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DYNAMICAL ANALYSIS AND OPTIMAL CONTROL PROBLEM OF IMPACT OF VACCINE AWARENESS PROGRAMS ON EPIDEMIC SYSTEM

PAYAL RANA, KULDEEP CHAUDHARY, SUDIPA CHAUHAN*

Amity Institute of Applied Science, Amity University, Sector-125, Noida, U.P, INDIA

Abstract. There has been an unprecedented global public health and economic crisis due to the coronavirus disease 2019 (COVID-19). For containing the infection and returning to normal routines, vaccination is an important foreseeable mean. But there are many people who do not have exposure to information about vaccinations or are either misinformed and this may take the form of vaccine hesitancy. Thus, positive vaccination awareness to even the most vulnerable section of society or remote areas of the country may be the need of the hour for full population inoculation. The role of awareness programs by government as a control to increase vaccination and control the infection is discussed in this paper. Thus we formulate a model consisting of unaware and aware population amid vaccination campaigns/awareness. The existence, local stability and global stability(through graph theoretic approach) of the equilibria are analyzed. Following our model we extend it to an optimal problem with the objective to maximise vaccination and minimise promotional costs in our system. With the help of Pontryagin's Maximum Principle, we then obtain the optimal awareness intensity as part of intervention for vaccination for our optimal control problem. Through numerical simulations, the paper shows that awareness among general public increases the number of vaccinated individuals. Sensitivity analysis is performed for the optimal control calculated using latin hypercube sampling method. Thus, the paper highlights the necessary and crucial role of vaccine awareness programs to fight a disease in epidemic dynamics.

Keywords: awareness; vaccination; local stability; graph theoretic approach; optimal control.

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*Corresponding author

E-mail address: sudipachauhan@gmail.com

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1. Introduction

During the course of the ongoing Covid-19 pandemic, several approaches, strategies and interventions are at play to control the transmission of infection. From masking, social distancing to emphasis to build strong immunity, general public has been depended on these to fight the killer virus (SARS-CoV-2). But since 2021, the world finally started the process of vaccinations. Vaccinations are being seen now the most important factor to suppress the infection. Awareness programs and strategies will lead to vaccine eagerness and even induce appropriate behavioral changes in the public. But reaching out to the most vulnerable individuals who do not even have access to information about vaccines and vaccination process, should be a prime objective too of responsible stakeholders. As myths and misinformation about the Covid-19 vaccine(s) may usher in resistance and hesitancy [1], government should disseminate correct and fact based information. This would enable the government to build trust and confidence among the public in the vaccine(s) and the vaccination process. A epidemic may eventually die out whenever some type of immunization/vaccine awareness and vaccination will be incorporated in the system [2]. If people are aware of the importance of immunization and its advantages, the epidemic can be controlled to a great extent. A timely and correct launch of the vaccination is extremely vital. If there are any delay in vaccinations due to hesitancy amid an intense epidemic then it may lead to an increase in hospitalisations [3]. If people are not eager to get vaccinated or not even aware of the said process i.e delay in vaccination during increasing infections, and if one compensates by increasing the vaccination rate it will make way to disproportionate expenditure of vaccination doses.

1.1. Communication Strategy For Covid-19 Vaccine.

Spreading of awareness about vaccination has to come through great communication strategies by the government. With respect to ongoing pandemic of Covid-19, a communication strategy as in [4] has been shared by Ministry of Health and Family Welfare, Government of India with emphasises on the need of vaccines and game plan for spread of correct information to increase vaccination. Some of the objectives for awareness includes:

- Throughout the roll out of vaccines the hesitancy of general public be addressed.
- Timely and correct knowledge about the vaccine(s) and the process be provided.

- Reduce influence of misinformation or misconceptions.

But there still needs to be an proactive approach to carry out the above strategies to vulnerable section of society or remote areas of the country. This is to endure maximum immunization of population to suppress the epidemic.

