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THE STABILITY OF PETRI NET MODEL FOR THE COVID-19 PATIENT SERVICE SYSTEM

MONIKA RIANTI HELMI*, NOVERINA ALFIANY, NOVA NOLIZA BAKAR, DINI RAHMADANI

Department of Mathematics, Andalas University, Padang City 25171, Indonesia

Abstract: Coronavirus Disease 2019 (COVID-19) is an infectious disease caused by SARS-CoV-2 and designated a pandemic by the World Health Organization (WHO) on March 11, 2020. As cases increase, the number of patients requiring services is more than the available staff and facilities, in queues resulting in longer patient waiting times. One way to analyze the queuing system is to model using max-plus algebra. Before forming the Max-Plus Algebra model, Petri Net was built, which is a graphical and mathematical modeling tool for analyzing a system, so in this study, the results obtained in the form of a model Petri Net and max-plus algebra in the treatment of COVID-19 patients, where is the service flow patients used are limited to referral patient services with positive and suspected cases Patients Under Surveillance (PDP) COVID-19.

Keywords: max-plus algebra; COVID-19; Petri net; Lyapunov stability theory.

2020 AMS Subject Classification: 90524, 93D05, 05C50, 65F15.

1. INTRODUCTION

Coronavirus Disease 2019 (COVID-19) is an infectious disease caused by a newly discovered type

*Corresponding author
E-mail address: monikariantihelmi@sci.unand.ac.id
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of coronavirus, namely Severe Acute Respiratory Syndrome Coronavirus 2 (SARS-CoV-2), and was first detected in Wuhan, China, in December 2019 [1]. The increase in the number of COVID-19 cases took place quite quickly and spread to various countries in a short time. On March 11, 2020, COVID-19 became a global health problem later designated a pandemic by the World Health Organization (WHO). Indonesia reported its first case on March 2, 2020, where initially two cases were found [2]. Many studies have been conducted on COVID-19 issues, including research conducted by Labzai et. al. [3], and Borah et. al [4]. They examined the stability of models related to COVID-19 from the point of view of system dynamics. Yanuar, et. Al. [5] determined the length of hospital stay for COVID-19 using quantile Bayesian. Hospitals, Health Centers, Clinics, Laboratories, health services, and blood transfusion units are healthcare facilities that have a big and central role in the effort to overcome COVID-19.

As COVID-19 cases increased, the number of patients in need more services than the officers and facilities available, so there was a long queue. Long queues result in longer waiting times for patients. Therefore, service performance from the Hospital needs to be optimized by analyzing the behavior and stability of the queuing system.

One way to analyze the behavior and stability of the queuing system is by modeling the queuing system using max-plus algebra [6], [7],[8], [9], [10], [11]. Queuing theory is a field of mathematics investigating the behavior of waiting lines or queues and examining such systems, characteristics, and properties. A queuing system can be defined as a dynamic system in which states evolve due to events occurring at regular or irregular intervals. The purpose of modeling queuing systems is to predict queues' lengths and waiting times. Queuing theory is primarily associated with operations research, a branch of mathematics. Traditionally, queuing theory has been described using continuous mathematical systems categorized by the number of servers involved. However, different perspectives, including those in mathematics, have emerged. Petri nets are commonly employed in the description of discrete problems, as evident in references [12], [13], and [14][15]. Petri nets are mathematical modeling tools used to represent the evolution of states in discrete event systems. Analysis often involves the utilization of max-plus or min-max-plus algebra.

Konigsberg is one expert who developed queuing theory models from a discrete event system perspective. In [11], he created models for various queuing system styles, including those involving break servers. His approach utilized Petri nets to describe the system's conditions and event occurrences. Timed Petri nets, an extension of Petri nets, were employed in the research. The stability of the model was also assessed. However, the previous version of the timed Petri net considered only two holding times and lacked an exact max-plus standard autonomous equation.

