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FRACTIONAL DYNAMICS OF CORONAVIRUS WITH COMORBIDITY VIA CAPUTO-FABRIZIO DERIVATIVE

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Abstract. Coronavirus and its associated comorbidities have been the drivers of many deaths across the globe in recent times. Individuals with underlying medical conditions are at higher risk of becoming critically ill and developing complications if they are infected with the Coronavirus. In this paper, a Caputo-Fabrizio fractional-order model of coronavirus disease with comorbidity is formulated to access the impact of comorbidity diseases on COVID-19 transmission using both a fractional-order as well as a stochastic approach. Exponential law is utilized to present the existence and uniqueness of solutions using the fixed-point theory. The fractional stochastic approach is adopted to examine the same model to explore the random effect. Numerical simulations are used to support the theoretical results and the simulation results suggest that the increase of comorbidity development and the fractional-order derivative factor simultaneously increases the prevalence of the infection and the spread of the disease. The fractional stochastic numerical results suggest that the prediction of infection rate is more stochastic than deterministic.

Keywords: infectious disease; coronavirus; fixed point theory; exponential law; comorbidity.

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1. Introduction

The deadly Coronavirus disease (COVID-19) which was first reported in the city of Wuhan in china [1] is one of the most deadly epidemics the world has ever experienced. From Wuhan, it rapidly spread throughout the globe, that on the 11^{th} of March 2020, it was declared a global pandemic by the World Health Organization (WHO) [2]. It is still spreading to date and has infected over 183 million and killed over 3.9 million people the world over. Infected individuals have symptoms like high fever, fatigue, muscle pains, loss or change of taste or smell, shortness of breath, dry cough, and sore throat.

The disease can be contracted by susceptible individuals when they come in contact with respiratory droplets from infected individuals and through direct contact with contaminated surfaces [3]. As of June 2021, there is no cure for COVID-19, affected nations only rely on protective measures such as wearing face masks in public places, social distancing, maintaining proper hygiene and ventilation, quarantine, contact tracing, and vaccination to control the disease spread. Optimal supportive care in hospitals includes oxygen for severely ill patients and those who are at risk of severe disease and more advanced respiratory support such as ventilation for patients who are critically ill. Corticosteroids like dexamethasone, prednisone, and methylprednisolone are also used to help reduce the length of time on a ventilator and save lives of patients with severe and critical illness [4].

Recently studies have revealed that individuals infected with diseases like diabetes, lung disease and heart disease, HIV/AIDS, hypertension have a compromised immune system and thus are at higher risk of contracting COVID-19 and an increased risk of severe illness upon infection [5]. Comorbidity is defined as a disease or medical condition unrelated in etiology or causality to the principal diagnosis that coexists with the disease of interest [6]. According to a research study in China which monitored 344 COVID-19 patients in the ICU. The majority of those that died from the disease had at least one comorbidity, about 144 of them having hypertension [7]. Another study conducted in China showed that 247 out of 633 COVID-19 patients had at least one comorbidity [7]. In the USA, the Centers for Disease Control and Prevention (CDC) used COVID-NET in 14 states to monitor the demographics of COVID-19 patients who were hospitalized [8]. The results obtained from March 1 to 30, 2020, showed that, out of the 180 patients on COVID-NET, 89.3% of them had an underlying comorbidity. The most common comorbidities found were obesity, hypertension, and diabetes mellitus [8]. These results, therefore, point towards the need to investigate the dynamics of COVID-19 and comorbidity co-infections.

Mathematical modelling has played a major role in controlling many epidemics on the globe because, in the absence of real data, models provide both qualitative and quantitative information that help in minimizing the spread of many diseases. Recently, several integer order mathematical models on COVID-19 and comorbidities have been developed to analyse the impact of various comorbidities on COVID-19 transmission [9, 10, 11, 12, 13]. However, integer-order models have a major setback: the lack of hereditary memory effect for accurate predictions. Fractional-order derivatives on the other hand have become a powerful tool in modeling in the recent times because of their characterization. These operators possess memory effect crossover property and have statistical interpretation which makes the operators efficient [14]. There are several different fractional-order derivatives but the most common one is the Caputo derivative which is just a power law with a local singular kernel. We also have the Caputo-Fabrizio (CF) fractional order derivative with non-singular Kernel proposed by Caputo and Fabrizio [15]. Further properties of the CF operators were later developed by Losada and Nieto [16]. The effectiveness of the CF operator has been illustrated in many fractional-order models [17, 21, 20, 19, 18, 22, 23].

The aim of this work is to analyse a fractional-order model of COVID-19 with comorbidity as well as a corresponding stochastic model of COVID-19 with comorbidity. The study further presents the numerical scheme for the fractional coronavirus model with comorbidity in global derivatives via exponential decay kernel, and use these schemes to simulate the model and make relevant conclusions from our results, to help curb the spread of the disease.

2. Mathematical Model Formulation

The total human population at time t is given by $N_h(t)$ and partitioned into the following; susceptible humans $S_h(t)$, comorbidity susceptible humans $S_{cm}(t)$, humans infected with COVID-19 $I_{cv}(t)$, humans recovered from COVID-19 $R_{cv}(t)$, humans co-infected with comorbidity and coronavirus $I_{vm}(t)$, humans recovered from comorbidity disease $R_{cm}(t)$. Susceptible humans are recruited at a rate is Λ_h and they get infected with COVID-19 at a rate $\beta_{cv} = \beta\left(I_{cv} + \delta_{cm} I_{vm}\right)$, where δ_{cm} is the modification parameter for infectiousness of the co-infected humans. The natural mortality rate is denoted by μ_h and susceptible humans contract the comorbidity disease at a rate ϕ_{cm}. The comorbidity humans get infect with COVID-19 at a rate $\omega_{cm}\beta_{cv}$, where $\omega_{cm} > 1$. The COVID-19 induced mortality rate is r and the rate of recovery of humans infected with COVID is δ_1. The rate at which recovered humans become infected again is $\eta_1\beta_{cv}$. The rate of recovery of co-infected humans from comorbidity is δ_2 and $\eta_2\beta_{cv}$ is the rate at which recovered comorbidity humans get co-infected again.

The model system of equations is as follows:

$$
\begin{aligned}
\frac{dS_h}{dt} &= \Lambda_h - \beta_{cv} S_h - \left(\phi_{cm} + \mu_h\right) S_{h,} \\
\frac{dS_{cm}}{dt} &= \phi_{cm} S_h - w_{cm}\beta_{cv} S_{cm} - \mu_h S_{cm,} \\
\frac{dI_{cv}}{dt} &= \beta_{cv} S_h - \left(\delta_1 + \mu_h\right) I_{cv} + \eta_1 \beta_{cv} R_{cv,} \\
\frac{dR_{cv}}{dt} &= \delta_1 I_{cv} - \mu_h R_{cv} - \eta_1 \beta_{cv} R_{cv,} \\
\frac{dI_{vm}}{dt} &= w_{cm}\beta_{cv} S_{cm} - \left(\delta_2 + \mu_h\right) I_{vm} + \eta_2 \beta_{cv} R_{cm,} \\
\frac{dR_{cm}}{dt} &= \delta_2 I_{vm} - \mu_h R_{cm} - \eta_2 \beta_{cv} R_{cm,} \\
\beta_{cv} &= \beta\left(I_{cv} + \delta_{cm} I_{vm}\right),
\end{aligned}
\tag{1}
$$

with the initial conditions $S_h(0) = \zeta_1,\ S_{cm}(0) = \zeta_2,\ I_{cv}(0) = \zeta_3,\ R_{cv}(0) = \zeta_4,\ I_{vm}(0) = \zeta_5,$ $R_{cm}(0) = \zeta_6$.

3. Analysis of the Fractional COVID-19 Model

3.1. Mathematical Preliminaries.
We give the following definitions that will be applied for proofs of the existence, uniqueness and positivity of the Caputo-Fabrizio (CF) model analysed in this work.

Definition 1. [15] *Assume* $\psi(t) \in \mathscr{H}^1(\ell_1, \ell_2)$, *for* $\ell_2 > \ell_1,\ p \in [0,1]$. *Then the CF fractional operator is given as*

$$
\begin{aligned}
D_t^p(\psi(t)) &= \frac{\mathscr{M}(p)}{(1-p)} \int_{\ell_1}^{\ell_2} \psi'(\Theta) e^{\left[-p\frac{t-\Theta}{1-p}\right]} d\Theta,\ 0 < p < 1, \\
&= \frac{d\psi}{dt},\ p = 1,
\end{aligned} \tag{2}
$$

where $\mathscr{M}(p)$ *represents a normality that satisfies the condition* $\mathscr{M}(0) = \mathscr{M}(1) = 1$.

Definition 2. *The integral operator of fractional order corresponding to the CF fractional derivative defined in* [16] *states that*

$$
{}^{CF}I_t^p \psi(t) = \frac{2(1-p)}{(2-p)\mathscr{M}(p)} \psi(t) + \frac{2p}{(2-p)\mathscr{M}(p)} \int_0^t \psi(\tau) d\tau.,\ p \in [0,1],\ t \geq 0 \tag{3}
$$

Definition 3. *The Laplace transform of* $({}^{CF}D_t^p \psi(t), \kappa)$ *is represented as follows*

$$
L\left[0^{CF} D_t^p \psi(t), \kappa\right] = \frac{\kappa\tilde{\psi}(x,\kappa) - \psi_0(x)}{\kappa + p(1-\kappa)}, \tag{4}
$$

where $\tilde{\psi}(x,\kappa)$ *is the Laplace transform* $L\big(\psi(x,t),\kappa\big)$ *of* $\psi(x,t)$.

Classical models have been noticed not to be effective capturing complex phenomena. Non-integer order models have the capability of capturing the memory effect for accurate prediction. Currently, Caputo-Fabrizio operator characterized by no singularity which has a capability of crossover properties. In natural occurring circumstances the exponential decay which is Caputo–Fabrizio is common.

