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A FRACTIONAL DYNAMICS OF A POTATO DISEASE MODEL

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Abstract. Globally, potato is one of the staple foods eaten by a lot of people. It processed into different kinds of food for mankind. Climate change has brought a lot of changes with respect to the output of global food stock due to problems such as drought, diseases, etc. In this study, a potato disease model is formulated in a fractional-order derivative with the nonlocal and nonsingular operator (AB). The reproduction number of the potato model and the steady states are determined. The existence and uniqueness of solutions are established using the Banach space approach, and Hyers-Ulam stability is carried out to determine if the existence and uniqueness solution is stable. A numerical simulation is carried out with and without stochastic components, which indicates a similar result. However, the stochastic aspect depicts a random effect. It is established that the fractional-order derivative has effect on the dynamics of the potato disease.

Keywords: Hyers-Ulam stability; existence and uniqueness; Mittag-Leffler function; stochastic; random effect.
2010 AMS Subject Classification: 92C80.

1. Introduction

Potato is a starchy rich food originated from Andes of South America. Today, potato related foods can be found in every continent using technology. This important crop is vulnerable to a number of diseases, including a viral one, which is more dangerous to the survival of the crop.

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One of the most common viral diseases of the crop is the Potato leaf roll [13]. The disease accounts for about 50% of the global loss of potato yield. The disease is found in all parts of the world where potato crop production is undertaken. In quantifying, the amount of volume of Potatoes tonnage lost annually due to this disease is roughly around 20 million tons [13].

The basic infection of potatoes occurs when they are bitten by a virus named Caring Aphid. This aphis obtains the virus through the infected Potatoes with leaf roll. So far as the plants have been infected by the aphid, they contain the disease throughout their life cycles. Another mode of infection is the planting of infectious plants, and this is known as the secondary infection as the tuber from the beginning becomes infectious. Mathematical models present qualitative information in the absence of experimental data, which is costly and time-consuming to carry out [9, 12]. Numerous plant-related mathematical models have been constructed to explain the dynamics of existing situations[19, 20, 21]. Modeling using fractional order derivative from literature is better than the classical derivative [4, 5]. Of the existing fractional-order operators, the Atangana-Baleanu operator based on the generalised Mittag-Leffler function is preferred because it is nonsingular and nonlocal [1, 4, 15, 16]. It has crossover property which allows it to stretch from one operator to one another, leading to good predictions [1].

The author in [6] constructed an epidemiological model including vector population dynamics with respect to Africa cassava mosaic virus disease. In [7], the authors developed a mathematical model solely for pest-insect control employing mating disruption and trapping. Chapwanya and Dumont [8] investigated crop vector-borne disease, specifically the impact of virus lifespan and contact rate on the traveling–wave speed of infective fronts. Authors in [10], constructed a mathematical model on guava plants and examined the pest control relationship using the fractional calculus concept. In [11], the authors developed a mathematical model of vector-borne plant disease incorporating the memory on the host and vector.

To the best of the researcher's knowledge, there is no existing study incorporating the Mittag-Leffler function in deterministic and stochastic forms. The study compares the two scenarios' numerical analysis results.

2. Preliminaries

This section presents few imperative mathematical concepts necessary to further analyse the rest of the work. These crucial concepts include the following definitions and theorems associated with the operators:

3. Mathematical Preliminaries

This section presents few imperative mathematical concepts necessary to further analyse the rest of the work. These crucial concepts include the following definitions and theorems associated with the operators:

(1)
$$ {}_{0}^{C}D_t^q f(t) = \frac{1}{T(1-q)} \int_0^t \frac{d}{d\varpi} f(\varpi)(t-\varpi)^{-q} d\varpi $$

(2)
$$ {}_{0}^{RL}D_t^q f(t) = \frac{1}{T(1-q)} \frac{d}{dt} \int_0^t f(\varpi)(t-\tau)^{-q} d\varpi $$

(3)
$$ {}_{0}^{CF}D_t^q f(t) = \frac{M(q)}{1-q} \int_0^t \frac{d}{d\varpi} f(\varpi) \exp\left[-\frac{q}{1-q}(t-\varpi)\right] d\varpi $$

(4)
$$ {}_{0}^{AB}D_t^q f(t) = \frac{AB(q)}{1-q} \int_0^t \frac{d}{dt} f(\varpi) E_q\left[-\frac{q}{1-q}(t-\varpi)^q\right] d\tau $$

(5)
$$ {}_{0}^{ABR}D_t^q f(t) = \frac{AB(q)}{1-q} \frac{d}{dt} \int_0^t f(\varpi) E_q\left[-\frac{q}{1-q}(t-\varpi)^q\right] d\varpi $$

where

$$ E_q(-t^q) = \sum_{k=0}^{\infty} \frac{(-t)^{qk}}{T(qk+1)} $$

4. Main Results

5. Mathematical Model Formulation

The model is a modified version of Gatachew et al. [13] in a fractionalised form which comprises potato and vector population respectively with bi-linear incidence rate. The total Potato population $(N_a(t))$ at time t, is subdivided into susceptible potato $(S_a(t))$ exposed potato $(E_a(t))$ and infected potato $(I_a(t))$. The total vector population $(N_b(t))$ is partitioned into susceptible vector $(S_b(t))$ and infected vector (I_b). The potato recruitment rate is denoted by Λ_a and force of infection of susceptible potato is given by $\frac{2bS_a(t)I_b(t)}{S_a(t)+I_b(t)}$. The rate of exposed class move into infectious individual class is denoted by ϕ. γ_1 is the induced mortality rate and γ_2 is the rate of removing potato infected with disease. The natural mortality rate of potato is μ_a. The rate of infection of susceptible vector is denoted by c and Λ_b is the recruitment rate of vector. μ_b is the natural mortality of the vector.

$$
\begin{cases}
{}^{AB}D^q_{0,t}[S_a(t)] = \Lambda_a - \frac{2bS_a(t)I_b(t)}{S_a(t)+I_b(t)} - \mu_a S_a(t), \\
{}^{AB}D^q_{0,t}[E_a(t)] = \frac{2bS_a(t)I_b(t)}{S_a(t)+I_b(t)} - (\mu_a+\phi)E_a(t), \\
{}^{AB}D^q_{0,t}[I_a(t)] = \phi E_a(t) - (\mu_a+\gamma_1+\gamma_2)I_a(t), \\
{}^{AB}D^q_{0,t}[S_b(t)] = \Lambda_b - cS_b(t)I_a(t) - \mu_b S_b(t), \\
{}^{AB}D^q_{0,t}[I_b(t)] = cS_b(t)I_a(t) - \mu_b I_b(t), \\
S_a(t) > 0, E_a(t) \geq 0, I_a(t) \geq 0, S_b(t) > 0, I_b(t) \geq 0.
\end{cases} \tag{6}
$$

6. Existence and Uniqueness of the Coupled Solutions

In this section, Banach space by $D(\Xi)$ with $\Xi = [0,b]$, which possesses a real valued continuous function with sup norm where $Q = D(\Xi)\times D(\Xi)\times D(\Xi)\times D(\Xi)\times D(\Xi)$ with norm $\|(S_a,E_a,I_a,S_b,I_b)\| = \|S_a\|+\|E_a\|+\|I_a\|+\|S_b\|+\|I_b\|$ where $\|S_a\| = \sup_{t\in J}|S_a(t)|, \|E_a\| =$

$\sup_{t\in J}|E_a(t)|, \|I_a\| = \sup_{t\in J}|I_a(t)|, \|S_b\| = \sup_{t\in J}|I_b(t)|, \|I_b\| = \sup_{t\in J}|I_b(t)|$ Utilising AB integral operator with respect to (6). The following results is obtained:

(7)
$$\begin{cases} S_a(t)-S_a(0) = {}_0^{AB}D_{0,t}^{q}[S_a(t)]\left\{\Lambda_a - \frac{2bS_a(t)I_b(t)}{S_a(t)+I_b(t)} - \mu_a S_a(t),\right\} \\ E_a(t)-E_a(0) = {}_0^{AB}D_{0,t}^{q}[E_a(t)]\left\{\frac{2bS_a(t)I_b(t)}{S_a(t)+I_b(t)} - (\mu_a+\phi)E_a(t),\right\} \\ I_a(t)-I_a(0) = {}_0^{AB}D_{0,t}^{q}[I_a(t)]\left\{\phi E_a(t) - (\mu_a+\gamma_1+\gamma_2)I_a(t),\right\} \\ S_b(t)-S_b(0) = {}_0^{AB}D_{0,t}^{q}[S_b(t)]\left\{\Lambda_b - cS_b(t)I_a(t) - \mu_b S_b(t),\right\} \\ I_b(t)-I_b(0) = {}_0^{AB}D_{0,t}^{q}[I_b(t)]\left\{cS_b(t)I_a(t) - \mu_b I_b(t).\right\} \end{cases}$$

Following the mathematical preliminaries, the following is arrived at:

