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# APPLICABILITY AND ANALYSIS OF TRIGONOMETRIC – EXPONENTIAL SINGLE-STEP METHOD FOR THE NUMERICAL SOLUTION OF HIV-1 MODEL

S.E. FADUGBA[1,2,3,*], V.J. SHAALINI[4], M.O. OLUWAYEMI[1,2], M.C. KEKANA[5]

[1]Department of Physical Sciences, Mathematics Programme, Landmark University, Omu-Aran, Nigeria

[2]Landmark University SDG 4: Quality Education Research Group, Omu-Aran, Nigeria

[3]Department of Mathematics, Ekiti State University, Ado Ekiti, 360001, Nigeria

[4]Department of Mathematics, Bishop Heber College, Trichy, India

[5]Department of Mathematics, Tshwane University of Technology, Pretoria, South Africa



**Abstract:** This paper proposes the Trigonometric Exponential Single-Step Method (TESSM) for the numerical solution of HIV-1 infection model by using an interpolating function that consists of both trigonometric and exponential functions. The delay argument was approximated using Lagrange interpolation. The analysis of TESSM such as order of accuracy, convergence, consistency and stability was presented. The applicability of TESSM was tested on the HIV-1 infection model. The results generated via TESSM were also presented.

**Keywords:** delay argument; delay differential equation; exponential function; Lagrange interpolation; trigonometric function.

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[*]Corresponding author
E-mail addresses: fadugba.sunday@lmu.edu.ng, sunday.fadugba@eksu.edu.ng
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## 1. INTRODUCTION

Most of the physical models in science and engineering are emanated from Differential Equations (DEs). Some of these DEs are difficult to solve or cannot be solved analytically. An alternative approach is to use numerical integration methods for approximating the solution of DEs using prescribed initial or boundary conditions [1]. There are many methods developed for the numerical solutions of the Initial Value Problems (IVPs) of the form

$$y'(t) = f\left(t, y(t), y\left(t - \tau\big(t, y(t)\big)\right)\right), \qquad t > t_0$$
$$y(t) = \Phi(t), \qquad t \le t_0 \tag{1}$$

where $\Phi(t)$ is the initial function. In [2], the authors developed a new one-step rational method of order four for solving stiff and non-stiff Delay Differential Equations (DDEs) via interpolating function which consists of rational functions. Niekerk [3] proposed first, second and third order explicit nonlinear methods for singular and stiff IVPs. The algorithms are based on the representation of the solution by finite continued fractions. Fadugba [4] developed an improved numerical integration method via the transcendental function of exponential form for the solution of IVPs in Ordinary Differential Equations (ODEs). Islam [5] compared the numerical solutions of IVPs for ODEs with Euler's method and Runge-Kutta method. Stefanov [6] studied the cases of inverse interpolation of monotone and non-monotone functions. Some applications of inverse interpolation, including approximate solutions of nonlinear equations (root-finding) and analysis of census data, are also considered. Numerical models of nitrogen compound measurements in a stream with a removal mechanism using Saulyev technique with cubic spline interpolation were considered by [7]. Several authors have also studied the solutions of IVPs in ODEs via developed and existing methods, see [8] – [26]. Over the last two decades, there has been extensive research on the area of HIV-1 infection invading the human immune system. Bonhoeffer et *al*. [27] introduced a population model representing long-term dynamics of HIV infection in response to available drug therapies. According to the Joint United Nations Programme on HIV/AIDS (UNAIDS), 37 million people worldwide are infected with HIV-1 today of whom 24 million are in developing countries, see [28]. Infection with HIV-1, degrading the human immune system and recent advances in drug therapy to arrest HIV-1 infection has generated considerable research interest in the area. Long-term dynamics in a mathematical model of HIV-1 infection with delay in different variants of the basic drug therapy model were considered, see [28]. In this paper, we propose a new numerical method "Trigonometric – Exponential Single-Step Method (TESSM)" to analyse a mathematical model of HIV-1 infection. The rest of the paper is organized as follows. Section 2 presents the derivation of TESSM. In Section 3, the properties of TESSM in terms of order of accuracy, consistency, stability and convergence are analyzed and investigated. Section 4 presents the numerical solution of the HIV-1 infection model via TESSM. Section 5 concludes the paper.

