Available online at http://scik.org
Commun. Math. Biol. Neurosci. 2023, 2023:66
https://doi.org/10.28919/cmbn/8002
ISSN: 2052-2541

MATHEMATICAL MODELING OF DRUG ABUSE, UNEMPLOYMENT AND MENTAL STRESS ON POPULATION DYNAMICS OF MENTAL ILLNESS

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Abstract. There has been a rise in the number of reported cases of mental illness in both High Income Countries (HICs) and Low and Middle Income Countries (LMICs). Non-communicable Diseases (NCDs) seldom make use of mathematical modeling. This research suggests eight first-order differential equations to form the basis of a mathematical model for psychiatric disorders. There are eight distinct categories created to reflect the public at large: the vulnerable, the working and jobless, drug addicts, the emotionally distraught, and the mentally ill. Theoretically, the well-possedness of the model equations is established by examining the positive, bounded, existing, unique solutions and the local and global stability. The eigenvalue approach was used to investigate local stability, and a Lyapunov function was created to analyze global behavior. In order to back up the analytical results, we performed a numerical investigation of the dynamical behavior of the model's equations using the fourth-order Runge-Kutta technique with the use of the MATLAB software package. To better understand the impact of environmental factors on mental disease, researchers have experimented with changing a number of variables related to mental stress, unemployment and drug addiction among certain groups. Based on visual representations, the prevalence of mental illness skyrocketed anytime variables related to psychological strain or substance (drug) addiction rose in severity. In conclusion, lowering the growing rates of mental illness may be accomplished through increasing options for employment, improving working conditions, and fostering a welcoming workplace.

Keywords: mathematical modeling; mental stress; population dynamics; mental illness.

2020 AMS Subject Classification: 92D25.

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Received April 27, 2023

1. INTRODUCTION

Patients, professionals, and members of the general public all have different actions and emotional responses based on how they think the world views their experiences with mental diseases. People's behavioral and emotional reactions are heavily influenced by their beliefs about the events that occur [1]. Alterations in one's emotional state, level of thought, or pattern of conduct are all symptoms of a mental disorder (or a combination of the three). They are non-communicable diseases (NCDs), according to Daud & Qing [2] related to distress and social, work, or family issues. Non-communicable diseases (NCDs) are presently the main health and development threat to people worldwide. Currently, the stratified heterogeneity of NCD fatalities is seldom addressed [3].

Through several studies conducted in High-Income Countries (HIC), there has been an adoption of effective systems and approaches to mental health. Several Low and Middle-Income Countries (LMIC) have attempted to address mental health challenges. The lack of enough resources and logistics to achieve a mental illness-free community remains a big challenge [4]. Mental disorders are distressing and disturbing and pose an enormous burden in terms of cost, morbidity, and mortality, according to Thakur & Roy [5], which delay the accomplishment of sustainable developmental goals for a country. Since addressing mental health is crucial to achieving universal health coverage, as outlined by the "Big Four Agenda" and necessary to realize Vision 2030, it has been thrust to the forefront of Kenyan policy priorities.

The World Health Organization (WHO) Global Mental Health Action Plan 2013-2020 alludes that mental health remains a key determinant of any country's overall health and socio-economic development. Regarding WHO, a variety of outcomes for individuals within a given society, such as healthier lifestyles; better physical health; improved recovery from illness; fewer limitations in daily living; higher education attainment; greater productivity, employment, and earnings; better family relationships; social cohesion and engagement and improved quality of life, largely depend on the state of mental health. Mathematical models have been utilized for years to study biological sciences to understand diverse aspects of non-communicable diseases such as diabetes mellitus [6].

Mathematical Modelling and simulation are effective tools for creating universal healthcare strategies [7]. Unlike infectious illnesses, mathematical Modelling is seldom utilized for NCDs, and a model that currently treats all aspects of the disease does not exist [8]. Recent research has demonstrated that linear systems may explain and better predict NCD population dynamics [9]. This research will concentrate on the qualitative examination of the population models [10]. The issue is a system that is supposed to be properly characterized by a set of constant coefficient ordinary differential equations (ODEs), specifically the population dynamics of mental disease.

2. MATHEMATICAL MODEL FORMULATION

Substantial numbers of people suffer from mental illness, and its root causes are complex [11]. It may seem hard to create a model that accounts for everything. This study suggests a S-M-P-Q-R-X-Y-Z mental illness model for compartmental population dynamics on mental illness by establishing well-thought assumptions in order to build a mathematical model that is both mathematically well-posed and physiologically useful for this study.

Let S(t) represent the susceptible humans who are eligible for working. This sub-population was assumed to have gone through the basic education system and had attained a tenable age (24 years) for either formal or informal employment. Taking the rate of recruitment to susceptible class at any given time t as λ , the study distributed S(t) class into employed and unemployed subpopulation Q(t) and P(t) respectively, where ω_1 is the portion of people who get employed joining Q(t) and $(1-\omega_1)$ transimmision rate to P(t) class at any given time t. The state of employment does not cause death. However, it's considered to trigger mental stress, which has an advanced impact on mental illness and can lead to heavy drug abuse. Thus, the employed subpopulation Q(t) was considered to have X(t) class, for those individuals suffering mental stress and Z(t) representing the employed sub-population abusing drugs at any given time t. Consequently, the study takes M(t) to represent the unemployed subpopulation suffering from mental stress and R(t) as the unemployed sub-population abusing drugs. θ_1 , θ_2 and θ_3 are transmission rates from the employed class Q(t) while τ_1, τ_2 and τ_3 transmission rates from the unemployed subpopulation P(t) to join either mental stress or drug abuse classes.

The study considered drug abuse and mental stress as the main stressors (factors) leading to mental illness. Though the state of employment has been captured, it only helps this study to identify the advanced effect it can cause on the "main stressors" leading to mental illness. Y(t) is thus taken as the total population of those individuals with mental illness at any given time t. the transmission rates to mental illness class Y(t) are α_1, ρ_1, β_1 and π_2 which are for those individuals suffering mental stress and abusing drugs regardless of the state of employment.

Mental illness does not cause death; it triggers the causes of death, such as violent actions and suicide, and the patient can recover from mental illness. This study considered that, mental illness patients only die through natural death ε and those individuals μ recovering from mental diseases have gone through proper and adequate counseling, undergo sufficient therapy, have completely healed from drug abuse, and are capable of accessing good health facilities with the right support system for patients as well as family care system and thus they cannot get mental disorders once again. The total population N(t) at any given time, t, is thus given by;

$$N(t) = S(t) + M(t) + P(t) + Q(t) + R(t) + X(t) + Y(t) + Z(t) \tag{1}$$

where t ϵ $[0, t]$ and $t > 0$.

Table 1: Parameter description

Parameters	Description
λ	Rate of recruitment at any given time t
ρ_1	Rate of employed population Z(t) abusing drug resulting to mental illness Y(t)
ρ_2	The rate of employed population Z(t) harming drugs results in mental stress due to unhealthy work conditions.
ρ_3	The rate of employed population Z(t) abusing drugs and losing their jobs results in mental stress.
β_1	The transmission rate of unemployed sub-population with mental stress to mental illness.
β_2	The transmission rate of the unemployed subpopulation with mental stress resulting in drug abuse
θ_1	Speed of transmission of the population with a job to mental stress
θ_2	Rate of information of sub-population with a job to people abusing drugs
θ_3	Rate of losing an employment
π_1	Rate of getting mental stress as a result of drug abuse due to unemployment
π_2	Rate of getting mental illness as a result of drug abuse due to unemployment
α_1	Rate of getting mental illness as a result of mental stress caused by unhealthy work condition
α_2	Rate of abusing drugs as a result of mental stress caused by harmful work condition
ε	Rate of natural death
μ	The recovery rate of mental illness
τ_1	Rate of unemployed class join the mental stress class
τ_2	Rate of unemployed class join the substance abuse class
τ_3	Rate of people eligible for working get employed
ω_1	The portion of people who get a job at any given time t.
$1-\omega_1$	The number of people without an appointment at any given time t

