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A NEW SIMPLE DISCRETE-TIME MODEL FOR THE DESCRIPTION OF EXCESSIVE ALCOHOL CONSUMPTION WITH n COMPLICATIONS

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Abstract. In this paper, we developed a new discrete-time mathematical model as a promising approach to understand and analyze the progression of disease complications related to excessive alcohol consumption, namely potential drinkers (P_i), moderate drinkers (M_i), heavy drinkers (H_i), and heavy drinkers $(C_{i,j})$ with different disease complications and quitters of drinking (Q_i) . First, we study the local stability using mathematical theories such as the Routh-Hurwitz criteria and propose three control measures. The Pontryagin maximum principle is employed to characterize the optimality of the controls. This new model makes it possible to optimize strategies aimed at reducing alcohol abuse with complications, prevention and treatment, taking into account the many factors that contribute to these complications. this model offers new perspectives for a better understanding of these disease complications and the development of more effective treatments to improve public health. Numerical simulations of the model are performed using MATLAB.

Keywords: discrete mathematical model; excessive alcohol consumption; complications; optimization; local stability; Pontryagin maximum principle.

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1. INTRODUCTION

Many people around the world are affected by addiction, which has become a veritable scourge in contemporary society. Addiction has devastating consequences on the lives of those affected, as well as on those around, whether through the consumption of substances such as alcohol, illicit drugs, prescription medications, tobacco or even certain food products, or through compulsive behaviors such as pathological gambling, compulsive shopping, video games or excessive use of social networks.The addict is recognizable by his actions and behavior, as he often seeks only what makes him happy and provides him with what he's addicted to, thus wasting his own and his body's right. He is careless, nervous, averse to others and won't accept advice. His body weakens and his behavior becomes somewhat aggressive. He wreaks havoc on society, his family and his environment because he doesn't play his part and can prevent others from playing theirs, and becomes a bad example to those around him, especially children [4].

In this work , we will focus on excessive alcohol consumption, which can lead to physical health complications such as liver cirrhosis, cardiovascular disease, and certain types of cancer. It can also contribute to mental and emotional problems such as memory loss, poor concentration, anxiety, depression and even cognitive impairment. It can also affect our relationships, work, and social lives. According to a report published by the World Health Organization (WHO), alcohol abuse caused more than three million deaths in 2016 (5.3 of all deaths), or one in 20 deaths. More than three-quarters of these deaths were among men. Alcohol misuse accounts for more than 5% of the global burden of disease. Among the population aged 15-49 years, alcohol consumption was the leading risk factor worldwide in 2016, with 3.8 of female deaths and 12.2% of male deaths attributable to Alcohol consumption [2].

There are many opportunities to reduce the harmful use of alcohol globally, including the global alcohol action plan 2022-2030 to strengthen the implementation of the Global Strategy to Reduce Harmful Use of Alcohol, increased health awareness among populations, reductions in youth alcohol use observed in many countries, confirmation of the role of alcohol control policies in reducing health and gender inequalities, and increasing evidence on the effectiveness and cost-effectiveness of a wide range of alcohol control measures[2, 3, 8].

The modeling and analysis of alcohol consumption has attracted the attention of several mathematical researchers who have done a lot of work to understand the dynamics and analysis of alcohol consumption to reduce its harmful effects on the drinker and society, as well as to minimize the number of dependents. For example, S. H. Ma et al. [15] modeled alcoholism as a contagious disease and used optimal control to study their mathematical model with awareness programs and time delays. A. Essounaini et al.[17] developed a mathematical model of excessive alcohol consumption with n complications and discussed the stability of the local and global equilibrium without and with excessive alcohol consumption and the sensitivity analysis of R_0. X. Y. Wang et al [1]proposed and analyzed a non-linear alcoholism model and used optimal control for the purpose of hindering interaction between susceptible individuals and infected individuals. H. F. Huo et al [10] proposed a new social epidemic model to depict alcoholism with media coverage which was proven to be an effective way in pushing people to quit drinking. I. k. Adu et al [14] used a non-linear SHT R mathematical model to study the dynamics of drinking epidemic,they divided their population into four classes: non-drinkers (S), heavy drinkers (H), drinkers receiving treatment (T) and recovered drinkers (R). They discussed the existence and stability of drinking-free and endemic equilibrium. Khajji et al. [13] proposed a discrete alcohol model that divided the population into six compartments. They formulated an optimal control problem with three controls: the first one represents awareness programs for potential drinkers,the second one is the effort to encourage the rich people to go to the private treatment center, and the third one is follow-up and psychological support for temporary quitters of drinking. The authors used Prontryagins maximum principle to find optimal strategies that minimize the number of drinkers and maximize the number of heavy drinkers who join an addiction treatment center. See also [4, 7, 9, 12, 19] and other references which are interested in mathematical models with discrete time. In this research, we adopt discrete-time modeling, motivated by the fact that relevant data are collected at discrete intervals, such as per day, week, month, or year, the utility for representing discrete events, mathematical simplicity, and ease of digital implementation. This approach can prove to be very important for the modeling and analysis of epidemiological and public health phenomena.