(8)
$$\begin{aligned} S_a(t)-S_a(0) &= \tfrac{1-q}{AB(q)}R_1(q,t,S_a(t)) + \tfrac{q}{AB(q)\Gamma(q)} \times \int_0^t (t-\varpi)^{q-1}R_1(q,\varpi,S_a(\varpi))d\varpi, \\ E_a(t)-E_a(0) &= \tfrac{1-q}{AB(q)}R_2(q,t,E_a(t)) + \tfrac{q}{AB(q)\Gamma(q)} \times \int_0^t (t-\varpi)^{q-1}R_2(q,\varpi,E_a(\varpi))d\varpi, \\ I_a(t)-I_a(0) &= \tfrac{1-q}{AB(q)}R_3(q,t,I_a(t)) + \tfrac{q}{AB(q)\Gamma(q)} \times \int_0^t (t-\varpi)^{q-1}R_3(q,\varpi,I_a(\varpi))d\varpi, \\ S_b(t)-S_b(0) &= \tfrac{1-q}{AB(q)}R_4(q,t,S_b(t)) + \tfrac{q}{AB(q)\Gamma(q)} \times \int_0^t (t-\varpi)^{q-1}R_4(q,\varpi,S_b(\varpi))d\varpi, \\ I_b(t)-I_b(0) &= \tfrac{1-q}{AB(q)}R_5(q,t,I_b(t)) + \tfrac{q}{AB(q)\Gamma(q)} \times \int_0^t (t-\varpi)^{q-1}R_5(q,\varpi,I_b(\varpi))d\varpi. \end{aligned}$$

where,

(9)
$$\begin{aligned} R_1(q,t,S_a(t)) &= \Lambda_a - \tfrac{2bS_a(t)I_b(t)}{S_a(t)+I_b(t)} - \mu_a S_a(t), \\ R_2(q,t,E_a(t)) &= \tfrac{2bS_a(t)I_b(t)}{S_a(t)+I_b(t)} - (\mu_a+\phi)E_a(t), \end{aligned}$$

$$R_3(q,t,I_a(t)) = \phi E_a(t) - (\mu_a + \gamma_1 + \gamma_2) I_a(t),$$

$$R_4(q,t,S_b(t)) = \Lambda_b - cS_b(t)I_a(t) - \mu_b S_b(t),$$

$$R_5(q,t,I_b(t)) = cS_b(t)I_a(t) - \mu_b I_b(t).$$

The R_1, R_2, R_3, R_4 and R_5 fulfil Lipschitz condition only whenever $S_a(t), E_a(t), I_a(t), S_b(t)$ and $I_b(t)$ have an upper bound. If $S_a(t)$ and $S_a^*(t)$ are considered as couple functions, then one has,

$$\|R_1(q,t,S_a(t)) - R_1(q,t,S_a^*(t))\| = \left\| -\left(\frac{2I_b(t)}{S_a(t)+I_a(t)} + \mu_a\right)(S_a(t) - S_a^*(t)) \right\| \tag{10}$$

Considering,

$$\psi_1 = \left\| -\left(\frac{2I_b(t)}{S_a(t)+I_a(t)} + \mu_a\right) \right\|$$

one arrives at the following,

$$\|R_1(q,t,S_a(t)) - R_1(q,t,S_a^*(t))\| \leq \psi_1 \|S_a(t) -, S_a^*(t)\| \tag{11}$$

Following a similar approach, one obtains the following:

$$\begin{aligned}
&\|R_2(q,t,E_a(t)) - R_2(q,t,E_a^*(t))\| \leq \psi_2 \|E_a(t) -, E_a^*(t)\|, \\
&\|R_3(q,t,I_a(t)) - R_3(q,t,I_a^*(t))\| \leq \psi_3 \|I_a(t) -, I_a^*(t)\|, \\
&\left\|R_4(q,t,S_b(t)) - R_2(q,t,S_b^*(t))\right\| \leq \psi_4 \left\|S_b(t) -, S_b^*(t)\right\|, \\
&\left\|R_5(q,t,I_b(t)) - R_5(q,t,I_b^*(t))\right\| \leq \psi_5 \left\|I_b(t) -, I_b^*(t)\right\|.
\end{aligned} \tag{12}$$

where,

$$\psi_2 = (\mu_a + \phi), \psi_3 = (\mu_a + \gamma_1 + \gamma_2), \psi_4 = \mu_b, \psi_5 = \mu_a$$

This suggests that the Lipschitz condition has embraced all the five functions and in a recursive manner the equation (7) gives:

(13)

$$S_{an}(t) - S_a(0) = \tfrac{1-q}{AB(q)} R_1(q,t,S_{an-1}(t)) + \tfrac{q}{AB(q)\Gamma(q)} \times \int_0^t (t-\varpi)^{q-1} R_1(q,\varpi,S_{an-1}(\varpi)) d\varpi,$$

$$E_{an}(t) - E_a(0) = \tfrac{1-q}{AB(q)} R_2(q,t,E_{an-1}(t)) + \tfrac{q}{AB(q)\Gamma(q)} \times \int_0^t (t-\varpi)^{q-1} R_2(q,\varpi,E_{an-1}(\varpi)) d\varpi,$$

$$I_{an}(t) - I_a(0) = \tfrac{1-q}{AB(q)} R_3(q,t,I_{an-1}(t)) + \tfrac{q}{AB(q)\Gamma(q)} \times \int_0^t (t-\varpi)^{q-1} R_3(q,\varpi,I_{an-1}(\varpi)) d\varpi,$$

$$S_{bn}(t)-S_b(0)=\frac{1-q}{AB(q)}R_4(q,t,S_{bn-1}(t))+\frac{q}{AB(q)\Gamma(q)}\times\int_0^t(t-\varpi)^{q-1}R_4(q,\varpi,S_{bn-1}(\varpi))d\varpi,$$
$$I_{bn}(t)-I_b(0)=\frac{1-q}{AB(q)}R_5(q,t,I_{bn-1}(t))+\frac{q}{AB(q)\Gamma(q)}\times\int_0^t(t-\varpi)^{q-1}R_5(q,\varpi,I_{bn-1}(\varpi))d\varpi.$$

in addition to $S_{a_0}(t)=S_a(0),E_{a_0}(t)=E_a(0),I_{a_0}(t)=I_a(0),S_{b_0}(t)=S_b(0),I_{b_0}(t)=I_b(0)$. By considering successive terms difference the following results are yielded:

$$\begin{aligned}
&\Phi_{S_a,n}(t)=S_{a_n}(t)-S_{a_{n-1}}(t)-\tfrac{1-q}{AB(q)}\left(R_1(q,t,S_{a_{n-1}}(t))-R_1(q,t,S_{a_{n-2}}(t))\right)\\
&+\tfrac{q}{AB(q)\Gamma(q)}\int_0^t(t-\varpi)^{q-1}\left(R_1(q,\varpi,S_{a_{n-1}}(t))-R_1(q,\varpi,S_{a_{n-2}}(\varpi))\right)d\varpi,\\
&\Phi_{E_a,n}(t)=E_{a_n}(t)-E_{a_{n-1}}(t)-\tfrac{1-q}{AB(q)}\left(R_2(q,t,E_{a_{n-1}}(t))-R_2(q,t,E_{a_{n-2}}(t))\right)\\
&+\tfrac{q}{AB(q)\Gamma(q)}\int_0^t(t-\varpi)^{q-1}\left(R_2(q,\varpi,E_{a_{n-1}}(t))-R_2(q,\varpi,E_{a_{n-2}}(\varpi))\right)d\varpi,\\
&\Phi_{I_a,n}(t)=I_{a_n}(t)-I_{a_{n-1}}(t)-\tfrac{1-q}{AB(q)}\left(R_3(q,t,I_{a_{n-1}}(t))-R_3(q,t,I_{a_{n-2}}(t))\right)\\
&+\tfrac{q}{AB(q)\Gamma(q)}\int_0^t(t-\varpi)^{q-1}\left(R_3(q,\varpi,I_{a_{n-1}}(t))-R_3(q,\varpi,I_{a_{n-2}}(\varpi))\right)d\varpi,\\
&\Phi_{S_b,n}(t)=S_{b_n}(t)-S_{b_{n-1}}(t)-\tfrac{1-q}{AB(q)}\left(R_4(q,t,S_{b_{n-1}}(t))-R_4(q,t,S_{b_{n-2}}(t))\right)\\
&+\tfrac{q}{AB(q)\Gamma(q)}\int_0^t(t-\varpi)^{q-1}\left(R_4(q,\varpi,S_{a_{n-1}}(t))-R_4(q,\varpi,S_{a_{n-2}}(\varpi))\right)d\varpi,\\
&\Phi_{I_b,n}(t)=I_{a_n}(t)-I_{a_{n-1}}(t)-\tfrac{1-q}{AB(q)}\left(R_5(q,t,I_{a_{n-1}}(t))-R_5(q,t,I_{a_{n-2}}(t))\right)\\
&+\tfrac{q}{AB(q)\Gamma(q)}\int_0^t(t-\varpi)^{q-1}\left(R_5(q,\varpi,I_{b_{n-1}}(t))-R_5(q,\varpi,I_{b_{n-2}}(\varpi))\right)d\varpi.
\end{aligned}\tag{14}$$

