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OPTIMAL MULLETS CONSUMPTION BY OSPREY POPULATION USING UTILITY FUNCTION

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Abstract. We consider the interaction between mullets and osprey populations. The main objective of this work is that the osprey must choose the predation intensity over time in a way that maximizes the present value of the utility stream derived by consuming mullets. The model has features of both convex and concave optimal control problems and therefore, phase plane analysis has to be combined with the problem of synthesis of bang-bang, singular and chattering solution pieces.

Keywords: prey-predator model; optimal control; utility function; mullets; osprey.
2020 AMS Subject Classification: 92B05, 49J30, 91B16, 15A03, 92B05.

1. Introduction

Mathematical ecology is one of the primeval studies in both biology and applied mathematics. To define any type of biological phenomena, such as competition between two species, predation, refugia, extinction of species, etc., it is necessary to have a good knowledge of mathematics. In these contexts, mathematical models can be used as an effective tool to realistically describe and analyze such phenomena. Moreover, an appropriate mathematical model can be

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utilized to accurately predict the future behavior of an ecosystem. Chiefly in optimization, the use of its methods in ecology has been widely advocated by K. E. F. Watt ([1], Chapter 13). One of the first and efficient optimization techniques to solve dynamic system is the Pontryagin's Maximum Principle. It is used in optimal control theory to find the best possible control for taking a dynamical system from one state to another. It has been very useful in the study of optimal rocket trajectories, [2]. A major reason for its success in this field is that Newton's Laws of Motion provide very accurate mathematical models of the dynamics of the rocket.

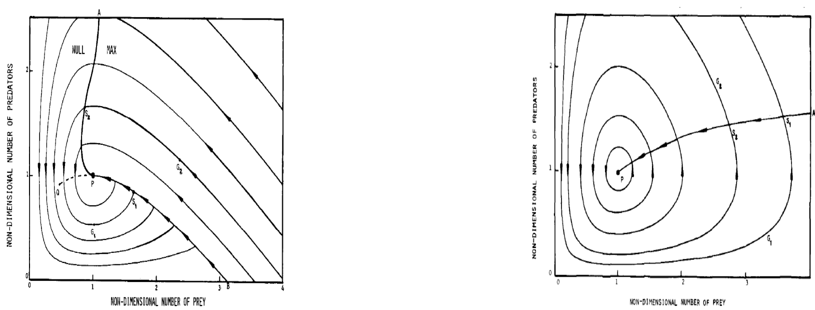

FIGURE 1. Optimal control of a prey-predator model. Source [6]