1.2. Literature Review.

There are several recent work which has been done till now. For instance, in [5] a dynamic set of SIR model of aware and unaware population is studied and the parameter properties on the epidemic awareness model is discussed. A epidemic vaccination model is studied in [6] with the influence of social interactions and have analytically investigate the model with numerical discussion. An epidemic model with time delay along with the effect of awareness programs in [7] of an infectious disease is studied. In the paper [8], an optimal control problem for a stochastic SIVR model with non-pharmaceutical interventions and vaccination controls is discussed using dynamic programming approach and obtained numerical simulations of the deterministic model. In [9, 2] impact of behavioral incentives for vaccination in epidemic models and dynamic analysis are discussed. In [10] a compartmental model is proposed consisting of the intensity of the media reports for mitigating spread of a disease and its local stability is discussed. *Thus, we shall propose a model consisting of aware-unaware population amid vaccination with awareness campaigning policy by government as control and aim to discuss its stability analysis(local and global) as well optimal control policy.*

1.3. Structure of our Study.

We will propose an epidemic system consisting of unaware-aware-vaccinated population with the impact of awareness campaigns in Section 3. Then in Section 4 we proceed with the general analysis of our system though local and global stability analysis. In Section 5, we will formulate an optimal control problem and discuss it using Pontryagin's maximum principle [11]. Then through numerical discussion we will aim to show the working of the system in the presence of the awareness initiatives. In Section 6, we will perform numerical simulation and sensitivity analysis of the optimal control to get proper understanding of mechanism of our system in the presence of the control.

2. Model Formulation

The assumptions for the model are the following:

- We assume that the total population of individuals i.e M remains constant over time t.
- The total population is eligible for vaccination.
- Unaware population may become aware due to interaction(positive word of mouth for vaccination) with aware individuals.
- Aware population get vaccinated.
- There are promotional efforts(external influence)/awareness for vaccination by the government as intervention.

To define the model, the following notations are adopted throughout this paper:

$N_1(t)$	:	Number of individuals who are unaware of the vaccination at time t,
$N_2(t)$	:	Number of individuals who are aware of the vaccination at time t but still have not been vaccinated,
$N_3(t)$	:	Number of current vaccinated individuals,
β	:	The rate at which the unaware population interacts with aware and vaccinated population say through word of mouth,
u	:	The rate of promotion effect/awareness as intervention by government for vaccination,
δ	:	The rate at which aware individual get vaccinated.
μ	:	The rate at which unaware people are becoming aware
α	:	The rate at which vaccinated people are again joining the unaware class. Either due to severe issues after vaccination or due to the misinformation regarding the vaccination they pretend to behave as they are unaware individuals.

3. The Model

We consider a government department/ marketing firm that needs to enhance the awareness about the vaccine to vaccinate the maximum population of individuals. To incorporate the stage of awareness of information and the stage of decision making, the evaluation of the unaware,

aware and vaccinated individuals described by the following dynamical equations:

$$\frac{dN_1}{dt} = -\beta N_1(N_2+N_3) - \mu u N_1 + \alpha N_3 \tag{1}$$

$$\frac{dN_2}{dt} = \beta N_1(N_2+N_3) + \mu u N_1 - \delta N_2 \tag{2}$$

$$\frac{dN_3}{dt} = \delta N_2 - \alpha N_3 \tag{3}$$

Where, $\beta N_1(N_2+N_3)$ term represents the increase in the number of aware individuals due to interaction between unaware individual and aware and vaccinated individuals; $\mu u N_1$ indicates the increase due to promotional efforts (external influences); δN_2 denotes the decrease in the number of aware individual due to transfer of aware indivduals to vaccinated individuals and term αN_3 indicates the decrease in the vaccinated individuals due to misinformation about the vaccine. Note that $N_1(t)+N_2(t)+N_3(t) = M$.

4. General Analysis

In the following subsections, we would be discussing the stability analysis of the system which would involve the local stability analysis, existence of no limit cycle by Bendixon Dulac criteria and global stability analysis by graph theoretic approach.