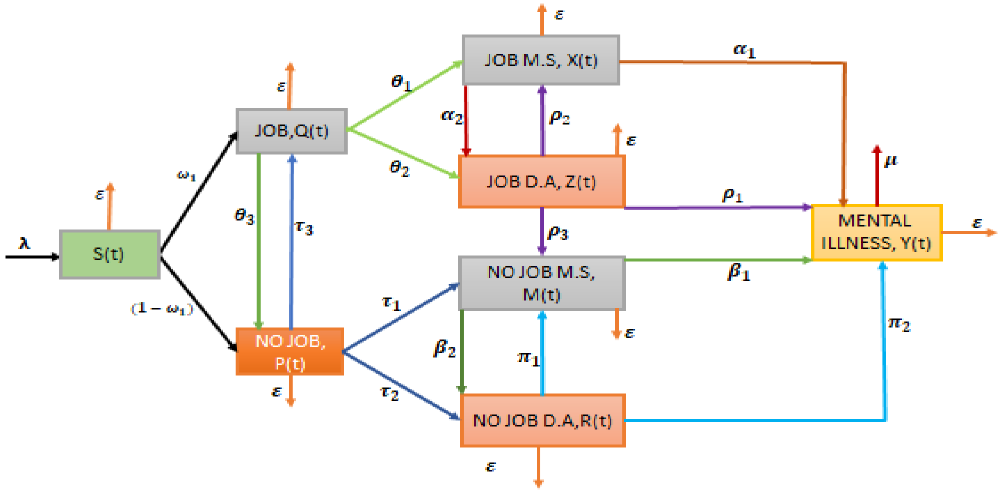

Figure 1: Mathematical model flow diagram

2.1 Modal Equation

The system of ordinary differential equations governing the S-M-P-Q-R-X-Y-Z mental illness model is given by the system of Equation (2) as;

$$\begin{cases} \dfrac{dS}{dt} = \lambda - \omega_1 S - (1-\omega_1)S - \varepsilon S \\ \dfrac{dM}{dt} = \tau_1 P + \pi_1 R - M(\beta_1 + \beta_2 + \varepsilon) + \rho_3 Z \\ \dfrac{dP}{dt} = (1-\omega_1)S + \theta_3 Q - P(\tau_1 + \tau_2 + \tau_3 + \varepsilon) \\ \dfrac{dQ}{dt} = \omega_1 S + \tau_3 P - Q(\theta_1 + \theta_2 + \theta_3 + \varepsilon) \\ \dfrac{dR}{dt} = \beta_2 M + \tau_2 P - R(\pi_1 + \pi_2 + \varepsilon) \\ \dfrac{dX}{dt} = \theta_1 Q - X(\alpha_1 + \alpha_2 + \varepsilon) + \rho_2 Z \\ \dfrac{dY}{dt} = \beta_1 M + \alpha_1 X + \pi_2 R - Y(\varepsilon + \mu) + \rho_1 Z \\ \dfrac{dZ}{dt} = \theta_2 Q + \alpha_2 X - Z(\rho_1 + \rho_2 + \rho_3 + \varepsilon) \end{cases} \tag{2}$$

3. Model Analysis

Mathematical analysis of the formulated model system (2) is presented to in this section. The study shows the system of ordinary differential equation (2) governing the model is well-posed.

3.1 Positivity of the SMPQRXYZ Model

The positivity of the SMPQRXYZ Model is determined to ascertain the existence of all state variables on the real domain, $\mathbb{R}$.

Theorem 1: Let $K(t)=\{S(t), M(t), P(t), Q(t), R(t), X(t), Y(t)$ and $Z(t)\in\mathbb{R}_+^8$:

$S(0)\geq 0, M(0)\geq 0, P(0)\geq 0, Q(0)\geq 0, R(0)\geq 0, X(0)\geq 0, Y(0)\geq 0, Z(0)\geq 0\}$ then the solution set of the modal system of equation (2) for the initial data set $(S, M, P, Q, R, X, Y, Z)(0)\geq 0$ is positive $\forall t > 0$.

Proof: Consider the equation on mental illness from the system of model equation (2):

$$\frac{dY}{dt}=\beta_1 M+\alpha_1 X+\pi_2 R-Y(\varepsilon+\mu)+\rho_1 Z \tag{3}$$

By inspecting, $\beta_1 M\geq 0$, $\alpha_1 X\geq 0$, $\pi_2 R\geq 0$ and $\rho_1 Z\geq 0$, and thus:

$$\frac{dY}{dt}\geq -Y(\varepsilon+\mu) \tag{4}$$

Equation (4) represents a first-order linear differential inequality and thus can be solved using the separation of variables to obtain:

$$\int\frac{dY}{Y}\geq\int-(\varepsilon+\mu)dt \tag{5}$$

$$In\ Y\geq -(\varepsilon+\mu)t+c$$

$$Y(t)\geq e^{-(\varepsilon+\mu)t}.e^{c} \tag{6}$$

In the absence of mental illness and applying initial conditions, (6) become,

$$Y(t)\geq Y(0)e^{-(\varepsilon+\mu)t},\ Y(0)\geq 0,\ \forall t\geq 0. \tag{7}$$

By considering the same approach in the remaining seven equations in the system of model equation (2), the results are;

$$S(t) \geq S(0)e^{-(1+\varepsilon)t}, \; S(0) \geq 0 \;, \forall t > 0 \quad (8)$$

$$M(t) \geq M(0)e^{-(\beta_1+\beta_2+\varepsilon)t}, \; M(0) \geq 0 \;, \forall t > 0 \quad (9)$$

$$P(t) \geq P(0)e^{-(\tau_1+\tau_2+\tau_3+\varepsilon)t}, \; P(0) \geq 0 \;, \forall t > 0 \quad (10)$$

$$Q(t) \geq Q(0)e^{-(\theta_1+\theta_2+\theta_3+\varepsilon)t}, \; Q(0) \geq 0, \forall t > 0 \quad (11)$$

$$R(t) \geq R(0)e^{-(\pi_1+\pi_2+\varepsilon)t}, \; R(0) \geq 0, \forall t > 0 \quad (12)$$

$$X(t) \geq X(0)e^{-(\alpha_1+\alpha_2+\varepsilon)t}, \; X(0) \geq 0 \;, \forall t > 0 \quad (13)$$

$$Z(t) \geq Z(0)e^{-(\rho_1+\rho_2+\rho_3+\varepsilon)t}, \; Z(0) \geq 0, \forall t > 0 \quad (14)$$

Therefore, the set solutions $S(t), M(t), P(t), Q(t), R(t), X(t), Y(t)$ and Z(t) lie in the positive quadrant $\forall t > 0$, which proves the theorem.

3.2: Boundedness of the solution

The consequences restricting a population's growth are vital in analogy to a dynamic population system [7]. To study the boundedness of the solution of the system around the steady states, all the state variables and parameters of the *S-M-P-Q-R-X-Y-Z Mental Illness* Modal are assumed to be positive $\forall t > 0$. In this regard, boundedness is determined by the following theorem:

Theorem 2: The set

$K = \left\{ (S, M, P, Q, R, X, Y, Z) : 0 \leq S + M + P + Q + R + X + Y + Z \leq \dfrac{\lambda}{\varepsilon}, 0 \leq K(t) \leq \dfrac{\lambda}{\varepsilon} \right\}$ is a region of attraction for all solutions in the system initiating the first quadrant (positively invariant and attracts all values, $\mathbb{R}^8_+$).