So, we will study the dynamics and analysis of a mathematical alcohol model $P_iM_iH_iC_{i,1}C_{i,2}C_{i,3}.....C_{i,n}Q_i$ which contains the following additions:

- discrete -time mathematical modeling.

- n Compartments (Ci;j) represents the number of the heavy drinkers with different disease complications associated with excessive alcohol consumption.

- Mortality induced by heavy drinkers and heavy drinkers with disease complications.

In this paper, we propose a new discrete-time mathematical model $P_iM_iH_iC_{i,1}C_{i,2}C_{i,3}.....C_{i,n}Q_i$ to understand the progression of disease complications related to excessive alcohol consumption in Section 2. In section 3, we discuss local stability without and with disease complications and in sectio 4, we present the optimal control problem for the proposed model where we give some results concerning the existence of the optimal controls and we characterize these optimal controls using Pontryagin's Maximum Principle. Also, numerical simulations Also numerical and discussion are given in Section 5. Finally, we conclude the paper in Section 6.

2. A Mathematical Model

We suggest a discrete model $P_iM_iH_iC_{i,1}C_{i,2}C_{i,3}.....C_{i,n}Q_i$ to characterize population dynamics and observe interactions between drinking classes. The population is separated into n+4 compartments, including potential drinkers P_i, moderate drinkers M_i, heavy drinkers H_i, and heavy drinkers with different complications Ci(t) and quitters of drinking Q(t).

The following diagram will show the direction of movement of individuals between compartments in Figure (1).

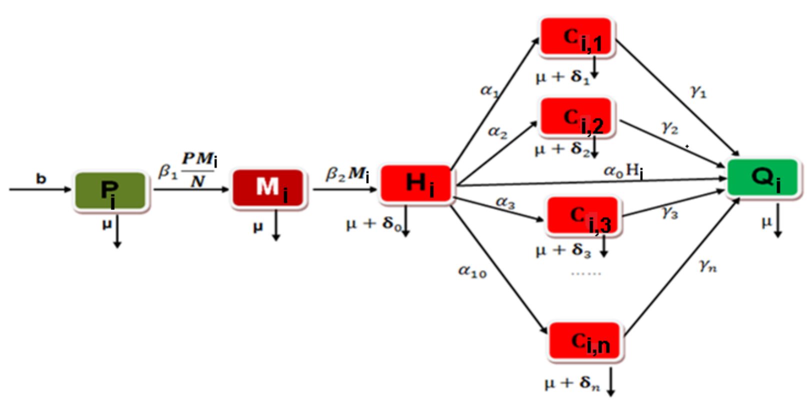

Figure 1. Schematic diagram of the n+4 drinking classes in the model

We examine the system of six non-linear differential equations(2.1):

$$\begin{cases} P_{i+1} = P_i + b - \beta_1 \frac{P_i M_i}{N} - \mu P_i \\ M_{i+1} = M_i + \beta_1 \frac{P_i M_i}{N} - (\beta_2 + \mu) M_i \\ H_{i+1} = H_i + \beta_2 M_i - (\mu + \alpha_0 + \delta_0 - \sum_{j=1}^{n} \alpha_j) H_i \\ C_{j,i+1} = C_{j,i} + \alpha_j H_i - \left(\mu + \gamma_j + \delta_j\right) C_{j,i} \\ Q_{i+1} = Q_i + \alpha_0 H_i + \sum_{j=1}^{n} \gamma_j C_{j,i} - \mu Q_i \end{cases} \tag{2.1}$$

Potential drinkers $\mathbf{P_i}$:

$$P_{i+1} = P_i + b - \beta_1 \frac{P_i M_i}{N} - \mu P_i \tag{2.2}$$

Potential drinkers P_i denote individuals who are over the age of majority, are augmented by the recruitment rate denotes b and diminished by the rates $\beta_1 \frac{P_i M_i}{N}$ and μP_i , where μ is the natural mortality rate, β_1 is the passing rate from P_i to M_i .

Moderate drinkers M_i:

$$M_{i+1} = M_i + \beta_1 \frac{P_i M_i}{N} - (\beta_2 + \mu) M_i \tag{2.3}$$

Moderate drinkers M_i are augmented by $\beta_1 \frac{PiM_i}{N}$ rates and diminished by $\beta_2 M_i$ and μM_i rates, where β_2 is the rate of passing from M_i to H_i.

Heavy drinkers H_i:

$$H_{i+1} = H_i + \beta_2 M_i - (\mu + \alpha_0 + \delta_0 - \sum_{j=1}^{n} \alpha_j) H_i \tag{2.4}$$

The heavy drinker number H_i includes dependent individuals. The compartment becomes bigger as the number of excessive drinkers increases at $\beta_2 M_i$ and decreases at $\alpha_0 H_i$, $\sum_{j=1}^{n} \alpha_j H_i$ and $(\mu + \delta_0) H_i$. Where δ_0 is the mortality rate induced by H_i.