4.1. Equilibrium points.

The equilibrium for our system (1-3) is given by $E^* = (N_1^*, N_2^*, N_3^*)$ given by

$$N_2^* = \frac{\beta M - \mu u}{\beta(1+\frac{\delta}{\alpha})^2} > 0 \quad if \quad \beta M > \mu u$$

$$N_3^* = \frac{\delta}{\alpha} N_2^*$$

where $N_1^* = M - N_2^* - N_3^*$

4.2. Local Stability Analysis. The jacobian corresponding to system with respect to E^* is as follows:

$$J = \begin{bmatrix} A & A \\ \delta & -\alpha \end{bmatrix}$$

where $A = (\beta M - \mu u)(\frac{\frac{\delta}{\alpha}-1}{\frac{\delta}{\alpha}+1})$. The characteristic equation at E^* is :

$$\lambda^2 - \lambda(A-\alpha) - \delta A = 0$$

The system is locally asymptotically stable node if $\alpha < \delta < \frac{\alpha^2(\frac{\delta}{\alpha})}{\beta M - \mu u} + \alpha$ and $\delta > \frac{1}{4}$. And if $\delta < \alpha$ then the system at E^* is an unstable saddle.

4.3. Global Stability Analysis.

Theorem 4.1. *The system does not have a limit cycle.*

Proof. Let us consider $D = \frac{1}{N_2}$. As $N_1 = M - N_2 - N_3$, if we denote the right hand sides of (2) and (3) by g_1 and g_2 respectively, then it follows:

$$\frac{\partial D g_1}{\partial N_2} + \frac{\partial D g_2}{\partial N_3} = -\frac{\beta N_1 N_3}{N_2} - \frac{u\mu N_1}{N_2^2} - \frac{\alpha}{N_2} < 0$$

Therefore, by Bendixon Dulac's criteria [12], our system does not have a limit cycle. Next we will be using the graph theoretic approach to construct a lyapunov function to prove the global stability of E^*. □

4.3.1. *Graph theoretic approach.* We shall establish GAS(globally asymptotically stable) of E^* using the graph-theoretic approach method referring to the terminologies used as in [13, 14, 15, 16] to construct the lyapunov function. A directed graph consists of (i, j)(a set of ordered pair) and vertices where (i, j) is known as arc to terminal vertex j from initial vertex i. $d^-(j)$ is called the in-degree of vertex j which denotes the number of arcs in the digraph whose terminal vertex is j. And $d^+(i)$ is called the out-degree of vertex i which denotes the number of arcs in the digraph whose initial vertex is i. Let us consider a weighted directed graph say $\xi(A)$ over a $d \times d$ weighted matrix A where the weights(a_{ij}) of each arc if they exist are $a_{ij} > 0$, and if otherwise then $a_{ij} = 0$. We consider c_i as the cofactor of l_{ij} of the laplacian of $\xi(A)$ which is given by:

$$l_{ij} = \begin{cases} -a_{ij} & i \neq j \\ \sum_{k \neq i} a_{ij} & i = j \end{cases}$$

If there is a directed to and fro path for the arcs in $\chi(M)$ i.e strongly connected, then $c_i > 0$ $\forall i = 1, 2..., q$. From Theorem 3.3 of [13]; if $a_{ij} > 0$ and $d^-(i) = 1$, for some i, j, then

$$c_i a_{ij} = \sum_{k=1}^{d} c_j a_{jk}$$

Similarly from Theorem 3.4 of [13]; if $a_{ij} > 0$ and $d^+(j) = 1$, for some i, j, then

$$c_i a_{ij} = \sum_{k=1}^{d} c_k a_{ki}$$

Therefore, we aim to construct a lyapunov function Z where we shall be using the graph theoretic technique as from theorem of [13] given below:

Theorem 4.2. [13] *Let us consider an open set $D \subset R^m$ and a function $f : D \to R^m$ for a system*

$$\dot{z} = f(z) \tag{4}$$

and assuming:

a) $\exists\, Z_i : D \to R$, $L_{ij} : D \to R$ and $a_{ij} \geq 0$ such that

$$Z_i' = Z_i'|_{(4)} \leq \textstyle\sum_{j=1}^{d} a_{ij} L_{ij}(z), \text{ with } z \in D,\ i = 1, \ldots, d$$

b) For $\xi(A)$ each directed cycle D_c satisfies:

$$\textstyle\sum_{(ij) \in G(D_c)} L_{ij}(z) \leq 0\ ,\ z \in D$$

where $G(D_c)$ is set of arcs in D_c

Then, for $c_i \geq 0, i = 1, \ldots, d$ the function is:

$$Z(z) = \sum_{i=1}^{d} c_i Z_i(z)$$

satisfies $Z'|_{(4)} \leq 0$, that is, $Z(z)$ is a Lyapunov function for 4.