Proof: Let (S, M, P, Q, R, X, Y, Z) be any solutions with positive initial conditions $K = S + M + P + Q + R + X + Y + Z$. Computing the time derivative of K along solutions of system equation (2), we have;

$$\frac{dK}{dt} = \frac{dS}{dt} + \frac{dP}{dt} + \frac{dQ}{dt} + \frac{dR}{dt} + \frac{dM}{dt} + \frac{dX}{dt} + \frac{dY}{dt} + \frac{dZ}{dt} \quad (15)$$

Substituting model equation (2) to equation (15) and simplifying further yields;

$$\frac{dK}{dt} = \lambda - \varepsilon\left(S + M + P + Q + R + X + Y + Z\right) - \mu Y \tag{16}$$

$$\frac{dK}{dt} = \lambda - \varepsilon K - \mu Y \tag{17}$$

If there is no mental illness at any given time $t, \mu = 0$ and thus (17) reduces to

$$\frac{dK}{dt} = \lambda - \varepsilon K, \Longrightarrow K' = \lambda - \varepsilon K \implies K' - \varepsilon K = \lambda \tag{18}$$

Using integrating factors method, we proceed as follows;

$$K'\mu - \varepsilon K\mu = \mu\lambda, \tag{19}$$

$$K'\mu - K\mu' = \mu\lambda \tag{20}$$

By product rule and taking $\mu' = \varepsilon\mu$;

$$\left(\mu K\right)' = \mu\lambda, \tag{21}$$

$$\int \frac{\mu'}{\mu} = \int \varepsilon \, dt \implies In\,\mu = \varepsilon t + c \text{ and thus } \mu = e^{\varepsilon t}.e^{c} \tag{22}$$

Substituting we have $\left(Ke^{\varepsilon t}\right)' = e^{\varepsilon t}\lambda,$ (23)

$$\int\left(Ke^{\varepsilon t}\right)' = \int \lambda e^{\varepsilon t} dt \implies Ke^{\varepsilon t} = \frac{1}{\varepsilon}\lambda e^{\varepsilon t} + C \text{ and thus}$$

$$K = Ce^{-\varepsilon t} + \frac{\lambda}{\varepsilon} \tag{24}$$

Where C is a constant and $t \Rightarrow \infty$, then $\lim_{t\to\infty} K \le \frac{\lambda}{\varepsilon}$ where λ is the rate of recruitmet and ε the rate of natural death. The *S-M-P-Q-R-X-Y-Z Mental Illness* Modal is therefore bounded as $K = \left[(S, M, P, Q, R, X, Y, Z) : 0 \le S + M + P + Q + R + X + Y + Z \le \frac{\lambda}{\varepsilon}, 0 \le K(t) \le \frac{\lambda}{\varepsilon}\right]$ the flow generated by the model can thus be considered for analysis since the S-M-P-Q-R-X-Y-Z Mental Illness Modal is mathematically well-posed and biologically meaningful.

3.3 Steady States

To study the stability of the proposed SMPQRXYZ model, the equilibrium points of the system need to be determined. Let $\psi_1 = \beta_1 + \beta_2 + \varepsilon$, $\psi_2 = \tau_1 + \tau_2 + \tau_3 + \varepsilon$, $\psi_3 = \theta_1 + \theta_2 + \theta_3 + \varepsilon$,

$\psi_4=\pi_1+\pi_2+\varepsilon$, $\psi_5=\alpha_1+\alpha_2+\varepsilon$, $\psi_6=\rho_1+\rho_2+\rho_3+\varepsilon$ and $\psi_7=\varepsilon+\mu$, then substitute in system (2) to form system (25). Based on different mental illness stressors considered in this study, the possible equilibrium points considered are:

Case 1: All stress factors of mental illness exist; $S(t)\neq 0,\ M(t)\neq 0,\ P(t)\neq 0,\ Q(t)\neq 0,\ R(t)\neq 0,$ $X(t)\neq 0,\ Y(t)\neq 0$ and $Z(t)\neq 0$.

The equilibrium point is $E_0=(S^0,M^0,P^0,Q^0,R^0,X^0,Y^0,Z^0)\ =\left(s,m,p,q,r,x,y,z\right)$ where;

$$s=\frac{\lambda}{1+\epsilon},\ m=\frac{r\pi_1+z\rho_3+p\tau_1}{\psi_1},\ p=\frac{\lambda+q\theta_3+q\epsilon\theta_3-\lambda\omega_1}{\left(1+\epsilon\right)\psi_2},\ q=\frac{\lambda\left(\tau_3\left(-1+\omega_1\right)-\psi_2\omega_1\right)}{\left(1+\epsilon\right)\left(\theta_3\tau_3-\psi_2\psi_3\right)},$$

$$r=\frac{-z\beta_2\rho_3-p\beta_2\tau_1-p\tau_2\psi_1}{\pi_1\beta_2-\psi_1\psi_4},\ x=\frac{q\left(\theta_2\rho_2+\theta_1\psi_6\right)}{-\alpha_2\rho_2+\psi_5\psi_6},\ y=\frac{-r\pi_2-x\alpha_1-m\beta_1-z\rho_1}{\psi_7}\text{ and}$$

$$z=\frac{-q\alpha_2\theta_1-q\theta_2\psi_5}{\alpha_2\rho_2-\psi_5\psi_6}.$$

Case 2: At unemployment-free equilibrium point E_1, all the people have a source of income; $Q(t)\neq 0$ then, $\Rightarrow P(t)=0$ and all P(t) in the system (25) is eliminated. The resulting equilibrium point is thus $E_1=(S^1,M^1,Q^1,R^1,X^1,Y^1,Z^1)\ =(a_1,a_2,a_3,a_4,a_5,a_6,a_7)$ where:

$$a_1=\frac{\lambda}{1+\epsilon},\ a_2=\frac{a_1\omega_1}{\psi_3},\ a_3=\frac{a_2\theta_2\rho_2+a_2\theta_1\psi_6}{-\alpha_2\rho_2+\psi_5\psi_6},\ a_4=\frac{a_3\alpha_2+a_2\theta_2}{\psi_6},\ a_5=\frac{a_4\beta_2\rho_3}{\pi_1\beta_2-\psi_1\psi_4},$$

$$a_6=\frac{-a_5\pi_1+a_4\rho_3}{\psi_1}\text{ and } a_7=\frac{-a_5\pi_2+a_3\alpha_1+a_6\beta_1+a_4\rho_1}{\psi_7}.$$

Case 3: If no individual is suffering from mental illness i.e. $Y(t)=0$, mental illness-free equilibrium point $E_2=(S^2,M^2,P^2,Q^2,R^2,X^2,Z^2)\ =(b_1,b_2,b_3,b_4,b_5,b_6,b_7)$ is obtained where;

$$b_1=\frac{\lambda}{1+\epsilon},\ b_2=\frac{-b_1\tau_3+b_1\tau_3\omega_1-b_1\psi_2\omega_1}{\theta_3\tau_3-\psi_2\psi_3},\ b_3=\frac{b_2\psi_3-b_1\omega_1}{\tau_3}\ b_4=\frac{-b_2\alpha_2\theta_1-b_2\theta_2\psi_5}{\alpha_2\rho_2-\psi_5\psi_6},$$

$$b_5=\frac{b_3\pi_1\tau_2+b_4\rho_3\psi_4+b_3\tau_1\psi_4}{-\pi_1\beta_2+\psi_1\psi_4},\ b_6=\frac{b_2\theta_1+b_4\rho_2}{\psi_5}\text{ and } b_7=\frac{b_6\beta_2+b_3\tau_2}{\psi_4}.$$

Case 4: If their exist no mental stress patients in the society, $X(t)=0$, $M(t)=0$ and mental illness result form drug abuse i.e. $R(t)\neq 0$, $Z(t)\neq 0$; the mental stress free-equilibrium point $E_3=(S^3,P^3,Q^3,R^3,Y^3,Z^3)$ $=(c_1,c_2,c_3,c_4,c_5,c_6)$ is obtained where;

$$c_1=\frac{\lambda}{1+\epsilon},\ c_2=\frac{c_1\psi_3-c_1\theta_3\omega_1-c_1\psi_3\omega_1}{\theta_3\tau_3+\psi_2\psi_3},\ c_3=\frac{c_2\tau_3+c_1\omega_1}{\psi_3},\ c_4=\frac{c_2\tau_1}{\psi_4},c_5=\frac{c_3\theta_2}{\psi_6}\ \text{and}\ c_6=\frac{c_4\pi_2+c_5\rho_1}{\psi_7}.$$