## 2. DERIVATION OF TESSM

Consider the interpolating function of the form

$$F(x) = ax^2 + be^{2x} + csinx \tag{2}$$

Evaluating (1) at the points $(x = x_n)$ and $(x = x_{n+1})$ yields, respectively

$$F(x_n) = a{x_n}^2 + be^{2x_n} + csinx_n \tag{3}$$

and

$$F(x_{n+1}) = a{x_{n+1}}^2 + be^{2x_{n+1}} + csinx_{n+1} \tag{4}$$

Subtracting (4) from (3), yields

$$F(x_{n+1}) - F(x_n) = a({x_{n+1}}^2 - {x_n}^2) + b(e^{2x_{n+1}} - e^{2x_n}) + c(sinx_{n+1} - sinx_n) \tag{5}$$

Using the fact that

$$x_n = nh \text{ and } x_{n+1} = (n+1)h = nh + h$$

Therefore,

$${x_{n+1}}^2 - {x_n}^2 = (2n+1)h^2 \tag{6}$$

$$e^{2x_{n+1}} - e^{2x_n} = e^{2nh}(e^{2h} - 1) \tag{7}$$

$$sinx_{n+1} - sinx_n = 2\sin\left(\frac{h}{2}(2n+1)\right)cos\left(\frac{h}{2}\right) \tag{8}$$

Substituting (6), (7) and (8) into (5) and the fact that

$$y_{n+1} - y_n \equiv F(x_{n+1}) - F(x_n) \tag{9}$$

yields

$$y_{n+1} - y_n = a(2n+1)h^2 + be^{2nh}(e^{2h} - 1) + c(sinx_{n+1} - sinx_n) \tag{10}$$

To get the values of *a, b* and *c,* differentiating (3) thrice and setting

$F'(x_n) = f_n,\ F''(x_n) = {f_n}^{(1)}$ and $F'''(x_n) = {f_n}^{(2)}$ , one gets

$$\begin{bmatrix} 2nh & 2e^{2nh} & \cos(nh) \\ 2 & 4e^{2nh} & -\sin(nh) \\ 0 & 8e^{2nh} & -\cos(nh) \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} f_n \\ {f_n}^{(1)} \\ {f_n}^{(2)} \end{bmatrix} \tag{11}$$

Solving (11) , one obtains

$$a = \frac{1}{2}\left\{\frac{4\sin(nh)f_n - \sin(nh){f_n}^{(1)} - 2\cos(nh)f_n + 5\cos(nh){f_n}^{(1)} - 2\cos(nh){f_n}^{(2)}}{4\sin(nh)nh - 2\cos(nh)nh + 5\cos(nh)}\right\} \tag{12}$$

$$b = \frac{1}{2}\left\{\frac{nh\sin(nh){f_n}^{(2)} - nh\cos(nh){f_n}^{(1)} + \cos(nh)f_n + \cos(nh){f_n}^{(2)}}{(4\sin(nh)nh - 2\cos(nh)nh + 5\cos(nh))e^{2nh}}\right\} \tag{13}$$

$$c = \frac{-4nh{f_n}^{(1)} - 2nh{f_n}^{(2)} + 4f_n - {f_n}^{(2)}}{4\sin(nh)nh - 2\cos(nh)nh + 5\cos(nh)} \tag{14}$$

Substituting (12), (13) and (14) into (10), yields

$$\begin{aligned} y_{n+1} - y_n = {} & \frac{(2n+1)}{2}\left\{\frac{4\sin(nh)f_n - \sin(nh){f_n}^{(1)} - 2\cos(nh)f_n + 5\cos(nh){f_n}^{(1)} - 2\cos(nh){f_n}^{(2)}}{4\sin(nh)nh - 2\cos(nh)nh + 5\cos(nh)}\right\} \\ & + \frac{(e^{2h}-1)}{2}\left\{\frac{nh\sin(nh){f_n}^{(2)} - nh\cos(nh){f_n}^{(1)} + \cos(nh)f_n + \cos(nh){f_n}^{(2)}}{(4\sin(nh)nh - 2\cos(nh)nh + 5\cos(nh))e^{2nh}}\right\} \\ & + (\sin(nh+h) - \sin(nh))\left\{\frac{-4nh{f_n}^{(1)} - 2nh{f_n}^{(2)} + 4f_n - {f_n}^{(2)}}{4\sin(nh)nh - 2\cos(nh)nh + 5\cos(nh)}\right\} \end{aligned} \tag{15}$$

Equation (15) is the new explicit one-step method “TESSM”.