3.2. The Caputo-Fabrizio model.
The Caputo-Fabrizio fractional-order model of COVID-19 disease with comorbidity is given as follows.

$$\begin{aligned}
&{}_{0}^{CF}D_t^q S_h = \Lambda_h - \beta_{cv} S_h - (\phi_{cm} + \mu_h) S_h,\\
&{}_{0}^{CF}D_t^q S_{cm} = \phi_{cm} S_h - w_{cm}\beta_{cv} S_{cm} - \mu_h S_{cm},\\
&{}_{0}^{CF}D_t^q I_{cv} = \beta_{cv} S_h - (\delta_1 + \mu_h) I_{cv} + \eta_1 \beta_{cv} R_{cv},\\
&{}_{0}^{CF}D_t^q R_{cv} = \delta_1 I_{cv} - \mu_h R_{cv} - \eta_1 \beta_{cv} R_{cv},\\
&{}_{0}^{CF}D_t^q I_{vm} = w_{cm}\beta_{cv} S_{cm} - (\delta_2 + \mu_h) I_{vm} + \eta_2 \beta_{cv} R_{cm},\\
&{}_{0}^{CF}D_t^q R_{cm} = \delta_2 I_{vm} - \mu_h R_{cm} - \eta_2 \beta_{cv} R_{cm},\\
&\beta_{cv} = \beta\left(I_{cv} + \delta_{cm} I_{vm}\right),
\end{aligned} \tag{5}$$

with the following initial conditions $S_h(0) = \zeta_1, S_{cm}(0) = \zeta_2, I_{cv}(0) = \zeta_3, R_{cv}(0) = \zeta_4, I_{vm}(0) = \zeta_5, R_{cm}(0) = \zeta_6$.

3.3. Model steady states and basic reproduction number. In this section, the equilibrium states of the fractional coronavirus with comorbidity model (5) are investigated and their stabilities anlyzed. The coronavirus with comorbidity free state of the system (1) is given by

$$E^0 = (S_h^0, S_{cm}^0, I_{cv}^0, R_{cv}^0, I_{vm}^0, R_{cm}^0) = \left(\frac{\Lambda_h}{\phi_{cm} + \mu_h}, \frac{\phi_{cm}\Lambda_h}{\mu_h(\phi_{cm} + \mu_h)}, 0, 0, 0, 0\right),$$

while the endemic equilibrium point is

$$E^* = (S_h^*, S_{cm}^*, I_{cv}^*, R_{cv}^*, I_{vm}^*, R_{cm}^*),$$

where

$$S_h^* = \frac{\Lambda_h}{\phi_{cm} + \beta_{cv} + \mu_h},$$

$$S_{cm}^* = \frac{\phi_{cm}\Lambda_h}{(\beta_{cm}\omega_{cm} + \mu_h)(\phi_{cm} + \beta_{cv} + \mu_h)},$$

$$I_{cv}^{*} = \frac{\beta_{cv}\Lambda_h\left(\eta_1\beta cv+\mu_h\right)}{\mu_h\left(\phi_{cm}+\beta_{cv}+\mu_h\right)\left(\eta_1\beta_{cv}+\mu_h+\delta_1\right)},$$
$$R_{cv}^{*} = \frac{\sigma_1\beta_{cv}\Lambda_h}{\mu_h\left(\phi_{cm}+\beta_{cv}+\mu_h\right)\left(\eta_1\beta_{cv}+\mu_h+\delta_1\right)},$$
$$I_{vm}^{*} = \frac{\omega_{cm}\phi_{cm}\beta_{cv}\Lambda_h\left(\eta_2\beta_{cv}+\mu_h\right)}{\mu_h\left(\beta_{cm}\omega_{cm}+\mu_h\right)\left(\phi_{cm}+\beta_{cv}+\mu_h\right)\left(\eta_2\beta_{cv}+\mu_h+\delta_2\right)},$$
$$R_{cm}^{*} = \frac{\sigma_2\omega_{cm}\phi_{cm}\beta_{cv}\Lambda_h}{\mu_h\left(\beta_{cm}\omega_{cm}+\mu_h\right)\left(\phi_{cm}+\beta_{cv}+\mu_h\right)\left(\eta_2\beta_{cv}+\mu_h+\delta_2\right)}.$$

Furthermore, the basic reproduction number R_0 of the model (5) in a susceptible population is expressed as:

$$R_0 = \frac{\beta\left(\delta_{cm}S_{cm}^{0}\omega_{cm}\mu_h+\delta_1\delta_{cm}S_{cm}^{0}\omega_{cm}+\mu_h S_h^{0}+\delta_2 S_h^{0}\right)}{\left(\mu_h+\delta_1\right)\left(\mu_h+\delta_2\right)} = R_{0cv}+R_{0vm} \tag{6}$$

where $R_{0cv} = \frac{\beta S_h^0}{\mu_h+\delta_1}$ and $R_{0vm} = \frac{\beta\delta_{cm}S_{cm}^{0}\omega_{cm}}{\mu_h+\delta_2}$.

Theorem 1. *The steady state E^0 is locally asymptotically stable if all of the eigenvalues ϕ_i of $J(E^0)$ satisfy;*

$$|\arg(\phi_i)| > \frac{q\pi}{2}, i = 1,2,3,4,5,6. \tag{7}$$

Proof. The Jacobian matrix $J(E^0)$ of the model (5) evaluated at the coronavirus with comorbidity free state E^0 is given by the following

(8)

$$J(E^0) = \begin{pmatrix} -\left(\mu_h+\phi_{cm}\right) & 0 & -\frac{\beta\Lambda_h}{\mu_h+\phi_{cm}} & 0 & -\frac{\delta_{cm}\beta\Lambda_h}{\mu_h+\phi_{cm}} & 0 \\ \phi_{cm} & -\mu_h & -\frac{\omega_{cm}\beta\phi_{cm}\Lambda_h}{\mu_h(\mu_h+\phi_{cm})} & 0 & -\frac{\omega_{cm}\delta_{cm}\beta\phi_{cm}\Lambda_h}{\mu_h(\mu_h+\phi_{cm})} & 0 \\ 0 & 0 & \frac{\beta\Lambda_h}{\mu_h+\phi_{cm}}-\left(\mu_h+\delta_1\right) & 0 & -\frac{\delta_{cm}\beta\Lambda_h}{\mu_h+\phi_{cm}} & 0 \\ 0 & 0 & \delta_1 & -\mu_h & 0 & 0 \\ 0 & 0 & \frac{\omega_{cm}\beta\phi_{cm}\Lambda_h}{\mu_h(\mu_h+\phi_{cm})} & 0 & \frac{\omega_{cm}\delta_{cm}\beta\phi_{cm}\Lambda_h}{\mu_h(\mu_h+\phi_{cm})}-\left(\mu_h+\delta_2\right) & 0 \\ 0 & 0 & 0 & 0 & \delta_2 & -\mu_h \end{pmatrix}$$

It is obvious that four of the eigenvalues are negatives which are $-\mu_h, -\mu_h, -\mu_h, -\left(\phi_{cm}+\mu_h\right)$ and the other two eigenvalues are be obtained from the 2×2 matrix given by:

$$J_1 = \begin{pmatrix} \frac{\beta\Lambda_h}{\mu_h+\phi_{cm}}-\left(\mu_h+\delta_1\right) & -\frac{\delta_{cm}\beta\Lambda_h}{\mu_h+\phi_{cm}} \\ \frac{\omega_{cm}\beta\phi_{cm}\Lambda_h}{\mu_h(\mu_h+\phi_{cm})} & \frac{\omega_{cm}\delta_{cm}\beta\phi_{cm}\Lambda_h}{\mu_h(\mu_h+\phi_{cm})}-\left(\mu_h+\delta_2\right) \end{pmatrix}. \tag{9}$$

The other eigenvalues are the roots of the following equation

$$\lambda^2 + a_1\lambda + a_2 = 0, \tag{10}$$

where

$$\begin{aligned} a_1 &= (\mu_h + \delta_1)\left[1 - R_{0cv}\right] + (\mu_h + \delta_2)\left[1 - R_{0vm}\right] \\ a_2 &= \frac{2\beta^2\Lambda_h^2\omega_{cm}\delta_{cm}\phi_{cm}}{\mu_h(\mu_h + \phi_{cm})^2} + (\mu_h + \delta_1)(\mu_h + \delta_2)\left[1 - R_0\right]. \end{aligned}$$

Using the Routh-Hurwitz criterion, the eigenvalues are either negative or have negative real parts if only if $a_1, a_2 > 0$. The coefficient $a_1 > 0$ if $R_{0cv} < 1$ and $R_{0vm} < 1$ are satisfied. Note that we have $R_{0cv} < R_0$ and $R_{0vm} < R_0$, while $a_2 > 0$ if $R_0 < 1$. The argument of the roots of Eq. (10) are all greater than $\frac{q\pi}{2}$ if $R_0 < 1$. Hence E^0 is locally asymptotically whenever $R_0 < 1$. □

3.4. Existence and Uniqueness of Solutions. It is vital to examine the existence and uniqueness of the solutions of the COVID-19 model (5) in the light of exponential decay law. The existence of a solution of the COVID-19 model is investigated here using the fixed point theory. The fractional integral (2) is applied to (5) to obtain

$$\begin{aligned} S_h(t) - S_h(0) &= {}_0^{CF}I_t^q\{\Lambda_h - \beta_{cv}S_h - (\phi_{cm} + \mu_h)S_h\}, \\ S_{cm}(t) - S_{cm}(0) &= {}_0^{CF}I_t^q\{\phi_{cm}S_h - w_{cm}\beta_{cv}S_{cm} - \mu_h S_{cm}\}, \\ I_{cv}(t) - I_{cv}(0) &= {}_0^{CF}I_t^q\{\beta_{cv}S_h - (\delta_1 + \mu_h)I_{cv} + \eta_1\beta_{cv}R_{cv}\}, \\ R_{cv}(t) - R_{cv}(0) &= {}_0^{CF}I_t^q\{\delta_1 I_{cv} - \mu_h R_{cv} - \eta_1\beta_{cv}R_{cv}\}, \\ I_{vm}(t) - I_{vm}(0) &= {}_0^{CF}I_t^q\{w_{cm}\beta_{cv}S_{cm} - (\delta_2 + \mu_h)I_{vm} + \eta_2\beta_{cv}R_{cm}\}, \\ R_{cm}(t) - R_{cm}(0) &= {}_0^{CF}I_t^q\{\delta_2 I_{vm} - \mu_h R_{cm} - \eta_2\beta_{cv}R_{cm}\}. \end{aligned} \tag{11}$$