Case 5: If non abuses substance(s) in the population considered in this study i.e. $M(t)\neq 0$ $X(t)\neq 0$ $R(t)=0$ and $Z(t)=0$; then drug abuse-free equilibrium point $E_4=(S^4,M^4,P^4,Q^4,X^4,Y^4)$ $=(d_1,d_2,d_3,d_4,d_5,d_6)$ is obtained where;

$$d_1=\frac{\lambda}{1+\epsilon},\ d_2=\frac{-d_1\tau_3+d_1\tau_3\omega_1-d_1\psi_2\omega_1}{\theta_3\tau_3-\psi_2\psi_3},\ d_3=\frac{d_1+d_2\theta_3-d_1\omega_1}{\psi_2},\ d_4=\frac{d_3\tau_1}{\psi_1},\ d_5=\frac{d_2\theta_1}{\psi_5}\ \text{and}$$

$$d_6=\frac{d_5\alpha_1+d_4\beta_1}{\psi_7}.$$

3.4 Stability analysis

The stability analysis of the equilibrium points is determined for the proposed model. The study considered the local stability of E_0,E_1,E_2,E_3 and E_4. The global stability of E_3 and E_4 is also studied to ascertain which stress factor(s) pose a greater impact on mental illness.

3.4.1 Local stability

A variation matrix is constructed, and the nature of the eigenvalues of the matrix is determined. A point is said to be locally asymptomatic stable if the eigenvalues of the variation matrix are negative, otherwise unstable.

Theorem 3: Consider the first equilibrium point $E_0=(S^0,M^0,P^0,Q^0,R^0,X^0,Y^0,Z^0)$ $=(s,m,p,q,r,x,y,z)$; the system is stable if $\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}<-(\psi_2+\psi_3)$ $\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2}<-(\psi_1+\psi_4)$, and $\sqrt{4\alpha_2\rho_2+\psi_5^2+2\psi_5\psi_6+\psi_6^2}<-(\psi_5-\psi_6)$ otherwise unstable.

Proof: Let $S(t)\neq 0, M(t)\neq 0, P(t)\neq 0, Q(t)\neq 0, R(t)\neq 0, X(t)\neq 0, Y(t)\neq 0$ and $Z(t)\neq 0$.

The variation matrix is constructed by differentiating the system of equation (25) concerning (w.r.t) S, M, P, Q, R, X, Y, and Z to get $Matrix\ M_0$ below;

$$M_0 = \begin{pmatrix} -1-\varepsilon & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\psi_1 & \tau_1 & 0 & \pi_1 & 0 & \rho_3 & 0 \\ 1-\omega_1 & 0 & -\psi_2 & \theta_3 & 0 & 0 & 0 & 0 \\ \omega_1 & 0 & \tau_3 & -\psi_3 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 & \tau_2 & 0 & -\psi_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta_1 & 0 & -\psi_5 & \rho_2 & 0 \\ 0 & 0 & 0 & \theta_2 & 0 & \alpha_2 & -\psi_6 & 0 \\ 0 & \beta_1 & 0 & 0 & \pi_2 & \alpha_1 & \rho_1 & -\psi_7 \end{pmatrix}$$

Solving by the help of Wolfram Mathematica Software, the resulting eigenvalues are;

$-1-\epsilon$, $\frac{1}{2}\left(-\psi_2-\psi_3-\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}\right)$, $\frac{1}{2}\left(-\psi_2-\psi_3+\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}\right)$,

$\frac{1}{2}\left(-\psi_1-\psi_4-\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2}\right)$, $\frac{1}{2}\left(-\psi_1-\psi_4+\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2}\right)$,

$\frac{1}{2}\left(-\psi_5+\psi_6-\sqrt{4\alpha_2\rho_2+\psi_5^2+2\psi_5\psi_6+\psi_6^2}\right)$, $\frac{1}{2}\left(-\psi_5+\psi_6+\sqrt{4\alpha_2\rho_2+\psi_5^2+2\psi_5\psi_6+\psi_6^2}\right)$ and $-\psi_7$.

Thus, the system $E_0=(S^0,M^0,P^0,Q^0,R^0,X^0,Y^0,Z^0)=(s,m,p,q,r,x,y,z)$ is stable if

$\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}<-(\psi_2+\psi_3)$ $\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2}<-(\psi_1+\psi_4)$, and

$\sqrt{4\alpha_2\rho_2+\psi_5^2+2\psi_5\psi_6+\psi_6^2}<-(\psi_5-\psi_6)$ otherwise unstable.

Theorem 4: At the second equilibrium point $E_1=(S^1,M^1,Q^1,R^1,X^1,Y^1,Z^1)$ $=(a_1,a_2,a_3,a_4,a_5,a_6,a_7)$ discussed in case 2, the system is considered locally asymptomatic stable if $\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2}<-(\psi_1+\psi_4)$ and $\sqrt{4\alpha_2\rho_2+\psi_5^2-2\psi_5\psi_6+\psi_6^2}<-(\psi_5+\psi_6)$;

Otherwise, unstable.

Proof: Let $Q(t)\neq 0$ then it implies that $P(t)=0$. Substituting this in equation 2, we have a system of equation 26. By differentiating system equation (26) described in case 2, the variation $Matrix\ M_1$ below is generated;

$$M_1 = \begin{pmatrix} -1-\varepsilon & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\psi_1 & 0 & \pi_1 & 0 & \rho_3 & 0 \\ \omega_1 & 0 & -\psi_3 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & -\psi_4 & 0 & 0 & 0 \\ 0 & 0 & \theta_1 & 0 & -\psi_5 & \rho_2 & 0 \\ 0 & 0 & \theta_2 & 0 & \alpha_2 & -\psi_6 & 0 \\ 0 & \beta_1 & 0 & \pi_2 & \alpha_1 & \rho_1 & -\psi_7 \end{pmatrix}$$

The resulting eigenvalues M_1 are;

$-1-\epsilon$, $-\psi_3$, $\frac{1}{2}\left(-\psi_1-\psi_4-\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2}\right)$, $\frac{1}{2}\left(-\psi_1-\psi_4+\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2}\right)$,

$\frac{1}{2}\left(-\psi_5-\psi_6-\sqrt{4\alpha_2\rho_2+\psi_5^2-2\psi_5\psi_6+\psi_6^2}\right)$, $\frac{1}{2}\left(-\psi_5-\psi_6+\sqrt{4\alpha_2\rho_2+\psi_5^2-2\psi_5\psi_6+\psi_6^2}\right)$, and $-\psi_7$.

The system E_2 is thus stable if; $\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2} < -(\psi_1+\psi_4)$ and $\sqrt{4\alpha_2\rho_2+\psi_5^2-2\psi_5\psi_6+\psi_6^2} < -(\psi_5+\psi_6)$ otherwise unstable.

Theorem 5: The system at the third equilibrium point $E_2 = (S^2, M^2, P^2, Q^2, R^2, X^2, Z^2)$ $= (b_1, b_2, b_3, b_4, b_5, b_6, b_7)$ is stable if $\sqrt{4\alpha_2\rho_2+\psi_5^2-2\psi_5\psi_6+\psi_6^2} < -(\psi_5+\psi_6)$ and $\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2} < -(\psi_1+\psi_4)$ otherwise unstable.