## 3. Analysis of the Properties of TESSM

The properties of the new method are analyzed as follows.

### 3.1 Convergence of TESSM

Using these facts in (15),

$$\begin{aligned} e^{2h} - 1 &= 1 + 2h + \frac{(2h)^2}{2!} + \cdots - 1 \\ &= 2h + \frac{(2h)^2}{2!} + \cdots = 2h + 2h^2 + \cdots = h(2 + 2h + \cdots) \end{aligned} \tag{16}$$

$$\begin{aligned} \sin(nh+h) - sinnh &= \sin(x_n + h) - sinx_n \\ &= sinx_n cosh + \cos x_n sinh - sinx_n \end{aligned}$$

$$= sinx_n(cosh-1)+cosx_n sinh \quad (17)$$

But

$$cosh = 1-\frac{h^2}{2!}+\frac{h^4}{4!}-\cdots,\ sinh = h-\frac{h^3}{3!}+\cdots \quad (18)$$

Therefore,

$$\sin(nh+h)-sinnh = \left(-\frac{h^2}{2!}+\frac{h^4}{4!}-\cdots\right)sinx_n+\left(h-\frac{h^3}{3!}+\cdots\right)cosx_n$$

$$= h(Asinx_n+Bcosx_n) \quad (19)$$

where

$$A = \left(-\frac{h^2}{2!}+\frac{h^4}{4!}-\cdots\right) \text{ and } B = \left(1-\frac{h^2}{3!}+\cdots\right) \quad (20)$$

Using (16) – (20), (15) becomes

$$y_{n+1}-y_n = h\{D+E(Asinx_n+Bcosx_n)c\} \quad (21)$$

with

$$D = \frac{(2n+1)h}{2}\left\{\frac{4\sin(nh)f_n-\sin(nh){f_n}^{(1)}-2\cos(nh)f_n+5\cos(nh){f_n}^{(1)}-2\cos(nh){f_n}^{(2)}}{4\sin(nh)nh-2\cos(nh)nh+5\cos(nh)}\right\} \quad (22)$$

$$E = \frac{(2+2h\ldots)}{2}\left\{\frac{nh\sin(nh){f_n}^{(2)}-nh\cos(nh){f_n}^{(1)}+\cos(nh)f_n+\cos(nh){f_n}^{(2)}}{(4\sin(nh)nh-2\cos(nh)nh+5\cos(nh))e^{2nh}}\right\} \quad (23)$$

From the general one-step method

$$y_{n+1}-y_n = h\phi(x_n,y_n:h) \quad (24)$$

Comparing (21) and (24), one obtains

$$\phi(x_n,y_n;h) = D+E+(Asinx_n+Bcosx_n)c \quad (25)$$

Using the first term of $\sin(nh)$, $\cos(nh)$, $e^{2h}$, $\sin(nh+h)$ and $\cos(nh+h)$, therefore (15) becomes

$$y_{n+1}-y_n = \left(n+\frac{1}{2}\right)h^2\left[\frac{-2f_n+5{f_n}^{(1)}-2{f_n}^{(2)}}{-2nh+5}\right]+h\left[\frac{f_n-nh{f_n}^{(1)}+{f_n}^{(2)}}{-2nh+5}\right]$$

$$+h\left[\frac{4f_n-4nh{f_n}^{(1)}+2nh{f_n}^{(2)}-{f_n}^{(2)}}{-2nh+5}\right]$$

Therefore

$$y_{n+1} - y_n = \frac{\left[f_n(h + 4h - h^2 - 2nh^2) + f_n^{(1)}\left(\frac{5}{2}h^2\right) + f_n^{(2)}(-h^2)\right]}{(-2nh + 5)}$$

But $(-2nh+5)^{-1} = (5-2nh)^{-1} = \left(\frac{1}{5}\right)\left(1-\frac{2}{5}nh\right)^{-1} = \left(\frac{1}{5}\right)\left(1+\frac{2}{5}nh\right)$

$$y_{n+1} - y_n = f_n\left(h - \frac{h^2}{5} - \frac{h^3(2n(2n+1))}{25}\right) + f_n^{(1)}\left(h^2 - \frac{h^2}{2} - \frac{h^3 n}{5}\right) + f_n^{(2)}\left(-\frac{h^2}{5} - \frac{h^3(2n)}{25}\right) \qquad (26)$$