It is imperative to note that,

$$\begin{aligned}
&S_{a_n}(t)=\sum_{i=0}^n\Phi_{S_a,i}(t),E_{a_n}(t)=\sum_{i=0}^n\Phi_{E_a,i}(t),I_{a_n}(t)=\sum_{i=0}^n\Phi_{I_a,i}(t),\\
&S_{b_n}(t)=\sum_{i=0}^n\Phi_{S_b,i}(t),I_{b_n}(t)=\sum_{i=0}^n\Phi_{I_b,i}(t).
\end{aligned}$$

Furthermore, utilizing Eqs. (11) and (12) and having in mind that,

$$\begin{aligned}
&\Phi_{S_a,n-1}(t)=S_{a_{n-1}}(t)-S_{a_{n-2}}(t),\Phi_{E_a,n-1}(t)=E_{a_{n-1}}(t)-E_{a_{n-2}}(t),\Phi_{I_a,n-1}(t)=I_{a_{n-1}}(t)-I_{a_{n-2}}(t)\\
&\Phi_{S_b,n-1}(t)=S_{b_{n-1}}(t)-S_{b_{n-2}}(t),\Phi_{I_b,n-1}(t)=I_{b_{n-1}}(t)-I_{b_{n-2}}(t).
\end{aligned}$$

This leads to

$$\begin{aligned}
&\left\|\Phi_{S_a,n}(t)\right\|\leq\tfrac{1-q}{AB(q)}\psi_1\left\|\Phi_{S_a,n-1}(t)\right\|\tfrac{q}{AB(q)\Gamma(q)}\psi_1\times\int_0^t(t-\varpi)^{q-1}\left\|\Phi_{S_a,n-1}(\varpi)\right\|d\varpi,\\
&\left\|\Phi_{E_a,n}(t)\right\|\leq\tfrac{1-q}{AB(q)}\psi_2\left\|\Phi_{E_a,n-1}(t)\right\|\tfrac{q}{AB(q)\Gamma(q)}\psi_2\times\int_0^t(t-\varpi)^{q-1}\left\|\Phi_{E_a,n-1}(\varpi)\right\|d\varpi,\\
&\left\|\Phi_{I_a,n}(t)\right\|\leq\tfrac{1-q}{AB(q)}\psi_3\left\|\Phi_{I_a,n-1}(t)\right\|\tfrac{q}{AB(q)\Gamma(q)}\psi_3\times\int_0^t(t-\varpi)^{q-1}\left\|\Phi_{I_a,n-1}(\varpi)\right\|d\varpi,
\end{aligned}\tag{15}$$

$$\left\|\Phi_{S_b,n}(t)\right\| \leq \tfrac{1-q}{AB(q)}\psi_4\left\|\Phi_{S_b,n-1}(t)\right\| \tfrac{q}{AB(q)\Gamma(q)}\psi_4 \times \int_0^t (t-\varpi)^{q-1}\left\|\Phi_{S_b,n-1}(\varpi)\right\| d\varpi,$$
$$\left\|\Phi_{I_b,n}(t)\right\| \leq \tfrac{1-q}{AB(q)}\psi_5\left\|\Phi_{I_b,n-1}(t)\right\| \tfrac{q}{AB(q)\Gamma(q)}\psi_5 \times \int_0^t (t-\varpi)^{q-1}\left\|\Phi_{I_b,n-1}(\varpi)\right\| d\varpi.$$

Theorem 1. *Considering that the following condition exists*

$$\frac{1-q}{AB(q)}\psi_i + \frac{q}{AB(q)\Gamma(q)}d^q\psi_i < 1,\ i,1,2,3,...,5. \tag{16}$$

Then, (6) possesses a distinctive solution for $t \in [0,b]$.

Proof. It is demonstrated, $S_a(t), E_a(t), I_a(t), S_b(t), I_b(t)$ are bounded functions. Furthermore, as can observe with respect to Eqs. (11) and (12), the symbols R_1, R_2, R_3, R_4 and R_5 represent or hold for the Lipchitz condition. Hence, making use of Eq. (15) together with recursive hypothesis yields

$$\begin{aligned}
\left\|\Phi_{S_a,n}(t)\right\| &\leq \|S_{a_0}(t)\|\left(\tfrac{1-q}{AB(q)}\psi_1 + \tfrac{qd^q}{AB(q)\Gamma(q)}\psi_1\right)^n,\\
\left\|\Phi_{E_a,n}(t)\right\| &\leq \|E_{a_0}(t)\|\left(\tfrac{1-q}{AB(q)}\psi_2 + \tfrac{qd^q}{AB(q)\Gamma(q)}\psi_2\right)^n,\\
\left\|\Phi_{I_a,n}(t)\right\| &\leq \|I_{a_0}(t)\|\left(\tfrac{1-q}{AB(q)}\psi_3 + \tfrac{qd^q}{AB(q)\Gamma(q)}\psi_3\right)^n,\\
\left\|\Phi_{S_b,n}(t)\right\| &\leq \left\|S_{b_0}(t)\right\|\left(\tfrac{1-q}{AB(q)}\psi_4 + \tfrac{qd^q}{AB(q)\Gamma(q)}\psi_4\right)^n,\\
\left\|\Phi_{I_b,n}(t)\right\| &\leq \left\|I_{b_0}(t)\right\|\left(\tfrac{1-q}{AB(q)}\psi_5 + \tfrac{qd^q}{AB(q)\Gamma(q)}\psi_5\right)^n.
\end{aligned} \tag{17}$$

□

Therefore, one can simply infer that these sequences exist and satisfy

$$\left\|\Phi_{S_a,n}(t)\right\| \to 0, \left\|\Phi_{E_a,n}(t)\right\| \to 0, \left\|\Phi_{I_a,n}(t)\right\| \to 0, \left\|\Phi_{S_b,n}(t)\right\| \to 0, \left\|\Phi_{I_b,n}(t)\right\| \to 0 \ as\ n \to \infty$$

In addtion, from Eq. (17) and making use of the triangle inequality, for any k , one arrives at the following

$$\begin{aligned}
\left\|S_{a_{n+k}}(t) - S_{a_n}(t)\right\| &\leq \sum_{j=n+1}^{n+k} U_1^j = \tfrac{U_1^{n+1}-U_1^{n+k+1}}{1-U_1}\\
\left\|E_{a_{n+k}}(t) - E_{a_n}(t)\right\| &\leq \sum_{j=n+1}^{n+k} U_2^j = \tfrac{U_2^{n+1}-U_2^{n+k+1}}{1-U_2}\\
\left\|I_{a_{n+k}}(t) - I_{a_n}(t)\right\| &\leq \sum_{j=n+1}^{n+k} U_3^j = \tfrac{U_3^{n+1}-U_3^{n+k+1}}{1-U_3}\\
\left\|S_{b_{n+k}}(t) - S_{b_n}(t)\right\| &\leq \sum_{j=n+1}^{n+k} U_4^j = \tfrac{U_4^{n+1}-U_4^{n+k+1}}{1-U_4}\\
\left\|I_{b_{n+k}}(t) - I_{b_n}(t)\right\| &\leq \sum_{j=n+1}^{n+k} U_5^j = \tfrac{U_5^{n+1}-U_5^{n+k+1}}{1-U_5}
\end{aligned} \tag{18}$$

with $U_i = \frac{1-q}{AB(q)}\psi_i + \frac{q}{AB(q)\Gamma(q)}d^q\psi_i < 1$ by an assumption. Hence, $S_{a_n}, E_{a_n}, I_{a_n}, S_{b_n}, I_{b_n}$ is considered as a Cauchy sequences in the light of Banach space B (J). This demonstrates that these sequences are uniformly convergent. Applying the limit theorem with regard to Eq. (14) as $n \to \infty$ supports the fact that the limit of these sequences possess the unique solution of (6). This presents the existence of a single solution for Eq. (6) under the condition established in (16).