(12)

$$\begin{aligned}
S_h(t)-S_h(0) &= \tfrac{2(1-q)}{(2-q)M(q)}\{\Lambda_h-\beta_{cv}(t)S_h(t)-(\phi_{cm}+\mu_h)S_h(t)\}\\
&\quad+\tfrac{2q}{(2-q)M(q)}\int_0^t\Big(\Lambda_h-\beta_{cv}(\varepsilon)S_h(\varepsilon)-(\phi_{cm}+\mu_h)S_h(\varepsilon)\Big)d\varepsilon,\\
S_{cm}(t)-S_{cm}(0) &= \tfrac{2(1-q)}{(2-q)M(q)}\{\phi_{cm}S_h(t)-w_{cm}\beta_{cv}(t)S_{cm}(t)-\mu_hS_{cm}(t)\}\\
&\quad+\tfrac{2q}{(2-q)M(q)}\int_0^t\{\phi_{cm}S_h(\varepsilon)-w_{cm}\beta_{cv}(\varepsilon)S_{cm}(\varepsilon)-\mu_hS_{cm,}(\varepsilon)\}d\varepsilon,\\
I_{cv}(t)-I_{cv}(0) &= \tfrac{2(1-q)}{(2-q)M(q)}\{\beta_{cv}(t)S_h(t)-(\delta_1+\mu_h)I_{cv}(t)+\eta_1\beta_{cv}R_{cv,}(t)\}\\
&\quad+\tfrac{2q}{(2-q)M(q)}\int_0^t\{\beta_{cv}(\varepsilon)S_h(\varepsilon)-(\delta_1+\mu_h)I_{cv}(\varepsilon)+\eta_1\beta_{cv}(\varepsilon)R_{cv,}(\varepsilon)\}d\varepsilon,\\
R_{cv}(t)-R_{cv}(0) &= \tfrac{2(1-q)}{(2-q)M(q)}\{\delta_1I_{cv}(t)-\mu_hR_{cv}(t)-\eta_1\beta_{cv}(t)R_{cv}(t)\}\\
&\quad+\tfrac{2q}{(2-q)M(q)}\int_0^t\{\delta_1I_{cv}(\varepsilon)-\mu_hR_{cv}(\varepsilon)-\eta_1\beta_{cv}(\varepsilon)R_{cv}(\varepsilon)\}d\varepsilon,\\
I_{vm}(t)-I_{vm}(0) &= \tfrac{2(1-q)}{(2-q)M(q)}\{w_{cm}\beta_{cv}(t)S_{cm}(t)-(\delta_2+\mu_h)I_{vm}(t)+\eta_2\beta_{cv}(t)R_{cm}(t)\}\\
&\quad+\tfrac{2q}{(2-q)M(q)}\int_0^t\{w_{cm}\beta_{cv}(\varepsilon)S_{cm}(\varepsilon)-(\delta_2+\mu_h)I_{vm}(\varepsilon)+\eta_2\beta_{cv}(\varepsilon)R_{cm}(\varepsilon)\}d\varepsilon,\\
R_{cm}(t)-R_{cm}(0) &= \tfrac{2(1-q)}{(2-q)M(q)}\{\delta_2I_{vm}(t)-\mu_hR_{cm}(t)-\eta_2\beta_{cv}(t)R_{cm}(t)\}\\
&\quad+\tfrac{2q}{(2-q)M(q)}\int\{\delta_2I_{vm}(\varepsilon)-\mu_hR_{cm}(\varepsilon)-\eta_2\beta_{cv}(\varepsilon)R_{cm}(\varepsilon)\}d\varepsilon.
\end{aligned}$$

We use the following notation for the sake of simplicity.

(13)

$$\Phi_1(t,S_h)=\Lambda_h-\beta_{cv}S_h-(\phi_{cm}+\mu_h)S_h,$$

$$\Phi_2(t,S_{cm})=\phi_{cm}S_h-w_{cm}\beta_{cv}S_{cm}-\mu_hS_{cm},$$

$$\Phi_3(t,I_{cv})=\beta_{cv}S_h-(\delta_1+\mu_h)I_{cv}+\eta_1\beta_{cv}R_{cv},$$

$$\Phi_4(t,R_{cv})=\delta_1I_{cv}-\mu_hR_{cv}-\eta_1\beta_{cv}R_{cv},$$

$$\Phi_5(t,I_{vm}) = w_{cm}\beta_{cv}S_{cm} - (\delta_2+\mu_h)I_{vm} + \eta_2\beta_{cv}R_{cm},$$

$$\Phi_6(t,R_{cm}) = \delta_2 I_{vm} - \mu_h R_{cm} - \eta_2\beta_{cv}R_{cm}.$$

Theorem 2. *The kernels* Φ_1, Φ_2, Φ_3, Φ_4, Φ_5, *and* Φ_6 *satisfy the lipschitz condition if*

$$0 \le \beta(l_1+\delta_{cm}l_2) + (\mu+\phi_{cm}) < 1. \tag{14}$$

Proof.

$$\begin{aligned} ||\Phi_1(t,S_h) - \Phi_1(t,S_{h1})|| &= ||-\beta_{cv}(S_h - S_{h1}) - (\phi_{cm}+\mu_h)(S_h - S_{h1})|| \\ &\le (\phi_{cm}+\mu_h)||(S_h - S_{h1})|| - ||\beta_{cv}(S_h - S_{h1})|| \\ &\le (\phi_{cm}+\mu_h) + \beta||I_{cv}+\delta_{cm}I_{vm}||||(S_h - S_{h1})|| \\ &\le \beta(l_1+\delta_{cm}l_2) + (\mu+\phi_{cm})||(S_h - S_{h1})||. \end{aligned} \tag{15}$$

Let $\tilde{\omega}_1 = \beta(l_1+\delta_{cm}l_2) + (\mu+\phi_{cm})$, where $l_1 = I_{cv}$ and $l_2 = \delta_{cm}I_{vm}$ are bounded functions, then we obtain

$$||\Phi_1(t,S_h) - \Phi_1(t,S_{h1})|| \le \tilde{\omega}_1||(S_h - S_{h1})||.$$

Therefore the lipschitz condition is satisfied for Φ_1. Also, if $0 \le \beta(l_1+\delta_{cm}l_2) + (\mu+\phi_{cm}) < 1$, then Φ_1 is a contraction.

Similarly,

Φ_2, Φ_3, Φ_4, Φ_5, Φ_6 fulfil the lipschitz conditions

$$\begin{aligned} ||\Phi_2(t,S_{cm}) - \Phi_1(t,S_{cm1})|| &\le \tilde{\omega}_2||(S_{cm} - S_{cm1})||. \\ ||\Phi_3(t,I_{cv}) - \Phi_3(t,I_{cv1})|| &\le \tilde{\omega}_3||(I_{cv} - I_{cv1})|| \\ ||\Phi_4(t,R_{cv}) - \Phi_4(t,R_{cv1})|| &\le \tilde{\omega}_4||(R_{cv} - R_{cv1})|| \\ ||\Phi_5(t,I_{vm}) - \Phi_5(t,I_{vm1})|| &\le \tilde{\omega}_5||(I_{vm} - I_{vm1})|| \end{aligned} \tag{16}$$

$$||\Phi_6(t,R_{cm}) - \Phi_6(t,R_{cm1})|| \leq \tilde{\omega}_6||(R_{vm} - R_{vm1})||.$$

□

Taking into consideration the above mentioned kernels, (12) yeilds

$$\begin{aligned}
S_h(t) &= S_h(0) + \tfrac{2(1-q)}{(2-q)M(q)}\Phi_1(t,S_h) + \tfrac{2q}{(2-q)M(q)}\int_0^t \Phi_1(\varepsilon,S_h)d\varepsilon,\\
S_{cm}(t) &= S_{cm}(0) + \tfrac{2(1-q)}{(2-q)M(q)}\Phi_2(t,S_{cm}) + \tfrac{2q}{(2-q)M(q)}\int_0^t \Phi_2(\varepsilon,S_{cm})d\varepsilon,\\
I_{cv}(t) &= I_{cv}(0) + \tfrac{2(1-q)}{(2-q)M(q)}\Phi_3(t,I_{cv}) + \tfrac{2q}{(2-q)M(q)}\int_0^t \Phi_3(\varepsilon,I_{cv})d\varepsilon,\\
R_{cv}(t) &= R_{cv}(0) + \tfrac{2(1-q)}{(2-q)M(q)}\Phi_4(t,R_{cv}) + \tfrac{2q}{(2-q)M(q)}\int_0^t \Phi_4(\varepsilon,R_{cv})d\varepsilon,\\
I_{vm}(t) &= I_{vm}(0) + \tfrac{2(1-q)}{(2-q)M(q)}\Phi_5(t,I_{vm}) + \tfrac{2q}{(2-q)M(q)}\int_0^t \Phi_5(\varepsilon,I_{vm})d\varepsilon,\\
R_{cm}(t) &= R_{cm}(0) + \tfrac{2(1-q)}{(2-q)M(q)}\Phi_6(t,R_{cm}) + \tfrac{2q}{(2-q)M(q)}\int \Phi_6(\varepsilon,R_{cm})d\varepsilon.
\end{aligned} \tag{17}$$

Now, the recursive formula is presented based on aforesaid kernels, and equation (17) transformed to the following:

$$\begin{aligned}
S_{h(n)}(t) &= \tfrac{2(1-q)}{(2-q)M(q)}\Phi_1(t,S_{h(n-1)}) + \tfrac{2q}{(2-q)M(q)}\int_0^t \Phi_1(\varepsilon,S_{h(n-1)})d\varepsilon,\\
S_{cm(n)}(t) &= \tfrac{2(1-q)}{(2-q)M(q)}\Phi_2(t,S_{cm(n-1)}) + \tfrac{2q}{(2-q)M(q)}\int_0^t \Phi_2(\varepsilon,S_{cm(n-1)})d\varepsilon,\\
I_{cv(n)}(t) &= \tfrac{2(1-q)}{(2-q)M(q)}\Phi_3(t,I_{cv(n-1)}) + \tfrac{2q}{(2-q)M(q)}\int_0^t \Phi_3(\varepsilon,I_{cv(n-1)})d\varepsilon,\\
R_{cv(n)}(t) &= \tfrac{2(1-q)}{(2-q)M(q)}\Phi_4(t,R_{cv(n-1)}) + \tfrac{2q}{(2-q)M(q)}\int_0^t \Phi_4(\varepsilon,R_{cv(n-1)})d\varepsilon,\\
I_{vm(n)}(t) &= \tfrac{2(1-q)}{(2-q)M(q)}\Phi_5(t,I_{vm(n-1)}) + \tfrac{2q}{(2-q)M(q)}\int_0^t \Phi_5(\varepsilon,I_{vm(n-1)})d\varepsilon,
\end{aligned} \tag{18}$$

$$R_{cm(n)}(t) \;=\; \frac{2(1-q)}{(2-q)M(q)}\Phi_6(t,R_{cm(n-1)})+\frac{2q}{(2-q)M(q)}\int\Phi_6(\varepsilon,R_{cm(n-1)})d\varepsilon.$$

The associated initial conditions are presented as follows:

$S_{h0}(t) = S_h(0)$, $S_{cm0}(t) = S_{cm}(0)$, $I_{cv0}(t) = I_{cv}(0)$, $R_{cv0}(t) = R_{cv}(0)$, $I_{vm0}(t) = I_{vm}(0)$, $R_{cm0}(t) = R_{cm}(0)$.