After simplifying the above equation, one obtains

$$y_{n+1} = y_n + hf_n + h^2\left[\frac{f_n^{(1)}}{2} - \frac{f_n}{5} - \frac{f_n^{(2)}}{5} + \frac{nhf_n^{(1)}}{5} - \frac{2n(2n+1)hf_n}{25} - \frac{2nhf_n^{(2)}}{25}\right]$$

Thus,

$$y_{n+1} = y_n + hf_n + h^2 B$$

where $B = \left[\frac{f_n^{(1)}}{2} - \frac{f_n}{5} - \frac{f_n^{(2)}}{5} + \frac{nhf_n^{(1)}}{5} - \frac{2n(2n+1)hf_n}{25} - \frac{2nhf_n^{(2)}}{25}\right]$

$$\Phi(x_n, y_n, z_n : h) = f_n + C = f\big(x_n, y_n, y(t_n - \tau)\big) + C = f(x_n, y_n, z_n) + C$$

Here $C = hB$ and $z_n = y(t_n - \tau)$

Similarly,

$$\Phi(x_n, y_n^*, z_n^* : h) = f(x_n, y_n^*, z_n^*) + C$$

Therefore,

$$\Phi(x_n, y_n^*, z_n^* : h) - \Phi(x_n, y_n, z_n : h) = f(x_n, y_n^*, z_n^*) + C - f(x_n, y_n, z_n) - C$$
$$= f(x_n, y_n^*, z_n^*) - f(x_n, y_n, z_n)$$

Let $\tilde{y}$ and $\tilde{y}^*$ be defined as a point in the interior of the interval whose end points $y_n$ and $y_n^*$ and $z_n$ and $z_n^*$ in the domain D, respectively. Applying the Mean Value Theorem, yields

$$f(x_n, y_n^*, z_n^*) - f(x_n, y_n, z_n)$$
$$= Max\left\{\frac{\partial f(x_n, \tilde{y})}{\partial y_n}, \frac{\partial f(x_n, \tilde{y}^*)}{\partial z_n}\right\}[(y_n^* - y_n) + (z_n^* - z_n)]$$

Define

$$M = Max\left\{\frac{\partial f(x_n, \tilde{y})}{\partial y_n}, \frac{\partial f(x_n, \tilde{y}^*)}{\partial z_n}\right\}$$

Thus,

$$\Phi(x_n, y_n^*, z_n^*: h) - \Phi\big(x_n, y_{n,} z_n: h\big)$$

$$= \left\{\frac{\partial f(x_n, \tilde{y})}{\partial y_n}, \frac{\partial f(x_n, \tilde{y}^*)}{\partial z_n}\right\}[(y_n^* - y_n) + (z_n^* - z_n)]$$

$$\Phi(x_n, y_n^*, z_n^*: h) - \Phi\big(x_n, y_{n,} z_n: h\big) = M[(y_n^* - y_n) + (z_n^* - z_n)]$$

Taking the absolute value of both sides of the last equation, we have

$$\left|\Phi(x_n, y_n^*, z_n^*: h) - \Phi\big(x_n, y_{n,} z_n: h\big)\right| = |M[(y_n^* - y_n) + (z_n^* - z_n)]|$$

$$\leq |M||(y_n^* - y_n) + (z_n^* - z_n)|$$

Therefore we can say that our derived method is convergent and hence $\Phi$ is Lipschitzian.

### 3.2 Order of Accuracy of TESSM

To determine the order of the new scheme, consider the Taylor's series expansion of the form

$$y(x_n + h) = y(x_n) + hf(x_n, y(x_n)) + \frac{h^2}{2!}f^{(1)}(x_n, y(x_n)) + \frac{h^3}{3!}f^{(2)}(x_n, y(x_n)) + O(h^4) \quad (27)$$