The main obstacle to the utilization of optimal control theory in ecology is the lack of accurate mathematical models of the dynamics of ecosystems. One ecosystem that has been the topic of a variety of mathematical modelings is the prey-predator system. Recently, T. Royama [3] made a comparative study of many of the mathematical models that are proposed for the prey-predator system. In this study, they used the Lotka-Volterra model. Utilizing such a simplified model of insect populations will facilitate the integration of the use of optimal control theory in ecology. There's no difficulty in extending this analysis to other prey-predator models or a model of the prey-predator system with age structures. Most works on the prey-predator system were done on the uncontrolled models. A first and original study of a controlled prey-predator system was introduced by G. Gause [4]. Later L. G. Slobodkin et al. [5] limited themselves to regulate variables that are a- priori proportional to the prey and predator numbers. Additionally, the category of control variables they employed is far more general. In an other work, B. S. Goh et al. [6] proposed the use of optimal control theory to obtain optimal strategies for the control of a prey-predator system. Here, two sorts of control variables were used: one control variable is the rate of release of predators or prey which are bred in laboratories, the second adverse variable is the rate of application of an insecticide. An interesting result of this contribution was the efficient regulation of a pest through an insecticide that destroys only the predators but leaves the pests unharmed. This is due to the fact that the prey-predator system may be a phase space. The extent of the control variable and therefore, the timing of its application are often manipulated to supply desired responses from the dynamical system. Another interesting result is that the system is often controlled by releasing pests that are bred in laboratories. These findings could also be useful in formulating an integrated control scheme for the management of a pest (see Figure 1). The optimal control has been widely studied by researchers from a spread of backgrounds, recently upscaled and varied in the literature. In [7], A. Gohary et al. discuss the problem of optimal control for the steady state of the Lotka-Volterra model. The conditions of the asymptotic stability of the steady state of this model aim to obtain the optimal control functions. In this study, the optimal Lyapunov function is employed to help to estimate the domain of attraction which is acquired by solving a partial differential equation that is not easily solvable for all the systems. In [8], D. Amalia. R. U discussed a predator prey mathematical model with infection and harvesting of prey that only occur in the prey population, additionally, it is assumed that the prey infection would not affect the predator population. In their work, they analyzed the mathematical model of predator-prey with infection and harvesting of prey. Optimal control is applied within the model to help prevent the prey infection, where the control aims to increase the susceptible prey. In [9], Simon, J. S. H. formulated an optimal control problem for a predator-prey model with the disease within the prey population. This model is an adapted Lotka-Volterra model but with an applied SI epidemic dynamics on the prey population. Two controls are then applied to the system: first, a separating control, that is intended to separate the sound prey from the infected prey population, while the other is a treatment control aims to decrease the speed of death caused by the disease. They then formulated a finite-time horizon optimal control problem by minimizing the infected prey population, and thus, the cost induced from the appliance of the controls. In [10], S. AL-Nassir proposed a two-dimensional continuous prey-predator model with first-order differential equations. All the possible equilibria and their local stability are investigated. This autonomous system is then extended to an optimal control model by proposing an impact variable, which reduces the danger of extinction of the prey population. In [11], I. Agmour et al. formulated a bioeconomic model of a prey and predator planktonic species and studied the positivity and boundedness of the solution. They also analyzed the possible equilibrium and their local stability. Additionally, the worldwide stability of the system around the interior equilibrium was investigated as well as the the optimal harvesting policy to debate the dynamical profit of the interacting planktonic species. To point out the impact of the toxicity coefficient, they needed to make analytical estimates that were validated using simulations. In [12], M. Lamlili E.N. et al. proposed an optimal control approach in their work where they applied the Pontryagin minimum principle to characterize the optimal control, to attenuate the population of susceptible individuals, and also to reduce the mortality rate of coronary heart disease. A numerical simulation was administered to show the impact of the proposed optimal control. In [13], M. Lamlili E.N. et al. showed that mathematical modeling remains one of the best ways to analyze the spread of the coronavirus and control its prevalence. In this context, they proposed a SIAR compartment model with control to reduce the reproduction number R0 and slow down the epidemic outbreak. Another lively area of research in the context of determining dynamics and interaction of predator-prey is the study of the utility function. Ecologists often utilize utility functions to check several hypotheses. In recent works, N. Serra [14] discussed a preliminary study on the possible connection between the Lotka-Volterra model and predator-prey utility functions. Also, they defined a generalization of the utility functions to the predator-prey population, seeing that the utility functions depend on parameters such as the adopted strategies, physical efficiency of the predator versus the prey, environmental conditions and prey prudence. In this context, N. Serra [15] proposed a mathematical model to elucidate the interacting behavior of predator and prey. This model is based on the utility function of the competing individuals. Such functions depend on various parameters that suitably describe animal instincts, considering both physical and environmental conditions. In A. Idmbarek et al. [16], we examined the connection between prey and predators by studying the interactive behavior of this model and by using the change of prey. the goal is to maximize the profit function of each predator by determining the strategy provided by each predator to maximize its profit. To this end, we maximized this utility function being constrained by balance equations between biomass and trophic. The novelty of this work is to combine two different approaches: the optimal control and the utility function, in order to model and to predict the prey-predator behavior. The main objective of our study is to obtain decision rules for the predator. The present investigation is organized as follows: In the next section 2, we define a biological model description for osprey and mullets interaction and then we transformed this last problem into the state variable optimal control problem. In Section 3, we apply standard control theory in current value terms on the model described in the first section to find the optimal consumption rules. In Section 4, we compute the possible equilibrium and we study their stability.

2. Model Description, Formulation, and Analysis

The osprey or more specifically the western osprey (Pandion Haliaetus) also called sea hawk and river hawk is a diurnal, fish-eating bird of prey with a cosmopolitan range. It is an outsized raptor, reaching quite 60 cm (24 in) long and 180 cm (71 in) across the wings (see Figure 2).