Heavy drinkers with complication $C_{i,j}$:

$$C_{i+1,j} = C_{i,j} + \alpha_i H_i - \left(\mu + \gamma_j + \delta_j\right) C_{i,j} \tag{2.5}$$

Excessive drinkers with liver complications related to prolonged and heavy alcohol consumption (alcoholic hepatitis and fibrosis and cirrhosis), is augmented by the rate $\alpha_i H_i$ and diminished by the rates $\gamma_j C_{i,j}$, $\mu C_{i,j}$ and $\mu \delta_i$. where δ_i is the death rate induced by complication j in excessive drinkers.

For more informations see article [9]

Quit drinking Q_i:

$$Q_{i+1}= Q_i+\alpha_0 H_i+\sum_{j=1}^{n}\gamma_j C_{i,j}-\mu Q_i \tag{2.6}$$

Q_i denotes individuals who definitively and temporally stop drinking, grows by the rates $\alpha_0 H_i$ and $\sum\limits_{j=1}^{n}\gamma_j C_{i,j}$ and diminishes by the rate μQ_i.

Total population size is given by N with

$$N = P_i+M_i+H_i+C_{i,1}+C_{i,2}+.....+C_{i,n}+Q_i.$$

3. Local Stability Analysis

3.1. The drinking-free equilibrium.

In this paragraph, we examine the local stability of the beverage-free equilibrium.

Theorem 3.1. *The beverage-free equilibrium $E^0\left(\frac{b}{\mu},0,0\right)$ of the system*(2.1) *is asymptotically stable if $R_0<1$ and unstable if $R_0>1$.*

Proof We now consider the stability local of the drinking-free equilibrium, for the system diffined by(2.1), the matrix jacobian is given by:

$$J(E)=\begin{pmatrix} 1-\beta_1\frac{M_i}{N}-\mu & -\beta_1\frac{P_i}{N} & 0 \\ \beta_1\frac{M_i}{N} & 1+\beta_1\frac{P_i}{N}-\beta_2-\mu & 0 \\ 0 & \beta_2 & 1-(\mu+\alpha_0+\delta_0+\sum\limits_{j=1}^{n}\alpha_j) \end{pmatrix} \tag{3.1}$$

The Jacobian matrix for equilibrium without alcohol consumption is given by the following formula

$$J(E^0)=\begin{pmatrix} 1-\mu & -\beta_1 & 0 \\ 0 & 1+\beta_1-\beta_2-\mu & 0 \\ 0 & \beta_2 & 1-(\mu+\alpha_0+\delta_0+\sum\limits_{j=1}^{n}\alpha_j) \end{pmatrix} \tag{3.2}$$

where $P_0=\frac{b}{\mu}=N$.

The equation characteristic of this matrix is given by

$$\det(J(E^0)-\lambda I_3)=0$$

where I_3 is a square identity matrix of oder 3.

The following eigenvalues have been obtained:

$$\begin{aligned} &\lambda_1 = 1-\mu \\ &\lambda_2 = 1-(\beta_2+\mu-\beta_1) = 1-(\mu+\beta_2)\left(1-\tfrac{\beta_1}{\mu+\beta_2}\right) \\ &\lambda_3 = 1-(\mu+\alpha_0+\delta_0+\sum_{j=1}^{n}\alpha_j) \end{aligned} \tag{3.3}$$

$$R_0 = \frac{\beta_1}{\mu+\beta_2} \tag{3.4}$$

Consequently, all the characteristic equation's eigenvalues are negative if $R_0 < 1$.

As a result, we conclude that the equilibrium without alcohol consumption is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

3.2. Endemic equilibrium. In this specific section, we examine the local stability of the equilibrium in the presence of drink.

To equilibrate the current consumption of the system in equation (1), setting $P_{i+1} = P_i$, $M_{i+1} = M_i$ and $H_{i+1} = H_i$. On condition that at last one of the compartments infected is not null. We test system equilibrium (2.1) by putting the right side of system equation (2.1) null, then solution for P_i^*, M_i^* and H_i^*.

We have obtained the following system (3.5)

$$\begin{cases} P_{i+1} = P_i + b - \beta_1\frac{P_iM_i}{N} - \mu P_i \\ M_{i+1} = M_i + \beta_1\frac{P_iM_i}{N} - (\beta_2+\mu)M_i \\ H_{i+1} = H_i + \beta_2 M_i - (\mu+\alpha_0+\delta_0+\sum_{j=1}^{n}\alpha_j)H_i \end{cases} \tag{3.5}$$

Using the equation for the turn in system (11), we have

$$M_i^* = \frac{\mu+\alpha_0+\delta_0+\sum_{j=1}^{n}\alpha_j-1}{\beta_2}H_i \tag{3.6}$$

According to the two equation of system (11), we have

$$P_i^* = N\left[\frac{(\mu+\beta_2)-1}{\beta_1}\right] \tag{3.7}$$

$$P_i^* = \frac{b}{\mu R_0} \tag{3.8}$$

From the first equation in the system (3.5), we have

$$M_i^* = \frac{b(R_0-1)}{\beta_1} \tag{3.9}$$

$$H_i^* = \frac{b\beta_2(R_0-1)}{\beta_1(\mu+\alpha_0+\delta_0+\sum_{j=1}^{n}\alpha_j)} \tag{3.10}$$

