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OPTIMAL STRATEGY OF VACCINATION AND PREVENTION MEASURES IN A SEQIRS PANDEMIC MODEL

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Abstract. In this paper, we formulate and analyse an optimal control problem for a SEQIRS pandemic model describing the transmission dynamics of the COVID-19 pandemic, given by a system of nonlinear differential equations. Optimal control strategies such as vaccination and preventive measures are adopted as control measures, to minimize the numbers of susceptible, exposed, and infected individuals. We prove the existence of optimal controls and characterization is established using Pontryagin's maximum principle. Numerical simulations are performed to analyse the effectiveness of optimization strategies by comparing the results obtained.

Keywords: COVID-19; optimal control; mathematical model.
2010 AMS Subject Classification: 92C60.

1. INTRODUCTION

The world is facing an unprecedented threat, as the COVID-19 virus pandemic quickly spread to the world. Because of this pandemic, suffering has pervaded, millions of lives have been disrupted, and the global economy is under threat. Even rich countries with strong health systems are under stress. At a time when the world was grappling with the spread of the COVID-19

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virus, several mutated strains of the virus appeared that seemed to be more rapid and able to spread, prolonging the pandemic in many regions of the world. The emergence of mutated copies of the virus is not surprising, but rather a natural process, because the virus mutates over time to ensure its survival. Among the worrisome variants of the coronavirus, according to the World Health Organization, we mention Alpha (known as the British strain), beta (known as the South African strain), Gamma (known as the Brazilian strain), Delta (known as the Indian strain) and Omicron. The latter is a new strain that was announced in South Africa on November 24, 2021, characterized by more mutations than other mutated strains, and maybe a more spreading infection. In addition to all this, scientific studies have shown that there is a possibility that those recovering from COVID-19 will contract the disease again, after diminishing immunity from the first infection with time, and this is what makes the matter worse and more complicated and makes the world in a closed spiral.

The World Health Organization noted the importance of following physical distancing procedures in all gatherings, maintaining a social distance of at least one meter, and an obligation to communicate remotely without touching or kissing hands. In particular, the infection is transmitted by nasal or oral secretions, which spread to the air during the speech, coughing, sneezing or breathing in general. Infection can also occur through physical contact with a surface contaminated with spray droplets and aerosols carrying viral particles. It also stressed the need to adopt vaccination against the novel coronavirus as an important strategy to prevent the threat of the pandemic to provide an opportunity to ensure individual and collective protection. The vaccine creates special protection against COVID-19, by allowing the immune system to maintain first contact with the antigen. In the case of subsequent contact with the virus, the speed of recognition and the intensity of the immune response will prevent infection. It also reduces the risk of infection and the transformation of the infection into a long-term COVID-19 syndrome. And according to many studies that have proven that the symptoms of infection of individuals with the coronavirus after taking the vaccination are milder and the likelihood of developing severe disease, hospitalization, and death will be lower. Those who have not been vaccinated may have more than twice the risk of infection again than those who have been vaccinated.

Infectious disease modeling is a tool used to study and analyse the mechanisms by which diseases are spread, characterize them, and predict the future course of epidemics. In the past few decades, several researchers have developed various mathematical models to investigate the transmission dynamics of infectious diseases and their control measures (see, e.g. [2, 8, 11, 14, 15]). Modeling helps to determine which intervention(s) should be avoided and which should be tried and thus helps to demonstrate the effectiveness of intervention strategies throughout the outbreak. Part of this discipline consists of modeling the evolution of the pandemic using mathematical tools. These tools can consist of a system of partial differential equations, graph theory, probability, or even data science. The field of mathematical epidemiology experienced, during the COVID-19 pandemic, a production coupled with exceptional media coverage of its work. Even though data and knowledge on the emerging disease were fragmented, a wide variety of models were developed and applied in unprecedented times, to estimate the number of reproductions, the date of the start of the pandemic or cumulative incidence, and also to explore different scenarios of pharmaceutical or non-pharmaceutical interventions. Their results have greatly contributed to epidemiological surveillance and informed public health policy decision-making to limit the spread of the virus.