7. Hyers–Ulam Stability and Approximation Technique

Definition 7.1. *The AB fractional integral Eq. (6) is considered to be Hyers–Ulam stable whenever there exists a constant $\Delta_i > 0, i \in N^5$ satisfying: Given that $q_i > 0, i \in N^5$, for*

$$\left| S_a(t) - \frac{1-q}{AB(q)}R_1(q,t,S_a(t)) + \frac{q}{AB(q)\Gamma(q)} \times \int_0^t (t-\varpi)^{q-1}R_1(q,\varpi,S_a(\varpi))d\varpi \right| \le \xi_1,$$

$$\left| E_a(t) - \frac{1-q}{AB(q)}R_2(q,t,E_a(t)) + \frac{q}{AB(q)\Gamma(q)} \times \int_0^t (t-\varpi)^{q-1}R_2(q,\varpi,E_a(\upsilon))d\varpi \right| \le \xi_2,$$

$$\left| I_a(t) - \frac{1-q}{AB(q)}R_3(q,t,I_a(t)) + \frac{q}{AB(q)\Gamma(q)} \times \int_0^t (t-\varpi)^{q-1}R_3(q,\varpi,I_a(\varpi))d\varpi \right| \le \xi_3, \tag{19}$$

$$\left| S_b(t) - \frac{1-q}{AB(q)}R_4(q,t,S_b(t)) + \frac{q}{AB(q)\Gamma(q)} \times \int_0^t (t-\varpi)^{q-1}R_4(q,\varpi,S_b(\varpi))d\varpi \right| \le \xi_4,$$

$$\left| I_b(t) - \frac{1-q}{AB(q)}R_5(q,t,I_b(t)) + \frac{q}{AB(q)\Gamma(q)} \times \int_0^t (t-\varpi)^{q-1}R_5(q,\varpi,I_b(\varpi))d\varpi \right| \le \xi_5.$$

there exist $\left(\dot{S}_a(t), \dot{E}_a(t), \dot{I}_a(t), \dot{S}_b(t), \dot{I}_b(t)\right)$ which are fulfilling:

$$\begin{aligned}
\dot{S}_a(t) &= \frac{1-q}{AB(q)}R_1\left(q,t,S_a(t)\right) + \frac{q}{AB(q)\Gamma(q)} \times \int_0^t (t-\varpi)^{q-1}R_1\left(q,\varpi,\dot{S}_a(\varpi)\right)d\varpi \\
\dot{E}_a(t) &= \frac{1-q}{AB(q)}R_1\left(q,t,E_a(t)\right) + \frac{q}{AB(q)\Gamma(q)} \times \int_0^t (t-\varpi)^{q-1}R_1\left(q,\varpi,\dot{E}_a(\varpi)\right)d\varpi \\
\dot{I}_a(t) &= \frac{1-q}{AB(q)}R_1\left(q,t,I_a(t)\right) + \frac{q}{AB(q)\Gamma(q)} \times \int_0^t (t-\varpi)^{q-1}R_1\left(q,\varpi,\dot{I}_a(\varpi)\right)d\varpi \\
\dot{S}_b(t) &= \frac{1-q}{AB(q)}R_1\left(q,t,S_b(t)\right) + \frac{q}{AB(q)\Gamma(q)} \times \int_0^t (t-\varpi)^{q-1}R_1\left(q,\varpi,\dot{S}_b(\varpi)\right)d\varpi \\
\dot{I}_b(t) &= \frac{1-q}{AB(q)}R_1\left(q,t,I_b(t)\right) + \frac{q}{AB(q)\Gamma(q)} \times \int_0^t (t-\upsilon)^{q-1}R_1\left(q,\varpi,\dot{I}_b(\varpi)\right)d\varpi
\end{aligned} \tag{20}$$

Indicating that

$$\begin{aligned} &\left|S_a(t)-\dot{S}_a(t)\right| \le \varpi_1\zeta_1, \left|S_a(t)-\dot{S}_a(t)\right| \le \varpi_2\zeta_2, \left|S_a(t)-\dot{S}_a(t)\right| \le \varpi_3\zeta_3, \\ &\left|S_a(t)-\dot{S}_a(t)\right| \le \varpi_4\zeta_4, \left|S_a(t)-\dot{S}_a(t)\right| \le \varpi_5\zeta_5 \end{aligned} \tag{21}$$

Theorem 2. *Using the assumption J, the proposed model of fractional order (6) is considered Hyers–Ulam stable*

Proof. Employing the Theorem (1), the proposed Potato disease AB fractional model (6) possesses a singular solution. Let $(S_a(t), E_a(t), I_a(t), S_b(t), I_b(t))$ represents an approximate solution of the model (6) fulfilling the conditions of Equation system (8) .

$$\begin{aligned}
&\left\|S_a(t)-\dot{S}_a(t)\right\| \le \tfrac{1-q}{AB(q)} \left\|R_1(q,t,S_a(t)) - R_1\left(q,t,\dot{S}_a(t)\right)\right\| \\
&\tfrac{1-q}{AB(q)} \int_0^t (t-\varpi)^{q-1} \left\|R_1(q,t,S_a(t)) - R_1\left(q,t,\dot{S}_a(t)\right)\right\| d\varpi \\
&\left[\tfrac{1-q}{AB(q)} + \tfrac{q}{AB(q)\Gamma(q)}\right] \Theta_1 \left\|S_a - \dot{S}_a\right\|, \\
&\left\|E_a(t)-\dot{E}_a(t)\right\| \le \tfrac{1-q}{AB(q)} \left\|R_2(q,t,E_a(t)) - R_2\left(q,t,\dot{E}_a(t)\right)\right\| \\
&\tfrac{1-q}{AB(q)} \int_0^t (t-\varpi)^{q-1} \left\|R_2(q,t,E_a(t)) - R_2\left(q,t,\dot{E}_a(t)\right)\right\| d\varpi \\
&\left[\tfrac{1-q}{AB(q)} + \tfrac{q}{AB(q)\Gamma(q)}\right] \Theta_1 \left\|E_a - \dot{E}_a\right\|, \\
&\left\|I_a(t)-\dot{I}_a(t)\right\| \le \tfrac{1-q}{AB(q)} \left\|R_3(q,t,I_a(t)) - R_3\left(q,t,\dot{I}_a(t)\right)\right\| \\
&\tfrac{1-q}{AB(q)} \int_0^t (t-\varpi)^{q-1} \left\|R_3(q,t,I_a(t)) - R_3\left(q,t,\dot{I}_a(t)\right)\right\| d\varpi \\
&\left[\tfrac{1-q}{AB(q)} + \tfrac{q}{AB(q)\Gamma(q)}\right] \Theta_1 \left\|I_a - \dot{I}_a\right\|, \\
&\left\|S_b(t)-\dot{S}_b(t)\right\| \le \tfrac{1-q}{AB(q)} \left\|R_4(q,t,S_b(t)) - R_4\left(q,t,\dot{S}_b(t)\right)\right\| \\
&\tfrac{1-q}{AB(q)} \int_0^t (t-\varpi)^{q-1} \left\|R_4(q,t,S_b(t)) - R_4\left(q,t,\dot{S}_b(t)\right)\right\| d\varpi \\
&\left[\tfrac{1-q}{AB(q)} + \tfrac{q}{AB(q)\Gamma(q)}\right] \Theta_1 \left\|S_b - \dot{S}_b\right\|, \\
&\left\|I_b(t)-\dot{I}_b(t)\right\| \le \tfrac{1-q}{AB(q)} \left\|R_5(q,t,I_b(t)) - R_5\left(q,t,\dot{I}_b(t)\right)\right\| \\
&\tfrac{1-q}{AB(q)} \int_0^t (t-\varpi)^{q-1} \left\|R_5(q,t,I_b(t)) - R_5\left(q,t,\dot{I}_b(t)\right)\right\| d\varpi \\
&\left[\tfrac{1-q}{AB(q)} + \tfrac{q}{AB(q)\Gamma(q)}\right] \Theta_1 \left\|I_b - \dot{I}_b\right\|.
\end{aligned} \tag{22}$$

In a similar manner, one obtains the following:

$$\begin{aligned} \left\|E_a(t)-\dot{E}_a(t)\right\| &\leq \xi\Delta_2, \\ \left\|I_a(t)-\dot{I}_a(t)\right\| &\leq \xi\Delta_3, \\ \left\|S_b(t)-\dot{S}_b(t)\right\| &\leq \xi\Delta_4, \\ \left\|I_a(t)-\dot{I}_a(t)\right\| &\leq \xi\Delta_5. \end{aligned} \tag{23}$$

Utilising the Eqs. (22) and (23) , the AB Potato disease model in fractional integral of (6) is Hyers–Ulam and also the AB-fractional order model (6) is Hyers–Ulam stable. This ends the proof. □

7.1. Model Equilibria.

7.1.1. *The Potato Disease Free Equilibrium Point.*

In the absence of potato infected or vector infected individual, that is $E_a^* = I_a^* = I_b^* = 0$. The system (6) leads to the following potato free equilibrium point denoted by D^0 and expressed as:

$$D^0 = (S_a^0, E_a^0, I_a^0, S_b^0, I_b^0) = (\frac{(2b\Lambda_b}{\mu_a}, 0, 0, \frac{c\Lambda_b}{\mu_b}, 0).$$

The long term behaviour of potato disease model will be examined by first of all obtaining the basic reproduction number via next generation matrix method in the subsequent subsection.