Theorem 3.2. *when $R_0 > 1$, E_i^* is locally asymptotically stable .*

Proof We define $E_i^*\,(P_i^*, M_i^*, H_i^*)$ as the system's endemic equilibrium (11) and $P_i^* \neq 0, M_i^* \neq 0$ $H_i^* \neq 0$

The Jacobian matrix is

$$J(E_i^*) = \begin{pmatrix} 1-\beta_1\frac{M_i^*}{N}-\mu & -\beta_1\frac{P_i^*}{N} & 0 \\ \beta_1\frac{M_i^*}{N} & 1+\beta_1\frac{P^*}{N}-\beta_2-\mu & 0 \\ 0 & \beta_2 & 1-(\mu+\alpha_0+\delta_0+\sum_{j=1}^{n}\alpha_j) \end{pmatrix} \tag{3.11}$$

The eigenvalues of the matrix $J(E_i^*)$ are $\lambda_1 = 1-(\mu+\alpha_0+\delta_0+\sum_{j=1}^{n}\alpha_j)$ and the others are determined with the characteristic equation of this matrix is

$$\det(J(E_i^*)-\lambda I_2) = 0$$

where I_2 is a square identity matrix of order 2.

$$P(\lambda) = \lambda^2 + a_1\lambda + a_2 \tag{3.12}$$

where

$$a_1 = \beta_1\frac{M_i^*}{N}+\mu+\beta_2+\mu-\beta_1\frac{P_i^*}{N}-2$$
$$a_2 = \left(\beta_1\frac{M_i^*}{N}+\mu-1\right)\left(\beta_2+\mu-\beta_1\frac{P_i^*}{N}-1\right)+\beta_1^2\frac{P_i^*}{N}\frac{M_i^*}{N}$$

When $R_0 \succ 1$,the calculation yields

$P(1)=1+a_1+a_2 \succ 0$

$P(-1)=1-a_1+a_2 \succ 0$

Fur thermore, the constant term satsifies $a_2 \succ 0$

By routh- Hurwitz Criterrion$[11]$, the system(1) is locally asymptotically stable if $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1a_2 > a_3$.

The jury criterion [11] implies that the two both roots $\lambda_2 and\ \lambda_3$, of the equation $P(\lambda) = 0$ satisfy $|\lambda_2| \prec 1 and\ \ |\lambda_3| \prec 1$ The of linearization theory implies that the positive equilibrium $E_i^*\,(P_i^*, M_i^*, H_i^*)$ of system (11) is locally asymptotically stable if $R_0 > 1$, i.e, the endemic equilibrium E_i^* of system (3.5) is locally asymptotically stable.

4. THE OPTIMAL CONTROL PROBLEM

Our objective in this proposed strategy of control is to minimize the number of heavy drinkers H_i and the heavy drinkers with i complication C_i, maximize the number of the quitters of drinking Q_i and also minimize the cost spent in an awereness program and treatment.

In the model (1) we include three controls u_i, v_i and w_i . The control u_i represents the awereness programs effort (education programs, media...) applied on the potential drinkers to protect the potential drinkers not to be drinkers. The second control v_i measures the effort of treatment applied on the heavy drinkers.We note that the control function $(1-\varepsilon)v_i$ represents the fraction of the heavy drinkers who will be treated and go to moderate drinkers and the fraction εv_i of those leaving the heavy drinkers who will be treated and quit drinking. Finaly, w_i measures the effort the treatment applied on the heavy drinkers with i complication.We note that the control function $(1-\chi)w_i$ represents the fraction of the heavy drinkers with i complications who will be treated and go to moderate drinkers and the fraction χw_i of those leaving the heavy drinkers with i complications who will be treated and quit drinking. So the controlled mathematical system is given by the following system of differential equations(4.1).

$$\begin{cases} P_{i+1} = P_i + b - \beta_1 \frac{P_i M_i}{N} - \mu P_i - u_i P_i \\ M_{i+1} = M_i + \beta_1 \frac{P_i M_i}{N} - \beta_2 M_i - \mu M_i + (1-\varepsilon) v_i H_i + (1-\chi) w_i C_{j,i} \\ H_{i+1} = H_i + \beta_2 M_i - (\mu + \alpha_0 + \delta_0 + \sum\limits_{j=1}^{n} \alpha_j) H_i - v_i H_i \\ C_{j,k+1} = C_{j,i} + \alpha_j H_i - \left(\mu + \delta_j + \gamma_j\right) C_{i,j} - w_i C_{j,i} \quad \text{and} \quad j = \{1, 2, \ldots\ldots, n\} \\ Q_{i+1} = Q_i + \alpha_0 H_i - \mu Q_i + u_i P_i + \varepsilon v_i H_i + \chi w_i C_{j,i} + \sum\limits_{j=1}^{n} \alpha_j\ H_i \end{cases} \tag{4.1}$$

where $P_i \geq 0$, $M_i \geq 0$, $H_i \geq 0$, $C_{ij} \geq 0$ and $Q_i \geq 0$ are the given initial states.