- **Lyapunov Contruction**

Now we shall follow the method of graph theoretic for the construction of lyapunov function for global stability: Construction: $Z_2 = \frac{(N_2-N_2^*)^2}{2}$ and $Z_3 = \frac{(N_3-N_3^*)^2}{2}$. Now by differentiation;

$$Z_2' = (N_2 - N_2^*)N_2' \leq \beta M N_2(N_2+N_3) + (u\mu M - \delta N_3^*)(N_2+N_2^*) = a_{32}L_{32} + a_{22}L_{22}$$

$$Z_3' = (N_3 - N_3^*)N_3' \leq \delta N_2 N_3 + \alpha N_3^* N_3(1+N_3) = a_{23}L_{23} + a_{33}L_{33}$$

with $a_{32} = \beta M$, $a_{22} = (u\mu M - \delta N_3^*)$ if $u\mu M > \delta N_3^*$, $a_{23} = \delta$ and $a_{33} = \alpha N_3^*$ and all other $a_{ij} = 0$. We get an associated weighted directed graph which has two vertices and one cycles

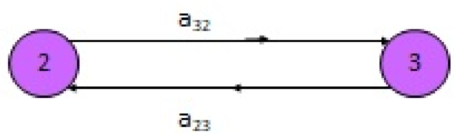


FIGURE 1. Directed Graph

in Fig 1. We can verify that for the cycle at $N_2 = -2N_3$ we verify $L_{32} + L_{23} = 0$. Then by Theorem3.5([13]) $\exists c_i's, 2 \leq i \leq 3$ such that $Z = \sum_{i=1}^{d} c_i Z_i$ is a lyapunov function. Using Theorem 3.3 we get the relation between c_i. For $a_{23} > 0$ and $d^-(2) = 1$ we get $c_2 a_{23} = c_3 a_{32}$. Hence, $c_3 = 1$ and $c_2 = \frac{M\beta}{\delta} c_3$. Thus, the lyapunov function is $K = \frac{M\beta}{\delta} K_2 + K_3$. And for K':

$$K' = \frac{M\beta}{\delta}(N_2 - N_2^*)N_2' + (N_3 - N_3^*)N_3'$$

If we consider the set $U = \{x \in R_+^2 \colon K' = 0\}$ then we see that $N_2 = N_2^*$ and $N_3 = N_3^*$. Hence, for the system we get a unique equilibrium point which is nothing but E^* and is GAS. Therefore we say that E^* is GAS using LaSalle's Invariance principle .

5. Optimal Control Promotional Effort Policy

In this section, we consider a government or firm that need to promote the vaccination for COVID-19 through promotional strategy to increase the number of vaccinated individual. The objective involves the maximising of the number of vaccinated individuals while minimizing the total promotional effort cost. As of time $t = 0$, the government may start to promote vaccination events e.g. in newpapers, internet, radio and televisions etc. This type of promotions are costly and we assume a quadratic promotional cost function [17, 18, 19]:

$$C(u) = \frac{b}{2}u^2(t) \tag{5}$$

where $b \geq 0$ is constant and value of the b denotes the magnitude of the promotional effort rates; $u(t) \geq 0$ is the promotional effort rate used by government. In this paper, a simple and reasonable example of the promotional effort cost function is provided by quadratic form with the assumption that $C(u)$ is twice continuously differentiable function of u such that $C_u > 0, C_u u > 0, C(0) = 0$.

Suppose that the planning horizon is not very larges, we do not discount the future. The objective of the government is :

- minimize the promotional effort cost in planning horizon
- maximize the numbers of vaccinated individuals.