7.1.2. *The Basic Reproduction Number.*

The basic reproduction number R_0 is obtained utilising the next-generation matrix method [14] and initially let F and V constitute the current infections and transfer matrices of the potato disease model (6) in that order (see [14]. The associated Jacobian matrices computed with respect to the DFE point is given by:

$$F = \begin{pmatrix} 0 & 0 & \frac{2b\Lambda_a}{\mu_a} \\ 0 & 0 & 0 \\ 0 & \frac{c\Lambda_b}{\mu_b} & 0 \end{pmatrix} \textit{ and } V = \begin{pmatrix} \phi+\mu_a & 0 & 0 \\ -\phi & \gamma_1+\gamma_2+\mu_a & 0 \\ 0 & 0 & \mu_b \end{pmatrix}.$$

The reproduction number R_0 is determined by the spectral radius of the corresponding next generation matrix, thus FV^{-1},

$$R_0 = \rho(FV^{-1}) = R_0 = \sqrt{\frac{2bc\phi^2\Lambda_a\Lambda_b}{\phi\mu_a\mu_b^2(\mu_a+\gamma_1+\gamma_2)(\mu_a+\phi)}}.$$

7.1.3. *The Potato Disease Endemic Equilibrium Point.* This subsection presents the endemic equilibrium solution of the Potato disease model (6) given by

$$E^1 = \left(S_a^*, E_a^*, I_a^*, S_b^*, I_a^*\right),$$

where,

$$S_a^* = \frac{\sqrt{\left(-I_b^*\mu_a + \Lambda_a - 2bI_b^*\right)^2 + 4I_b^*\Lambda_a\mu_a} + I_b^*\mu_a - \Lambda_a + 2bI_b^*}{2\mu_a}$$

$$E_a^* = \frac{2bI_b^*S_a^*}{\left(S_a^* + I_b^*\right)\left(\mu_a + \phi\right)}$$

$$I_a^* = \frac{E_a^*\phi}{\mu_a + \gamma_1 + \gamma_2}$$

$$S_b^* = \frac{\Lambda_b}{\mu_b + I_a^*c}$$

$$I_b^* = \frac{I_a^*c\Lambda_b}{\mu_b\left(\mu_b + I_a^*c\right)}$$

7.2. Local Stability of the DFE.

Theorem 3. *The DFE is considered locally asymptotically stable whenever $R_0 < 1$ and unstable otherwise.*

Proof. The Theorem 3 is proved by showing that the eigenvalues of the corresponding Jacobian matrix of system model (6) fulfils the condition $|arg(q_i)| > \frac{p\pi}{2}$, where q_i represents the eigenvalues evaluated at DFE on the Jacobian matrix $J(E^0)$, for $i \in \{1, 2, ..., 5\}$. The results of the Jacobian matrix $J(E^0)$ in the fractional order context of the system model (6) is evaluated at the DFE is given by

$$J(E^0) = \begin{pmatrix} -\mu_a & 0 & 0 & 0 & -\frac{2b\Lambda_a}{\mu_a} \\ \\ 0 & -(\phi+\mu_a) & 0 & 0 & \frac{2b\Lambda_a}{\mu_a} \\ \\ 0 & \phi & -(\gamma_1+\gamma_2+\mu_a) & 0 & 0 \\ \\ 0 & 0 & -\frac{c\Lambda_b}{\mu_b} & -\mu_b & 0 \\ \\ 0 & 0 & \frac{c\Lambda_b}{\mu_b} & 0 & -\mu_b \\ \end{pmatrix}.$$

Evidently, $-\mu_a$, $-\mu_b$ are eigenvalues of $J(E^0)$ are negatives, therefore, satisfies the condition $|arg(q_i)| > \frac{p\pi}{2}$. The rest of the eigenvalues of $J(E^0)$ obtained from the sub matrix

$$J_0 = \begin{pmatrix} -(\phi+\mu_a) & 0 & \frac{2b\Lambda_a}{\mu_a} \\ \\ \phi & -(\gamma_1+\gamma_2+\mu_a) & 0 \\ \\ 0 & \frac{c\Lambda_b}{\mu_b} & -\mu_b \\ \end{pmatrix}.$$

The associated characteristic polynomial equation of J_0 is expressed as

$$q(m) = -q_3m^3 + q_2m^2 + q_1m + q_0,$$

where,

$$q_3 = -\mu_a\mu_b, \tag{24}$$

$$q_2 = -\mu_a\mu_b\left(2\mu_a+\mu_b+\gamma_1+\gamma_2+\phi\right), \tag{25}$$

$$q_1 = -\mu_a\mu_b\left(\left(\mu_a+\gamma_1+\gamma_2\right)\left(\left(\mu_a+\phi\right)\right)-\mu_b\left(2\mu_a+\gamma_1+\gamma_2+\phi\right)\right), \tag{26}$$

$$q_0 = (R_o-1), \tag{27}$$

q_3, q_2, q_1 are negative and q_0 depicts negative whenever $R_0 < 1$. It can be observed that no sign change occurs in the coefficients of the polynomial $m(q)$. Based on Descartes law of signs, conclusion can be drawn that the roots of the polynomial are negative.Therefore, the matrix J_0 possesses only negative eigenvalues whenever $R_0 < 1$ and the eigenvalues therefore fulfil the condition $|arg(q_i)| > \frac{p\pi}{2}$, for $i \in \{1,2,...,5\}$. Therefore, the DFE is locally asymptotically stable whenever $R_0 < 1$ and unstable otherwise. □

8. Numerical Scheme for Fractional Stochastic Model

In this section, we present a numerical scheme of the fractional stochastic model based on the Atangana-Baleanu operator.

(28)
$$\begin{aligned}
S_a(t) &= \left(\Lambda_a - \tfrac{2bS_aI_b}{S_a+I_b} - \mu_a S_a(t)\right)dt + \theta_1 S_a(t)dX_1(t)\\
E_a(t) &= \left(\tfrac{2bS_aI_b}{S_a+I_b} - (\phi+\mu_a)E_a(t)\right)dt + \theta_2 E_a(t)dX_2(t),\\
I_a(t) &= (\phi E_a - (\gamma_1+\gamma_2+\mu_a)I_a)\,dt + \theta_3 I_a(t)dX_3(t)\\
S_b(t) &= (\Lambda_b - cS_bI_a - \mu_b S_b.)\,dt + \theta_4 S_b(t)dX_4(t)\\
I_b(t) &= (cS_bI_a - \mu_b I_b)\,.dt + \theta_5 I_b(t)dX_5(t)
\end{aligned}$$

where $X_i(t), i = 1,2,3,4,5$ constitute the standard Brownian motion and $\theta_i, i = 1;2,3,4,5$ depict the stochastic constant. This fractional stochastic model in the Atangana-Baleanu operator is solved utilising a numerical scheme with Newton polynomial with varying fractional derivative order as in [18]. In order to execute this, the model 28 is reorganised in Atangana-Baleanu operator in Caputo sense as follows:

(29)
$$\begin{aligned}
{}^{AB}_{0}D^q_t S_a(t) &= \left(\Lambda_a - \tfrac{2bS_aI_b}{S_a+I_b} - \mu_a S_a\right) + \theta_1 Q_1(t,S_a)X_1'(t),\\
{}^{AB}_{0}D^q_t E_a(t) &= \left(\tfrac{2bS_aI_b}{S_a+I_b} - (\phi+\mu_a)E_a\right) + \theta_2 Q_2(t,E_a)X_2'(t),
\end{aligned}$$

$$ {}_{0}^{AB}D_t^q I_a(t) = (\phi E_a - (\gamma_1 + \gamma_2 + \mu_a) I_a) + \theta_3 Q_3(t, I_a) X_3'(t), $$

$$ {}_{0}^{AB}D_t^q S_b(t) = (\Lambda_b - cS_b I_a - \mu_b S_b.) + \theta_4 Q_4(t, S_b) X_4'(t), $$

$$ {}_{0}^{AB}D_t^q I_b(t) = (cS_b I_a - \mu_b I_b) . + \theta_5 Q_5(t, I_b) X_5'(t). $$

In making the model 29 easy to work with, the above system equation is rearranged as follows:

$$ {}_{0}^{AB}D_t^q S_a(t) = S_a(t, S_a, E_a, I_a, S_b, I_b) + \theta_1 Q_1(t, S_a) X_1'(t), $$

$$ {}_{0}^{AB}D_t^q E_a(t) = E_a(t, S_a, E_a, I_a, S_b, I_b) + \theta_2 Q_2(t, E_a) X_2'(t), $$

(30)
$$ {}_{0}^{AB}D_t^q I_a(t) = I_a(t, S_a, E_a, I_a, S_b, I_b) + \theta_3 Q_3(t, I_a) X_3'(t), $$

$$ {}_{0}^{AB}D_t^q S_b(t) = S_b(t, S_a, E_a, I_a, S_b, I_b) + \theta_4 Q_4(t, S_b) X_4'(t), $$

$$ {}_{0}^{AB}D_t^q I_b(t) = I_b(t, S_a, E_a, I_a, S_b, I_b) + \theta_5 Q_5(t, I_b) X_5'(t). $$