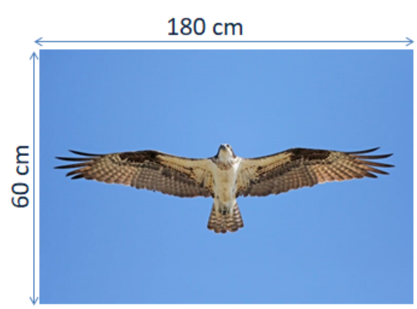

FIGURE 2. The length and width of osprey. Source: https://www.allaboutbirds.org/guide/Osprey/photo-gallery/60320591

It is brown on the upper parts and predominantly greyish on the highest and underparts. The osprey tolerates an honest sort of habitat, nesting in any location near water providing an adequate food supply. It can be found on all continents except Antarctica, although in South America it lives only as a non-breeding migrant. As its other common names suggest, the osprey's diet consists almost exclusively of fish. It possesses specialized physical characteristics and exhibits unique behavior to assist in hunting and catching prey. As a result of those unique characteristics, it has been given its taxonomic genus.

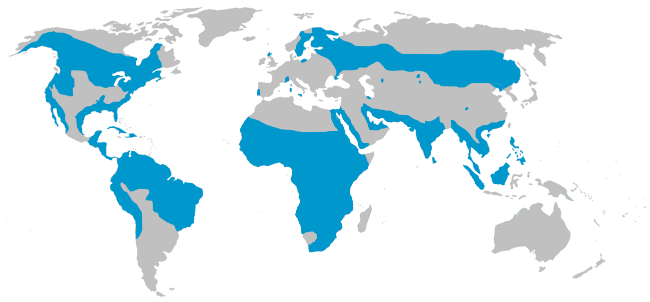

FIGURE 3. Global distribution of osprey. Source: https://commons.wikimedia.org/wiki/File:Pandion_global_range.svg

Over the last three generations (29 years), there has been a 108% increase in the species population in North America [17]. Other estimates provide a way sharper increase between 1965 and 2007 (1,100% increase over 40 years, equating to an 84.2% increase per decade; data from Breeding Bird Survey and/or Christmas Bird Count [18]). Note, however, that these surveys cover only 50% of the species home in North America. the ECU population is estimated to be increasing (BirdLife International in prep). In North Africa, the population decreased sharply in recent decades with a reported 35.7% decline in Morocco between 1990 and 2013 [19], though the population is now believed to be stable (Garrido et al. in prep). The species also appear to be undergoing a decline in India (State of India's Birds 2021). But overall, the species global population is estimated to be increasing (see Figure 3). Among the prey of osprey, there are mullets or grey mullets (see Figure 4) which are a family (Mugilidae) of ray-finned fish found worldwide in coastal temperate and tropical waters, and a few species in fresh water.

FIGURE 4. An osprey captures a mullet. Source: https://www.allaboutbirds.org/guide/Osprey/photo-gallery/60320571

Mullets have served as a crucial source of food in Mediterranean Europe since Roman times. The family includes about 78 species in 20 genders, also it is found within the North Sea, the English Channel, the Atlantic to the coast of Morocco in the south, and along the Mediterranean coasts. There are quite 80 species. The standard noticeable behavior in mullets is the tendency to leap out of the water. There are two distinguishable sorts of leaps: a straight, clean slice out of the water to flee predators and a slower, lower jump while turning to its side that leads to a bigger and more distinguishable splash. It is believed that lower jump helps to release the oxygen rich air for gas exchange during a small organ above the pharynx. Mullets species are widespread in the coastal waters of the tropical and subtropical zones of all seas. In the eastern Pacific, it can be found in California to Chile. In the western Atlantic, it is present in the area from Nova Scotia to Brazil, from Cape Cod to the Gulf of Mexico, absent from the Bahamas islands, and in large a part of the West Indies and therefore the Caribbean. In the eastern Atlantic, it meets from the Bay of Biscay to South Africa. Present in the Mediterranean and the Black Sea basins. (see Figure 5).

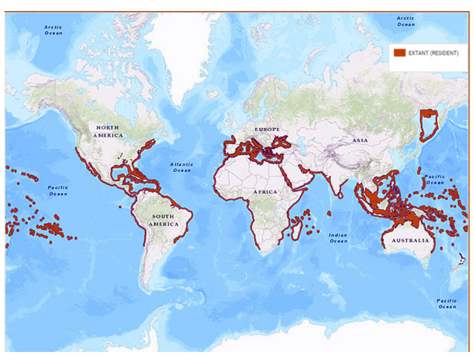

FIGURE 5. Global distribution of mullets. Source: http://www.ittiofauna.org/web museum/pesciossei/perciformes/mugilidae/mugil/mugilcephalus/index.htm