Many specialists and researchers have applied optimal control techniques to understand and develop ways to limit the spread of the novel coronavirus, by proposing the most effective mitigation, strategy to reduce the number of infected individuals, for example, washing hands, wearing masks, isolating patients, closing public places and vaccinate the population, etc. [3] used the optimal control to control a mathematical model of COVID-19 based on some strategies such as closure, quarantine, and self-isolation, as he aimed in his work to find optimal control strategies that reduce the asymptomatic individuals, and the infected individuals who have not been reported. Whereas in the article [4], a nonlinear deterministic model is designed to describe the transmission dynamics of COVID-19. The authors use four COVID-19 controls representing the practice of physical or social distancing protocols, the practice of personal hygiene by cleaning contaminated surfaces with alcohol-based detergents, the practice of proper and safety measures by exposed, symptomatic infected, and asymptomatic infected persons, and fumigation schools at all levels of education, sports and commercial areas such as markets and public toilet facilities. By formulating an optimal control model for the proposed model, numerical simulations were performed to study and analyse the effectiveness of control strategies and suggest the optimal strategy to control the emerging coronavirus. Many other results related to optimal strategies for controlling the COVID-19 pandemic have been established and can be found in numerous articles (see, for example, [5, 9, 6, 13, 12]).

The objective of my work is to apply the theory of optimal control to a SEQIRS pandemic model for the transmission of COVID-19 and to determine the optimal strategies that allow the pandemic to be controlled and reduce as much as possible the number of susceptible, exposed, and infected individuals with the virus. we associate our model with two measures of control, one represents confinement, the aim of which is to reduce the rate of contact between infected or susceptible people and healthy people, and the other represents vaccination with all the booster doses to reduce virus infection or protect against severe symptoms that lead to resuscitation or death. The structure of this paper is organized as follows. In the next section, we propose the SEQIRS pandemic model with control measures. In Section 3, an optimal control problem is formulated and studied analytically using Pontryagin's Maximum Principle. In Section 4, we present numerical simulations and discussions for the optimality systems. Finally, we give a brief conclusion in Section 5.

2. COVID-19 MODEL WITH CONTROLS

The model describes the pandemic dynamics of COVID-19 in a population N where individuals can belong to five compartments identified as susceptible $S(t)$, exposed $E(t)$, confined $Q(t)$, infected $I(t)$ and recovered $R(t)$. Infection is spread by direct contact between a susceptible person and an infected person at the rate of β, or between a susceptible person and an exposed person at the rate of α. Therfore, more new cases initially move from compartment S to compartment E. To reduce this contact, we have implemented a quarantine and isolation strategy for susceptible and exposed people to prevent their movement and thus limit the spread of the virus in the population. When testing quarantined individuals in compartment Q, they are either transferred to compartment R after confirmation that they are not infected at the rate of r_1 or transferred to compartment I when they test positive to receive necessary medical care at the rate of δ. While individuals exposed after symptom onset move to compartment I at the rate of

σ and then to compartment R when fully recovered and all symptoms have disappeared at the rate of r_2. Susceptible people are sent directly to compartment R after receiving the vaccination, the latter helps to create special protection against COVID-19, recovered individuals can return to compartment S by loss of immunity and reinfect with the virus at the rate of γ.
The parameters Ξ, μ and η represent rates of human birth, natural death and disease-induced death, respectively. In the model, we introduce two control functions $u_1(\cdot)$ and $u_2(\cdot)$. The first control u_1 represents preventive measures such as quatrain and isolation that help reduce the contact rate. the u_2 control represents vaccination to minimize the number of susceptible individuals and the number of infected individuals and therefore maximize the number of recovered and protected individuals.
The system diagram for the transmission of COVID-19 is shown in Figure 1.

