_n(s) + (\sigma + m)\, I_n(s) \right] ds, \\
Q_{n+1}(t) =& Q_n(t) - \int_0^t \left[\frac{dQ_n(s)}{ds} - gE_n(s) - \sigma I_n(s) + (\theta + m)\, Q_n(s) \right] ds, \\
R_{n+1}(t) =& R_n(t) - \int_0^t \left[\frac{dR_n(s)}{ds} - \theta Q_n(s) + mR_n(s) \right] ds.
\end{aligned} \tag{9}
$$

where $n = 0, 1, 2, \ldots$ and S_0, E_0, I_0, Q_0 and R_0 are given by the initial conditions (4).

We notice that the VIM (9) is constructed by applying restricted variation for both linear and nonlinear terms. Based on He's variational iteration method, we will improve the variational iteration (9) by reducing the restrictions in the correction functionals as follows

$$
\begin{aligned}
S_{n+1}(t) =& S_n(t) + \int_0^t \lambda_1(s) \left[\frac{dS_n(s)}{ds} - m + mS_n(s) + \beta \tilde{S}_n(s)\left(\tilde{E}_n(s) + \tilde{I}_n(s)\right) \right] ds, \\
E_{n+1}(t) =& E_n(t) + \int_0^t \lambda_2(s) \left[\frac{dE_n(s)}{ds} - \beta \tilde{S}_n(s)\left(\tilde{E}_n(s) + \tilde{I}_n(s)\right) + (p+m+g)\, E_n(s) \right] ds, \\
I_{n+1}(t) =& I_n(t) + \int_0^t \lambda_3(s) \left[\frac{dI_n(s)}{ds} - p\tilde{E}_n(s) + (\sigma + m)\, I_n(s) \right] ds, \\
Q_{n+1}(t) =& Q_n(t) + \int_0^t \lambda_4(s) \left[\frac{dQ_n(s)}{ds} - g\tilde{E}_n(s) - \sigma \tilde{I}_n(s) + (\theta + m)\, Q_n(s) \right] ds, \\
R_{n+1}(t) =& R_n(t) + \int_0^t \lambda_5(s) \left[\frac{dR_n(s)}{ds} - \theta \tilde{Q}_n(s) + mR_n(s) \right] ds.
\end{aligned} \tag{10}
$$

We note that similar improvement, namely the reduction of restriction variations, was also suggested in [21, 25, 26]. As previously, we take variations of system (10) to get

$$
\begin{aligned}
\delta S_{n+1}(t) =& \delta S_n(t) + \delta \int_0^t \lambda_1(s) \left(\frac{dS_n(s)}{ds} + mS_n(s) \right) ds, \\
=& \delta \left[\left(1 + \lambda_1(t)\right) S_n(t) \right] - \delta \int_0^t (\lambda_1'(s) - m\lambda_1(s)) S_n(s)\, ds,
\end{aligned}
$$

$$
\begin{aligned}
\delta E_{n+1}(t) =& \delta E_n(t) + \delta \int_0^t \lambda_2(s) \left(\frac{dE_n(s)}{ds} + (p+m+g)E_n(s) \right) ds, \\
=& \delta \left[(1+\lambda_2(t))\, E_n(t) \right] - \delta \int_0^t (\lambda_2'(s) - (p+m+g)\lambda_2(s)) E_n(s)\, ds, \\
\delta I_{n+1}(t) =& \delta I_n(t) + \delta \int_0^t \lambda_3(s) \left(\frac{dI_n(s)}{ds} + (\sigma+m)I_n(s) \right) ds, \\
=& \delta \left[(1+\lambda_3(t))\, I_n(t) \right] - \delta \int_0^t (\lambda_3'(s) - (\sigma+m)\lambda_3(s)) I_n(s)\, ds, \\
\delta Q_{n+1}(t) =& \delta Q_n(t) + \delta \int_0^t \lambda_4(s) \left(\frac{dQ_n(s)}{ds} + (\theta+m)Q_n(s) \right) ds \\
=& \delta \left[(1+\lambda_4(t))\, Q_n(t) \right] - \delta \int_0^t (\lambda_4'(s) - (\theta+m)\lambda_4(s)) I_n(s)\, ds, \\
\delta R_{n+1}(t) =& \delta R_n(t) + \delta \int_0^t \lambda_5(s) \left(\frac{dR_n(s)}{ds} + mR_n(s) \right) ds \\
=& \delta \left[(1+\lambda_5(t))\, R_n(t) \right] - \delta \int_0^t (\lambda_5'(s) - m\lambda_5(s)) R_n(s)\, ds
\end{aligned} \tag{11}
$$