The local truncation error is defined as

$$T_{n+1} = y(x_n + h) - y_{n+1} \quad (28)$$

Substituting (27) and (28) into (26), one obtains

$$T_{n+1} = y(x_n) + hf(x_n, y(x_n)) + \frac{h^2}{2!}f^{(1)}(x_n, y(x_n)) + \frac{h^3}{3!}f^{(2)}(x_n, y(x_n)) + O(h^4)$$
$$-\left[y_n + f_n\left(h - \frac{h^2}{5} - \frac{h^3\big(2n(2n+1)\big)}{25}\right) + f_n^{(1)}\left(h^2 - \frac{h^2}{2} - \frac{h^3 n}{5}\right) + f_n^{(2)}\left(-\frac{h^2}{5} - \frac{h^3(2n)}{25}\right)\right] \quad (29)$$

By means of the localizing assumptions $f_n = f\big(x_n, y(x_n), y(t_n - \tau)\big)$, then (29) becomes

$$T_{n+1} = f^{(2)}(x_n, y(x_n))\left(\frac{h^3}{6} + \frac{h^2}{5} + \frac{h^3(2n)}{25}\right) - f(x_n, y(x_n))\left(-\frac{h^2}{5} - \frac{h^3\big(2n(2n+1)\big)}{25}\right)$$
$$-f^{(1)}(x_n, y(x_n))\left(\frac{h^2}{2} - \frac{h^3 n}{5}\right) + O(h^4)$$

Hence, the local truncation error (LTE) is

$$T_{n+1} = f^{(2)}(x_n, y(x_n))\left(\frac{h^3}{6} + \frac{h^2}{5}\left(1 + \frac{2nh}{5}\right)\right) + f(x_n, y(x_n))\left(\frac{h^2}{5}\left(1 + \frac{2nh(1+2n)}{25}\right)\right)$$
$$+f^{(1)}(x_n, y(x_n))\left(h^2\left(\frac{1}{2} - \frac{nh}{5}\right)\right) + O(h^4) \quad (30)$$

The order of accuracy of the new scheme is 2.

**3.3 Consistency Property of TESSM**

A general one-step method is said to be consistent if and only if

$$\phi(x_n, y_n, z_n; 0) = f_n$$

From (25), Setting h = 0, yields

$$D = 0 \tag{31}$$

$$E = \frac{f_n + f_n^{(2)}}{5} \tag{32}$$

$$(Asinx_n + Bcosx_n)c = \frac{4f_n - f_n^{(2)}}{5} \tag{33}$$

Using (31) – (33), (25) becomes

$$\phi(x_n, y_n, z_n; 0) = \frac{5f_n + f_n^{(2)} - f_n^{(2)}}{5} = f_n$$

Hence, $\phi(x_n, y_n, z_n; 0) = f_n$ (34)

Equation (34) shows that TESSM is consistent.

**3.4 Stability of the TESSM**

**Theorem 1**

Let $y_n = y(x_n)$ and $u_n = u(x_n)$ denote two different numerical solutions of DDE with the initial conditions specified as $y(x_0) = \alpha$ and $u(x_0) = \alpha^*$ respectively such that $|\alpha\text{-}\alpha^*| < \varepsilon$, $\varepsilon$ >0. If the numerical estimates are generated by the interpolation scheme (1), we have

$$y_{n+1} = y_n + h\,\Phi(x_n, y_n, z_n; h) \tag{35}$$

$$u_{n+1} = u_n + h\,\Phi(x_n, u_n, v_n; h) \tag{36}$$

The condition that

$$|y_{n+1} - u_{n+1}| < k|\beta - \beta^*| \tag{37}$$

is the necessary and sufficient condition that TESSM (15) be stable and convergent.

**Proof:** Let

$$y_{n+1} = y_n + hf(x_n, y_n, z_n) + C \tag{38}$$

$$u_{n+1} = u_n + hf(x_n, u_n, v_n) + C \tag{39}$$

Thus,

$$y_{n+1} - u_{n+1} = y_n + h\,f(x_n, y_n)\text{-}l_n - hf(x_n, v_n) \tag{40}$$

$$y_{n+1} - u_{n+1} = y_n - u_n + h[f(x_n, y_n) - f(x_n, v_n)] \quad (41)$$

Applying the Mean Value Theorem with the assumption that $\tilde{y}$ and $\tilde{y}^*$ are the points in the interior of the interval whose end points are $y_n$ and $u_n$ and $z_n$ and $v_n$ respectively, we have

$$y_{n+1} - u_{n+1} = y_n - u_n + h\left\{\frac{\partial f(x_n,\tilde{y})}{\partial y_n}, \frac{\partial f(x_n,\tilde{y}^*)}{\partial z_n}\right\}[(y_n - u_n) + (z_n - v_n)]$$