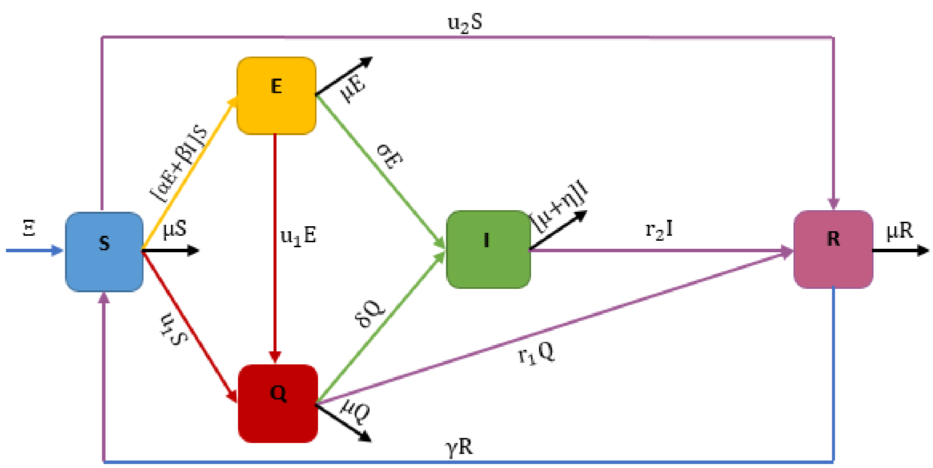


FIGURE 1. Compartmental diagram for the transmission dynamics of COVID-19

The total population is given by

$$N(t) = S(t) + E(t) + Q(t) + I(t) + R(t).$$

Through the schematic diagram in Figure 1, the system of non-linear differential equations is expressed as follows:

$$\frac{dS(t)}{dt} = \Xi - (\alpha E(t) + \beta I(t))S(t) - (u_1(t) + u_2(t) + \mu)S(t) + \gamma R(t),$$

$$\frac{dE(t)}{dt} = (\alpha E(t) + \beta I(t))S(t) - (u_1(t) + \sigma + \mu))E(t),$$

$$\frac{dQ(t)}{dt} = u_1(t)\,(S(t)+E(t)) - (\delta + r_1 + \mu)\,Q(t),$$

$$\frac{dI(t)}{dt} = \sigma E(t) + \delta Q(t) - (r_2 + \eta + \mu)\,I(t), \tag{1}$$

$$\frac{dR(t)}{dt} = u_2(t)S(t) + r_1 Q(t) + r_2 I(t) - (\gamma + \mu)\,R(t),$$

with $S(0) \geq 0$, $E(0) \geq 0$, $Q(0) \geq 0$, $I(0) \geq 0$, and $R(0) \geq 0$ as the initial conditions.

3. Optimal Control Problem

In this section, we present the optimal control problem to minimize the objective function that takes into account the number of susceptible, exposed, infected individuals, and the cost of implementing the strategies associated with controls u_i, $i = 1,2$.

We define the objective function J as follows:

$$J(u_1,u_2) = \int_0^{t_f} \left[C_1 S(t) + C_2 E(t) + C_3 I(t) + \frac{1}{2}\sum_{i=1}^{i=2} \rho_i u_i^2(t) \right] dt, \tag{2}$$

where $C_i > 0$, $i = 1,2,3$ are the balancing weight constants of the susceptible, exposed and infected individuals respectively, whereas $\rho_i > 0$ are the balancing cost factors on the respective controls u_i, for $i = 1,2$, and t_f is the final time.

Then, our aim is to find an optimal control pair $u^* = (u_1^*, u_2^*)$ such that

$$J(u_1^*, u_2^*) = \min_{\Omega}\; J(u_1,u_2), \tag{3}$$

where

$$\Omega = \left\{ (u_1,u_2) : u_i(t) \text{ is lebesgue measurable, } 0 \leq u_i(t) < 1,\; t \in [0,t_f], \text{for } i = 1,2 \right\} \tag{4}$$

is the set of admissible controls.

Theorem 1. *Consider the optimal control problem (3) associated with the system (1), then there exists an optimal control pair $u^* = (u_1^*, u_2^*)$ in Ω such that*

$$J(u_1^*, u_2^*) = \min\; \{J(u_1,u_2), (u_1,u_2) \in \Omega\}.$$

Proof. To prove the existence of an optimal control pair, we use the following conditions given in [7]:

1. The set of solutions to the system (1) with control variables in Ω is non-empty.
2. Convexity and closure of the set Ω.
3. Convexity of the integrand of the objective functional on Ω.
4. The state system can be written as linear fonction of control variables with coefficients depending on time and state variables.
5. There exist constants $\upsilon_1, \upsilon_2 > 0$ and $\upsilon_3 > 1$ such that the integrand of (2) is bounded below by $\upsilon_1 \left(\mid u_1 \mid^2 + \mid u_2 \mid^2\right)^{\upsilon_3 \backslash 2} - \upsilon_2$.

We verify the first condition thanks to a result of Lukes [10] which ensures the existence of solutions for the state system (1) with constant coefficients.

By definition, the controls set Ω is closed. Furthermore, let $v, w \in \Omega$, where $v = (v_1, v_2)$ and $w = (w_1, w_2)$. It follows, for $\varepsilon \in [0,1]$ we have $\varepsilon v_i + (1-\varepsilon) w_i \in \Omega$, $i = 1,2$, satisfiying the convexity proprety of the controls set Ω.