In this work we study the interaction between these two species

$$
\begin{cases}
\dot{M}(t) = rM(t)\left(1 - \dfrac{M(t)}{K}\right) - \delta M(t)O(t) \\
\dot{O}(t) = -dO(t) + \delta M(t)O(t)
\end{cases}
\tag{1}
$$

The mathematical model 1 describes the evolution of the biomass of osprey and mullets fish populations. We assume that the mullets are the prey of the osprey population. The mullet's abundance is denoted by $M(t)$. It grows according to a logistic equation where r is the natural growth rate and K is the carrying capacity. The parameter δ represents the capture coefficients of mullets fish populations by the osprey and the abundance of the latter is denoted by $O(t)$. The natural death is given by d and the parameter δ denotes the conservation coefficient of osprey biomass in mullets populations (see Figure 6).

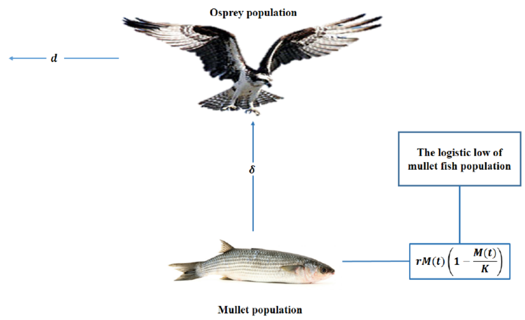

FIGURE 6. Interaction resulting between osprey and mullet populations.

The main objective of our study is to get decision rules for the osprey. The problem is to choose the consumption rate per osprey $v(t)$ over time in a way that maximizes the entire benefit received over an infinite horizon

$$\int_0^\infty e^{-nt} U[v(t)]dt, \tag{2}$$

where U denotes the osprey's utility function and n is the constant discount rate.

The system therefore becomes

$$\begin{cases} \dot{M}(t) = rM(t)\left(1-\frac{M(t)}{K}\right) - v(t)O(t), \\ \dot{O}(t) = -dO(t) + v(t)O(t). \end{cases} \tag{3}$$

The control problem which maximizes 2 subject to 3, $v \geq 0$ and $O \geq 0$ can be transformed to the following one state variable optimal control problem using the following transformation technique frequently used in resource models: We define the mullets-osprey ratio as

$$y(t) = \frac{O(t)}{M(t)}, \tag{4}$$

then we obtain the new state equation for y

$$\dot{y}(t) = -\left[d + v(t) - r\left(1-\frac{M}{K}\right)\right] y(t) + v(t)y^2(t). \tag{5}$$

If the utility function $U(v(t))$ in 2 is concave then the problem which maximize 2, subject to 5, $y \geq 0$ and $v \geq 0$ is similar to the classical growth models.

For $v(t) = 0$, we will have $U(0) = 0$ which gives $U' > 0$.

For $v(t) > 0$, if $v(t)$ is larger than a certain $\widetilde{v}(t)$, we obtain that U'' is negative ($U'' < 0$), and if $v(t)$ is less than a certain $\widetilde{v}(t)$, we obtain that U'' is positive ($U'' > 0$) which assumes that U is convex-concave. Note that $U'(0)$ can vanish.

Thus the average Osprey described is an insatiable one with increasing marginal utility for small levels of consumption. For higher values of v, the marginal utility is diminishing. To simplify the question under consideration, let us replace the state variable inequality constraint $y \geq 0$ with the terminal constraint

$$\lim_{t \to \infty} y(t) \geq 0$$

which is according to equation 5, equivalent to $y(t) \geq 0$.

3. Optimal Consumption Rules

Apply the standard control theory in current value terms to the model posed in the first section of this paper, we obtain the following set of necessary conditions for $v(t)$ to be optimal: there exists a continuous adjoint function $q(t)$ such that the Hamiltonian:

$$H = U(v(t)) + q(t)\left[\left(-d + v(t) - r\left(1 - \frac{M}{K}\right)\right) y(t) + vy^2(t)\right] \tag{6}$$

is maximized by $v = v(t)$ and the adjoint function satisfies

$$\begin{aligned} \dot{q}(t) &= nq(t) - H_y \\ &= \left[n - d - v(t) + r\left(1 - \frac{k}{M}\right) + 2v(t)y(t)\right] q(t). \end{aligned} \tag{7}$$

The transversality conditions are expressed as follows

$$\lim_{t \to \infty} e^{-nt} q(t) \geq 0, \ \lim_{t \to \infty} e^{-nt} q(t) y(t) = 0. \tag{8}$$

Note that the usual arrow-type sufficiency condition which assumes the concavity of the maximized Hamiltonian is not satisfied here. However, using a state transformation, it can be shown that conditions 6-8 are sufficient for the optimality of v.