Accordingly, we obtain the following stationary conditions

$$
\begin{aligned}
1+\lambda_1(s)|_{s=t} =& 0, \ \left(\lambda_1'(s) - m\lambda_1(s) \right) |_{s=t} = 0, \\
1+\lambda_2(s)|_{s=t} =& 0, \ \left(\lambda_2'(s) - (p+m+g)\lambda_2(s) \right) |_{s=t} = 0, \\
1+\lambda_3(s)|_{s=t} =& 0, \ \left(\lambda_3'(s) - (\sigma+m)\lambda_3(s) \right) |_{s=t} = 0, \\
1+\lambda_4(s)|_{s=t} =& 0, \ \left(\lambda_4'(s) - (\theta+m)\lambda_4(s) \right) |_{s=t} = 0, \\
1+\lambda_5(s)|_{s=t} =& 0, \ \left(\lambda_5'(s) - m\lambda_5(s) \right) |_{s=t} = 0.
\end{aligned} \tag{12}
$$

From the stationary conditions (12), the Lagrange multipliers for the variational iteration (10) are identified as follows

$$
\begin{aligned}
\lambda_1(s) =& -\exp(m(s-t)), \\
\lambda_2(s) =& -\exp((p+m+g)(s-t)), \\
\lambda_3(s) =& -\exp((\sigma+m)(s-t)), \\
\lambda_4(s) =& -\exp((\theta+m)(s-t)), \\
\lambda_5(s) =& -\exp(m(s-t)).
\end{aligned} \tag{13}
$$

Thus, the improved variational iteration method (IVIM) for the system (3) can be written as

(14)

$$
\begin{aligned}
S_{n+1}(t) =& S_n(t) - \int_0^t \exp(m(s-t)) \left[\frac{dS_n(s)}{ds} - m + mS_n(s) + \beta S_n(s)\left(E_n(s) + I_n(s)\right) \right] ds, \\
E_{n+1}(t) =& E_n(t) - \int_0^t \exp((p+m+g)(s-t)) \left[\frac{dE_n(s)}{ds} - \beta S_n(s)\left(E_n(s) + I_n(s)\right) + (p+m+g)\, E_n(s) \right] ds, \\
I_{n+1}(t) =& I_n(t) - \int_0^t \exp((\sigma+m)(s-t)) \left[\frac{dI_n(s)}{ds} - pE_n(s) + (\sigma+m)\, I_n(s) \right] ds, \\
Q_{n+1}(t) =& Q_n(t) - \int_0^t \exp((\theta+m)(s-t)) \left[\frac{dQ_n(s)}{ds} - gE_n(s) - \sigma I_n(s) + (\theta+m)\, Q_n(s) \right] ds, \\
R_{n+1}(t) =& R_n(t) - \int_0^t \exp(m(s-t)) \left[\frac{dR_n(s)}{ds} - \theta Q_n(s) + mR_n(s) \right] ds.,
\end{aligned}
$$

where $n = 0, 1, 2, \ldots$

It is noticed that Rangkuti et al. [21] have considered reducing the restrictions in the correction functionals for their problem. Using this procedure, they also obtained Lagrange multipliers of the form of exponential functions as in (13). However, they only implemented the first order of their Lagrange multiplier, i.e. $\lambda = -1$, which is the same as if we use restricted variations for both linear and nonlinear terms in the correction functionals.

4. NUMERICAL RESULTS AND DISCUSSION

In this Section, we compare the solution of the system (3) obtained by the VIM (9) with that obtained by the IVIM (14). For the numerical test, we take the following parameter value

$$
A = \frac{28}{100}, m = \frac{3}{10}, b = \frac{1}{2}, p = \frac{3}{10}, g = \frac{8}{100}, \sigma = \frac{2}{10}, \text{ and } \theta = \frac{1}{10}, \tag{15}
$$

with initial value

$$
S_0 = \frac{1}{10}, E_0 = \frac{3}{10}, I_0 = \frac{2}{10}, \text{ and } Q_0 = \frac{4}{10}. \tag{16}
$$