Let $\mathscr{X} = (S,E,Q,I,R)$ and $u = (u_1, u_2) \in \Omega$, the objective functional J in (2) has an integrand of the Lagrangian form defined as

$$\mathscr{L}(\mathscr{X}, u) = C_1 S(t) + C_2 E(t) + C_3 I(t) + \tfrac{1}{2} \sum_{i=1}^{i=2} \rho_i u_i^2(t).$$

Let $v = (v_1, v_2) \in \Omega$ and $w = (w_1, w_2) \in \Omega$. Then, for $\theta \in [0,1]$ we have

$$\mathscr{L}(\mathscr{X}, \theta v + (1-\theta) w) = C_1 S(t) + C_2 E(t) + C_3 I(t) + \tfrac{1}{2} \textstyle\sum_{i=1}^{i=2} \rho_i \left(\theta v_i + (1-\theta) w_i\right)^2,$$

and

$$\theta \mathscr{L}(\mathscr{X}, v) + (1-\theta) \mathscr{L}(\mathscr{X}, w) = C_1 S(t) + C_2 E(t) + C_3 I(t) + \frac{1}{2} \theta \sum_{i=1}^{i=2} \rho_i v_i^2 + \frac{1}{2} (1-\theta) \sum_{i=1}^{i=2} \rho_i w_i^2.$$

Therefore,

$$\begin{aligned} \mathscr{L}(\mathscr{X}, \theta v + (1-\theta) w) - (\theta \mathscr{L}(\mathscr{X}, v) + (1-\theta) \mathscr{L}(\mathscr{X}, w)) &= \tfrac{1}{2} \left(\theta^2 - \theta\right) \sum_{i=1}^{i=2} \rho_i (v_i - w_i)^2 \\ &\leq 0, \quad \text{since } \theta \in [0,1], \end{aligned}$$

implying that the integrand $\mathscr{L}(\mathscr{X}, u)$ of the objective functional J is convex on Ω.

Also, the state system (1) is clearly linear in control variables u_1 and u_2 with coeffecient depending on state variables.

Lastly, the fifth property is verified as follows:

$$\begin{aligned}\mathscr{L}(\mathscr{X},u) &\geq \frac{1}{2}\sum_{i=1}^{i=2}\rho_i u_i^2\\ &\geq \upsilon_1\left(|\,u_1\,|^2+|\,u_2\,|^2\right)^{\upsilon_3\backslash 2}-\upsilon_2,\end{aligned}$$

where $\upsilon_1=\frac{1}{2}\min\{\rho_1,\rho_2\}$, $\upsilon_2>0$ and $\upsilon_3=2$. □

3.1. Characterization of optimal controls.

After establishing the existence of the optimal control that minimizes the objective functional J, we will characterize this optimal control by applying the Pontryagin's Maximum Principle to the Hamiltonian.

Let $\mathscr{X}=(S,E,Q,I,R)$, $u=(u_1,u_2)\in\Omega$ and $\Lambda=(\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5)$ the adjoint variable. The Hamiltonian function is defined as

$$\begin{aligned}\mathrm{H}(\mathscr{X},u,\Lambda,t)= \;& C_1S(t)+C_2E(t)+C_3I(t)+\frac{1}{2}\sum_{i=1}^{i=2}\rho_i u_i^2(t)+\lambda_1(t)\frac{dS(t)}{dt}+\lambda_2(t)\frac{dE(t)}{dt}\\ &+\lambda_3(t)\frac{dQ(t)}{dt}+\lambda_4(t)\frac{dI(t)}{dt}+\lambda_5(t)\frac{dR(t)}{dt}.\end{aligned} \tag{5}$$

Necessary condition that $(\mathscr{X}^*(t),u^*(t))$ be can optimal solution for the optimal control problem is the existence of a non-trivial vector function $\Lambda(t)=(\lambda_1(t),\lambda_2(t),\lambda_3(t),\lambda_4(t),\lambda_5(t))$ such that

$$\begin{aligned}\frac{d\mathscr{X}}{dt} &= \frac{\partial \mathrm{H}(\mathscr{X}^*(t),u^*(t),\Lambda(t),t)}{\partial\Lambda}\\ 0 &= \frac{\partial \mathrm{H}(\mathscr{X}^*(t),u^*(t),\Lambda(t),t)}{\partial u}\\ \frac{d\Lambda(t)}{dt} &= -\frac{\partial \mathrm{H}(\mathscr{X}^*(t),u^*(t),\Lambda(t),t)}{\partial\mathscr{X}}.\end{aligned} \tag{6}$$