Let $\overline{v}$ so that $v > \widetilde{v}$ be level of consumption for which the elasticity of the utility function is given by

$$U'(\overline{v})\overline{v} = U(\overline{v}), \tag{9}$$

then the maximization of H with respect to v implies that

$$v(t) = \begin{cases} 0 \\ 0 \\ v^* \end{cases} \text{ or } v(t) = v^* \text{ if } U'(\overline{v}) = \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} q(1+y) \tag{10}$$

where $v^* = v^*(y,q) > \overline{v}$ is the solution of

$$U'(v^*) = q(1+y),$$

Note that because of 9 a value of $0 < v < \overline{v}$ cannot be optimal and that $U'(\overline{v}) \gtrless q(1+y)$ is equivalent to $H(v=0) > H(v=\overline{v})$. The policy above is illustrated in Figure 7

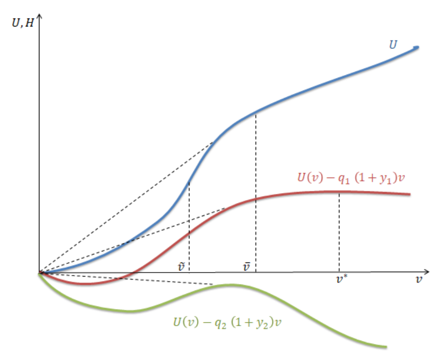

FIGURE 7. Convex-concave utility function and maximization of Hamiltonian

4. LOCAL STABILITY

The steady states of the system 1 are obtained by solving the system of equations 11

$$\begin{cases} rM(t)\left(1 - \dfrac{M(t)}{K}\right) - \delta M(t)O(t) = 0 \\ -dO(t) + \delta M(t)O(t) = 0 \end{cases} \tag{11}$$

we find that the system admits four biologically feasible steady states as follows

$$\begin{cases} P_1 = (0,0) \\ P_2 = \left(\dfrac{d}{\delta},0\right) \\ P_4 = \left(0,\dfrac{r}{\delta}\right) \\ P_3 = \left(\dfrac{d}{\delta},-\dfrac{rd-\delta rK}{\delta^2 K}\right) \end{cases}$$

The Jacobian matrix for system 1 is given by

$$J = \begin{bmatrix} r\left(1-\dfrac{M}{K}\right) - O\delta - \dfrac{r}{K}M & -M\delta \\ \delta O & \delta M - d \end{bmatrix}.$$

- For the equilibrium point $P_1 = (0,0)$ the Jacobian matrix is given by

$$J(P_1) = \begin{bmatrix} r & 0 \\ 0 & -d \end{bmatrix},$$

  then the eigenvalues are $\lambda_1 = -d < 0$, and $\lambda_2 = r > 0$. We deduce that this point is unstable.

- For the equilibrium point $P_2 = \left(\dfrac{d}{\delta},0\right)$ the Jacobian matrix is given by

$$J(P_2) = \begin{bmatrix} \dfrac{r}{\delta K}(\delta K - 2d) & -d \\ 0 & 0 \end{bmatrix},$$

  the eigenvalues are $\lambda_1 = -\frac{1}{K}d\frac{r}{\delta} < 0$ and $\lambda_2 = 0$. In this case, nothing can be concluded.

- For the equilibrium point P_3 the Jacobian matrix is given by

$$J(P_3) = \begin{bmatrix} 0 & 0 \\ r & -d \end{bmatrix},$$

  the eigenvalues are $\lambda_1 = -d < 0, \lambda_2 = 0$. Therefore, nothing can be concluded.