The difference between the successive terms are presented as follow:

(19)

$$\begin{aligned} E_n(t)=S_{h(n)}(t)-S_{h(n-1)}(t) \;&=\; \frac{2(1-q)}{(2-q)M(q)}\Big[\Phi_1(t,S_{h(n)})-\Phi_1(t,S_{h(n-1)})\Big] \\ &\quad+\frac{2q}{(2-q)M(q)}\int_0^t\Big[\Phi_1(\varepsilon,S_{h(n)})-\Phi_1(\varepsilon,S_{h(n-1)})\Big]d\varepsilon, \end{aligned}$$

$$\begin{aligned} F_n(t)=S_{cm(n)}(t)-S_{cm(n-1)}(t) \;&=\; \frac{2(1-q)}{(2-q)M(q)}\big[\Phi_2(t,S_{cm(n)})-\Phi_2(t,S_{cm(n-1)})\Big] \\ &\quad+\frac{2q}{(2-q)M(q)}\int_0^t\big[\Phi_2(\varepsilon,S_{cm(n)})-\Phi_2(\varepsilon,S_{cm(n-1)})\big]d\varepsilon, \end{aligned}$$

$$\begin{aligned} G_n(t)=I_{cv(n)}(t)-I_{cv(n-1)}(t) \;&=\; \frac{2(1-q)}{(2-q)M(q)}\big[\Phi_3(t,I_{cv(n)})-\Phi_3(t,I_{cv(n-1)})\big] \\ &\quad+\frac{2q}{(2-q)M(q)}\int_0^t\big[\Phi_3(\varepsilon,I_{cv(n)})-\Phi_3(\varepsilon,I_{cv(n-1)})\big]d\varepsilon, \end{aligned}$$

$$\begin{aligned} H_n(t)=R_{cv(n)}(t)-R_{cv(n-1)}(t) \;&=\; \frac{2(1-q)}{(2-q)M(q)}\big[\Phi_4(t,R_{cv(n)})-\Phi_4(t,R_{cv(n-1)})\big] \\ &\quad+\frac{2q}{(2-q)M(q)}\int_0^t\big[\Phi_4(\varepsilon,R_{cv(n)})-\Phi_4(\varepsilon,R_{cv(n-1)})\big]d\varepsilon, \end{aligned}$$

$$\begin{aligned} I_n(t)=I_{vm(n)}(t)-I_{vm(n-1)}(t) \;&=\; \frac{2(1-q)}{(2-q)M(q)}\big[\Phi_5(t,I_{vm(n)})-\Phi_5(t,I_{vm(n-1)})\Big] \\ &\quad+\frac{2q}{(2-q)M(q)}\int_0^t\Big[\Phi_5(\varepsilon,I_{vm(n)})-\Phi_5(\varepsilon,I_{vm(n-1)})\Big]d\varepsilon, \end{aligned}$$

$$\begin{aligned} J_n(t)=R_{cm(n)}(t)-R_{cm(n-1)}(t) \;&=\; \frac{2(1-q)}{(2-q)M(q)}\Big[\Phi_6(t,R_{cm(n)})-\Phi_6(t,R_{cm(n-1)})\Big] \\ &\quad+\frac{2q}{(2-q)M(q)}\int\Big[\Phi_6(\varepsilon,R_{cm(n)})-\Phi_6(\varepsilon,R_{cm(n-1)})\Big]d\varepsilon. \end{aligned}$$

Note that

$$S_{h(n)}(t)=\textstyle\sum_i^n E_i(t), \qquad S_{cm(n)}(t)=\sum_i^n F_i(t),$$

$$I_{cv(n)}(t)=\sum_i^n G_i(t), \qquad R_{cv(n)}(t)=\sum_i^n H_i(t),$$

$$I_{vm(n)}(t)=\sum_i^n I_i(t), \qquad R_{vm(n)}(t)=\sum_i^n J_i(t).$$

On the other hand,

$$\begin{aligned}
||E_n(t)||=||S_{h(n)}(t)-S_{h(n-1)}(t)|| &= \Bigg\|\frac{2(1-q)}{(2-q)M(q)}\Big[\Phi_1(t,S_{h(n)})-\Phi_1(t,S_{h(n-1)})\Big]\\
&\quad+\frac{2q}{(2-q)M(q)}\int_0^t\Big[\Phi_1(\varepsilon,S_{h(n)})-\Phi_1(\varepsilon,S_{h(n-1)})\Big]d\varepsilon\Bigg\|\\
&\leq \frac{2(1-q)}{(2-q)M(q)}\Big[\Phi_1(t,S_{h(n)})-\Phi_1(t,S_{h(n-1)})\Big]\\
&\quad+\frac{2q}{(2-q)M(q)}\Bigg\|\int_0^t\Big[\Phi_1(\varepsilon,S_{h(n)})-\Phi_1(\varepsilon,S_{h(n-1)})\Big]d\varepsilon\Bigg\|\\
&\leq \frac{2(1-q)}{(2-q)M(q)}\tilde{\omega}_1||S_{h(n-1)}(t)-S_{h(n-2)}(t)||\\
&\quad+\frac{2q}{(2-q)M(q)}\tilde{\omega}_1\int_0^t\Big\|S_{h(n-1)}(\varepsilon)-S_{h(n-2)}(\varepsilon)\Big\|d\varepsilon.
\end{aligned}$$

Therefore,

$$\|E_n(t)\|\leq\frac{2(1-q)}{(2-q)M(q)}\tilde{\omega}_1\|E_{n-1}(t)\|+\frac{2q}{(2-q)M(q)}\tilde{\omega}_1\int_0^t\Big\|E_{n-1}(\varepsilon)\Big\|d\varepsilon.$$

In a similar manner, we obtain the following:

$$\begin{aligned}
\|F_n(t)\| &\leq \tfrac{2(1-q)}{(2-q)M(q)}\tilde{\omega}_2\|F_{n-1}(t)\|+\frac{2q}{(2-q)M(q)}\tilde{\omega}_2\int_0^t\Big\|F_{n-1}(\varepsilon)\Big\|d\varepsilon,\\
\|G_n(t)\| &\leq \tfrac{2(1-q)}{(2-q)M(q)}\tilde{\omega}_3\|G_{n-1}(t)\|+\frac{2q}{(2-q)M(q)}\tilde{\omega}_3\int_0^t\Big\|G_{n-1}(\varepsilon)\Big\|d\varepsilon, \qquad (20)\\
\|H_n(t)\| &\leq \tfrac{2(1-q)}{(2-q)M(q)}\tilde{\omega}_4\|H_{n-1}(t)\|+\frac{2q}{(2-q)M(q)}\tilde{\omega}_4\int_0^t\Big\|H_{n-1}(\varepsilon)\Big\|d\varepsilon,
\end{aligned}$$

$$\begin{aligned}
\|I_n(t)\| &\leq \tfrac{2(1-q)}{(2-q)M(q)}\tilde{\omega}_5\|I_{n-1}(t)\| + \frac{2q}{(2-q)M(q)}\tilde{\omega}_5\int_0^t \left\|I_{n-1}(\varepsilon)\right\| d\varepsilon, \\
\|J_n(t)\| &\leq \tfrac{2(1-q)}{(2-q)M(q)}\tilde{\omega}_6\|J_{n-1}(t)\| + \frac{2q}{(2-q)M(q)}\tilde{\omega}_6\int_0^t \left\|J_{n-1}(\varepsilon)\right\| d\varepsilon.
\end{aligned}$$

We thus have the following theorem

Theorem 3. *Fractional COVID-19 model (13) possesses a system of solutions if there exist* t_0 *such that*

$$\frac{2(1-q)}{(2-q)M(q)}\tilde{\omega}_1 + \frac{2q}{(2-q)M(q)}\tilde{\omega}_1 t_0 \leq 1. \tag{21}$$

Proof. Consider the following bounded functions: $S_h(t), S_{cm}(t), I_{cv}(t), R_{cv}(t), I_{vm}, R_{vm}(t)$. Further, we have shown that the kernels satisfy the Lipschitz condition. using the results of equations (19) and (20) respectively and adopting the recursive method, we obtain the following relations:

$$\begin{aligned}
\|E_n(t)\| &\leq \|S_{hn}(0)\|\left[\tfrac{2(1-q)}{(2-q)M(q)}\tilde{\omega}_1 + \tfrac{2q}{(2-q)M(q)}\tilde{\omega}_1 t\right]^n, \\
\|F_n(t)\| &\leq \|S_{cmn}(0)\|\left[\tfrac{2(1-q)}{(2-q)M(q)}\tilde{\omega}_2 + \tfrac{2q}{(2-q)M(q)}\tilde{\omega}_2 t\right]^n, \\
\|G_n(t)\| &\leq \|S_{nh}(0)\|\left[\tfrac{2(1-q)}{(2-q)M(q)}\tilde{\omega}_3 + \tfrac{2q}{(2-q)M(q)}\tilde{\omega}_3 t\right]^n, \\
\|H_n(t)\| &\leq \|R_{cvn}(0)\|\left[\tfrac{2(1-q)}{(2-q)M(q)}\tilde{\omega}_4 + \tfrac{2q}{(2-q)M(q)}\tilde{\omega}_4 t\right]^n, \\
\|I_n(t)\| &\leq \|I_{vmn}(0)\|\left[\tfrac{2(1-q)}{(2-q)M(q)}\tilde{\omega}_5 + \tfrac{2q}{(2-q)M(q)}\tilde{\omega}_5 t\right]^n, \\
\|J_n(t)\| &\leq \|R_{cmn}(0)\|\left[\tfrac{2(1-q)}{(2-q)M(q)}\tilde{\omega}_6 + \tfrac{2q}{(2-q)M(q)}\tilde{\omega}_6 t\right]^n.
\end{aligned} \tag{22}$$