$$y_{n+1} - u_{n+1} = y_n - l_n + h\{ L(y_n - u_n) + (z_n - v_n)\}$$

$$y_{n+1} - u_{n+1} = y_n - l_n + hL(y_n - u_n) + (z_n - v_n) \quad (42)$$

Taking the absolute value of the both side

$$|y_{n+1} - u_{n+1}| = |y_n - u_n + hL(y_n - u_n) + (z_n - v_n)|$$

$$\leq |1 + hL||(y_n - u_n) + (z_n - v_n)| \quad (43)$$

Given,

$$k = |1 + h(L)|, \quad y_n(x_0) = \beta \; and \; u_n(x_0) = \beta^* \quad (44)$$

Then we have,

$$|y_{n+1} - u_{n+1}| \leq k|\beta - \beta^*| \quad (45)$$

We therefore conclude that TESSM is stable and hence convergent. This completes the proof.

## 4. NUMERICAL EXAMPLES

In this section, we analyse a mathematical model of HIV-1 infection to CD4+ T cells including the inhibitor drug via TESSM.

Consider a mathematical model of HIV-1 infection to CD4+ T cells including the inhibitor drug discussed in the paper [28]. Let $x(t)$ be the number of infected cells and $y(t)$ be the number of virus producing cells and $z(t)$ be the density of the Cytotoxic T-Lymphocyte (CTL) responses against virus-infected cells.

**Model 1**

In this basic delay HIV-1 infection model, we assume that the virus producing cells are killed by CTL instantaneously. When the delay $\tau$ is small, this model can be represented by the following set of equations

$$\begin{cases} \frac{dx}{dt} = \lambda - dx - \beta x(t-\tau)y(t-\tau) \\ \frac{dy}{dt} = \beta x(t-\tau)y(t-\tau) - ay - pyz \\ \frac{dz}{dt} = ky - bz \end{cases} \tag{46}$$

Subject to initial condition (IC)

x(θ) = 280.0, y(θ) = 18.5189 and z(θ) = 185.1893, θ∈(-τ,0].

**Model 2**

In reality, there is a latency period during the process of killing of virus-producing cells by CTL. (i.e. not instantaneous as in Model 1). Hence we include a delay in the terms representing killing of virus-producing cells by CTL and in the stimulation of CTL. The model equations are given by

$$\begin{cases} \frac{dx}{dt} = \lambda - dx - \beta xy \\ \frac{dy}{dt} = \beta xy - ay - py(t-\tau)z \\ \frac{dz}{dt} = ky(t-\tau) - bz \end{cases} \tag{47}$$

Subject to the IC

x(θ) = 280.0, y(θ) = 18.5189 and z(θ) = 185.1893, θ∈(-τ,0].

**Model 3**

In this model, we include that the delays exist in the process of infection of healthy T cells and also in the terms representing killing of virus-producing cells by CTL and in the stimulation of CTL together. The model can be represented by the following set of equations

$$\begin{cases} \frac{dx}{dt} = \lambda - dx - \beta x(t-\tau_1)y(t-\tau_1) \\ \frac{dy}{dt} = \beta x(t-\tau_1)y(t-\tau_1) - ay - py(t-\tau_2)z \\ \frac{dz}{dt} = ky(t-\tau_2) - bz \end{cases} \tag{48}$$

Subject to the IC

x(θ) = 230.0, y(θ) = 18.5189 and z(θ) = 185.1893, θ∈(-τ,0].

The variables and parameters used in these three models are given in Table 1. The numerical simulations of these models by TESSM using Table 1 are shown in Figs. 1 - 3.