The first iteration of VIM (9) gives the following solution

$$\begin{aligned}
S_1^{VIM}(t) &= \frac{1}{10} + \frac{37}{150}t, \\
E_1^{VIM}(t) &= \frac{3}{10} - \frac{271}{1500}t, \\
I_1^{VIM}(t) &= \frac{1}{5} - \frac{1}{100}t, \\
Q_1^{VIM}(t) &= \frac{2}{5} - \frac{12}{125}t, \\
R_1^{VIM}(t) &= \frac{1}{25}t,
\end{aligned} \tag{17}$$

while the second iteration produces the following solution

$$\begin{aligned}
S_2^{VIM}(t) &= \frac{1}{10} + \frac{37}{150}t - \frac{13799}{225000}t^2 + \frac{37037}{5062500}t^3, \\
E_2^{VIM}(t) &= \frac{3}{10} - \frac{271}{1500}t + \frac{3859}{45000}t^2 - \frac{37037}{5062500}t^3, \\
I_2^{VIM}(t) &= \frac{1}{5} - \frac{1}{100}t - \frac{123}{5000}t^2, \\
Q_2^{VIM}(t) &= \frac{2}{5} - \frac{12}{125}t + \frac{823}{75000}t^2, \\
R_2^{VIM}(t) &= \frac{1}{25}t - \frac{27}{2500}t^2.
\end{aligned} \tag{18}$$

As usual, the iteration for VIM can be continued to get more accurate solutions.

In Figure 1, we plot the approximate solution obtained from the fourth iteration of VIM (9). Such approximate solution should be compared to a reference solution. Since there is no exact solution, we use the numerical solution obtained by the fourth-order Runge-Kutta (RK4) method with $\Delta t = 10^{-6}$ as the reference solution. Figure 1 shows that the VIM solution is only accurate for relatively small time interval. Indeed, the VIM solution for all variables almost coincides with the reference solution only for $t \in [0,1]$. As t gets bigger, the VIM solution deviates more from the reference solution. This behaviour is more apparent in Figure 2, in which we plot both the VIM solution and the reference solution over a larger time interval. Figure 2 shows that at $t = 6$, the VIM leads to an inaccurate solution of the normalized SEIQRD model. Furthermore, the size of some normalized subpopulations is more than one, and others have negative values, which shows that the VIM solution is unrealistic.

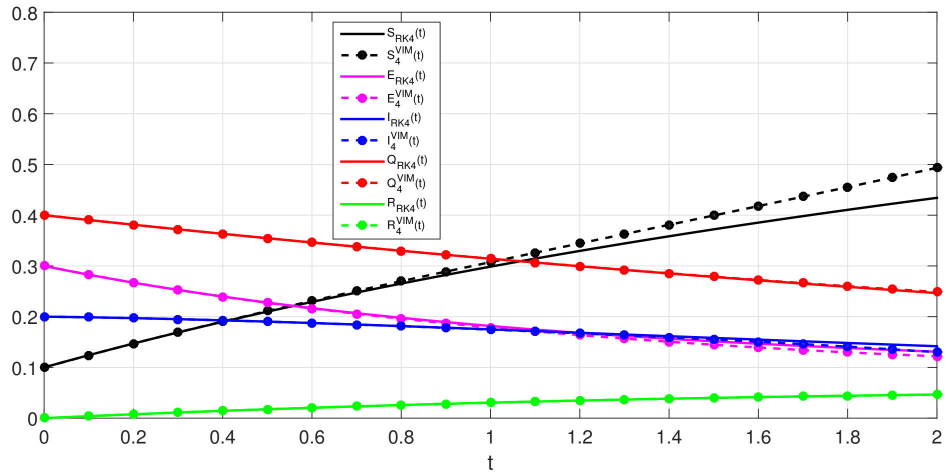

FIGURE 1. RK-4 and VIM solutions for $t \in [0,2]$. The VIM solution, in particular for variables S, E, and I, is reliable only for small interval of t.