In this paper, we will describe the flow of services for COVID-19 patients using Petri Net, a graph modeling tool and mathematics to analyze the system so that information can be obtained about the system's structure and dynamic behavior. The purpose of this study is to get the max-plus algebra model on the design of the COVID-19 service system based on the Petri net model that has been obtained. Furthermore, we also analyze the stability of the Petri net model using the Lyapunov stability theory

2. PRELIMINARIES

Max Plus Algebra and Some Related Notation

We will briefly introduce max plus algebra, which will use in the following discussion. Max plus algebra is a discrete algebraic system used to represent the behavior of a class of discrete event systems by simple linear equations, allowing for modeling, analyzing, and controlling these systems. It is an idempotent semiring and dioid used in the modeling of timed systems, and it is typically used in the completion time of a production system that has "max" occurring in the equations. The max-plus algebra is based on the set of real numbers extended by the upper reals, with addition and multiplication, and it is isomorphic to the min-plus algebra, which is based on the set of real numbers developed by the lower reals, with addition and multiplication. A more detailed explanation of max plus algebra can be found in various sources. [16][17][18][19]

Definition 1. *Given* $\mathbb{R}_\varepsilon \coloneqq \mathbb{R} \cup \{\varepsilon\}$ *where* $\mathbb{R}$ *is the set of all real numbers and* $\varepsilon \coloneqq -\infty$. *At* $\mathbb{R}_\varepsilon$ *the following operations are defined:* $\forall x, y \in \mathbb{R}_\varepsilon$, $x \oplus y \coloneqq max\{x, y\}$ *and* $x \otimes y \coloneqq x + y$. $(\mathbb{R}_\varepsilon,\ \oplus,\ \otimes)$ is called max-plus algebra and is denoted by $\mathbb{R}_{max}$. Like in regular algebra, the $\otimes$

operation in max-plus algebra has priority over the $\oplus$ operation.

Matrices on Max-Plus Algebra

The set of $n \times m$ matrices in max-plus algebra is denoted by $\mathbb{R}_{\varepsilon}^{n\times m}$. For $n, m \in \mathbb{N}$, defined

$$\overline{n} \coloneqq \{1, 2, \dots, n\} \text{ dan } \overline{m} \coloneqq \{1, 2, \dots, m\}.$$

Element $A \in \mathbb{R}_{\varepsilon}^{n\times m}$ i −th row, j −th column denoted by $a_{i,j}$ for $i \in \overline{n}$ and $\in \overline{m}$. Matrices A is written as

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,m} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,m} \end{bmatrix}.$$

The elements $a_{i,j}$ are also denoted as

$$[A]_{i,j}\,,\ \ i \in \overline{n}, j \in \overline{m}.$$

Definition 2. [max plus at work] *Let* $\mathbb{R}_{\varepsilon}^{n\times m} := \{A\}$, *where* A *is a matrix whose elements the* ij *−th is denoted by* $[A]_{i,j}$ *, where* $[A]_{i,j} \in \mathbb{R}_{\varepsilon}, i \in \overline{n}, j \in \overline{m}$, *holds:*

a) *for scalar* $\alpha \in \mathbb{R}_{\varepsilon}, A, B \in \mathbb{R}_{\varepsilon}^{n\times m}, i \in \overline{n}$, *and* $j \in \overline{m}$ *is defined*

$\alpha \otimes A$ *is a matrix whose* ij *−th element is :*$\alpha \otimes [A]_{i,j} = \alpha \otimes a_{i,j}$

and

$A \oplus B$ *is a matrix whose* ij *−th element :* $[A \oplus B]_{i,j} = a_{i,j} \oplus b_{i,j}$,

b) *for* $A \in \mathbb{R}_{\varepsilon}^{n\times p}, B \in \mathbb{R}_{\varepsilon}^{p\times m}$, $i \in \overline{n}$, *and* $j \in \overline{m}$ *is defined* $A \otimes B$ *is a matrix whose* ij *−th element is:*

$[A \otimes B]_{i,j} = {}_{k=1}^{p}\!\oplus a_{i,k} \otimes b_{k,j} = max\{a_{i,k} + b_{k,j}\}.$

Petri Net

A petri net is a directed bipartite graph and first developed by a German mathematician, Carl Adam Petri in early 1960. The advantages of system modeling using Petri nets include the ability to graphically depict a system model, allowing for easy visualization of complex systems. Petri net can model the hierarchy of a system in detail so that with analytical techniques from Petrinet, it is possible to develop a good system with systematic and qualitative analysis [12], [20], [13][10]. *Petri net consists of 4 tuples (P, T, A, w) with:*