- Similarly, for point P_4 the Jacobian matrix $J(P_4)$ is given as follows

$$J\left(P_4\right)=\left[\begin{array}{cc} -d\frac{r}{\delta}\frac{1}{K} & -d \\ -\frac{1}{\delta K}\left(rd-r\delta K\right) & 0 \end{array}\right],$$

the eigenvalues are

$$\begin{cases} \lambda_1=\dfrac{1}{2\delta K}\left(\sqrt{rd\left(-4\delta K\left(\delta K-d\right)+rd\right)}-rd\right), \\ \lambda_2=-\dfrac{1}{2\delta K}\left(\sqrt{rd\left(-4\delta K\left(\delta K-d\right)+rd\right)}+rd\right). \end{cases}$$

We distinguish the following cases:

(1) If $rd\left(-4\delta K\left(\delta K-d\right)+rd\right) > 0$, then $\delta K < d$ with $\delta K > 0$ and $\sqrt{rd\left(-4\delta K\left(\delta K-d\right)+rd\right)}-rd > 0$. We obtain that $\lambda_1 > 0$ and $\lambda_2 < 0$. In this case, the point P_4 is unstable.

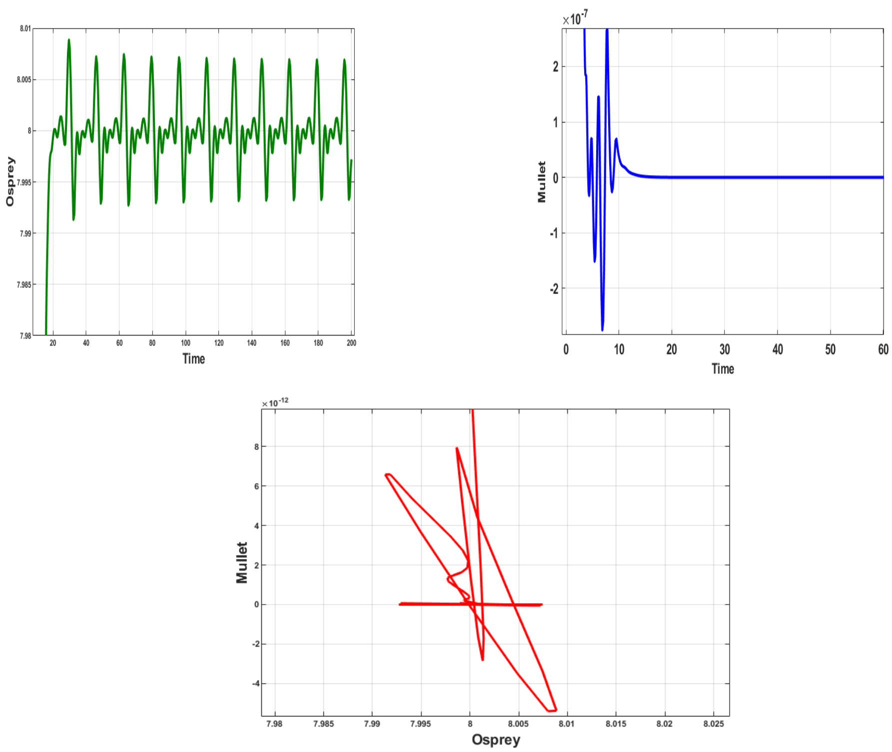

FIGURE 8. Dynamical behaviour and phase portraits of osprey and mullet populations for $r=0.8$, $K=8$, $\delta=0.3$ and $d=3$.

The Figure 8 illustrates the instability of the equilibrium point P_4. By choosing as initial value the point $P_4(0.01, 0.01,)$, we find that this last one tend to the point

$(7.997, 0.01)$. Note that to plot the trajectories of the three marine populations at this equilibrium, we considered the following values for the parameters $r = 0.8$, $K = 8$, $\delta = 0.3$ and $d = 3$.

(2) If $rd(-4\delta K(\delta K - d) + rd) = 0$ then $rd = 0$ or $-4\delta K(\delta K - d) + rd = 0$. For $rd \neq 0$ we have $\delta K = \frac{rd}{4\delta K} + d$. In this case, $\lambda_1 < 0$ and $\lambda_2 < 0$, and hence, the point P_4 is stable.