Then, the problem is to minimize the objective functional

$$J(u,v,w) = H_T - Q_T + \sum_{j=1}^{n} C_{j,T} + \sum_{k=1}^{n} \left(H_k + \sum_{j=1}^{n} C_{j,k} - Q_k + \frac{A_{1,k}}{2} u_k^2 + \frac{A_{2,k}}{2} v_k^2 + \sum_{j=1}^{n} \frac{B_{j,k}}{2} w_{j,k}^2 \right) \tag{4.2}$$

Where the parameters $A_{1,k}$,$A_{2,k}$ and $A_{3,k}$ are the strictly positive cost coefficients. They are selected to weigh the relative importance of u_i, v_i and w_i .

In other words, we seek the optimal controls u_k, v_k and w_k such that:

$$J(u_k^*, v_k^*, w_k^*) = \min_{(u,v,w) \in U_{ad}^3} J(u_k, v_k, w_{,k}), \tag{4.3}$$

where U_{ad} is the set of admissible controls defined by:where Uad is the set of admissible control defined by

$$\begin{aligned} U_{ad} &= \{(u,v,w) : u = (u_0, u_1, ..., u_{T-1}), v = (v_0, v_1, ..., v_{T-1}), w = (w_0, w_1, ..., w_{T-1}) : \\ a_i &\leq u_{i,k} \leq b_i \; ; \; c_i \leq v_{i,k} \leq d_i ; e_i \leq w_{i,k} \leq f_i ; k = 0,1,2...T-1\} \end{aligned} \tag{4.4}$$

4.1. Characterization of optimal controls. We apply the Pontryagin's Maximum Principle[5, 6, 7]. The key idea is introducing the adjoint function to attach the system of difference equations to the objective functional resulting in the formation of a function called the Hamiltonian. This principle converts the problem of finding the control to optimize the objective functional subject to the state difference equations with initial condition to find the control to optimize Hamiltonian pointwise (with respect to the control).

Now we have the Hamiltonian $\hat{H}_k$, defined by:

$$\hat{H}_k = H_k + \sum_{j=1}^{n} C_{j,k} - Q_k + \frac{A_1}{2} u_k^2 + \frac{A_2}{2} v_k^2 + \sum_{j=1}^{n} \frac{B_{j,k}}{2} w_{j,k}^2 + \sum_{i=1}^{n+4} \lambda_{i,k+1} f_{i,k+1} \tag{4.5}$$

where f_k is the right side of the system of of difference equations (4.1) of the k^{th} state variable at time step $k+1$.

Theorem 4.1. *Given an optimal control* $(u_k^*, v_k^*, w_k^*) \in U_{ad}^3$ *and the solutions* P_k^*, M_k^* , H_k^* , $C_{k,j}^*$ *and* Q_k^*,*of corresponding state system* (2.1), *there exist adjoint functions* $\lambda_{1,k}$, $\lambda_{2,k}$, $\lambda_{3,k}$, $\lambda_{j+3,k}$ *and* , $\lambda_{n+4,k}$ *satisfying the equations*

$$\begin{cases}
\lambda_{1,k} = \frac{\partial \hat{H}_k}{\partial Pk} = (\lambda_{2,k+1} - \lambda_{1,k+2})\beta_1 \frac{M_{ik}}{N_k} + (\lambda_{n+4,k+1} - \lambda_{1,k+1})u_k + \lambda_{1,k+1}(1-\mu)\lambda_{1,k+1} \\
\lambda_{2,k} = \frac{\partial \hat{H}_k}{\partial M_k} = (\lambda_{2,k+1} - \lambda_{1,k+1})\beta_1 \frac{P_k}{N_k} + \beta_2 \left(\lambda_{3,k+1} - \lambda_{2,k+1}\right) + \lambda_{2,k+1}(1-\ \mu). \\
\lambda_{3,k} = \frac{\partial \hat{H}_{ik}}{\partial H_k} = 1 + (\lambda_{n+4,k+1} - \lambda_{2,k+1})\varepsilon v_i + \alpha_j(\lambda_{2,k+1} - \lambda_{3,k+1}) + \alpha_0 \lambda_{n+4} \\
\qquad - (\mu + \alpha_0 + \delta_0 + \sum_{j=1}^{n} \alpha_j)\lambda_{3,k+1} + \sum_{j=1}^{n} \alpha_j \lambda_{j+3,k+1} + \lambda_{2,k+1} \\
\lambda_{j+3,k} = \frac{\partial \hat{H}_k}{\partial C_{k,j}} = 1 - (\mu + \gamma_j + \delta_j)\lambda_{j+3,k+1} + (\chi \lambda_{n+4,k+1} - \lambda_{j+3,k+1})w_k \\
\qquad + \gamma_j \lambda_{n+4,k+1} + \lambda_{2,k+1}(1-\chi)w_j + \lambda_{j+3,k+1} \\
\lambda_{n+4,k} = \frac{\partial \hat{H}_k}{\partial Q_k} = -1 + \mu \lambda_{n+4,k+1} + \lambda_{n+4,k+1}.
\end{cases} \tag{4.6}$$

with the transversality conditions

$$\lambda_1(T) = \lambda_2(T) = 0,\ \lambda_3(T) = 1,\ \lambda_{j+3}(T) = 1 \ \textit{ and } \ \lambda_{n+4}(T) = -1 \qquad j = \{1,2,.....,n\} \tag{4.7}$$