Employing the AB definition, the following numerical scheme based on Newton polynomial is obtained as:

$$ S_a^{n+1} = S_a^n + \frac{1-q}{AB(q)} S_a\left(t_n, S_a^n, E_a^n, I_a^n, S_b^n, I_b^n\right) $$

$$ + \frac{q(\Delta t)^q}{AB(q)\Gamma(q+1)} \sum_{j=2}^{n} S_a\left(t_{j-2}, S_a^{j-2}, E_a^{j-2}, I_a^{j-2}, S_b^{j-2}, I_b^{j-2}\right) \times \Psi $$

$$ + \frac{q(\Delta t)^q}{AB(q)\Gamma(q+1)} \sum_{j=2}^{n} \theta_1 Q_1(t_{j-2}, S_a^{j-2}) \left(X_1(t_{j-1}) - X_1(t_{j-2})\right) \times \Psi $$

$$+\frac{q(\Delta t)^q}{AB(q)\Gamma(q+2)}\sum_{j=2}^{n}\left[\begin{array}{l}\theta_1 Q_1(t_{j-1},S_a^{j-1})\left(X_1(t_j)-X_1(t_{j-1})\right)\\ \\ -\theta_1 Q_1(t_{j-2},S_a^{j-2})\left(X_1(t_{j-1})-X_1(t_{j-2})\right)\\ \end{array}\right]\times\Sigma$$

$$\frac{q(\Delta t)^q}{AB(q)\Gamma(q+3)}\sum_{j=2}^{n}\left[\begin{array}{l}\theta_1 Q_1(t_j,S_a^{j})\left(X_1(t_{j-1})-X_1(t_j)\right)\\ \\ -2\theta_1 Q_1(t_{j-1},S_a^{j-1})\left(X_1(t_j)-X_1(t_{j-1})\right)\\ \\ +\theta_1 Q_1(t_{j-2},S_a^{j-2})\left(X_1(t_{j-1})-X_1(t_{j-2})\right)\\ \end{array}\right]\times\Delta$$

$$+\frac{q(\Delta t)^q}{AB(q)\Gamma(q+2)}\sum_{j=2}^{n}\left[\begin{array}{l}S_a\left(t_{j-1},S_a^{j-1},E_a^{j-1},I_a^{j-1},S_b^{j-1},I_b^{j-1}\right)\\ \\ -S_a\left(t_{j-2},S_a^{j-2},E_a^{j-2},I_a^{j-2},S_b^{j-2},I_b^{j-2}\right)\\ \end{array}\right]\times\Sigma$$

$$+\frac{q(\Delta t)^q}{2AB(q)\Gamma(q+3)}\sum_{j=2}^{n}\left[\begin{array}{l}S_a\left(t_j,S_a^{j},E_a^{j},I_a^{j},S_b^{j},I_b^{j}\right)\\ \\ -2S_a\left(t_{j-1},S_a^{j-1},E_a^{j-1},I_a^{j-1},S_b^{j-1},I_b^{j-1}\right)\\ \\ +S_a\left(t_{j-2},S_a^{j-2},E_a^{j-2},I_a^{j-2},S_b^{j-2},I_b^{j-2}\right)\\ \end{array}\right]\times\Delta$$

$$E_a^{n+1} = E_a^n + \frac{1-q}{AB(q)} E_a\left(t_n, S_a^n, E_a^n, I_a^n, S_b^n, I_b^n\right)$$

$$+\frac{q(\Delta t)^q}{AB(q)\Gamma(q+1)} \sum_{j=2}^{n} E_a\left(t_{j-2}, S_a^{j-2}, E_a^{j-2}, I_a^{j-2}, S_b^{j-2}, I_b^{j-2}\right) \times \Psi$$

$$+\frac{q(\Delta t)^q}{AB(q)\Gamma(q+1)} \sum_{j=2}^{n} \theta_2 Q_2(t_{j-2}, E_a^{j-2})\left(X_2(t_{j-1}) - X_2(t_{j-2})\right) \times \Psi$$

$$+\frac{q(\Delta t)^q}{AB(q)\Gamma(q+2)} \sum_{j=2}^{n} \left[\begin{array}{l} \theta_2 Q_2(t_{j-1}, E_a^{j-1})\left(X_2(t_j) - X_2(t_{j-1})\right) \\ \\ -\theta_2 Q_2(t_{j-2}, E_a^{j-2})\left(X_2(t_{j-1}) - X_2(t_{j-2})\right) \\ \end{array}\right] \times \Sigma$$

$$\frac{q(\Delta t)^q}{AB(q)\Gamma(q+3)} \sum_{j=2}^{n} \left[\begin{array}{l} \theta_2 Q_2(t_j, E_a^{j})\left(X_2(t_{j-1}) - X_2(t_j)\right) \\ \\ -2\theta_2 Q_2(t_{j-1}, E_a^{j-1})\left(X_2(t_j) - X_2(t_{j-1})\right) \\ \\ +\theta_2 Q_2(t_{j-2}, E_a^{j-2})\left(X_2(t_{j-1}) - X_2(t_{j-2})\right) \\ \end{array}\right] \times \Delta$$

$$+\frac{q(\Delta t)^q}{AB(q)\Gamma(q+2)} \sum_{j=2}^{n} \left[\begin{array}{l} E_a\left(t_{j-1}, S_a^{j-1}, E_a^{j-1}, I_a^{j-1}, S_b^{j-1}, I_b^{j-1}\right) \\ \\ -E_a\left(t_{j-2}, S_a^{j-2}, E_a^{j-2}, I_a^{j-2}, S_b^{j-2}, I_b^{j-2}\right) \\ \end{array}\right] \times \Sigma$$

$$+\frac{q(\Delta t)^q}{2AB(q)\Gamma(q+3)} \sum_{j=2}^{n} \left[\begin{array}{l} E_a\left(t_j, S_a^j, E_a^j, I_a^j, S_b^j, I_b^j\right) \\ \\ -2E_a\left(t_{j-1}, S_a^{j-1}, E_a^{j-1}, I_a^{j-1}, S_b^{j-1}, I_b^{j-1}\right) \\ \\ +E_a\left(t_{j-2}, S_a^{j-2}, E_a^{j-2}, I_a^{j-2}, S_b^{j-2}, I_b^{j-2}\right) \\ \end{array}\right] \times \Delta$$

$$I_a^{n+1} = I_a^n + \frac{1-q}{AB(q)} I_a\left(t_n, S_a^n, E_a^n, I_a^n, S_b^n, I_b^n\right)$$

$$+\frac{q(\Delta t)^q}{AB(q)\Gamma(q+1)} \sum_{j=2}^{n} I_a\left(t_{j-2}, S_a^{j-2}, E_a^{j-2}, I_a^{j-2}, S_b^{j-2}, I_b^{j-2}\right) \times \Psi$$

$$+\frac{q(\Delta t)^q}{AB(q)\Gamma(q+1)} \sum_{j=2}^{n} \theta_3 Q_3(t_{j-2}, I_a^{j-2})\left(X_3(t_{j-1}) - X_3(t_{j-2})\right) \times \Psi$$

$$+\frac{q(\Delta t)^q}{AB(q)\Gamma(q+2)} \sum_{j=2}^{n} \begin{bmatrix} \theta_3 Q_3(t_{j-1}, I_a^{j-1})\left(X_3(t_j) - X_3(t_{j-1})\right) \\ \\ -\theta_3 Q_3(t_{j-2}, I_a^{j-2})\left(X_3(t_{j-1}) - X_3(t_{j-2})\right) \\ \end{bmatrix} \times \Sigma$$

$$\frac{q(\Delta t)^q}{AB(q)\Gamma(q+3)} \sum_{j=2}^{n} \begin{bmatrix} \theta_3 Q_3(t_j, I_a^j)\left(X_3(t_{j-1}) - X_3(t_j)\right) \\ \\ -2\theta_3 Q_3(t_{j-1}, I_a^{j-1})\left(X_3(t_j) - X_3(t_{j-1})\right) \\ \\ +\theta_3 Q_3(t_{j-2}, I_a^{j-2})\left(X_3(t_{j-1}) - X_3(t_{j-2})\right) \\ \end{bmatrix} \times \Delta$$

$$+\frac{q(\Delta t)^q}{AB(q)\Gamma(q+2)} \sum_{j=2}^{n} \begin{bmatrix} I_a\left(t_{j-1}, S_a^{j-1}, E_a^{j-1}, I_a^{j-1}, S_b^{j-1}, I_b^{j-1}\right) \\ \\ -I_a\left(t_{j-2}, S_a^{j-2}, E_a^{j-2}, I_a^{j-2}, S_b^{j-2}, I_b^{j-2}\right) \\ \end{bmatrix} \times \Sigma$$

$$+\frac{q(\Delta t)^q}{2AB(q)\Gamma(q+3)} \sum_{j=2}^{n} \begin{bmatrix} I_a\left(t_j, S_a^j, E_a^j, I_a^j, S_b^j, I_b^j\right) \\ \\ -2I_a\left(t_{j-1}, S_a^{j-1}, E_a^{j-1}, I_a^{j-1}, S_b^{j-1}, I_b^{j-1}\right) \\ \\ +I_a^*\left(t_{j-2}, S_a^{j-2}, E_a^{j-2}, I_a^{j-2}, S_b^{j-2}, I_b^{j-2}\right) \\ \end{bmatrix} \times \Delta$$