Theorem 2. *Given an optimal control* $u^*=(u_1^*,u_2^*)$ *and corresponding solutions* $\mathscr{X}^*=(S^*,E^*,Q^*,I^*,R^*)$ *that minimize* $J(u)$ *over* Ω. *Then, there exist adjoint variables* λ_i, $i=1,2,\cdots,5$ *satisfying*

$$\begin{aligned}\frac{d\lambda_1(t)}{dt} &= -C_1+[\alpha E(t)+\beta I(t)]\,[\lambda_1(t)-\lambda_2(t)]+u_1(t)\,(\lambda_1(t)-\lambda_3(t))+u_2(t)\,(\lambda_1(t)-\lambda_5(t))\\ &\quad+\mu\lambda_1(t),\end{aligned}$$

$$\frac{d\lambda_2(t)}{dt} = -C_2+\alpha S(t)\,[\lambda_1(t)-\lambda_2(t)]+u_1(t)\,(\lambda_2(t)-\lambda_3(t))+\sigma\,(\lambda_2(t)-\lambda_4(t))+\mu\lambda_2(t),$$

$$(7)\qquad \begin{aligned} \frac{d\lambda_3(t)}{dt} &= \delta\left(\lambda_3(t)-\lambda_4(t)\right)+r_1\left(\lambda_3(t)-\lambda_5(t)\right)+\mu\lambda_3(t), \\ \frac{d\lambda_4(t)}{dt} &= -C_3+\beta S(t)[\lambda_1(t)-\lambda_2(t)]+r_2\left(\lambda_4(t)-\lambda_5(t)\right)+(\eta+\mu)\,\lambda_4(t), \\ \frac{d\lambda_5(t)}{dt} &= -\gamma\lambda_1(t)+(\gamma+\mu)\,\lambda_5(t), \end{aligned}$$

where the transversality conditions $\lambda_i(t_f)=0$, $i=1,2,\cdots,5$. Moreover, the following characterization holds:

$$(8)\qquad \begin{cases} u_1^*(t) = \max\{\,\min\{\,1,\ \dfrac{S^*(t)\,(\lambda_1(t)-\lambda_3(t))+E^*(t)\,(\lambda_2(t)-\lambda_3(t))}{\rho_1}\,\},\ 0\,\}, \\ u_2^*(t) = \max\{\,\min\{\,1,\ \dfrac{[\lambda_1(t)-\lambda_5(t)]S^*(t)}{\rho_2}\,\},\ 0\,\}, \end{cases}$$

Proof. The adjoint equations (7) is derived from the Pontryagin's Maximum Principle by taking the partial derivatives of the Hamiltonian (5) with respect to the corresponding state variables, so that

$$\frac{d\lambda_1}{dt}=-\frac{\partial \mathrm{H}}{\partial S} \quad , \quad \frac{d\lambda_2}{dt}=-\frac{\partial \mathrm{H}}{\partial E}$$

$$\frac{d\lambda_3}{dt}=-\frac{\partial \mathrm{H}}{\partial Q} \quad , \quad \frac{d\lambda_4}{dt}=-\frac{\partial \mathrm{H}}{\partial I}$$

$$\frac{d\lambda_5}{dt}=-\frac{\partial \mathrm{H}}{\partial R}.$$

with transversality terminal conditions $\lambda_k(t_f)=0, k=1,2,3,4,5$. Furthermore, to get characterization of the optimal controls given by (8) we solve the following partiel differential equations on the interior of the control set Ω:

$$\frac{\partial \mathrm{H}}{\partial u_1}=0 \quad for \quad u_1^*$$

$$\frac{\partial \mathrm{H}}{\partial u_2}=0 \quad for \quad u_2^*$$

we use the optimality conditions, then

$$u_i^*(t) = \begin{cases} 0 & if \quad v_i^* < 0 \\ v_i^* & if \quad 0 \le v_i^* \le 1 \\ 1 & if \quad v_i^* > 1 \end{cases}$$

for $i = 1,2$ where

$$\begin{aligned} v_1^* &= \frac{S^*(t)\,(\lambda_1(t) - \lambda_3(t)) + E^*(t)\,(\lambda_2(t) - \lambda_3(t))}{\rho_1} \\ v_2^* &= \frac{[\lambda_1(t) - \lambda_5(t)]S^*(t)}{\rho_2} \end{aligned}$$