Thus, the system (17) exists and is smooth. In order to demonstrate that the above functions constitute the solutions of the system equation (5), we make the assumption that

$$
\begin{aligned}
S_h(t)-S_h(0) &= S_{hn}(t)-L_n(t),\\
S_{cm}(t)-S_{cm}(0) &= S_{cmn}(t)-M_n(t),\\
I_{cv}(t)-I_{cv}(0) &= I_{cvn}(t)-N_n(t),\\
R_{cv}(t)-R_{cv}(0) &= R_{cvn}(t)-O_n(t),\\
I_{vm}(t)-I_{vm}(0) &= I_{vmn}(t)-P_n(t),\\
R_{cm}(t)-R_{cm}(0) &= R_{vmn}(t)-Q_n(t),
\end{aligned}
\tag{23}
$$

where

$$
\begin{aligned}
||L_n(t)||=||S_h(t)-S_{h(n-1)}(t)|| &= \left\| \frac{2(1-q)}{(2-q)M(q)}\left[\Phi_1(t,S_h)-\Phi_1(t,S_{h(n-1)})\right] \right.\\
&\quad \left. +\tfrac{2q}{(2-q)M(q)}\int_0^t \left[\Phi_1(\varepsilon,S_h)-\Phi_1(\varepsilon,S_{h(n-1)})\right]d\varepsilon \right\|\\
&\leq \tfrac{2(1-q)}{(2-q)M(q)}||\Phi_1(t,S_h)-\Phi_1(t,S_{h(n-1)})||\\
&\quad +\tfrac{2q}{(2-q)M(q)}\left\|\int_0^t \left[\Phi_1(\varepsilon,S_h)-\Phi_1(\varepsilon,S_{h(n-1)})\right]d\varepsilon\right\|\\
&\leq \tfrac{2(1-q)}{(2-q)M(q)}\tilde{\omega}_1||S_h-S_{h(n-1)}||\\
&\quad +\tfrac{2q}{(2-q)M(q)}\tilde{\omega}_1\left\|S_h-S_{h(n-1)}\right\|t.
\end{aligned}
\tag{24}
$$

Carrying out this process recursively we have

$$
\|L_n(t)\| \leq \left[\frac{2(1-q)}{(2-q)M(q)}\tilde{\omega}_1+\frac{2q}{(2-q)M(q)}\tilde{\omega}_1 t\right]^{n+1}\tilde{\omega}_1{}^{n+1}k.
\tag{25}
$$

Taking limits as $n \longrightarrow \infty$, we have $\left\|L_n(t)\right\| \longrightarrow 0$. Similarly, we obtain

$$\left\|M_n(t)\right\| \longrightarrow 0, \quad \left\|N_n(t)\right\| \longrightarrow 0, \quad \left\|O_n(t)\right\| \longrightarrow 0, \quad \left\|P_n(t)\right\| \longrightarrow 0. \tag{26}$$

□

Now we show that the model has a unique solution. Assume that the system has another solution $S_h^*(t)$, $S_{cm}^*(t)$, $I_{cv}^*(t)$, $R_{cv}^*(t)$, $I_{vm}^*(t)$, $R_{cm}^*(t)$.

Then, exploring the properties of norm in equation (16) gives

$$S_h(t)-S_h^*(t) = \frac{2(1-q)}{(2-q)M(q)}\left[\Phi_1(t,S_h)-\Phi_1(t,S_h^*)\right] + \frac{2q}{(2-q)M(q)}\int_0^t \left[\Phi_1(\varepsilon,S_h)-\Phi_1(\varepsilon,S_h^*)\right]d\varepsilon \tag{27}$$

and utilizing the Lipschitz condition, we have

$$\|S_h(t)-S_h^*(t)\| \leq \frac{2(1-q)}{(2-q)M(q)}\tilde{\omega}_1||S_h-S_h^*(t)|| + \frac{2q}{(2-q)M(q)}\tilde{\omega}_1\int_0^t \left\|S_h(\varepsilon)-S_h^*(\varepsilon)\right\|d\varepsilon, \tag{28}$$

which leads to

$$\|S_h(t)-S_h^*(t)\|\left[1-\frac{2(1-q)}{(2-q)M(q)}\tilde{\omega}_1-\frac{2q}{(2-q)M(q)}\tilde{\omega}_1 t\right] \leq 0. \tag{29}$$

Theorem 4. *The fractional order model system (13) has a unique solution provided that*

$$\|S_h(t)-S_h^*(t)\|\left[1-\frac{2(1-q)}{(2-q)M(q)}\tilde{\omega}_1-\frac{2q}{(2-q)M(q)}\tilde{\omega}_1 t\right] > 0. \tag{30}$$

Proof. Following equation (31),

$$\|S_h(t)-S_h^*(t)\|\left[1-\frac{2(1-q)}{(2-q)M(q)}\tilde{\omega}_1-\frac{2q}{(2-q)M(q)}\tilde{\omega}_1 t\right] \leq 0. \tag{31}$$

, and from equation (31),

$$\|S_h(t)-S_h^*(t)\|\left[1-\frac{2(1-q)}{(2-q)M(q)}\tilde{\omega}_1-\frac{2q}{(2-q)M(q)}\tilde{\omega}_1 t\right] > 0, \tag{32}$$

with $\|S_h(t)-S_h^*(t)\| = 0$. Therefore, $S_h(t) = S_h^*(t)$. Similarly, $S_{cm}(t) = S_{cm}^*(t)$, $I_{cv}(t) = I_{cv}^*(t)$, $R_{cv}(t) = R_{cv}^*(t)$, $I_{vm}(t) = I_{vm}^*(t)$, and $R_{cm}(t) = R_{cm}^*(t)$. □

4. The Stochastic Coronavirus Model with Comorbidity

In this section, the coronavirus model with comorbidity is expressed in global derivative with a stochastic component. The stochastic aspect is introduced in the global derivative model (13) as follows:

$$
\begin{aligned}
S_h(t) &= S_h(0)+\int_0^t E_1(\theta,S_h)d\theta+L_1(\theta,S_h)dA_1(\theta)\\
S_{cm}(t) &= S_{cm}(0)+\int_0^t E_2(\theta,S_{cm})d\theta+L_2(\theta,S_{cm})dA_2(\theta)\\
I_{cv}(t) &= I_{cv}(0)+\int_0^t E_3(\theta,I_{cv})d\theta+L_3(\theta,I_{cv})dA_3(\theta)\\
R_{cv}(t) &= R_{cv}(0)+\int_0^t E_4(\theta,R_{cv})d\theta+L_4(\theta,R_{cv})dA_4(\theta)\\
I_{vm}(t) &= I_{vm}(0)+\int_0^t E_5(\theta,I_{vm})d\theta+L_5(\theta,I_{vm})dA_5(\theta)\\
R_{cm}(t) &= R_{cm}(0)+\int_0^t E_6(\theta,R_{cm})d\theta+L_6(\theta,R_{cm})dA_6(\theta),
\end{aligned}
\tag{33}
$$

where

$$
\begin{aligned}
E_1(\theta,S_h) &= h'(\theta)(\Lambda_h-\beta_{cv}S_h-(\phi_{cm}+\mu_h)S_h),\\
E_2(\theta,S_{cm}) &= h'(\theta)(\phi_{cm}S_h-w_{cm}\beta_{cv}S_{cm}-\mu_hS_{cm}),\\
E_3(\theta,I_{cv}) &= h'(\theta)(\beta_{cv}S_h-(\delta_1+\mu_h)I_{cv}+\eta_1\beta_{cv}R_{cv}),\\
E_4(\theta,R_{cv}) &= h'(\theta)(\delta_1I_{cv}-\mu_hR_{cv}-\eta_1\beta_{cv}R_{cv}),\\
E_5(\theta,I_{vm}) &= h'(\theta)(w_{cm}\beta_{cv}S_{cm}-(\delta_2+\mu_h)I_{vm}+\eta_2\beta_{cv}R_{cm}),\\
E_6(\theta,R_{cm}) &= h'(\theta)(\delta_2I_{vm}-\mu_hR_{cm}-\eta_2\beta_{cv}R_{cm}).
\end{aligned}
\tag{34}
$$

5. NUMERICAL SCHEME FOR CORONAVIRUS WITH COMORBIDITY MODEL

This section presents the numerical scheme for the coronavirus model (5) with comorbidity in global derivatives via exponential decay Kernel.

By making the assumption that $C_1, C_2, C_3, C_4, C_5, C_6, C_7$ are differentiable, and substituting the functions by Newton Polynomial interpolation we have the following scheme.