Furthermore, The optimal controls u_i^*, v_i^* and w_i^* are given by:

$$u_k^* = \min\left(1, \max\left(0, \frac{1}{A_1}\left(\lambda_{1,k+1} - \lambda_{n+4,k+1}\right) P_k\right)\right). \tag{4.8}$$

$$v_k^* = \min\left(1, \max\left(0, \frac{1}{A_2}\left\{\lambda_{3,k+1} - \varepsilon \lambda_{n+4,k+1} - (1-\varepsilon)\lambda_{2,k+1}\right\} H_k\right)\right). \tag{4.9}$$

$$w_k^* = \min\left(1, \max\left(0, \frac{1}{B_j}\left\{\left(\lambda_{i+3,k+1} - \chi \lambda_{n+4,k+1} - (1-\chi)\right)\lambda_{2,k+1}\right\} C_{j,k}\right)\right). \tag{4.10}$$

Proof The Hamiltonian $\hat{H}_k$ is given by

$$\begin{aligned}
\hat{H}_k = \ & H_k + \sum_{j=1}^{n} C_{j,k} - Q_k + \frac{A_1}{2}u_k^2 + \frac{A_2}{2}v_k^2 + \sum_{j=1}^{n} \frac{B_{j,k}}{2} w_{j,k}^2 \\
& + \lambda_{1,k+1}\left\{P_k + b - \beta_1 \frac{P_k M_k}{N} - \mu P_k - u_k P_k\right\} \\
& + \lambda_{2,k+1}\left\{M_k + \beta_1 \frac{P_k M_k}{N} - \beta_2 M_k - \mu M_k + (1-\varepsilon)v_k H_k + (1-\chi)w_k C_{k,j}\right\} \\
& + \lambda_{3,k+1}\left\{H_k + \beta_2 M_k - (\mu + \alpha_0 + \delta_0 + \sum_{j=1}^{n} \alpha_j)H_k + v_k H_k\right\} \\
& + \lambda_{j+3,k+1}\left\{C_{i,j} + \alpha_j H_k + \left(\delta_j + \gamma_j\right) H_k - w_k C_{j,k}\ \right\} \qquad j = \{1,2,.....,n\} \\
& + \lambda_{n+4,k+1}\left\{Q_i + \alpha_0 H_k - \mu Q_k + u_k P_k + \varepsilon v_k H_k + \chi w_k C_{j,k} + \sum_{j=1}^{n} \alpha_j\ H_k\right\}
\end{aligned}$$

The adjoint equations and transversality conditions can be obtained by using Pontryagin's Maximum Principle given in [13] such that

$$\begin{cases} \lambda_{1,k} = \frac{\partial \hat{H}_k}{\partial P_k}, \ \lambda_1(T) = 0 \\ \lambda_{2,k} = \frac{\partial \hat{H}_k}{\partial M_k}, \ \lambda_2(T) = 0 \\ \lambda_{3,k} = \frac{\partial \hat{H}_k}{\partial H_k}, \ \lambda_3(T) = 1 \\ \lambda_{j+3,k} = \frac{\partial \hat{H}_k}{\partial C_{j,k}}, \ \lambda_{j+4}(T) = 1 \\ \lambda_{n+4,k} = \frac{\partial \hat{H}_k}{\partial Q_k}, \ \lambda_{n+4}(T) = -1 \end{cases} \tag{4.11}$$

The optimal controls u_i^* ,v_i^* and w_i^* can be solved from the optimality condition,

$$\frac{\partial \hat{H}_k}{\partial u_k} = 0, \ \frac{\partial \hat{H}_k}{\partial v_k} = 0 \text{ and } \frac{\partial \hat{H}_k}{\partial w_k} = 0. \tag{4.12}$$

that is

$$\begin{aligned} \frac{\partial \hat{H}_k}{\partial u_k} &= A_{1,k}u_{1,k} + \left(\lambda_{n+4,k+1} - \lambda_{1,k+1}\right) P_k = 0. \\ \frac{\partial \hat{H}_i}{\partial v_i} &= A_{2,k}v_{1,k} + \left\{-\lambda_{3,k+1} + \varepsilon\lambda_{2,k+1} + (1-\varepsilon)\lambda_{5,k+1}\right\} H_k = 0. \\ \frac{\partial \hat{H}_k}{\partial w_{j,k}} &= B_{j,k}w_{j,k} + \left(\lambda_{5,k+1} - \lambda_{4,k+1}\right) C_{i,j,k} = 0. \end{aligned}$$

So, we have

$$u_k^* = \min\left(1, \max\left(0, \frac{1}{A_{1,k}}\left(\lambda_{1,k+1} - \lambda_{n+4,k+1}\right) P_k\right)\right). \tag{4.13}$$

$$v_k^* = \min\left(1, \max\left(0, \frac{1}{A_{2,k}}\left\{\lambda_{3,k+1} - \varepsilon\lambda_{n+4,k+1} - (1-\varepsilon)\lambda_{2,k+1}\right\} H_k\right)\right). \tag{4.14}$$

$$w_k^* = \min\left(1, \max\left(0, \frac{1}{B_j,k}\left\{\lambda_{j+3,k+1} - \chi\lambda_{n+4,k+1} - (1-\chi)\lambda_{2,k+1}\right\} C_{i,j,k}\right)\right). \tag{4.15}$$

By the bounds in $(U_{ad} \times V_{ad} \times W_{ad})$ of the controls, it is easy to obtain u_k^*, v_k^* and $w_{j,k}^*$ in the form of (33-34-35).