Proof: Using the same approach used in theorem 4 above, a variation matrix M_3 is generated;

$$M_2 = \begin{pmatrix} -1-\varepsilon & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\psi_1 & 0 & \pi_1 & 0 & \rho_3 & 0 \\ 1-\omega_1 & 0 & -\psi_2 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 & \tau_2 & -\psi_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\psi_5 & \rho_2 & 0 \\ 0 & 0 & 0 & 0 & \alpha_2 & -\psi_6 & 0 \\ 0 & \beta_1 & 0 & \pi_2 & \alpha_1 & \rho_1 & -\psi_7 \end{pmatrix}$$

Solving, the resulting eigenvalues are; $-1-\epsilon$, $-\psi_2$, $\frac{1}{2}\left(-\psi_1-\psi_4-\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2}\right)$,

$\frac{1}{2}\left(-\psi_1-\psi_4+\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2}\right)$, $\frac{1}{2}(-\psi_5-\psi_6-\sqrt{4\alpha_2\rho_2+\psi_5^2-2\psi_5\psi_6+\psi_6^2}$,

$\frac{1}{2}\left(-\psi_5-\psi_6+\sqrt{4\alpha_2\rho_2+\psi_5^2-2\psi_5\psi_6+\psi_6^2}\right)$ and $-\psi_7$. The system is stable if

$\sqrt{4\alpha_2\rho_2+\psi_5^2-2\psi_5\psi_6+\psi_6^2}<-(\psi_5+\psi_6)$ and $\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2}<-(\psi_1+\psi_4)$ otherwise unstable.

Theorem 6: The system $E_3=(S^3,P^3,Q^3,R^3,Y^3,Z^3)=(c_1,c_2,c_3,c_4,c_5,c_6)$ is stable if $\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}\geq 0$ and $\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}<-(\psi_2+\psi_3)$ otherwise unstable.

Proof: The variation matrix M_3 below is generated using the same approach used in the above theorem;

$$M_3=\begin{pmatrix} -1-\varepsilon & 0 & 0 & 0 & 0 & 0 \\ 1-\omega_1 & -\psi_2 & \theta_3 & 0 & 0 & 0 \\ \omega_1 & \tau_3 & -\psi_3 & 0 & 0 & 0 \\ 0 & \tau_2 & 0 & -\psi_4 & 0 & 0 \\ 0 & 0 & \theta_2 & 0 & -\psi_6 & 0 \\ 0 & 0 & 0 & \pi_2 & \rho_1 & -\psi_7 \end{pmatrix}$$

The resulting eigenvalues are; $-1-\epsilon$, $\frac{1}{2}\left(-\psi_2-\psi_3-\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}\right)$, $-\psi_4$, $-\psi_6$, and $-\psi_7$. the system is stable if $\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}\geq 0$ and $\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}<-(\psi_2+\psi_3)$ otherwise unstable.

Theorem 7: In case 5, the equilibrium point $E_4=(S^4,M^4,P^4,Q^4,X^4,Y^4)=(d_1,d_2,d_3,d_4,d_5,d_6)$ is stable if $\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}\geq 0$ and $\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}<-(\psi_2+\psi_3)$ otherwise unstable.

Proof: Let's consider R(t) =0 and Z(t)=0 . Substituting in the differential equation (2) system reduces the system to equation (29). Differentiating all the differential equations in the system (29) concerning S, M, P, Q, X, and Y, respectively, the variation matrix M_4 below is generated;

$$M_4=\begin{pmatrix} -1-\varepsilon & 0 & 0 & 0 & 0 & 0 \\ 0 & -\psi_1 & \tau_1 & 0 & 0 & 0 \\ 1-\omega_1 & 0 & -\psi_2 & \theta_3 & 0 & 0 \\ \omega_1 & 0 & \tau_3 & -\psi_3 & 0 & 0 \\ 0 & 0 & 0 & \theta_1 & -\psi_5 & 0 \\ 0 & \beta_1 & 0 & 0 & \alpha_1 & -\psi_7 \end{pmatrix}$$

At this equilibrium point, the eigenvalues obtained are; $-1-\epsilon$, $-\psi_1$,

$\frac{1}{2}\left(-\psi_2-\psi_3-\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}\right)$, $\frac{1}{2}\left(-\psi_2-\psi_3+\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}\right)$, $-\psi_5$, and

$-\psi_7$. The system is stable if $\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}\geq 0$ and

$\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}<-(\psi_2+\psi_3)$ otherwise unstable.

4.5.2: Global Stability

The Lyapunov function below is proposed to determine the global stability by performing analysis near the equilibrium points of the system. Considering mental illness caused by mental stress and drug abuse, two scenarios were considered ignored to study for global stability.

Theorem 8: Let $K_1=\frac{1}{2}\left(S^2+P^2+Q^2+R^2+Y^2+Z^2\right)$ the first Lyapunov function for the system correspond to the equilibrium point $E_3=(S^3,P^3,Q^3,R^3,Y^3,Z^3)$ $=(c_1,c_2,c_3,c_4,c_5,c_6)$ the system is globally asymptotically stable if $\frac{dK_1}{dt}<0$ and just tough if $\frac{dK_1}{dt}=0$, otherwise unstable.

Proof

$$\frac{dK_1}{dt}=S\frac{dS}{dt}+P\frac{dP}{dt}+Q\frac{dQ}{dt}+R\frac{dR}{dt}+Y\frac{dY}{dt}+Z\frac{dZ}{dt}$$

By substitution;

$$\frac{dK_1}{dt}=\left(\lambda-S-\epsilon S\right)+P\left(S-\omega_1 S-\psi_2 P+\theta_3 Q\right)+Q\left(\omega_1 S+\tau_3 P-\psi_3 Q\right)+Y\left(\pi_2 R-\psi_7 Y+\rho_1 Z\right)$$

$$+R\left(\tau_2 P-\psi_4 R\right)+Z\left(\theta_2 Q-\psi_6 Z\right)$$

Expanding and simplifying;

$$\frac{dK_1}{dt}=S\left(-S-S\epsilon+\lambda\right)+R\left(P\tau_2-R\psi_4\right)+Z\left(Q\theta_2-Z\psi_6\right)+Y\left(R\pi_2+Z\rho_1-Y\psi_7\right)$$

$$\frac{dK_1}{dt}=PS-S^2-S^2\epsilon+S\lambda+RY\pi_2+QZ\theta_2+PQ\theta_3+YZ\rho_1+PR\tau_2+PQ\tau_3-P^2\psi_2-Q^2\psi_3$$

$$-R^2\psi_4-Z^2\psi_6-Y^2\psi_7-PS\omega_1+QS\omega_1$$

The system will be globally asymptotically stable if $\frac{dK_1}{dt}<0$ and just stable if $\frac{dK_1}{dt}=0$, otherwise unstable. Numerical values will be used to verify this theorem.

Theorem 9: Let $K_2 = \frac{1}{2}\left(S^2 + M^2 + P^2 + Q^2 + X^2 + Y^2\right)$ the second Lyapunov function for the linear system. Consider the equilibrium point $E_4 = (S^4, M^4, P^4, Q^4, X^4, Y^4)$ $= (d_1, d_2, d_3, d_4, d_5, d_6)$; the system is globally asymptotically stable if and tough if otherwise unstable.

Proof

$$\frac{dK_2}{dt} = S\frac{dS}{dt} + M\frac{dM}{dt} + P\frac{dP}{dt} + Q\frac{dQ}{dt} + X\frac{dX}{dt} + Y\frac{dY}{dt}$$

$$= S\left(-S\left(1+\epsilon\right)+\lambda\right) + M\left(P\tau_1 - M\psi_1\right) + P\left(S + Q\theta_3 - P\psi_2 - S\omega_1\right) + Q\left(P\tau_3 - Q\psi_3 + S\omega_1\right)$$

$$+ X\left(Q\theta_1 - X\psi_5\right) + Y\left(X\alpha_1 + M\beta_1 - Y\psi_7\right)$$

Expanding we have:

$$\frac{dK_2}{dt} = PS\left(1-\omega_1\right) - S^2\left(1+\epsilon\right) + S\lambda + XY\alpha_1 + MY\beta_1 + QX\theta_1 + PQ\left(\theta_3 + \tau_3\right) + MP\tau_1$$

$$-M^2\psi_1 - P^2\psi_2 - Q^2\psi_3 - X^2\psi_5 - Y^2\psi_7 + QS\omega_1 .$$

The system will be globally asymptotically stable if $\frac{dK_2}{dt} < 0$ and just stable if $\frac{dK_2}{dt} = 0$, otherwise unstable. Numerical values will be used later in this chapter to verify this theorem.