P *: a finite set of places,* $P= \{p_1, p_2, \dots, p_n\}$*,*

T *: finite transition set,* $T = \{t_1, t_2, \dots, t_n\}$*,*

A *: set of arcs,* $A \subset (P \times T) \cup (T \times P)$*,*

w *: weight function,* $w : A \to \{1, 2, 3, \dots\}$*.*

The Petri net graph consists of two nodes: circles and lines. Circle represent places, while lines represent transitions. The place can serve as the input or output of a transition. Place as input states conditions that must be met for the transition to occur. After the transition occurs, then things will change. The place that expresses this condition is the output of the transition. Arcs are symbolized by arrows connecting place and transition. If the arc weight from place p_i to transition t_j is k, write $\omega(p_i, t_j) = k$, then there are k arcs from place p_i to transition t_j.

The marked Petri net is a 5-tuple (P, T, A, w, x_0) *where* (P, T, A, w) *is the Petri net and* x_0 *is the starting marker, and the state of marked Petri net is* $x = [x(p_1), x(p_2), \dots, x(p_n)]^T$ [2].

The number of elements x is equals to the number of places in the Petri net. $x(p_i)$ shows the number of tokens in the i −th place, so $x(pi) \in \{0, 1, 2, \cdot, n\}$. Token used in determining whether a transition is enabled or not. The transition $t_j \in T$ in a marked Petri net is said to be enabled if

$$x(p_i) \geq w(p_i, t_j) \tag{1}$$

The Petri net can be represented in an incidence matrix of size $n \times m$, where n is the number of places, and m is the number of transitions.

Definition 3. *Matrices of backward incidence and forward incidence that represents the Petri net is a matrix of size* $n \times m$ *with row elements* i *−th,* j *−th column is*

$$\begin{aligned} (\boldsymbol{A_b})_{\boldsymbol{i,j}} &\coloneqq \boldsymbol{w}(\boldsymbol{p_i}, \boldsymbol{t_j}), \\ (\boldsymbol{A_f})_{\boldsymbol{i,j}} &\coloneqq \boldsymbol{w}(\boldsymbol{t_j}, \boldsymbol{p_i}), \end{aligned} \tag{2}$$

where

A_f *: forward incidence matrices,*

A_b *: backward incidence matrices*

$w(p_i, t_j\ )$*: arc weight from i-th place to* $j-th$ *transition,*

$w(t_j, p_i)$*: arc weight from jth transition to* $i-th$ *place.*

The element in the backward incidence matrix is the arc weight that connects the place to the transition, while the elements in the matrix forward incidence is the arc weight that relates the transition to the place. If not, there is an arc that connects the place to the transition or vice versa, then the arc weight is filled with zero .

The incidence matrix $\dot{A}$ is the difference between the forward incidence matrix and backward incidence.

$$\dot{A} = A_f - A_b \tag{3}$$

One of the uses of the backward incidence matrices is to determine the transition enabled. Look again at Equation 2, the equation is valid only for place inputs. If p_i is not the input place of the transition t_j i.e. $p_i \notin I(t_j)$, then the arc weight from place p_i to transition t_j is zero because there is no arc connecting them, is written $w(p_i, t_j\ ) = 0$. Here is the equation to determine where the next token is

$$x' = x + \dot{A}u \tag{4}$$

where u represents the column vector having m elements obtained from the identity matrices column .

Coverability Tree

A coverability tree is a technique used to solve some aspects of analysis on discrete event systems and can be built from a Petri net with the initial state. The initial state of Petri Net is defined as the root node. The child of the root node is a state that can be reached from the initial state by firing a transition. These states are linked to the root node with edges. Each edge in the coverability tree has a transition weight, i.e., transitions fired to reach that state.

The coverability tree of Petri net (P, T, A, w) can be expressed by 3 tuple i.e. (S, E, v), which each represent the set of states, the set of edges and weight function. The members of the state set are n with n is the number of places on the Petri net [21][22][23][16][17].