(35)

$$\begin{aligned} S_h(t_{k+1}) &= S_h(t_k)+\tfrac{1-q}{M(q)}\Bigg[\tfrac{h(t_{k+1}-h(t_k))}{!t}\Bigg(\Lambda_h-\beta I_{cv}^{k+1}S_h^{k+1}+\beta\delta_{cm}I_{cv}^{k+1}S_h^{k+1} \\ &\quad -(\phi_{cm}+\mu_h)S_h^{k+1}\Bigg)-\tfrac{h(t_k-h(t_{k-1}))}{!t}\Bigg(\Lambda_h-\beta I_{cv}^{k}S_h^{k}+\beta\delta_{cm}I_{cv}^{k}S_h^{k}-(\phi_{cm}+\mu_h)S_h^{k}\Bigg] \\ &\quad +\tfrac{1-q}{M(q)}\Bigg[(C_1(t_{k+1})-C_1(t_k))\tfrac{h(t_{k+1}-h(t_k))}{!t}P_1(t_{k+1},S_h^{k+1})-(C_1(t_k) \\ &\quad -C_1(t_{k-1}))\tfrac{h(t_k-h(t_{k-1}))}{!t}P_1(t_k,S_h^{k})\Bigg] \\ &\quad +\tfrac{q}{M(q)}\Bigg[\tfrac{5}{12}\Bigg(\Lambda_h-\beta I_{cv}^{k-2}S_h^{k-2}+\beta\delta_{cm}I_{cv}^{k-2}S_h^{k-2} \\ &\quad -(\phi_{cm}+\mu_h)S_h^{k-2}\Bigg)\times\Bigg(h(t_{k-1}-h(t_{k-2}))\Bigg) \\ &\quad -\tfrac{4}{3}\Bigg(\Lambda_h-\beta I_{cv}^{k-1}S_h^{k-1}+\beta\delta_{cm}I_{cv}^{k-1}S_h^{k-1} \\ &\quad -(\phi_{cm}+\mu_h)S_h^{k-1}\Bigg)\times\Bigg(h(t_k-h(t_{k-1}))\Bigg) \\ &\quad +\tfrac{23}{12}\Bigg(\Lambda_h-\beta I_{cv}^{k}S_h^{k}+\beta\delta_{cm}I_{cv}^{k}S_h^{k} \\ &\quad -(\phi_{cm}+\mu_h)S_h^{k}\Bigg)\times\Bigg(h(t_{k+1}-h(t_k))\Bigg)\Bigg] \\ &\quad +\tfrac{q}{M(q)}\Bigg[\tfrac{5}{12}P_1(t_{k-2},S_h^{k-2})h(t_{k-1}-h(t_{k-2}))(C_1(t_{k-1})-C_1(t_{k-2})) \\ &\quad -\tfrac{4}{3}P_1(t_{k-1},S_h^{k-1})h(t_k-h(t_{k-1}))(C_1(t_k)-C_1(t_{k-1})) \\ &\quad \tfrac{23}{12}P_1(t_k,S_h^{k})h(t_{k+1}-h(t_k))(C_1(t_{k+1})-C_1(t_k))\Bigg]. \end{aligned}$$

(36)

$$
\begin{aligned}
S_{cm}(t_{k+1}) \;=\;& S_{cm}(t_k)+\frac{1-q}{M(q)}\left[\frac{h(t_{k+1}-h(t_k))}{!t}\left(\phi_{cm}S_h^{k+1}-w_{cm}\beta I_{cv}^{k+1}S_{cm}^{k+1}+\delta_{cm}w_{cm}\beta I_{vm}^{k+1}\right.\right.\\
&\left.-\mu_h S_{cm}^{k+1}\right)-\frac{h(t_k-h(t_{k-1}))}{!t}\left.\left(\phi_{cm}S_h^{k}-w_{cm}\beta I_{cv}^{k}S_{cm}^{k}+\delta_{cm}w_{cm}\beta I_{vm}^{k}-\mu_h S_{cm}^{k}\right)\right]\\
&+\frac{1-q}{M(q)}\left[(C_2(t_{k+1})-C_2(t_k))\frac{h(t_{k+1}-h(t_k))}{!t}P_2(t_{k+1},S_{cm}^{k+1})-(C_1(t_k)\right.\\
&\left.-C_2(t_{k-1}))\frac{h(t_k-h(t_{k-1}))}{!t}P_2(t_k,S_{cm}^{k})\right]\\
&+\frac{q}{M(q)}\left[\frac{5}{12}\left(\left(\phi_{cm}S_h^{k-2}-w_{cm}\beta I_{cv}^{k-2}S_{cm}^{k-2}+\delta_{cm}w_{cm}\beta I_{vm}^{k-2}-\mu_h S_{cm}^{k-2}\right)\right)\right.\\
&\times\left(h(t_{k-1}-h(t_{k-2}))\right)-\frac{4}{3}\left(\left(\phi_{cm}S_h^{k-1}-w_{cm}\beta I_{cv}^{k-1}S_{cm}^{k-1}+\delta_{cm}w_{cm}\beta I_{vm}^{k-1}\right.\right.\\
&\left.\left.-\mu_h S_{cm}^{k-1}\right)\right)\times\left(h(t_k-h(t_{k-1}))\right)+\frac{23}{12}\left(\left(\phi_{cm}S_h^{k}-w_{cm}\beta I_{cv}^{k}S_{cm}^{k}+\delta_{cm}w_{cm}\beta I_{vm}^{k}\right.\right.\\
&\left.\left.\left.-\mu_h S_{cm}^{k}\right)\right)\times\left(h(t_{k+1}-h(t_k))\right)\right]\\
&+\frac{q}{M(q)}\left[\frac{5}{12}P_2(t_{k-2},S_{cm}^{k-2})h(t_{k-1}-h(t_{k-2}))(C_2(t_{k-1})-C_2(t_{k-2}))\right.\\
&-\frac{4}{3}P_2(t_{k-1},S_{cm}^{k-1})h(t_k-h(t_{k-1}))(C_2(t_k)-C_2(t_{k-1}))\\
&\left.\frac{23}{12}P_2(t_k,S_{cm}^{k})h(t_{k+1}-h(t_k))(C_2(t_{k+1})-C_2(t_k))\right].
\end{aligned}
$$

(37)

$$
\begin{aligned}
I_{cv}(t_{k+1}) \;=\;& I_{cv}(t_k)+\frac{1-q}{M(q)}\left[\frac{h(t_{k+1}-h(t_k))}{!t}\left(\beta I_{cv}^{k+1}S_h^{k+1}-\delta_{cv}\beta I_{vm}^{k+1}+(\delta_1+\mu_h)I_{cv}^{k+1}R_{cv}^{k+1}\right.\right.\\
&\left.+\eta_1\beta I_{cv}^{k+1}-\eta_1\beta\delta_{cv}I_{vm}^{k+1}\right)-\frac{h(t_k-h(t_{k-1}))}{!t}\left(\beta I_{cv}^{k}S_h^{k}-\delta_{cv}\beta I_{vm}^{k}+(\delta_1+\mu_h)I_{cv}^{k}R_{cv}^{k}\right.\\
&\left.\left.+\eta_1\beta I_{cv}^{k}-\eta_1\beta\delta_{cv}I_{vm}^{k}\right)\right]\\
&+\frac{1-q}{M(q)}\left[(C_3(t_{k+1})-C_3(t_k))\frac{h(t_{k+1}-h(t_k))}{!t}P_3(t_{k+1},I_{cv}^{k+1})-(C_3(t_k)\right.\\
&\left.-C_3(t_{k-1}))\frac{h(t_k-h(t_{k-1}))}{!t}P_3(t_k,I_{cv}^{k})\right]\\
&+\frac{q}{M(q)}\left[\frac{5}{12}\left(\beta I_{cv}^{k-2}S_h^{k-2}-\delta_{cv}\beta I_{vm}^{k-2}+(\beta I_{cv}^{k-2}S_h^{k-2}-\delta_{cv}\beta I_{vm}^{k-2}\right.\right.\\
&\left.+(\delta_1+\mu_h)I_{cv}^{k-2}R_{cv}^{k-2}+\eta_1\beta I_{cv}^{k-2}-\eta_1\beta\delta_{cv}I_{vm}^{k-2}\right)\times\left(h(t_{k-1}-h(t_{k-2}))\right)\\
&-\frac{4}{3}\left(\beta I_{cv}^{k-1}S_h^{k-1}-\delta_{cv}\beta I_{vm}^{k-1}+(\delta_1+\mu_h)I_{cv}^{k-1}R_{cv}^{k-1}+\eta_1\beta I_{cv}^{k-1}-\eta_1\beta\delta_{cv}I_{vm}^{k-1}\right)\\
&\times\left(h(t_k-h(t_{k-1}))\right)+\frac{23}{12}\left(\beta I_{cv}^{k}S_h^{k}-\delta_{cv}\beta I_{vm}^{k}+(\delta_1+\mu_h)I_{cv}^{k}R_{cv}^{k}\right.
\end{aligned}
$$

$$+\eta_1\beta I_{cv}^{k}-\eta_1\beta\delta_{cv}I_{vm}^{k}\Big)\times\Big(h(t_{k+1}-h(t_k))\Big)\Big]$$
$$+\tfrac{q}{M(q)}\Big[\tfrac{5}{12}P_3(t_{k-2},I_{cv}^{k-2})h(t_{k-1}-h(t_{k-2}))(C_3(t_{k-1})-C_3(t_{k-2}))$$
$$-\tfrac{4}{3}P_3(t_{k-1},I_{cv}^{k-1})h(t_k-h(t_{k-1}))(C_3(t_k)-C_3(t_{k-1}))$$
$$\tfrac{23}{12}P_3(t_k,I_{cv}^{k})h(t_{k+1}-h(t_k))(C_3(t_{k+1})-C_3(t_k))\Big].$$