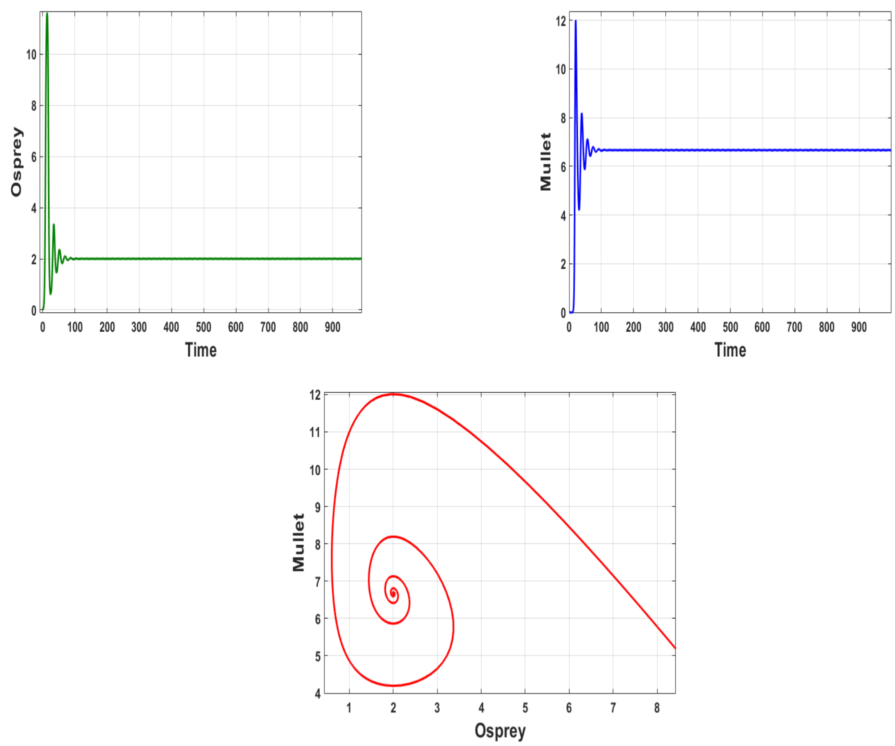

FIGURE 9. Dynamical behaviour and phase portraits of osprey and mullet populations for $r = 1$, $K = 7$, $\delta = 0.1$ and $d = 0.52$.

In Figure 9, we show the stability of the equilibrium point P_4. The initial value in this case is the point $P_4(0.01, 0.01,)$. It is clear from the figure that this point tend to $(2.001, 6.99)$. Note that to plot the trajectories of the three marine populations in this case, we considered the following values for the parameters $r = 1$, $K = 7$, $\delta = 0.1$ and $d = 0.52$.

For $rd = 0$, we have$-4\delta K(\delta K - d) + rd \neq 0$ then we obtain $\lambda_1 = \lambda_2 = 0$. In this case, nothing can be concluded.

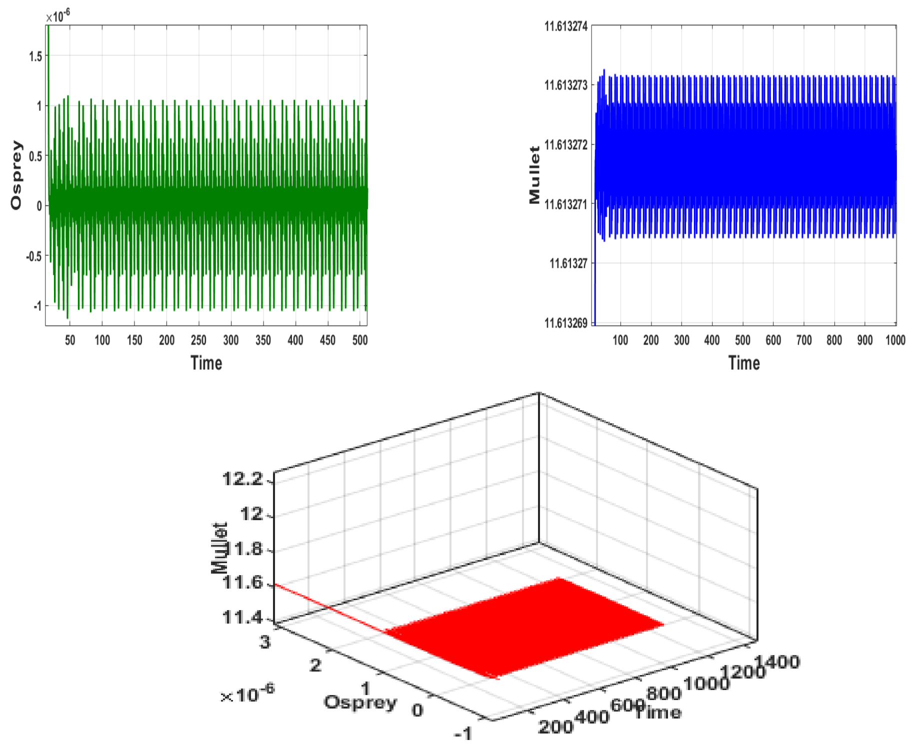

FIGURE 10. Dynamical behaviour and phase portraits of osprey and mullet populations for $r = 0.8$, $K = 12$, $\delta = 0.3$ and $d = 0$.