Lyapunov Stability of Petri Net

A system modeled with Petri Net is said to be stable if there exists an m-vector with all its elements being positive, denoted as Φ, or in other words, a strictly positive vector Φ can be found such that the following holds: $\Delta v = e^T A^T \Phi \leq 0$. Since the firing vector e is always non-negative (at least one element of vector ee is non-zero), this inequality can be proved as follows: $A^T \Phi \leq 0$. Furthermore, a system modeled with Petri Net can be stabilized if a firing vector for transitions can be found, denoted as e, such that the following holds : $A\mathrm{e} \leq 0$. More detailed information regarding the Lyapunov stability of a Petri Net can be found in [21], [24], [25], and [26], [27],[28].

3. MAIN RESULTS

COVID-19 Patient Services System

In serving patients, different paths are determined depending on the patient's cases [5]. A referral patient service flow can be formed via SISRUTE (System Integrated Hospital Referral Information) and FKTP (First Level Health Facility) referrals with positive or suspected PDP COVID-19 along with variables, as shown in Figure 1.

Based on Figure 1, patients who tested positive for COVID-19 treated in the positive room for COVID-19 and then managed according to the clinical path of COVID-19. Patients with negative COVID-19 swab test results are transferred to the usual inpatient room for further management according to PPK (Clinical Practice Manual) Basic Disease.