The objective function is :

$$maxJ = \int_0^T \left(\dot{N}_3(t) - \frac{b}{2}u^2(t)\right)dt \tag{6}$$

Combining the objective function and state equations, the optimization problem can be expressed as an optimal control problem :

$$\begin{cases} Max_{\mu(t)} J = \int_0^T \left((\delta N_2 - \alpha N_3) - \frac{b}{2}u^2(t)\right)dt, \\ s.t \frac{dN_1}{dt} = \frac{dN_2}{dt} - \frac{dN_3}{dt}, N_1(0) = N_{10} \\ \frac{N_2}{dt} = \left(\beta N_1(M - N_1) + \mu u(t) N_1\right) - \delta N_2(t), N_2(0) = N_{20} \\ \frac{N_3}{dt} = \delta N_2 - \alpha N_3, N_3(0) = N_{30} \end{cases} \tag{7}$$

5.1. Maximum principle. As seen in Section 5, optimal control problem is to maximize J subjected to the state equations $(1), (2)$ and (3) by using Pontryagin's maximum principle [11]. The control variable $u(t)$ is subjected to the constraint $0 \leq u(t) \leq \bar{U}$, where $\bar{U}$ is feasible upper limit for the promotional effort rate. The assumption of the upper limit for promotional effort rate $u(t)$ reflects the more realistic scenario on the maximum rate at which the promotional strategy may be employed. Such a restriction $\bar{U}$ is determined by the advertising budget, media limitations etc. The Hamiltonian function of the optimal control problem is:

$$H = \left((\delta N_2(t) - \alpha N_3) - \frac{b}{2}u^2(t) \right) + \lambda_1 \left(-\frac{dN_2}{dt} - \frac{dN_3}{dt} \right) + \lambda_2 \left(\beta N_1(M - N_1) + \mu u(t) N_1 - \delta N_2(t) \right) + \lambda_3(\delta N_2(t) - \alpha N_3) \tag{8}$$

Where, $\lambda_1, \lambda_2, \lambda_3$ are the adjoint variables. Hamiltonian function is sum of two parts: the first part is the integrand of the objective function which denotes the direct contribution, and the second parts consists of adjoint variables multiplied by the right-hand side of the equations (6) which denotes the indirect contribution to the objective functional from. By the maximum principle, the necessary optimality conditions stated in Seierstad and Sydsaeter [20], for optimal control variable $u^*(t)$, with the corresponding state trajectories $N_1,\ N_2,\ N_3$, to be an optimal control are existence of continuous and piecewise continuously differentiable functions $\lambda_1,\ \lambda_2,\ \lambda_3$ for all $t \in [0, T]$ such that the following conditions holds:

$$\frac{\partial H}{\partial u} = 0 \tag{9}$$

and for the adjoint variables.

$$\frac{d\lambda_1}{dt} = -\frac{\partial H}{\partial N_1} = (\lambda_1 + \lambda_2)\left(\beta(M - 2N_1) + \mu u(t) \right), \quad \lambda_1(T) = 0 \tag{10}$$

$$\frac{d\lambda_2}{dt} = -\frac{\partial H}{\partial N_2} = (\lambda_2 - \lambda_3 - 1)\delta, \quad \lambda_2(T) = 0 \tag{11}$$

$$\frac{d\lambda_3}{dt} = -\frac{\partial H}{\partial N_3} = (\lambda_3 - \lambda_1 + 1)\alpha, \quad \lambda_3(T) = 0 \tag{12}$$

with the transversality conditions $\lambda_i(T) = 0 \quad \forall i = 1, 2, 3$. Since $b > 0$ is positive parameter, for the sufficient optimality conditions [20] for optimal control problem (6) the following inequality hold

$$\frac{\partial^2 H}{\partial^2 u} = -b < 0$$

Parameter	β	α	δ	M	μ	b
Value	0.001	0.008	0.05	1000	0.04	150

TABLE 1. Values of Parameters

This implies that there exists a unique optimal control variable. From equation (7), the optimal value of promotional effort rate is

$$u^*(t) = \frac{1}{b}\big((\lambda_2 - \lambda_1)\mu N_1\big) \tag{13}$$

If there is no misinformation or unavailability of vaccine(for high-income country or developed country) i.e. $\alpha = 0$, then optimal promotional effort rate becomes