4. Numerical Simulations

In this section, the numerical analysis is done with the help of Wolfram Mathematica to substantiate the analytical findings. Values for several parameters were calculated based on the literature study, and these estimates are cited in Table 2. The study relied heavily on previously publish ed works related to this study due to difficulty and unavailability of data on mental illness cases. The initial values of the variables used to generate graphs in figure 2-9 are; S(0)=490,500, M(0)=150,000, P(0)=120,000, Q(0)=260,000, R(0)=95,500, X(0)=250,000, Y(0)=80,000 and Z(0)=32,000. Further, we extracted more data from the 2019 national census statistical data in Kenya.

From the numerical analysis, we found that one equilibrium point E_0 was unstable and all other four equilibrium points E_1, E_2, E_3 and E_4 were stable and since our model lie in the first quadrant, we will not verify the imaginary equilibrium point. For *Theorem 3*, the eigenvalues corresponding to the equilibrium point $E_0 = (S^0, M^0, P^0, Q^0, R^0, X^0, Y^0, Z^0)$ are; -1.6, -491897, -218180, -88006.8, -15654.3,-331609, 335605, and -0.605, thus the system is unstable. Similarly, the eigenvalues corresponding to E_1, E_2, E_3 and E_4 are; [-1.6, -380921, -88006.8, -15654.3, -469103, -50376.6, -0.605], [-1.6, -329157, -88006.8, -15654.3, -469103, -50376.6,-0.605], [-1.6, -491897, -218180, -75050.6, -261738, -0.605] and [-1.6, -28610, -491897, -218180, -257742, -0.605] respectively and hence the four equilibrium points are locally asymptotically stable.

Table 2: Parameter and values of the model

Parameter	**Value**	**Source**
λ	418,184	Kenya national bureau of Statistics(KNBS)
ω_1	291,892	[12]
θ_1	257,741	[12], [13]
θ_2	34,152	[12]
θ_3	89027	Assumed
τ_1	14,776	[12], [13]
τ_2	111,515	[12], [14]
τ_3	202,865	[12], [15]
α_1	27,321	[16]
α_2	230420	Assumed
π_1	45833.96	Assumed
π_2	29,216	[17], [15]

μ	0.005	[7]
ε	0.6	[7]
ρ_1	67,528	[17]
ρ_2	190213	Assumed
ρ_3	3996	Assumed
β_1	11,820	[16]
β_2	16790	Assumed

Using the MATLABR2019a Software package, the authors numerically investigated the dynamical behavior of the model linear ODEs in the system (2) in order to corroborate the results of the analytic part of the research [7], [18].

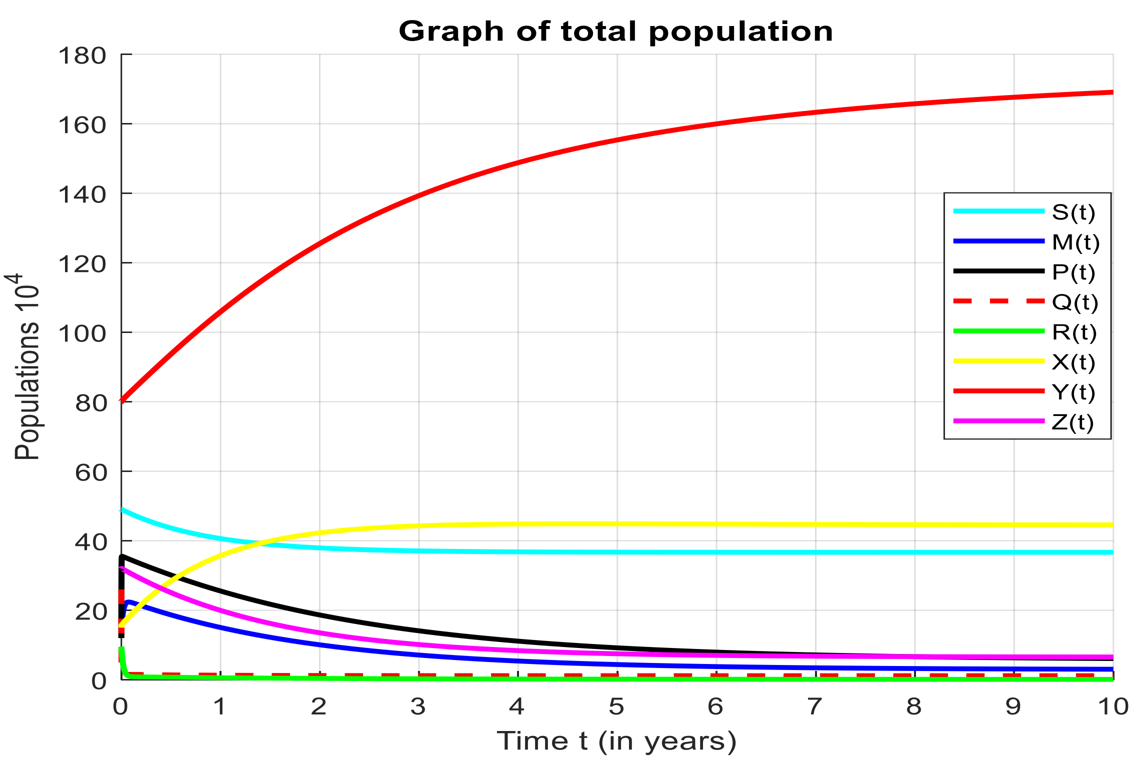

Figure 2: Dynamics of total population with respect to time, t

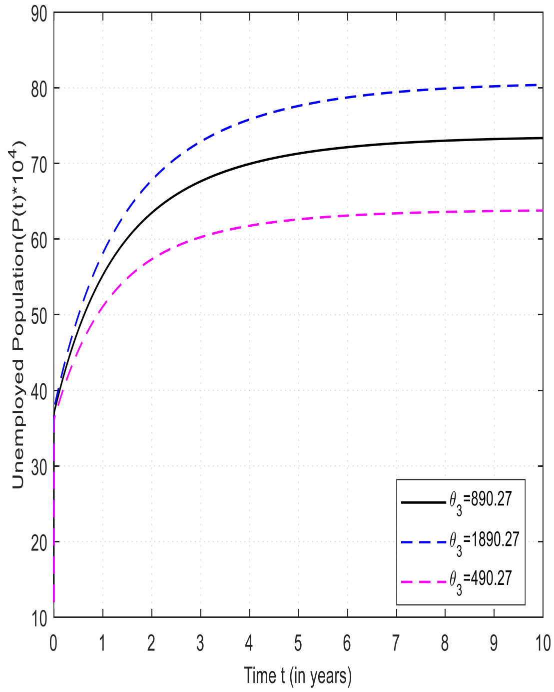

Figure 3: Dynamics of unemployed population P(t) with respect to θ_3.

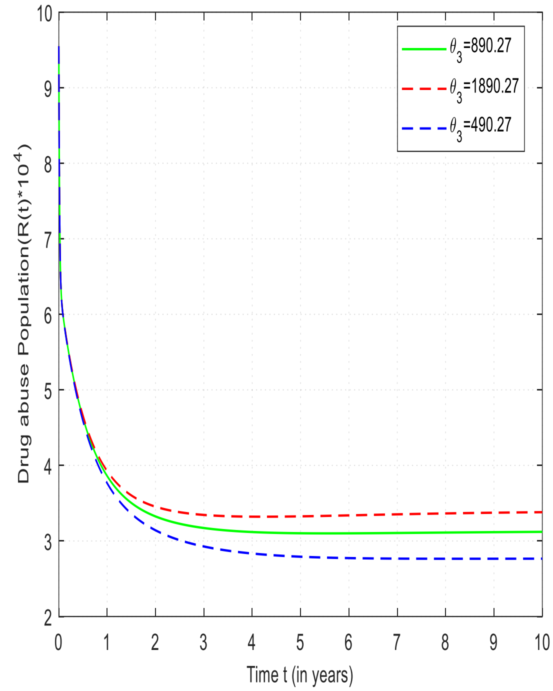

Figure 4:Dynamics of substance abuse with respect to θ_3.