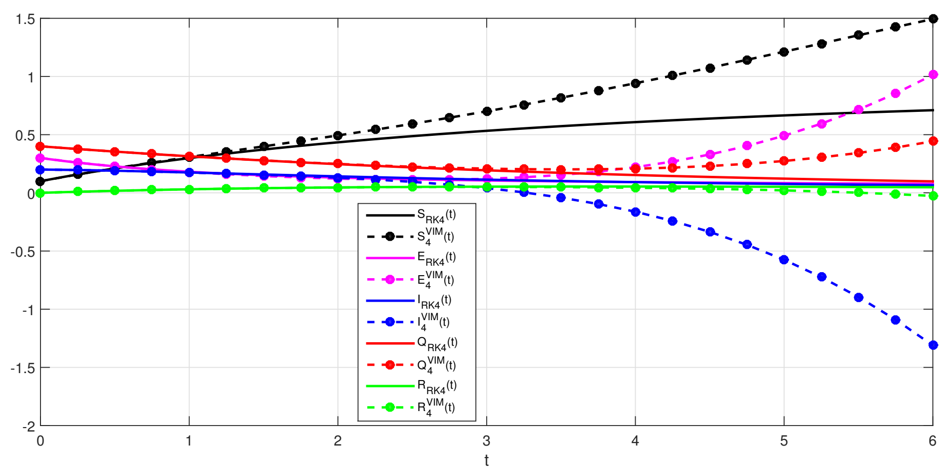

FIGURE 2. RK-4 and VIM solutions for $t \in [0,6]$. The VIM solution for all variables $(S, E, I, Q,$ and $R)$ becomes erroneous and unrealistic as t gets further from the initial position.

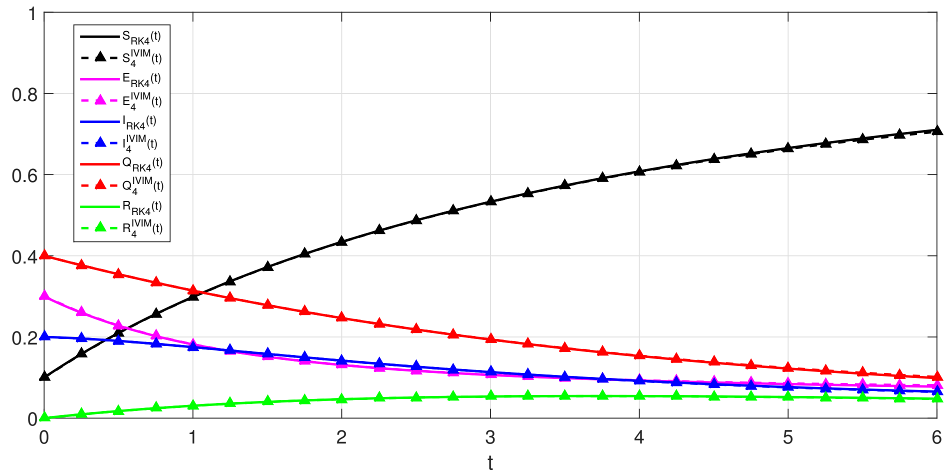

FIGURE 3. RK-4 and IVIM solutions for $t \in [0,6]$. The IVIM solution for all variables is very compatible with the RK-4 solution. This shows that IVIM is more efficient and accurate than VIM.

To obtain better results, we will implement the IVIM (14) to solve system (3). Using parameters (15) and initial values (16), the IVIM (14) leads to the following first iteration solution

$$
\begin{aligned}
S_1^{IVIM}(t) =& \frac{83}{90} - \frac{37}{45}\exp(-\frac{3}{10}t),\\
E_1^{IVIM}(t) =& \frac{7}{204} + \frac{271}{1020}\exp(-\frac{17}{25}t),\\
I_1^{IVIM}(t) =& \frac{9}{50} + \frac{1}{50}\exp(-\frac{1}{2}t),\\
Q_1^{IVIM}(t) =& \frac{4}{25} + \frac{6}{25}\exp(-\frac{2}{5}t),\\
R_1^{IVIM}(t) =& \frac{2}{15} - \frac{2}{15}\exp(-\frac{3}{10}t),
\end{aligned} \tag{19}
$$

and the following second iteration solution

$$
\begin{aligned}
S_2^{IVIM}(t) =& \frac{1430467}{2065500} - \frac{1286321783}{1667891250}\exp(-\frac{3}{10}t) + \frac{157451}{523260}\exp(-\frac{17}{25}t) + \frac{581}{13500}\exp(-\frac{1}{2}t)\\
&+ \frac{283087}{3442500}\exp(-\frac{3}{10}t)t - \frac{70189}{468180}\exp(-\frac{49}{50}t) - \frac{259}{16875}\exp(-\frac{4}{5}t),
\end{aligned}
$$