5. Numerical Simulation

In this section, we will numerically solve the optimal control problem for our $P_iM_iH_iC_{i,1}C_{i,2}C_{i,3}.....C_{i,n}Q_i$ model. Here, we obtain the optimality system from the state and adjoint equations. The proposed optimal control strategy is obtained by solving the optimal system which consists of five differential equations and boundary conditions. The optimality system can be solved using an iterative method.

Using an initial estimate for the control variables,$u, v$and w, the state variables $P; M; H; C_{i,j}$ and Q are solved forward and the adjoint variables for $j = 1; 2; 3; 4; 5$ are solved backward at time step $k = t_0$ and $k = t_f$. If the new values of the state and adjoint variables differ from the previous values, the new values are used to update $w_{1k}, w_{2k} and w_{3k}$, and the process is repeated until the system converges. We present some numerical simulations to illustrate our theoretical results, we consider the system (1) with the following parameter values $b = 65, N = 1000, d = 0,002; q = 0,02; m = 0,065; b_1 = 0,75;\ b_2 = 0,14; a_1 = 0,001; a_2 = 0,001; a_3 = 0,001; g_1 = 0,001; g_2 = 0,002$, and the initial values $P_0 = 600; M - 0 = 200; H_O = 100; C_0 = 60$ and $Q_O = 40$.

5.1. Discussion. Any large-scale expansion of alcohol consumption would be disastrous because of the impact of epidemics on global health systems and economies. This is why we are developing a vision to counter this phenomenon and avoid its human and economic impacts through the four control strategies described above (section (6)). These strategies have proved effective in reducing the number of people suffering from serious health problems (see figures (2)(c) and (d) and (f) and (g) and (8)(h)), especially the combination between the two controls u and v (see figures (i) and (l)), but the best strategy is the combination between the three controls u and v and w (see figures (e) and (k)), which reduces the burden on the health system and avoids this phenomenon. Our control programs (combining the three controls) have also proved capable of reducing the number of cancer-related complications (see figure (e)), thereby reducing any potential personal and economic impact of this phenomenon.

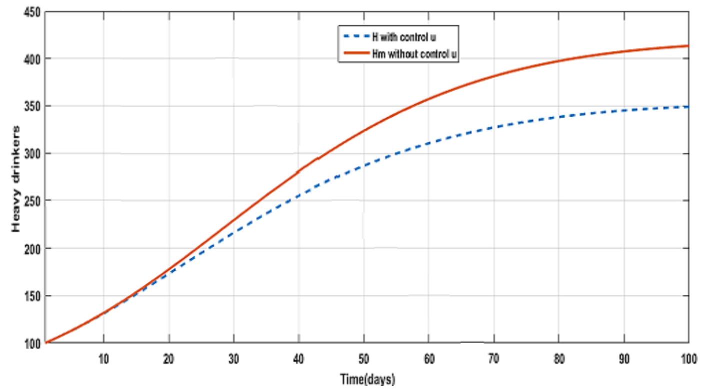

Figure 2. The Number of heady drinkers with and without control u

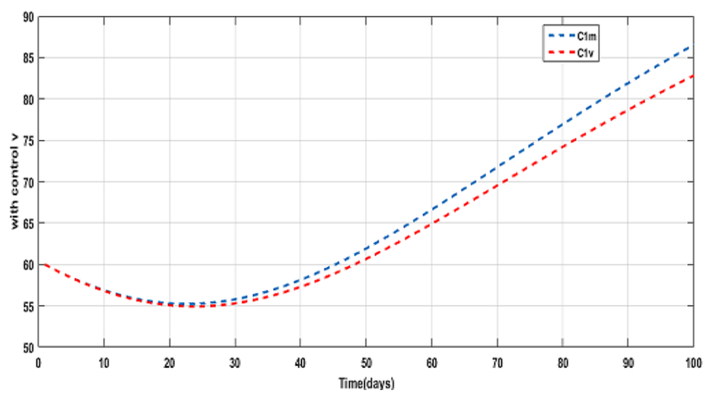

Figure 3. The number of heady drinkers with complications C_1 with and without control v