(38)

$$\begin{aligned}R_{cv}(t_{k+1}) &= R_{cv}(t_k)+\tfrac{1-q}{M(q)}\Big[\tfrac{h(t_{k+1}-h(t_k))}{!t}\Big(\delta_1 I_{cv}^{k+1}-\mu_h R_{cv}^{k+1}-\eta_1\beta I_{cv}^{k+1}R_{cv}^{k+1}\\&-\eta_1\delta_{cv}\beta I_{vm}^{k+1}R_{cv}^{k+1}\Big)-\tfrac{h(t_k-h(t_{k-1}))}{!t}\Big(\delta_1 I_{cv}^{k}-\mu_h R_{cv}^{k}-\eta_1\beta I_{cv}^{k}R_{cv}^{k}-\eta_1\delta_{cv}\beta I_{vm}^{k}R_{cv}^{k}\Big)\Big]\end{aligned}$$

$$+\tfrac{1-q}{M(q)}\Big[(C_4(t_{k+1})-C_4(t_k))\tfrac{h(t_{k+1}-h(t_k))}{!t}P_4(t_{k+1},R_{cv}^{k+1})-(C_4(t_k)$$
$$-C_4(t_{k-1}))\tfrac{h(t_k-h(t_{k-1}))}{!t}P_4(t_k,R_{cv}^{k})\Big]$$
$$+\tfrac{q}{M(q)}\Big[\tfrac{5}{12}\Big(\delta_1 I_{cv}^{k-2}-\mu_h R_{cv}^{k-2}-\eta_1\beta I_{cv}^{k-2}R_{cv}^{k-2}-\eta_1\delta_{cv}\beta I_{vm}^{k-2}R_{cv}^{k-2}\Big)$$
$$\times\Big(h(t_{k-1}-h(t_{k-2}))\Big)-\tfrac{4}{3}\Big(\delta_1 I_{cv}^{k-1}-\mu_h R_{cv}^{k-1}-\eta_1\beta I_{cv}^{k-1}R_{cv}^{k-1}$$
$$-\eta_1\delta_{cv}\beta I_{vm}^{k-1}R_{cv}^{k-1}\Big)\times\Big(h(t_k-h(t_{k-1}))\Big)+\tfrac{23}{12}\Big(\delta_1 I_{cv}^{k}-\mu_h R_{cv}^{k}-\eta_1\beta I_{cv}^{k}R_{cv}^{k}$$
$$-\eta_1\delta_{cv}\beta I_{vm}^{k}R_{cv}^{k}\Big)\times\Big(h(t_{k+1}-h(t_k))\Big)\Big]$$

$$+\tfrac{q}{M(q)}\Big[\tfrac{5}{12}P_4(t_{k-2},R_{cv}^{k-2})h(t_{k-1}-h(t_{k-2}))(C_4(t_{k-1})-C_4(t_{k-2}))$$
$$-\tfrac{4}{3}P_4(t_{k-1},R_{cv}^{k-1})h(t_k-h(t_{k-1}))(C_4(t_k)-C_4(t_{k-1}))$$
$$\tfrac{23}{12}P_4(t_k,R_{cv}^{k})h(t_{k+1}-h(t_k))(C_4(t_{k+1})-C_4(t_k))\Big].$$

(39)

$$\begin{aligned}I_{vm}(t_{k+1}) &= I_{vm}(t_k)+\tfrac{1-q}{M(q)}\Big[\tfrac{h(t_{k+1}-h(t_k))}{!t}\Big(w_{cm}\beta I_{cv}^{k+1}S_{cm}^{k+1}-w_{cm}\delta_{cm}\beta I_{vm}^{k+1}S_{cm}^{k+1}\\&+(\delta_2+\mu_h)I_{vm}^{k+1}+\eta_2\beta I_{cv}^{k+1}R_{cm}^{k+1}+\eta_2\beta I_{vm}^{k+1}R_{cm}^{k+1}\Big)\\&-\tfrac{h(t_k-h(t_{k-1}))}{!t}\Big(w_{cm}\beta I_{cv}^{k}S_{cm}^{k}-w_{cm}\delta_{cm}\beta I_{vm}^{k}S_{cm}^{k}+(\delta_2+\mu_h)I_{vm}^{k}\\&+\eta_2\beta I_{cv}^{k}R_{cm}^{k}+\eta_2\beta I_{vm}^{k}R_{cm}^{k}\Big)\Big]\end{aligned}$$

$$+\frac{1-q}{M(q)}\Bigg[(C_5(t_{k+1})-C_5(t_k))\frac{h(t_{k+1}-h(t_k))}{!t}P_5(t_{k+1},I_{vm}^{k+1})-(C_5(t_k)$$
$$-C_5(t_{k-1}))\frac{h(t_k-h(t_{k-1}))}{!t}P_5(t_k,I_{vm}^{k})\Bigg]$$
$$+\frac{q}{M(q)}\Bigg[\frac{5}{12}\Bigg(w_{cm}\beta I_{cv}^{k-2}S_{cm}^{k-2}-w_{cm}\delta_{cm}\beta I_{vm}^{k-2}S_{cm}^{k-2}+(\delta_2+\mu_h)I_{vm}^{k-2}$$
$$+\eta_2\beta I_{cv}^{k-2}R_{cm}^{k-2}+\eta_2\beta I_{vm}^{k-2}R_{cm}^{k-2}\Bigg)-\frac{4}{3}\Bigg(w_{cm}\beta I_{cv}^{k-1}S_{cm}^{k-1}-w_{cm}\delta_{cm}\beta I_{vm}^{k-1}S_{cm}^{k-1}$$
$$+(\delta_2+\mu_h)I_{vm}^{k-1}+\eta_2\beta I_{cv}^{k-1}R_{cm}^{k-1}+\eta_2\beta I_{vm}^{k-1}R_{cm}^{k-1}\Bigg)\times\Bigg(h(t_k-h(t_{k-1}))\Bigg)$$
$$+\frac{23}{12}\Bigg(w_{cm}\beta I_{cv}^{k}S_{cm}^{k}-w_{cm}\delta_{cm}\beta I_{vm}^{k}S_{cm}^{k}+(\delta_2+\mu_h)I_{vm}^{k}+\eta_2\beta I_{cv}^{k}R_{cm}^{k}$$
$$+\eta_2\beta I_{vm}^{k}R_{cm}^{k}\Bigg)\times\Bigg(h(t_{k+1}-h(t_k))\Bigg)\Bigg]$$
$$+\frac{q}{M(q)}\Bigg[\frac{5}{12}P_5(t_{k-2},I_{vm}^{k-2})h(t_{k-1}-h(t_{k-2}))(C_5(t_{k-1})-C_5(t_{k-2}))$$
$$-\frac{4}{3}P_5(t_{k-1},I_{vm}^{k-1})h(t_k-h(t_{k-1}))(C_5(t_k)-C_5(t_{k-1}))$$
$$\frac{23}{12}P_5(t_k,I_{vm}^{k})h(t_{k+1}-h(t_k))(C_5(t_{k+1})-C_5(t_k))\Bigg].$$

(40)

$$R_{cm}(t_{k+1}) = R_{cm}(t_k)+\frac{1-q}{M(q)}\Bigg[\frac{h(t_{k+1}-h(t_k))}{!t}\Bigg(\delta_2 I_{vm}^{k+1}-\mu_h R_{cm}^{k+1}-\eta_2\beta I_{cv}^{k+1}R_{cm}^{k+1}$$
$$-\eta_2\beta I_{vm}^{k+1}R_{cm}^{k+1}\Bigg)-\frac{h(t_k-h(t_{k-1}))}{!t}\Bigg(\delta_2 I_{vm}^{k}-\mu_h R_{cm}^{k}-\eta_2\beta I_{cv}^{k}R_{cm}^{k}-\eta_2\beta I_{vm}^{k}R_{cm}^{k}\Bigg)\Bigg]$$
$$+\frac{1-q}{M(q)}\Bigg[(C_6(t_{k+1})-C_6(t_k))\frac{h(t_{k+1}-h(t_k))}{!t}P_6(t_{k+1},R_{cm}^{k+1})-(C_6(t_k)$$
$$-C_6(t_{k-1}))\frac{h(t_k-h(t_{k-1}))}{!t}P_6(t_k,R_{cm}^{k})\Bigg]$$
$$+\frac{q}{M(q)}\Bigg[\frac{5}{12}\Bigg(\delta_2 I_{vm}^{k-2}-\mu_h R_{cm}^{k-2}-\eta_2\beta I_{cv}^{k-2}R_{cm}^{k-2}-\eta_2\beta I_{vm}^{k-2}R_{cm}^{k-2}\Bigg)$$
$$\times\Bigg(h(t_{k-1}-h(t_{k-2}))\Bigg)-\frac{4}{3}\Bigg(\delta_2 I_{vm}^{k-1}-\mu_h R_{cm}^{k-1}-\eta_2\beta I_{cv}^{k-1}R_{cm}^{k-1}$$
$$-\eta_2\beta I_{vm}^{k-1}R_{cm}^{k-1}\Bigg)\times\Bigg(h(t_k-h(t_{k-1}))\Bigg)+\frac{23}{12}\Bigg(\delta_2 I_{vm}^{k}-\mu_h R_{cm}^{k}-\eta_2\beta I_{cv}^{k}R_{cm}^{k}$$
$$-\eta_2\beta I_{vm}^{k}R_{cm}^{k}\Bigg)\times\Bigg(h(t_{k+1}-h(t_k))\Bigg)\Bigg]$$
$$+\frac{q}{M(q)}\Bigg[\frac{5}{12}P_6(t_{k-2},R_{cm}^{k-2})h(t_{k-1}-h(t_{k-2}))(C_6(t_{k-1})-C_6(t_{k-2}))$$
$$-\frac{4}{3}P_6(t_{k-1},R_{cm}^{k-1})h(t_k-h(t_{k-1}))(C_6(t_k)-C_6(t_{k-1}))$$
$$\frac{23}{12}P_6(t_k,R_{cm}^{k})h(t_{k+1}-h(t_k))(C_6(t_{k+1})-C_6(t_k))\Bigg].$$

6. Numerical Simulation and Discussions

In this section we present the numerical simulation of the coronavirus with comorbiidity model (5) hinged on the exponential law. The Adams–Bashforth scheme as extensively presented in [24] was utilized in solving the model (5). The step size used for this study is 0.001 with time interval [0,120]. The following parameter values used for the simulations are $\Lambda_h = 0.8$, $\beta_{cv} = 0.5$, $\phi_{cm} = 0.6$, $\mu_h = 0.0001$, $w_{cm} = 0.04$, $\delta_1 = 0.05$, $\eta_1 = 0.04$, $\delta_2 = 0.6$, $\eta_2 = 0.5$, $\delta_{cm} = 0.6$.

Figure 1(a) captures the dynamics of the susceptible individuals $S_h(t)$ and as the fractional order derivative increases the number of susceptible individuals decreases. This is reduction of virgin population which is common to many epidemiological models. Figure 1(b) depicts the susceptible individuals with comorbidity $S_{cm}(t)$ in the community. The number of individuals in this class decreases as the fractional order derivative increases. In Figure 1(c) the number of individuals infected with coronavirus $I_{cv}(t)$ increases as the fractional derivative also increases. Figure 1(d) shows the number of individuals recovered from coronavirus and they increase as the fractional order derivative increases. Figure 1(e) represents the co-infected individuals with coronavirus and comorbidity $I_{vm}(t)$ and the number individuals in this class increases as the fractional order increases from 0.75 to 1. Figure 1(f) shows the number of individuals recovered from comorbidity $R_{cm}(t)$ and they increase as the fractional order increases. This result predicts that humans with such complications are taking the right health decisions.