$$S_b^{n+1} = S_b^n + \frac{1-q}{AB(q)} S_b\left(t_n, S_a^n, E_a^n, I_a^n, S_b^n, I_b^n\right)$$

$$+\frac{q(\Delta t)^q}{AB(q)\Gamma(q+1)} \sum_{j=2}^{n} S_b\left(t_{j-2}, S_a^{j-2}, E_a^{j-2}, I_a^{j-2}, S_b^{j-2}, I_b^{j-2}\right) \times \Psi$$

$$+\frac{q(\Delta t)^q}{AB(q)\Gamma(q+1)} \sum_{j=2}^{n} \theta_4 Q_4(t_{j-2}, S_b^{j-2})\left(X_5(t_{j-1}) - X_5(t_{j-2})\right) \times \Psi$$

$$+\frac{q(\Delta t)^q}{AB(q)\Gamma(q+2)} \sum_{j=2}^{n} \left[\begin{array}{l} \theta_4 Q_4(t_{j-1}, S_b^{j-1})\left(X_5(t_j) - X_5(t_{j-1})\right) \\ \\ -\theta_4 Q_4(t_{j-2}, S_b^{j-2})\left(X_5(t_{j-1}) - X_5(t_{j-2})\right) \\ \end{array}\right] \times \Sigma$$

$$\frac{q(\Delta t)^q}{AB(q)\Gamma(q+3)} \sum_{j=2}^{n} \left[\begin{array}{l} \theta_4 Q_4(t_j, S_b^{j})\left(X_5(t_{j-1}) - X_5(t_j)\right) \\ \\ -2\theta_4 Q_4(t_{j-1}, S_b^{j-1})\left(X_5(t_j) - X_5(t_{j-1})\right) \\ \\ +\theta_4 Q_4(t_{j-2}, S_b^{j-2})\left(X_5(t_{j-1}) - X_5(t_{j-2})\right) \\ \end{array}\right] \times \Delta$$

$$+\frac{q(\Delta t)^q}{AB(q)\Gamma(q+2)} \sum_{j=2}^{n} \left[\begin{array}{l} S_b\left(t_{j-1}, S_a^{j-1}, E_a^{j-1}, I_a^{j-1}, S_b^{j-1}, I_b^{j-1}\right) \\ \\ -S_b\left(t_{j-2}, S_a^{j-2}, E_a^{j-2}, I_a^{j-2}, S_b^{j-2}, I_b^{j-2}\right) \\ \end{array}\right] \times \Sigma$$

$$+\frac{q(\Delta t)^q}{2AB(q)\Gamma(q+3)} \sum_{j=2}^{n} \left[\begin{array}{l} S_b\left(t_j, S_a^j, E_a^j, I_a^j, S_b^j, I_b^j\right) \\ \\ -2S_b\left(t_{j-1}, S_a^{j-1}, E_a^{j-1}, I_a^{j-1}, S_b^{j-1}, I_b^{j-1}\right) \\ \\ +S_b\left(t_{j-2}, S_a^{j-2}, E_a^{j-2}, I_a^{j-2}, S_b^{j-2}, I_b^{j-2}\right) \\ \end{array}\right] \times \Delta$$

$$I_b^{n+1} = I_b^n + \frac{1-q}{AB(q)} I_b\left(t_n, S_a^n, E_a^n, I_a^n, S_b^n, I_b^n\right)$$

$$+\frac{q(\Delta t)^q}{AB(q)\Gamma(q+1)} \sum_{j=2}^{n} I_b\left(t_{j-2}, S_a^{j-2}, E_a^{j-2}, I_a^{j-2}, S_b^{j-2}, I_b^{j-2}\right) \times \Psi$$

$$+\frac{q(\Delta t)^q}{AB(q)\Gamma(q+1)} \sum_{j=2}^{n} \theta_5 Q_5(t_{j-2}, I_b^{j-2})\left(X_5(t_{j-1}) - X_5(t_{j-2})\right) \times \Psi$$

$$+\frac{q(\Delta t)^q}{AB(q)\Gamma(q+2)} \sum_{j=2}^{n} \begin{bmatrix} \theta_5 Q_5(t_{j-1}, I_b^{j-1})\left(X_5(t_j) - X_5(t_{j-1})\right) \\ \\ -\theta_5 Q_5(t_{j-2}, I_b^{j-2})\left(X_5(t_{j-1}) - X_5(t_{j-2})\right) \\ \end{bmatrix} \times \Sigma$$

$$\frac{q(\Delta t)^q}{AB(q)\Gamma(q+3)} \sum_{j=2}^{n} \begin{bmatrix} \theta_5 Q_5(t_j, I_b^j)\left(X_5(t_{j-1}) - X_5(t_j)\right) \\ \\ -2\theta_5 Q_5(t_{j-1}, I_b^{j-1})\left(X_5(t_j) - X_5(t_{j-1})\right) \\ \\ +\theta_5 Q_5(t_{j-2}, I_b^{j-2})\left(X_5(t_{j-1}) - X_5(t_{j-2})\right) \\ \end{bmatrix} \times \Delta$$

$$+\frac{q(\Delta t)^q}{AB(q)\Gamma(q+2)} \sum_{j=2}^{n} \begin{bmatrix} I_b\left(t_{j-1}, S_a^{j-1}, E_a^{j-1}, I_a^{j-1}, S_b^{j-1}, I_b^{j-1}\right) \\ \\ -I_b\left(t_{j-2}, S_a^{j-2}, E_a^{j-2}, I_a^{j-2}, S_b^{j-2}, I_b^{j-2}\right) \\ \end{bmatrix} \times \Sigma$$

$$+\frac{q(\Delta t)^q}{2AB(q)\Gamma(q+3)} \sum_{j=2}^{n} \begin{bmatrix} I_b\left(t_j, S_a^j, E_a^j, I_a^j, S_b^j, I_b^j\right) \\ \\ -2I_b\left(t_{j-1}, S_a^{j-1}, E_a^{j-1}, I_a^{j-1}, S_b^{j-1}, I_b^{j-1}\right) \\ \\ +I_b\left(t_{j-2}, S_a^{j-2}, E_a^{j-2}, I_a^{j-2}, S_b^{j-2}, I_b^{j-2}\right) \\ \end{bmatrix} \times \Delta$$

where

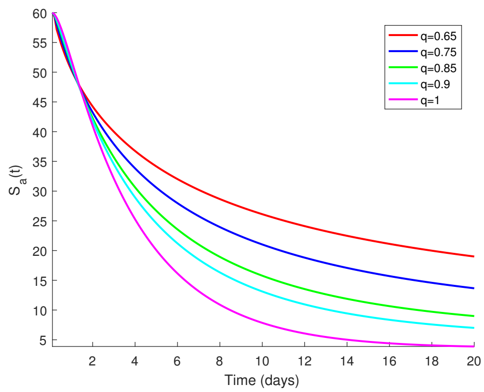
$$\Psi = [(n-j+1)^q-(n-j)^q],$$

$$\Sigma = \begin{bmatrix} (n-j+1)^q-(n-j+3+2q) \\ \\ -(n-j)^q(n-j+3+3q) \end{bmatrix},$$

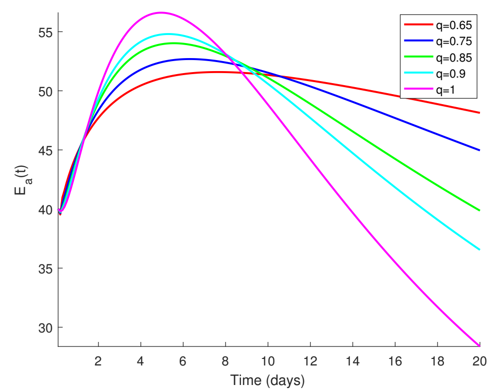
$$\Delta = \begin{bmatrix} (n-j+1)^q \begin{bmatrix} 2(n-j)^2+(3q+10)(n-j) \\ +2q^2+9q+12 \end{bmatrix} \\ \\ -(n-j) \begin{bmatrix} 2(n-j)^2+(5q+10)(n-j) \\ +6q^2+18q+12 \end{bmatrix} \end{bmatrix}$$