(3) The last case, if $rd\left(-4\delta K\left(\delta K-d\right)+rd\right)<0$, we have $\delta K > d$ with $\delta K > 0$ and $-4\delta K\left(\delta K-d\right) > rd$. Then, the eigenvalues are given by:

$$\begin{cases} \lambda_1 = \dfrac{-rd}{2\delta K} + i\dfrac{rd\left(-4\delta K\left(\delta K-d\right)+rd\right)}{2\delta K}, \\ \lambda_2 = -\dfrac{rd}{2\delta K} + i\dfrac{rd\left(-4\delta K\left(\delta K-d\right)+rd\right)}{2\delta K}. \end{cases}$$

We observe that

$$\lambda_1 = \lambda_2 = \frac{-rd}{2\delta K} + i\frac{rd\left(-4\delta K\left(\delta K-d\right)+rd\right)}{2\delta K}$$

with $\mathrm{Re}\left(\lambda\right) < 0$. Therefore, we conclude that the point P_4 in this case is stable.

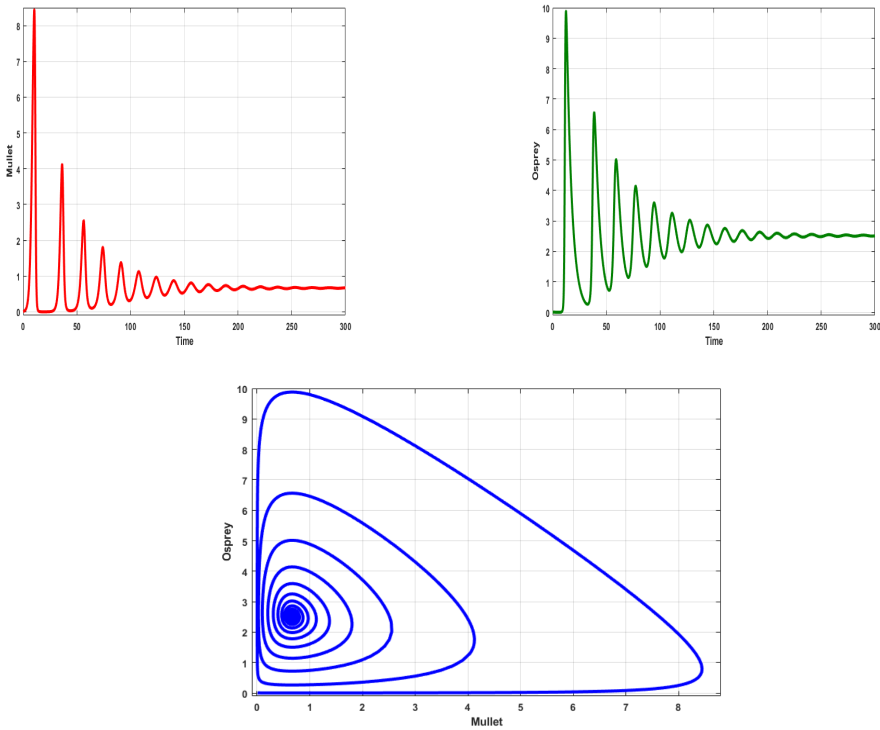

FIGURE 11. Dynamical behaviour and phase portraits of osprey and mullet populations for $r = 0.8$, $K = 12$, $\delta = 0.3$ and $d = 0.2$.

The Figure 11, illustrates the local stability of the equilibrium point P_4 in the last case. We note the trajectory of the populations remains in the vicinity of $(0.872, 6.53)$ with initial value $P_4(0.01, 0.01,)$.

We conclude that the point P_4 can be stable under certain conditions in the second case and the third case.

5. CONCLUSION

In this work, we have transformed the dynamic system describing the interaction between osprey and mullets into an optimal control problem. Solving this problem with a general convex-concave utility function, we combined two different approaches to the analysis of phase diagrams (of concave models) and the synthesis of bang-bang, singular solution parts (for convex and linear models, respectively), and we studied the existence of equilibrium points and their stability. Thus, we showed the potential of using the utility function for solving optimal control problems associated to ecological systems.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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