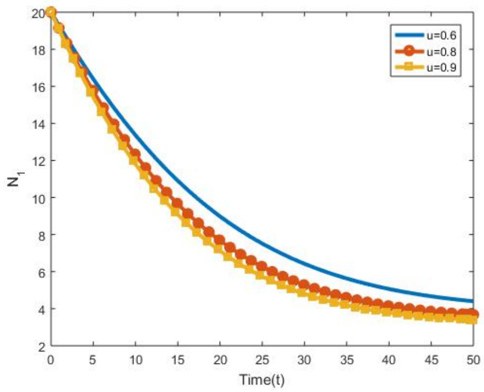
$$u^*(t) = \frac{1}{b}\left[\mu(1 - e^{-\delta(T-t)})(M - N_3 - N_2)\right] \tag{14}$$

From above we see that as $N_2 + N_3$ approaches towards M, $u^*(t)$ will decrease. This means that there will be no need of promotions of vaccination.

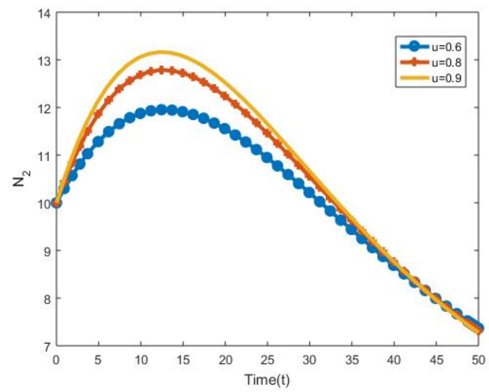
6. NUMERICAL EXAMPLE

The hypothetical numerical values [21] considered for the validation of analytic results are mentioned in Table 1. We have shown the sensitivity analysis of a few important parameters of the system. As shown in Fig 2, we have shown the influence of variation of u and δ on the system dynamics with respect to values in Table 1.

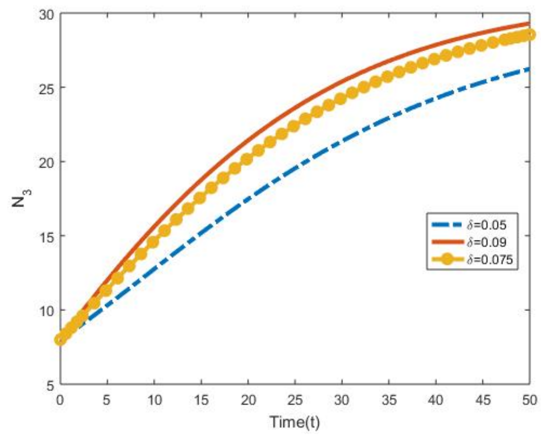
- Starting at $u = 0.6$ to $u = 0.8$, the number of unaware individuals tends to decrease over time due to the positive nudge by media in the form of awareness. Then as u increases to 0.9 the unaware population decreases further thus showing the immediate need of fact based and positive communication strategies by responsible stakeholders. Awareness among people will increase vaccination and help in curbing the disease outbreak.
- It is clear from Fig 2b that increase in u caters to increase in the aware population. And this aware population is the one a government strives for as the individuals are ready for vaccination and even follow Covid-appropriate behaviour. This ideal aware population aims to suppress the infection spread through vaccination.



(A) Impact of u on N_1



(B) Impact of u on N_2



(C) Impact of δ on N_3

FIGURE 2. Dynamics of system

- When the rate of vaccination is increased from 0.05 to 0.075 then to 0.09, the aware vaccinated population increases as well. Therefore, the strategies involved with the awareness campaigns needs to be evaluated thoroughly to increase vaccinations and control the epidemic.
- For numerical simulation of the optimal control we consider same values from Table 1, $T = 365$ with initial conditions $(890, 100, 10)$. Fig 3 shows system dynamics subject to the optimal control at $(8.93, 139.97, 856.12)$. The extreme behaviour of $u(t)$ is shown in Fig 4. The intensity of the control is initiated from 1 as at the time of start of vaccination the system would require maximum promotional awareness strategies. And

as people start get vaccinated we would see a decline in the magnitude of the awareness efforts. Now as more and more vaccinations take place, in order to minimise costs the rate of awareness would reduce further. Thus, we see that starting with inoculation we require maximum awareness interventions by government with the goal to see maximum individuals getting aware and moving to vaccination class. Later with good number of population getting vaccinated, the priority shifts to minimise costs incurred for awareness campaigns and thus awareness efforts reduce over time.