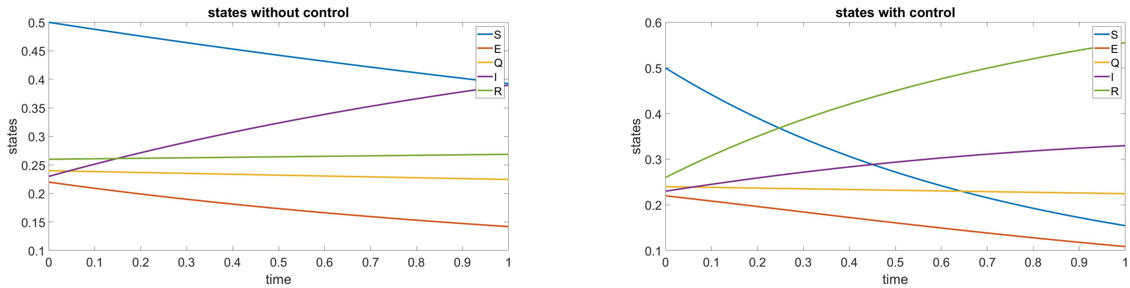
This completes the proof. □

4. SIMULATION

In this section, we give numerical results of the optimal control of the COVID-19 pandemic model (1). The simulations are carried out using MATLAB. The results are simulated for one year, divided into 1 month sections. The data is collected based heavily on the model parameters and coefficients values in [1].

The evolution of the five states without and with controls are represented in figures (2a) and (2b), respectively. The number of susceptible individuals (S) (blue) decreases less rapidly in the case without control, because the recovered individuals may also be susceptible in our SEQIRS model, while the number of infected individuals increases due to the uncontrolled spread of the virus. On the other hand, by introducing intervention strategies that combine the effort of preventive measures (quarantine and isolation) and vaccination. As shown in Figure (2b), the number of susceptible (S) (blue), exposed (E) (red), and infected (I) (purple) individuals reduces more rapidly when controls are in use than the case without controls and the number of recoveries (R) (green) increases to a high level.

To find the best control strategy, we simulate the evolution of each state with controls applied separately. Our goal is to determine the optimal control strategies for each state to reduce or limit the spread of viruses.



(a) States without controls

(b) States with controls

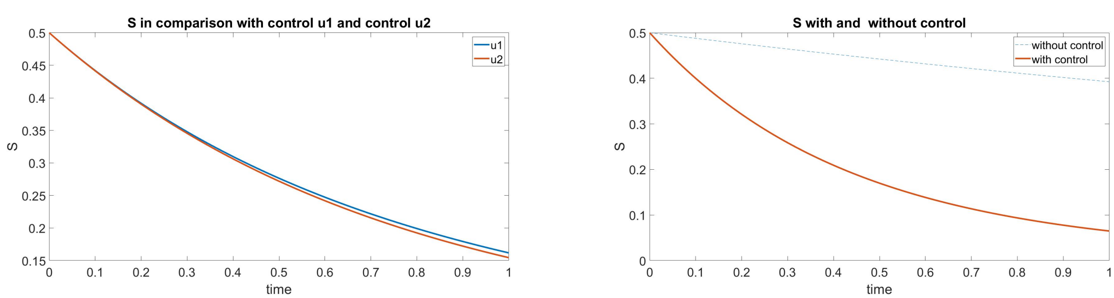
FIGURE 2. Comparison of states with and without controls

In what follows, we consider always strategies using controls u_1 and u_2 separately as shown in figures (3a), (4a) and (5a) and combined as shown in figures (3b), (4b) and (5b), to minimize the number of susceptible, exposed and infected individuals.

It can be observed in all the different cases considered that the number of exposed, susceptible, and infected decrease more rapidly in the presence of the optimal control strategies than when the controls are absent.

Figure (3a) shows the effect of vaccination u_1 and preventive measures u_2 separately on susceptible individuals, by comparing figures (3a) and (3b) we notice that the optimal control strategy which combines the use of u_1 and u_2 is more efficient.