Figure 2(a) is the susceptible humans $S_h(t)$ and as the fractional-order derivative increases the number of susceptible humans decreases. The result is similar to that of Figure 1(a), however, the random effect can be observed in this situation. Figure 2(b) shows the susceptible humans with comorbidity $S_{cm}(t)$ in the community. The number of comorbidity susceptible humans decreases as the fractional-order derivative increases. Figure 2(c) represents the number of individuals infected with coronavirus $I_{cv}(t)$ and it increases as the fractional derivative also increases. The random-effects suggest that the daily infection is not constant. This rise of infection perceived in Figure 2(c) is mostly characterized by epidemiological models of this nature. Figure 2(d) indicates the number of individuals recovered from coronavirus $R_{cv}(t)$ and the number of individuals increases as the fractional-order derivative increases. Figure 2(e) shows the dynamics of the co-infected humans with coronavirus and comorbidity $I_{vm}(t)$ and the number of individuals move up as the fractional-order derivatives increase from 0.75 to 1. Figure 2(f) represents that the number of individuals recovered from comorbidity $R_{cm}(t)$ and increases as the fractional order increases. This may suggest that individuals with comorbidity are undertaking the appropriate health decisions within the community.

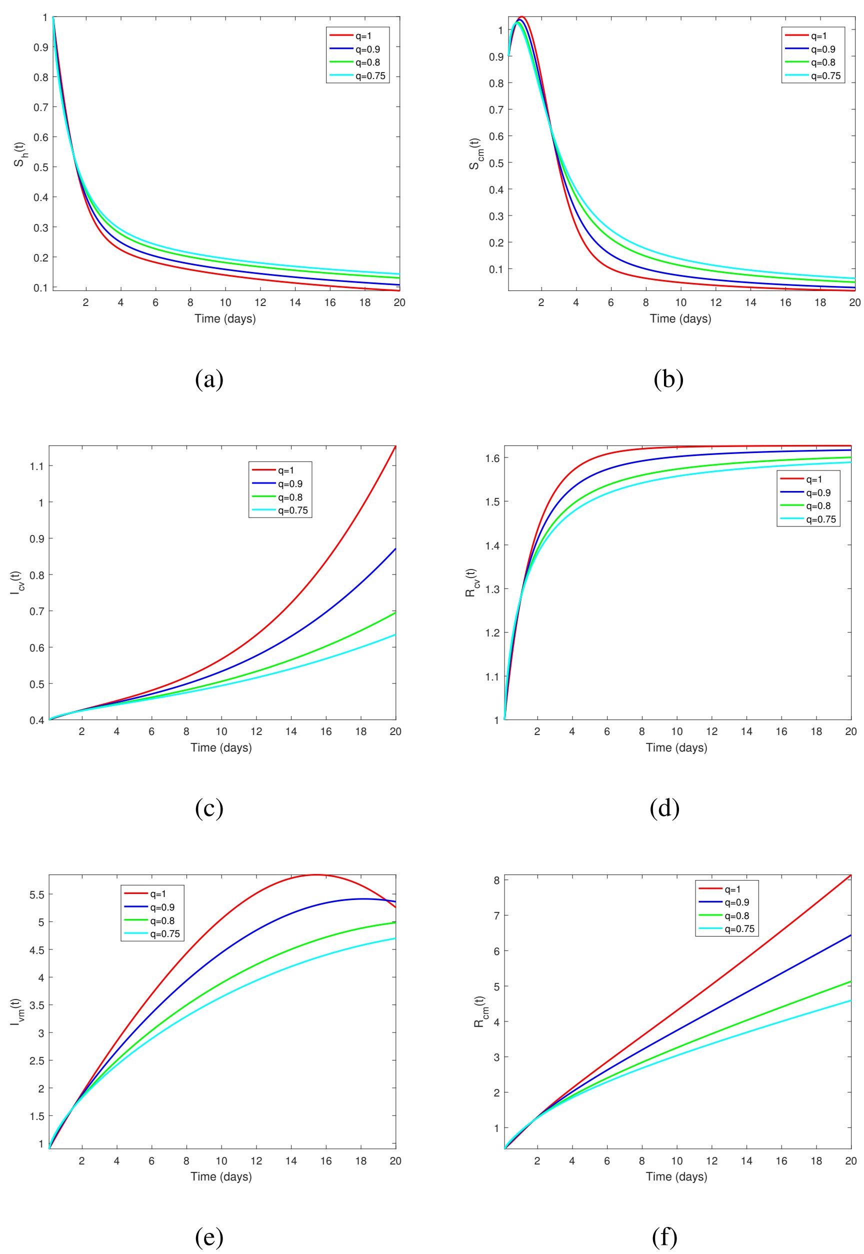

Figure 1. Simulation results for model (5), exponential law at $q = 1, 0.9, 0.80, 0.75$

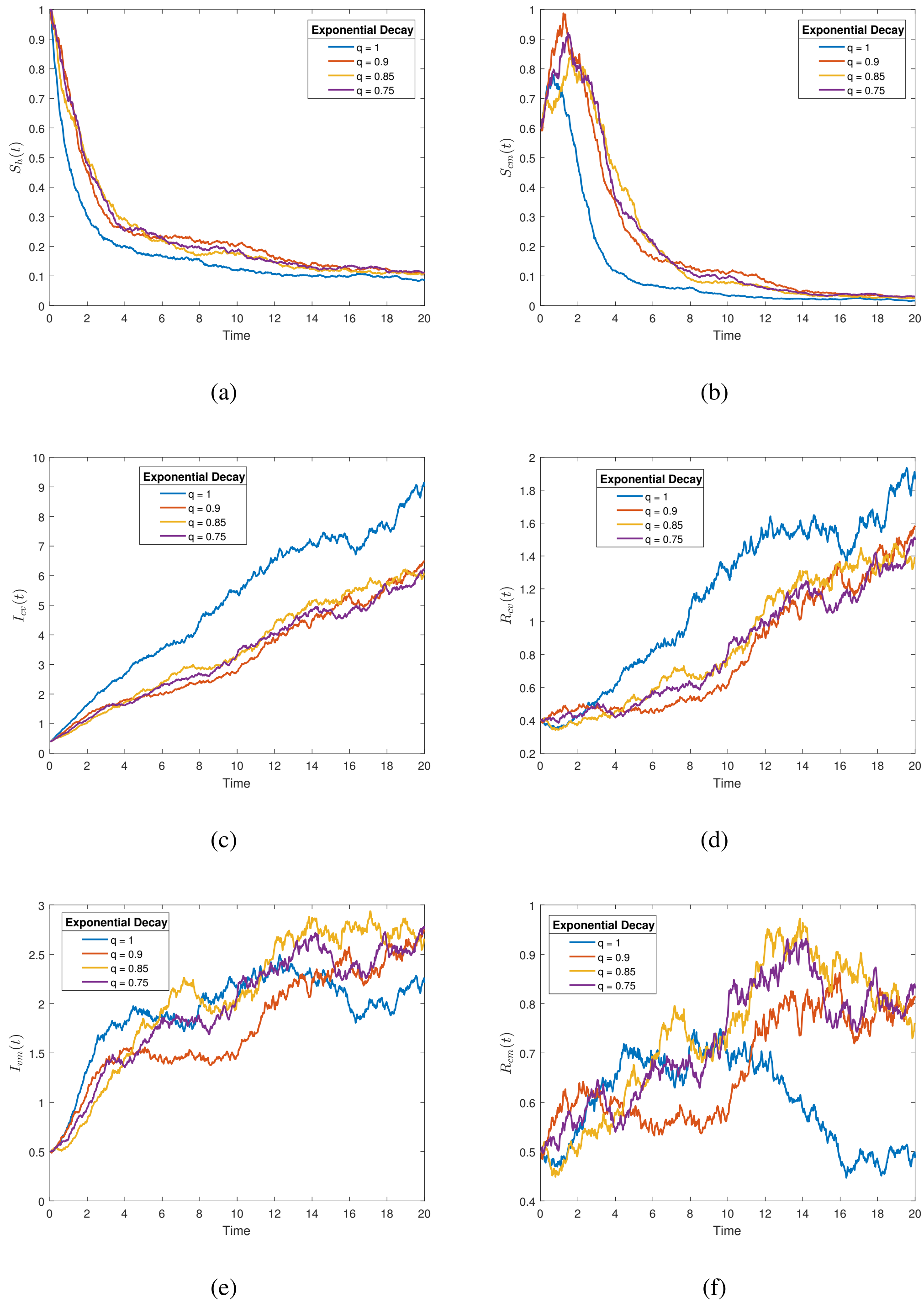

Figure 2. Simulations for fractional stochastic COVID-19 comorbidity model (33) via exponential decay law at $q = 1, 0.9, 0.85, 0.75$

7. Conclusion

In this work, we examined the dynamics of coronavirus with comorbidity in a community. The steady states of the model were established and reproduction number was also determined. Exponential law was applied to study the dynamics of the coronavirus with comorbidity by establishing the existence and uniqueness of solutions of the model using fixed point theory. A fractional stochastic approach in the light of exponential decay law was employed to analyse the same model. The numerical simulation results suggested that fractional-order derivative and parameter values have a serious impact on the dynamics of the fractional coronavirus with a comorbidity model. Similar results were obtained for the stochastic model, However, unlike the fractional-order model, the stochastic model exhibits some random effect. From an epidemiological viewpoint, comorbidity individuals acquire more re-infection due to lack of surveillance and precautions like wearing masks, social. Also, the increase of comorbidity development and the fractional- order derivative factor simultaneously increases the prevalence of the infection which can lead to a disaster situation. The disease can thus be controlled if comorbidity individuals observe all the above mentioned precautionary measures. The fractional stochastic numerical simulation results show that the random nature of the infection is not fixed as in the fractional deterministic model. It is recommended that complex phenomena be investigated using the fractional stochastic perspectives in order to present the randomness nature of the spread of many diseases.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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