9. NUMERICAL SIMULATION AND DISCUSSION

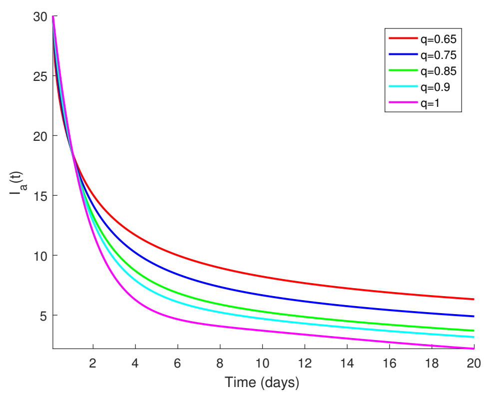
In this section, a numerical simulation is undertaken to provide support to the analytical results obtained. The parameter values were adopted in Getachew et al.(2021) $\Lambda_1 = 0.8, \Lambda_2 = 0.19, b = 0.00022, \gamma_1 = 0.033, \gamma_2 = 0.01, \mu_a = 0.04, \mu_b = 0.0028, \phi = 0.05, c = 0.005$ and the intial condition utilized in this work are $S_{a0} = 600, E_{a0} = 400, I_{a0} = 200, S_{b0} = 100, I_{b0} = 10$ In Figure 1, the numerical simulation is based on Newton polynomial with AB without stochastic component. Figure 1(a) is the number of susceptible potatoes class $(S_a(t))$ grown in a given environment, and as the fractional-order derivative increases from 0.65 towards one, the number of the susceptible reduces. Figure 1(b) depicts the number of exposed class $(E_a(t))$ and as the fractional-order derivative increases within the first six days, the number of the exposed class increases. In the subsequent days, the number of the exposed class begins to decrease as the fractional-order derivative increases from 0.65 to 1. In Figure 1(c), the number of infected potatoes class $(I_a(t))$ decreases as the fractional-order derivative increases from 0.65 to 1. Figure 1(d) represents the susceptible vector $(S_b(t))$ in the environment, and the number decreases as the fractional-order derivative increases from 0.65 to 1. The infected vector class $(I_b(t))$ in Figure 1 (e) indicates that the number of infected vector increase within the first three days as the fractional order increases and subsequently decreases as the fractional-order derivative increases from 0.65 to 1.



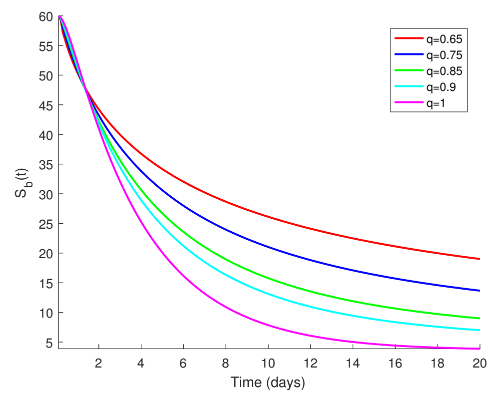
(a)



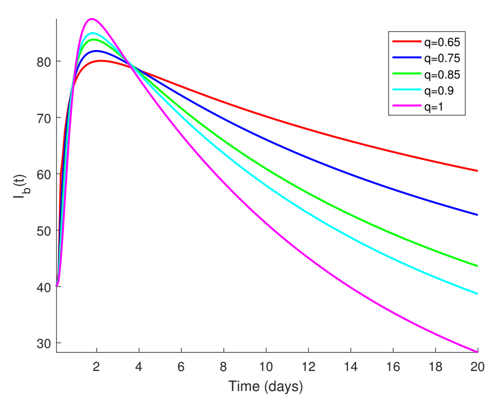
(b)



(c)



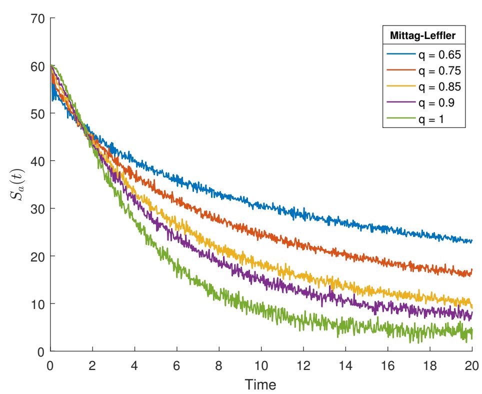
(d)



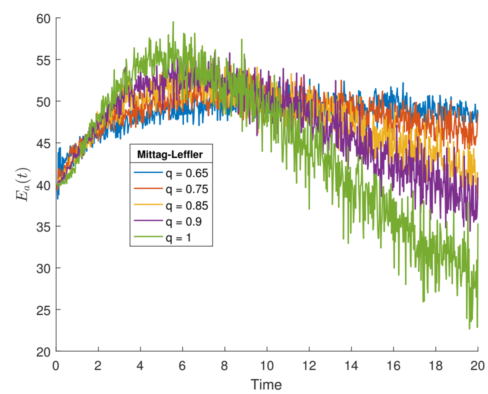
(e)

FIGURE 1. Simulation results of model (28), Mittag-Leffler function at $q = 0.65, 0.75, 0.80, 0.90, 1$

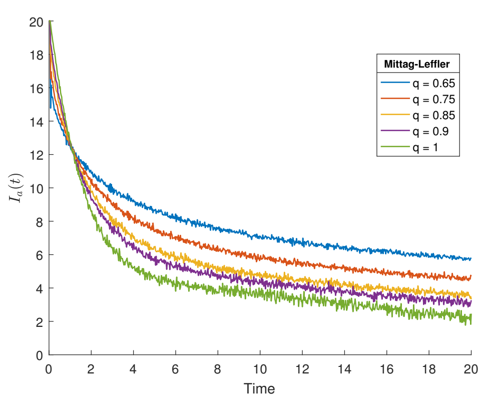
Figure 2 is the numerical simulation results based on Newton polynomial with AB with a stochastic component. In Figure 2(a), the number of susceptible potatoes class $(S_a(t))$ decreases as the fractional-order derivative increases from 0.65 to 1. Figure 2(b) shows that as the fractional-order derivative increases from 0.65 to 1 within the first six days, the number of exposed class $(E_a(t))$ increases. However, the exposed class decreases as the fractional-order derivative increases from 0.65 to 1 after the 6th day. In Figure 2(c), the number of infected potatoes $(I_a(t))$ decreases as the fractional-order derivative increases from 0.65 to 1. Figure 2 (d) is the susceptible vector class $(S_b(t))$ in which as the fractional-order derivative increases from 0.65 to 1, the number of susceptible vectors decreases in the environment. In Figure 2 (e), the number of infected vectors $(I_b(t))$ increase within the first three days as the fractional-order derivative increases from 0.65 to 1, and the number of infected vectors decreases subsequently as the fractional-order derivative increases from 0.65 to 1.



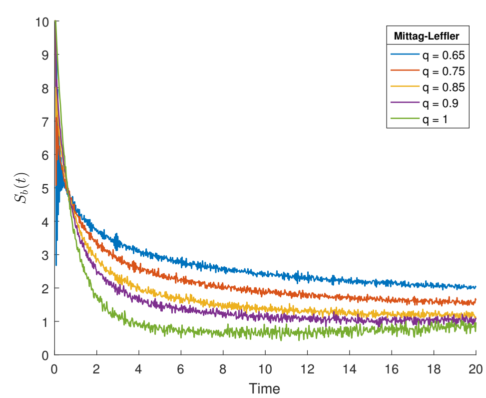
(a)



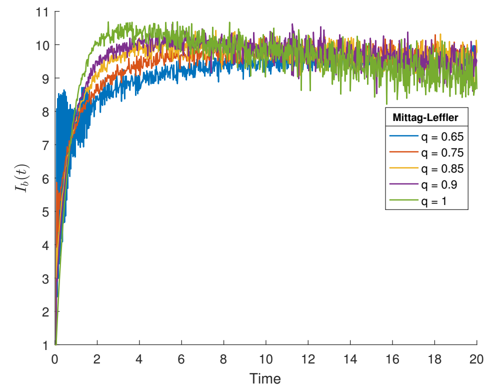
(b)



(c)



(d)



(e)

FIGURE 2. Simulation results of model (28), Mittag-Leffler function at $q = 0.65, 0.75, 0.80, 0.90, 1$ and stochastic constants $\theta = 0.2, 0.4, 0.6, 0.8, 0.9$.

10. CONCLUSION

In this work, a potato disease model was formulated in the concept of nonlocal and nonsingular operators perspectives. The reproduction number of the potato model had been computed, and steady states of the model were determined. The local stability analysis was carried out and found to be locally asymptotically stable. The study established the existence and uniqueness of solutions of the potato model using the Banach space approach. Hyers- Ulam stability analysis was carried out to determine the robust nature of ABC analysis of the existence and uniqueness of solutions of the potato model. A stochastic component was included in the AB operator potato model and numerically solved. Two numerical results were obtained with and without stochastic components. Similar results were obtained in each situation. However, the fractional stochastic model depicted some random effects indicating fluctuations in the increases or decreases at the various compartments. The stochastic numerical simulation results indicated the movement of the dynamics of the potato disease which is not fixed or linear. The stochastic component provided enough evidence of fluctuations in the spread of epidemics unlike a deterministic model. It is established from the numerical simulation results that the fractional-order derivative played a crucial role in the dynamics of the potato model. It can be suggested that since it was highly uncertain to predict the exact number of individuals that may add up or leave a compartment, the fractional stochastic approach ought to be considered in examining the dynamics of complex models.

Conflict of Interests

The author declares that there is no conflict of interests.

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