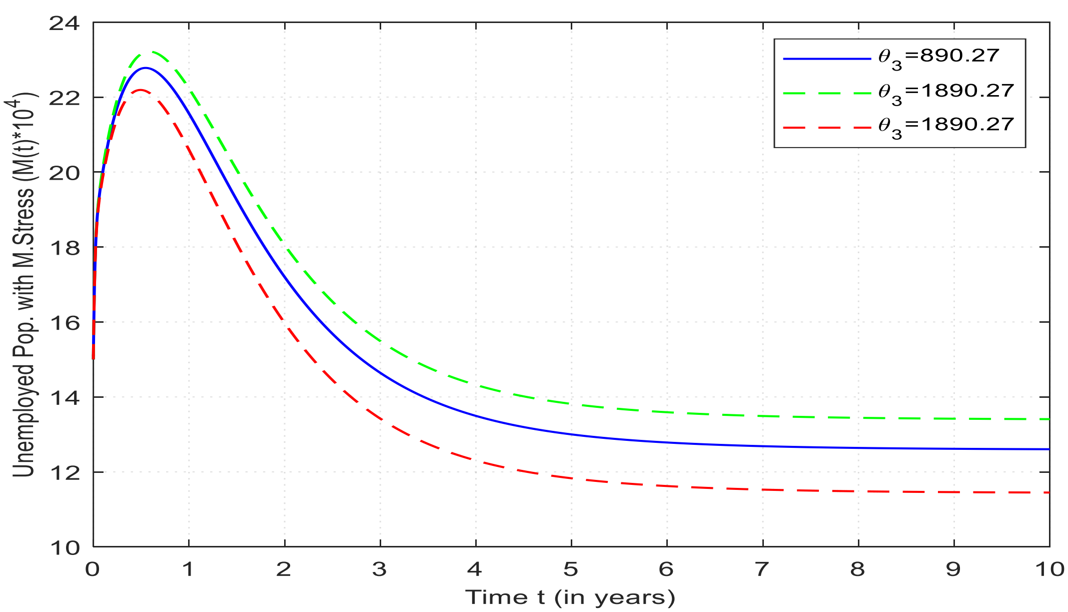

Figure 5: Effects of unemployed population with mental stress M(t) with respect to θ_3 over time

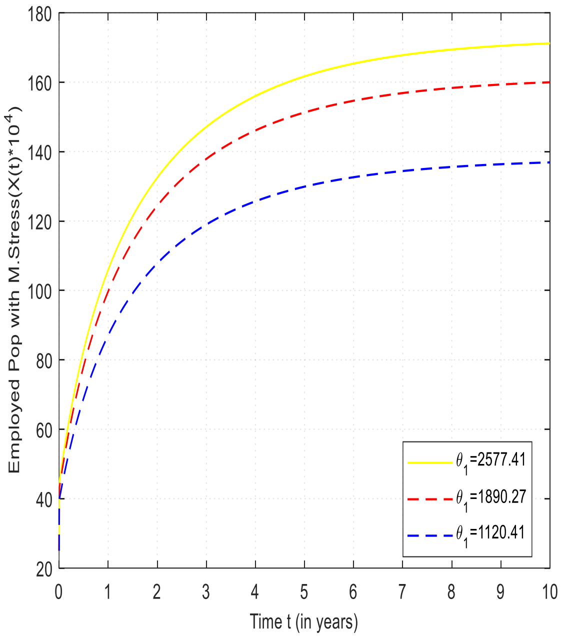

Figure 6: Effects of θ_1 *on* X(t) class with respect to time

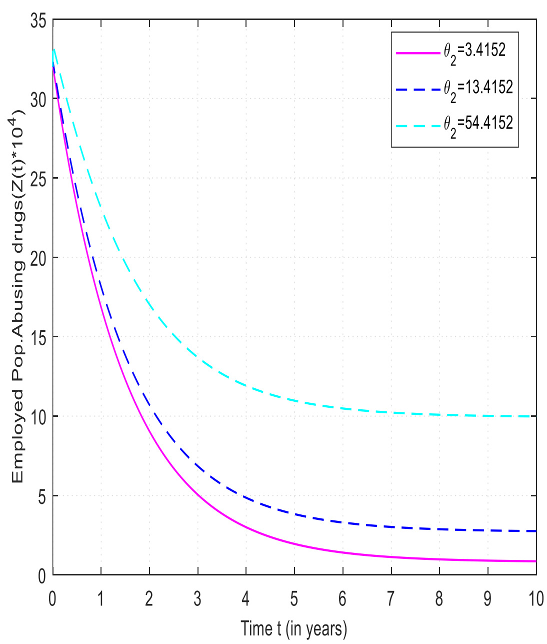

Figure 7: Effects of θ_2 on Z(t) class with respect to over time.

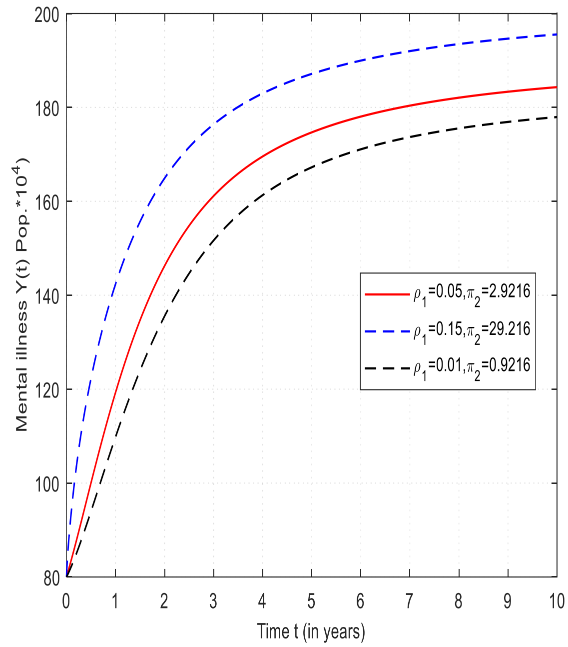

Figure 8: Effects of mental stress on mental illness Y (t) with respect to time

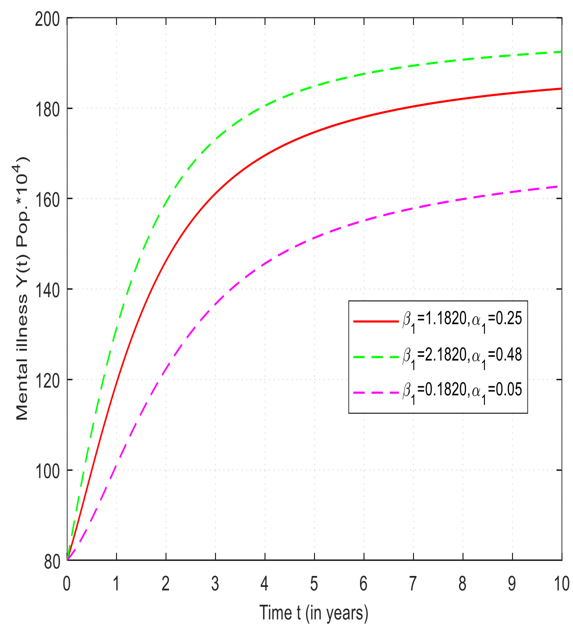

Figure 9:Effects of substance (drug) abuse on Y(t) with respect to time

5. Discussion of the Results

In this study, state of employment, substance abuse and mental stress are selected as the main factors triggering mental illnesses. Figure 2 describes the dynamic simulation of the total population with respect to time for our modal using the original parameters cited in Table 2. The population of those suffering mental illness *Y(t)* and the employed population with mental stress *X(t)* increased exponentially until the stationary point was attained at the top end. The unemployed population *P(t)* decreased significantly due to new recruitment from the susceptible class *S(t)*. It was noted that the subpopulation of those abusing drugs, regardless of the state of employment *R(t)* and Z(t), decreased gradually until to a point where equilibrium point E_3 was attained. The subpopulations *M(t), P(t),* and *Z(t)* was observed to increase sharply to a maximum point and then decrease slowly to attain their equilibrium points at the lowest point. At some point, the subpopulations R(t) and Q(t) were observed to decrease steadily, attaining their equilibrium point and remaining constant as time t increases.