**Table 1: Variables and Parameters used in the Models**

| **Parameters** | **Definition** | **Default values assigned** |
|---|---|---|
| $\lambda$ | production rate of CD4+ T cells | $10.0\text{mm}^{-3}\ \text{day}^{-1}$ |
| $d$ | Death rate of susceptible CD4+ T cells | $0.01\text{day}^{-1}$ |
| $\beta$ | Rate of contact between x and y | $0.002\text{mm}^{-3}\ \text{day}^{-1}$ |
| $a$ | Death rate of virus-producing cells | $0.24\text{day}^{-1}$ |
| $k$ | Rate of stimulation of CTL | $0.2\text{day}^{-1}$ |
| $b$ | Death rate of CTL | $0.02\text{day}^{-1}$ |
| $p$ | Killing rate of virus-producing cells by CTL | $0.001\text{mm}^{-3}\ \text{day}^{-1}$ |

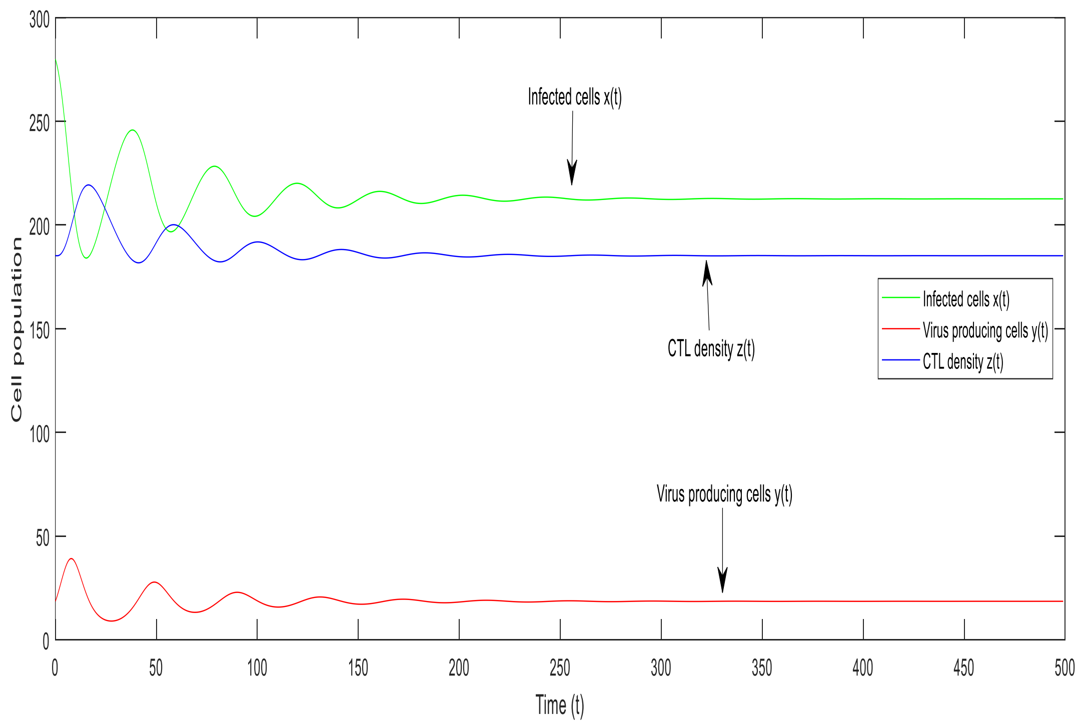


**Fig. 1: The numerical simulations of model 1 via TESSM**

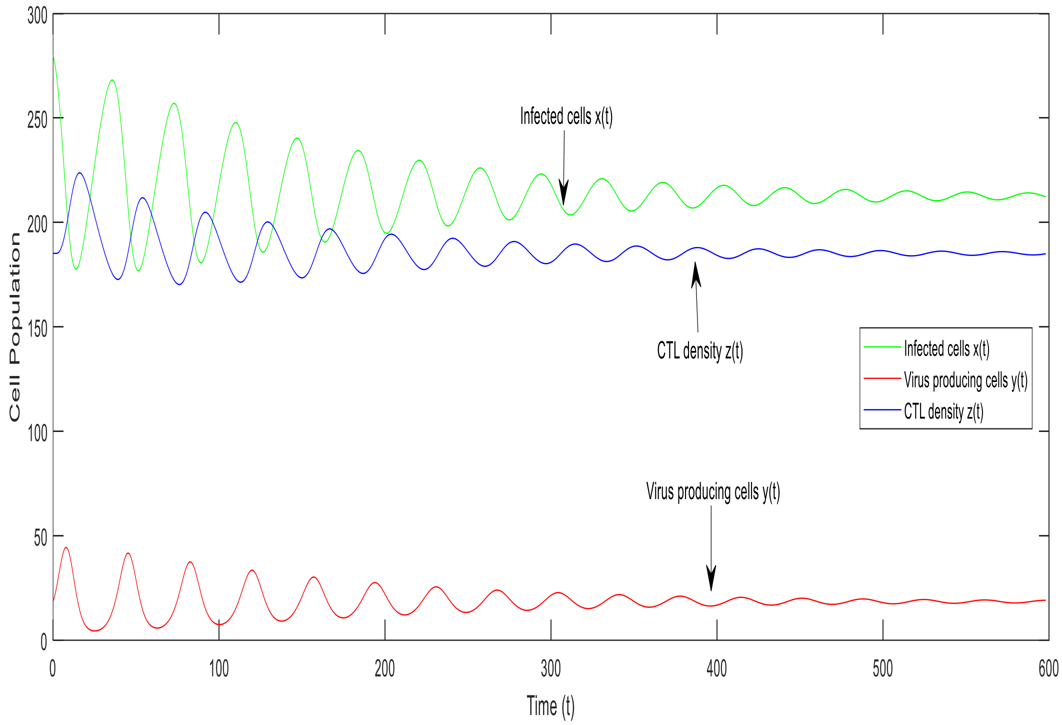


**Fig 2: The numerical simulations of model 2 via TESSM**

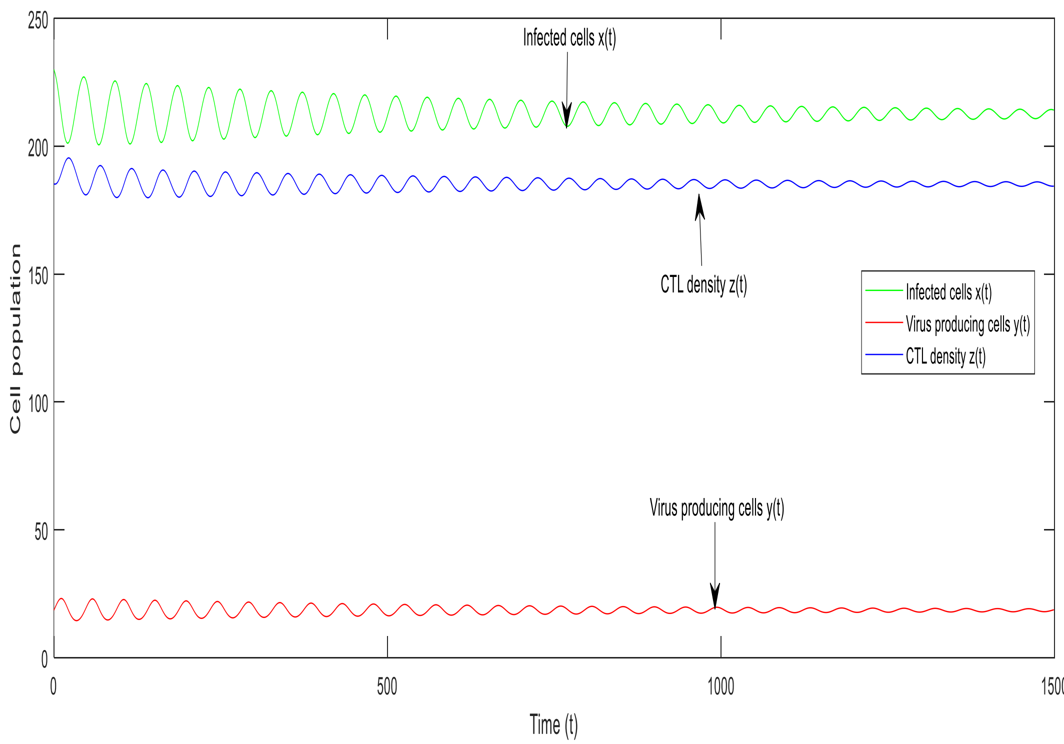


**Fig 3: The numerical simulations of model 3 via TESSM**

## 5. Conclusion

In this paper, we developed the trigonometric-exponential single-step method via an interpolating function of transcendental type for the numerical solution of the HIV-1 infection model. We also investigated and discussed the analysis of the properties of the derived method. The solution graphs of the results generated via TESSM for HIV-1 infection models 1-3 are well comparable with the numerical simulations given in [28]. Hence, it is noteworthy to conclude that TESSM is suitable for solving physical phenomena that led to DDEs in various fields of science and engineering.

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## Conflict of Interests

The authors declare that there is no conflict of interest.

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