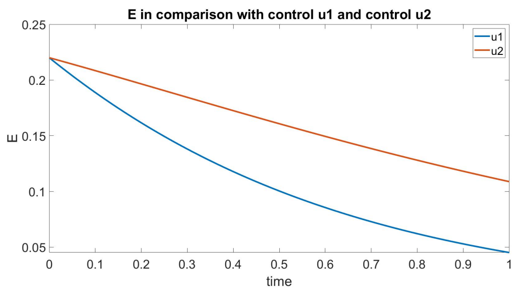
From figures (4a) and (5a), it can easily be seen that when the preventive measures u_1 are applied, the number of exposed and infected individuals is considerably reduced compared to the exposed and infected individuals having the measure of combined control of u_1 and u_2.



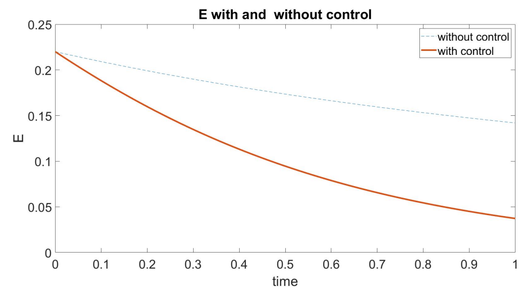
(a) Comparison of two controls

(b) With and without controls

FIGURE 3. Susceptible individuals S

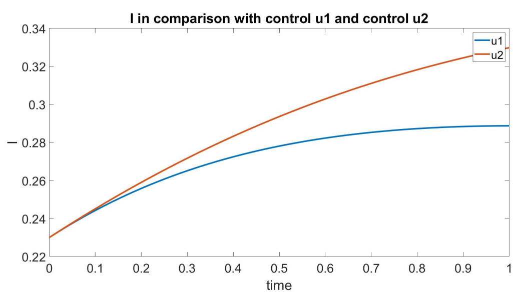


(a) Comparison of two controls

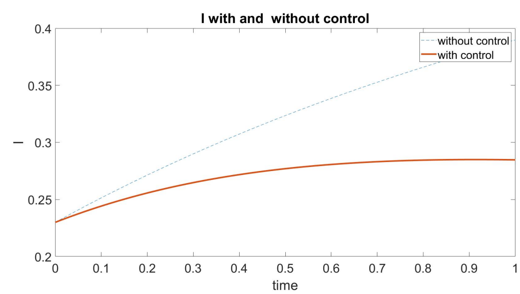


(b) With and without controls

FIGURE 4. Exposed individuals E



(a) Comparison of two controls



(b) With and without controls

FIGURE 5. Infected individuals I

Finally, we simulated separate and combined controls to examine the evolution of recovered individuals (R) as shown in Fig (6). The vaccination strategy u_2 influences significantly and sufficiently to maximize recovered individuals compared to individuals without optimal controls. However, u_1 is not a preferable strategy since it doesn't have any reel impact on the state (R).

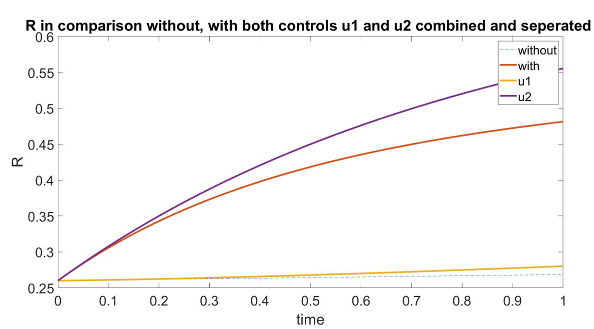


FIGURE 6. Recoverd individuals R states

5. Conclusion

This study aimed to formulate a model of COVID-19 transmission by considering two control variables as vaccination and preventive measures such as confinement and isolation. Also, its objective was to determine the best optimal control strategy to minimize the spread of COVID-19, limiting as much as possible the number of deaths and infections due to this pandemic. We have shown the existence of optimal controls by minimizing the number of susceptible, exposed, and infected individuals taking into account the cost of implementation. Using Pontryagin's Maximum Principle the optimal control strategies are determined. In addition, numerical simulations have been performed to illustrate and verify the analytical results. We found that each of the optimal control strategies was effective, necessary, and had a positive impact on minimizing the number of susceptible, exposed, and infected individuals compared with the uncontrolled system, therefore maximizing the number of recovered individuals.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

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Conflict of Interests

The authors declare that there is no conflict of interests.

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