From figure 3, the unemployed population increases as the rate of losing a job (θ_3) increases from 890.27 to 1890.27. There was a significant decrease as the value of (θ_3) is reduced further to 490.27. By varying the rate of losing a job (θ_3) on the drug abuse class R(t), as shown in figure 4, this study noticed similar effects. The population of those abusing drugs increased significantly as the value of (θ_3) increased and vice versa. Unemployed population was found to be more susceptible to drugs(substance) abuse due to idleness, peer pressure, and criminal cases.

To study the effects of unemployment on mental stress, the study varied the value of (θ_3), which directly affected the number of those with a source of income, as shown in figure 5 below. There was a sharp increase in the unemployed population with mental stress M(t) as time t increased, but this trend was seen to attain a maximum point and then dropping significantly with time t. Moreover, a slight change in (θ_3) showed the similar trend with numbers in M(t) class vary proportionally as (θ_3) increased. This indicated an increase in the mentally stressed population as more people lost a source of income.

Mental stress for employed population X(t) increased with time t, as shown in figure 6 at $\theta_1 = 2577.41$. When θ_1 was decreased from 2577.41 to 1890.41 and consequently, to 1120.41, a significant decrease in M(t) was noted. As the rate of the employed population (θ_2) increased, the graph flattened, indicating that, as more people get employed, those abusing drugs reduce significantly over time t. This indicates that if job opportunities are created, many people are engaged, reducing the chances of abusing drugs, leading to reduced cases of mental illness and stable families.

From figure 7, the study investigated substance(drug) abuse's impact on mental illness class Y(t). The population of those who had mental illness increased as the rate of the state of employment increases (regardless of whether an individual is employed or not employed). By varying the value of the rate of the working class abusing drugs(ρ_1)and also the unemployed population abusing drugs (π_2) on the mental illness population class, the results show a significant increase in mental illness within the population as time t increases until a maximum point where the equilibrium point is attained. An increase in (ρ_1) and (π_2) from 0.05 and 2.9216 to 0.15 and 29.216, respectively, as shown in figure 7, shows an increase in the population of individuals with mental illness. This is an indication that mental illness can be controlled by reducing the value of those abusing drugs (ρ_1) and (π_2) respectively.

Figure 9 shows the extent to which mental stress affects the state of mental illness within the population setup. From the graph, the trend indicates that mental stress harms mental illness. Varying the values of β_1 and α_1, which describe the transmission rate of people suffering mental stress in the general population regardless of the state of employment, showed that, mental illness can be controlled if β_1 and α_1 are under controlled. In this study, an increase β_1 from 1.1820 to 2.182 and α_1 from 0.25 to 0.48 shows a significant increase in the cases of mental illness within the community. Varying the same parameters downward reflected a downward trend in the cases of mental illness.

6. CONCLUSION

Based on eight differential equations(ODE's) described in system (2), an SMPQRXYZ mathematical model to study mental disease was established. Eight diverse categories are established to reflect distinct subpopulations, including the vulnerable, the employed, the jobless, drug abusers, the psychologically stressed, and the mentally sick. Employment status is often believed to be a primary determinant in substance addiction and mental health problems. Positiveness, boundedness, and local and global stability analyses of the equilibrium point were determined to establish the well-possedness of the mental disease model's equations.

The study constructed five equilibrium points of the system $E_0, E_1, E_2, E_3,$ and E_4. The local stability of the equilibrium points was studied using the eigenvalue method. The regional stability of each equilibrium point is then analyzed by constructing a variation matrix and determining the eigenvalues in each case. At the DFE point, the equilibrium point E_0 was unstable. All other equilibrium points were stable due to the existence of negative eigenvalues. Lyapunov function was used to study the global stability at the main stressors (drug abuse and mental stress-free equilibrium points). On the finding, the equilibrium points $E_1, E_2, E_3,$ and E_4 were locally asymptotically stable and globally stable.

MATLAB2019a software was used to run a numerical simulation that plotted out data on the relationship between unemployment, substance addiction, and mental illness. According to the findings in this study, an increase in the number of individuals who are unemployed either because they have lost their jobs or have never had them in the first place leads to an equally dramatic rise in the number of drug abusers and persons experiencing mental health problems. The effect of mental stress and substance addiction on mental illness was shown to spread in plots as the underlying stress factors for mental disease moved higher. This research recommends that governing bodies on mental health raise the working population's understanding of mental health issues so that fewer workers quit their jobs because of stress, hostile bosses, and other negative aspects of their workplaces. Taking a look at the staff's working environment as a whole is necessary.

The study suggests that parameters causing mental stress ($\theta_1, \tau_1, \pi_1, \rho_2, \rho_3$) and substance abuse($\alpha_2, \tau_2, \beta_2, \theta_2, \theta_3$) within a given population set up to be controlled for their great impact on mental illness. A clear policy on employment for those graduating from formal education and improvement in the general working condition for the employees will lead to reduced cases of drug abuse and mental stress which will help reduce mental illness within the community. Doing real data fitting, Modelling, and simulating the competition of mental stress and substance abuse on mental illness using Holing type II response would be an interesting area for future studies.

7. RECOMMENDATION

There are not enough records for people with mental illness for several reasons. Disconcerting follow-up visit costs, wasted time in overcrowded mental health clinics, and a rise in inaccurate diagnosis—especially from so-called "quacks" in the psychiatric and medical communities—have made patients wary of seeing their psychiatrists for treatment updates. Classifiers identify instances as normal, addiction-free, dependence-with-conditions, or disorder-free. On the other hand, there has been a recent increase in cases of personality disorders, mental stress, and drug misuse.

While the government of Kenya has made some attempts to improve mental health via the ministry of health (MOH), as seen by the 'KENYA MENTAL HEALTH ACTION PLAN 2021-2025(KMHAP)', researchers still lack access to adequate data on mental health. As one of the LMICs, Kenya's Ministry of Health (MOH) is urged by this study to make its mental health data publicly available without compromising patients' right to privacy, to fully implement the policies outlined in the Kenya Mental Health Action Plan (KMHAP) for the years 2021 through 2025, and to make available sufficient capitation to fund researchers engaged in mathematical and biological modeling. This will be a nice way to inspire more people to study mental health in order to provide a precise mathematical forecast of future trends, which can then be used to inform improved mental health policies that will help the community as a whole thrive.

Mathematical modeling and simulation of drug addiction and mental stress on population dynamics of mental disease need more study in a number of areas. In the first place, we need further study into the connection between substance abuse and the development, prevalence, and intensity of mental diseases. Stress on the mind may have an effect on the beginning, development, and severity of mental disease, hence studies examining this connection are necessary.

Second, there is a need for further study into the population dynamics of mental disease, especially with regards to the manifestation and variation of mental illnesses among various demographic groups and geographic locations. Population-level variables connected to the frequency and severity of mental illness might be determined by studying the prevalence and severity of mental illnesses in various geographical locations and demographic groups.

Lastly, greater study is required to enhance current models of mental disease and investigate new models that more accurately portray the intricacies of mental health issue etiology, clinical presentation, and treatment. To better understand the interaction between many variables affecting mental health at the individual, group, or regional levels, such models might combine elements of epidemiology, system dynamics, and population-level dynamics.

ACKNOWLEDGMENT

The authors thank the University of Embu for their kind support throughout the research.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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