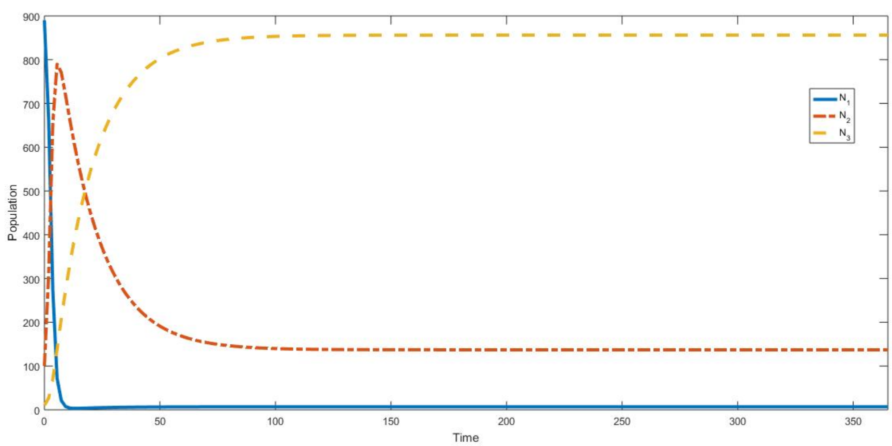


FIGURE 3. System with Optimal Control

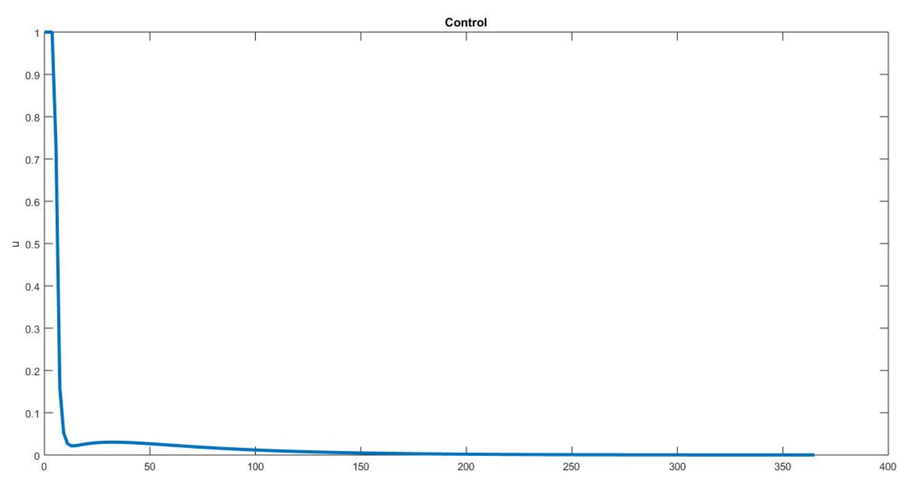


FIGURE 4. Optimal Control Profile

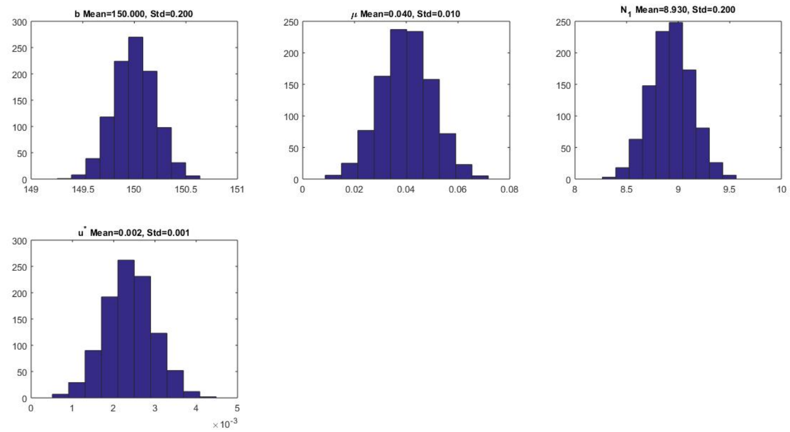


FIGURE 5. Distribution of parameters for u^*

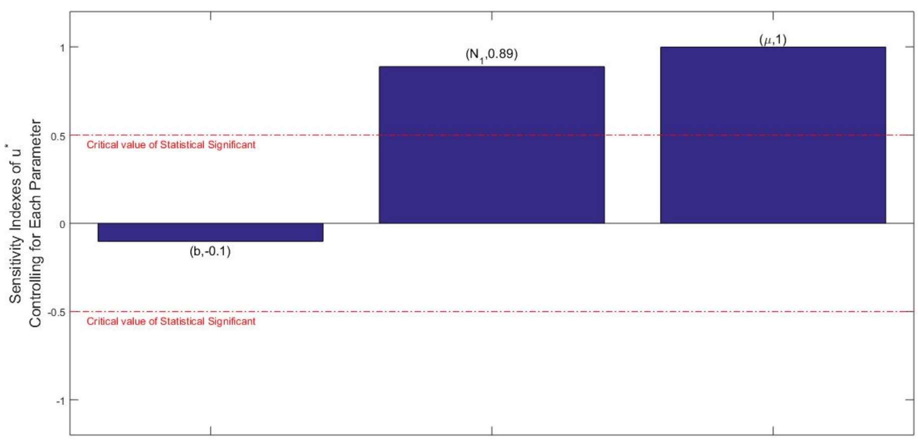


FIGURE 6. PRCC: input variables

6.1. Sensitivity Analysis of u^*. Now with the aim to quantify the uncertainty of input factors we shall be using a sensitivity analysis technique i.e PRCC (partial rank correlation coefficient) ([22], [23]). With the addition of the control u^*, we shall use PRCC to identify and quantify how the inputs associated with the control's uncertainty may impact the awareness. For the control term we calculated (13) we shall check the sensitivity at the equilibrium of the system found previously. We take in account the three parameters used in our model and we find the PRCC values using Matlab with the following pdfs and shown in Fig 5 and are sampled 1000 times.

$b \sim \quad Normal(150, 0.2)$

$\mu \sim \quad Normal(0.04, 0.01)$

$N_1 \sim \quad Normal(8.93, 0.2)$

We get the PRCC values for our input parameters which can be seen in Fig 6. We get the following indexes for the parameters: $b = -0.1, \mu = 1$ and $N_1 = 0.89$. The PRCC indices satisfies the same set of correlation as per the optimal policy formulated i.e μ and N_1 are positively correlated. We see that more the number of unaware population(N_1), more is the urgency for involvement of awareness control strategy. Since the value μ parameter is close to 1, it indicates a strong correlation to change in u^* i.e μ is strongly positively correlated to u^*. This suggests that when the optimal control is used as an intervention it would be increased as the rate of of unaware becoming aware i.e μ will increase. And the effect of the parameter b will bring about an opposite change in the optimal control policy as it is negatively correlated. Thus, this hints at the fact that as our promotional effort costs increase then the optimal control will decline in order to minimise the costs.