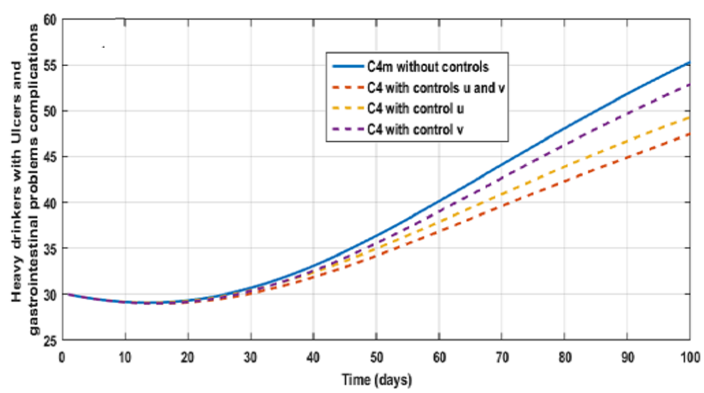

Figure 4. The number of heady drinkers with complications C_1 with and without controls u and v

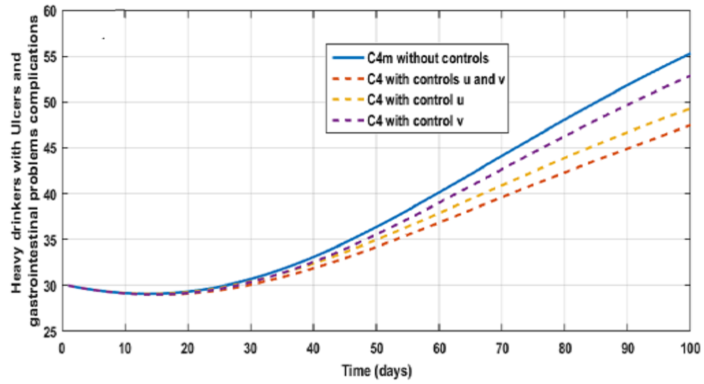

Figure 5. The number of heady drinkers with complications C_1 with both controls u and v

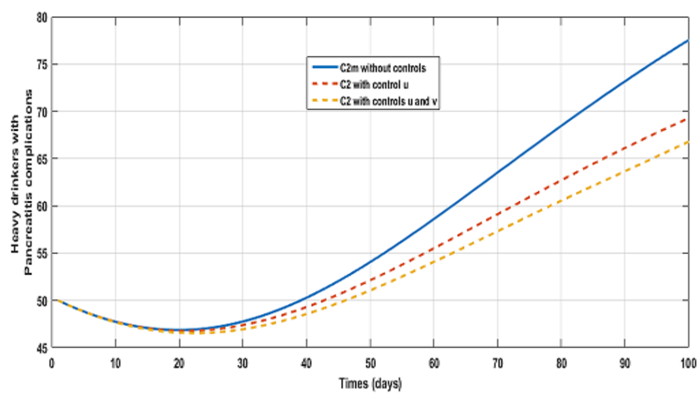

Figure 6. Number of heady drinkers with complications C_2 with and without controls u and v

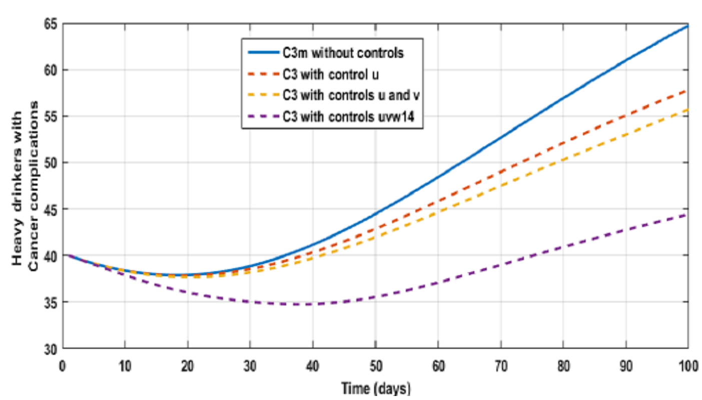

Figure 7. The number of heady drinkers with complications C_3 with and without control controls u and v

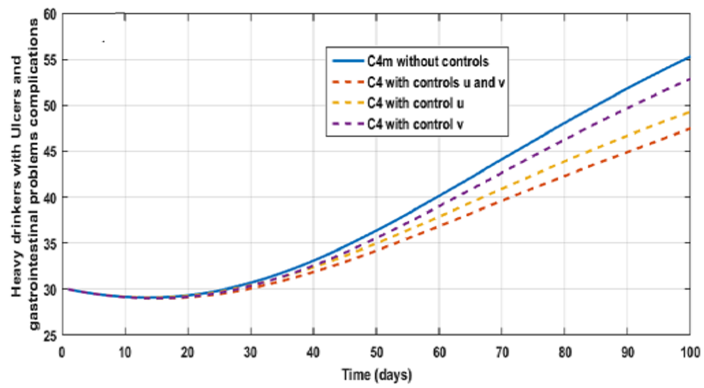

Figure 8. The number of heady drinkers with complications C_4 with and without control controls u and v

6. Conclusion

In this study, we propose a scenario for combating the effect and dynamics of disease development in excessive alcohol consumers of this model, drawing on lessons learned from Alcohol drinking to minimize the diseases serious effects on humanity and global economies. This is accomplished through the use of four different control strategies. The optimal controls are characterized using Pontryagin maximum principle, and the optimality system is resolved using an iterative approach. Finally, using MATLAB, numerical simulations are run to verify the theoretical analysis.

Conflict of Interests

The authors declare that there is no conflict of interests.

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