(20)

$$\begin{aligned}
E_2^{IVIM}(t) =& \frac{635033}{4681800} - \frac{18899653}{266862600}\exp(-\frac{17}{25}t) + \frac{157451}{1377000}\exp(-\frac{17}{25}t)t + \frac{581}{12150}\exp(-\frac{1}{2}t), \\
& - \frac{283087}{1308150}\exp(-\frac{3}{10}t) + \frac{70189}{206550}\exp(-\frac{49}{50}t) + \frac{259}{4050}\exp(-\frac{4}{5}t), \\
I_2^{IVIM}(t) =& \frac{7}{340} + \frac{28}{45}\exp(-\frac{1}{2}t) - \frac{271}{612}\exp(-\frac{17}{25}t), \\
Q_2^{IVIM}(t) =& \frac{247}{2550} + \frac{44}{105}\exp(-\frac{2}{5}t) - \frac{271}{3570}\exp(-\frac{17}{25}t) - \frac{1}{25}\exp(-\frac{1}{2}t), \\
R_2^{IVIM}(t) =& \frac{4}{75} + \frac{14}{75}\exp(-\frac{3}{10}t) - \frac{6}{25}\exp(-\frac{2}{5}t).
\end{aligned}$$

As in the case of VIM, the iteration of IVIM can be continued further. If the number of iteration steps increases, then the solutions should be convergent to more accurate results. In Figure 3 we show the solution of system (3) obtained from the fourth iteration of IVIM (14). It is found that the IVIM produces a much more accurate solution compared to the VIM (9). We observe that the IVIM solution is in excellent agreement with the reference solution for $t \in [0,6]$.

In Table 2 and Table 3 we show the mean absolute errors of VIM and IVIM solutions on specific domain for each dependent variable. Here, $\mathscr{E}_{S_4}^{VIM}$ is the mean absolute error of $S(t)$ obtained from the 4^{th}-iteration in VIM. For the other mean absolute errors of the other dependent variables are denoted analogously. The mean absolute error is calculated using the following formula

$$\mathscr{E}_X^{approx} = \frac{1}{M}\sum_{j=1}^{M} |X_j^{ref} - X_j^{approx}|, \tag{21}$$

where X_j^{ref} and X_j^{approx} are the reference and approximate solutions at t_j, respectively. The solutions are evaluated at a uniform grid with $\Delta t = t_{j+1} - t_j = 0.01$. Here M is the number of evaluated points. Table 2 and Table 3 show that at the same number of iteration steps, which in our case is the fourth iteration, the IVIM leads to a much more accurate solution compared to the VIM.

TABLE 2. Error of the fourth iteration VIM and IVIM solutions for variable S, E and I.

Domain	$\mathscr{E}_{S_4}^{VIM}$	$\mathscr{E}_{S_4}^{IVIM}$	$\mathscr{E}_{E_4}^{VIM}$	$\mathscr{E}_{E_4}^{IVIM}$	$\mathscr{E}_{I_4}^{VIM}$	$\mathscr{E}_{I_4}^{IVIM}$
$[0,1]$	2.336e-03	5.119e-07	8.283e-04	4.843e-07	8.090e-05	6.529e-07
$[0,2]$	1.601e-02	9.963e-06	3.660e-03	8.938e-06	2.103e-03	1.209e-05
$[0,3]$	4.687e-02	5.239e-05	4.528e-03	4.482e-05	1.332e-02	5.445e-05
$[0,6]$	2.485e-01	8.815e-04	1.623e-01	6.751e-04	2.676e-01	3.288e-04

TABLE 3. Error of the fourth iteration VIM and IVIM solutions for variable Q and R.

Domain	$\mathscr{E}_{Q_4}^{VIM}$	$\mathscr{E}_{Q_4}^{IVIM}$	$\mathscr{E}_{R_4}^{VIM}$	$\mathscr{E}_{R_4}^{IVIM}$
$[0,1]$	1.285e-05	3.904e-07	2.055e-06	1.833e-07
$[0,2]$	3.697e-04	8.240e-06	6.202e-05	3.762e-06
$[0,3]$	2.542e-03	4.354e-05	4.490e-04	1.844e-05
$[0,6]$	6.254e-02	5.975e-04	1.282e-02	1.614e-04

5. CONCLUSIONS

We have solved the SEIQR COVID-19 epidemic model using the VIM. Two different VIM implementations have been considered. The difference between the two implementations lies in the number of restricted variations used in the correction functionals. The first implementation, i.e. the restricted variations are applied for both linear and nonlinear parts of the correction functionals, leads to an approximate solution that is unrealistic and is correct only for relatively small time domains. To improve the accuracy of the first VIM, we have reduced the number of the restricted variation in the correction functionals. We have shown that the improved VIM significantly increases the accuracy of the first VIM.

ACKNOWLEDGEMENTS

This research was funded by FMIPA-UB via PNBP-University of Brawijaya according to DIPA-UB No. DIPA-023.17.2.677512/2021 and Contract of Professor and Doctoral Research Grant No. 1584/UN10.F09/PN/2021.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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