7. CONCLUSION

Awareness programs for Covid-19 by responsible stakeholders have the power to change the pattern of vaccination progress and reduce disease spread. Either through positive word of mouth among general public or awareness promotional interventions by government, the dynamics of epidemic system can change for the best. In this paper we have presented mathematical analysis of the role of awareness programs intervention by governments as control to increase vaccination considering aware-unaware population. For equilibrium point we did the stability analysis, showed the existence of no limit cycle and then proceeded to show global stability through graph theoretic approach. Later for our system, we consider an optimal control promotional effort policy for maximising vaccinations and minimising promotional costs and have found a unique optimal value of promotional effort rate. We carried out numerical simulations to investigate how awareness spreading and vaccination can help curb an epidemic. We saw that our disease-behavior dynamics improved as promotion effect and vaccination rates increased. The numerical simulation of the optimal control indicated that the awareness about vaccinations and Covid-appropriate behavior can suppress the infection as vaccinated individuals increase phenomenally. This is possible as awareness promotes the general public to take protection and look forward to vaccination for a safer future. Using PRCC the sensitivity analysis was performed for u^* which inferred the need of the control for unaware populated since it had a positive correlation. Thus, control programs that follows the awareness/promotional strategies effectively can help bring a safer environment through vaccination to even the most remote and vulnerable areas.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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