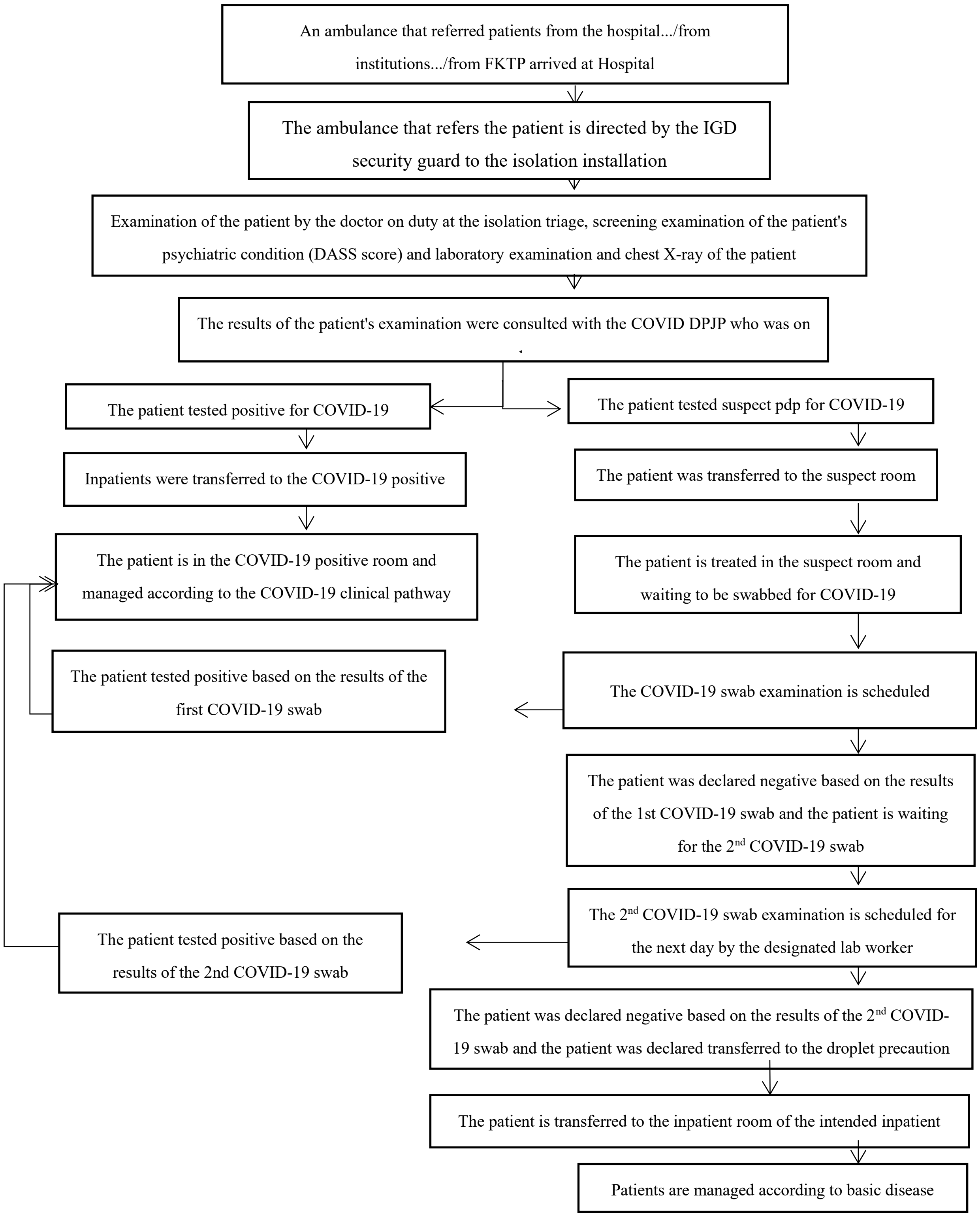

Figure. 1 The Flow of Service for COVID-19 Patients

The Petri Net Model for COVID-19 Patient Services

Based on the patient service flow data for COVID-19 in Figure 1, the Petri Net model can be formed as follows.

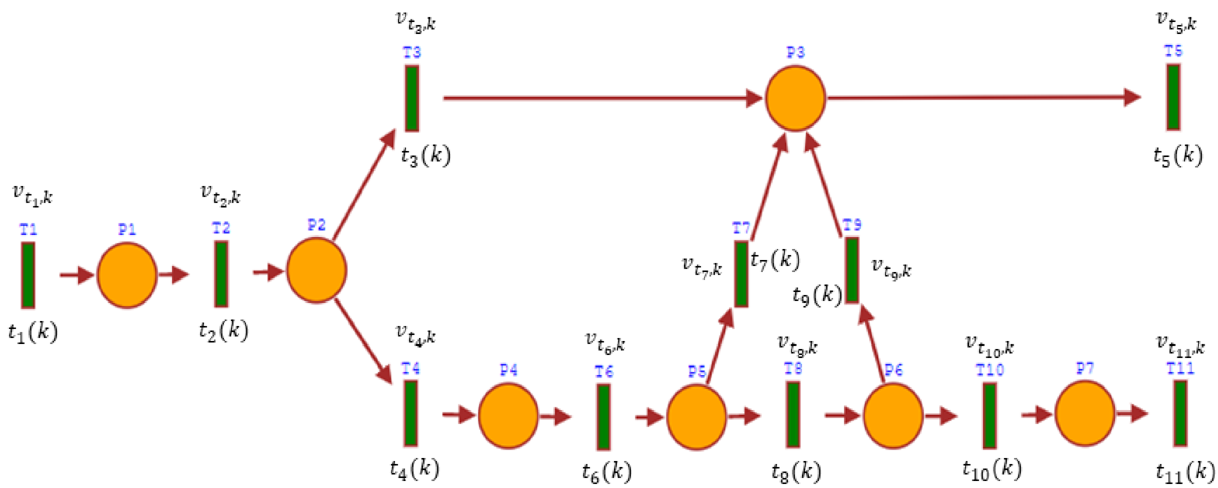

Figure 2 Petri Net Model for COVID-19 Patient Services

Based on Figure 2 above, a variable that shows time is defined as follows.

t_1 : patient was directed to the room isolation after the ambulance that refers patients from the hospital · · · / from agency · · · / from FKTP arrived at the hospital,

t_2 : the examination was completed by the doctor in charge of isolation triage, mental condition screening test (DASS score), and examination laboratory and chest x-ray

t_3: patient tested positive for COVID-19,

t_4: patient was named as a PDP suspect,

t_5: patient was treated in the COVID-19 positive room for further management by the clinical pathway of COVID-19,

t_6 : patient waits to be swabbed for COVID-19,

t_7: patient tested positive based on the results of the first COVID-19 swab,

t_8 : the patient was declared negative based on the results of the first COVID-19 swab and waiting to do the 2nd COVID-19 swab,

t_9 : the patient was declared positive based on the results of the COVID-19 swab,

t_{10} : the patient was declared negative by the DPJP room based on the results of the 2nd COVID-19 swab, so the patient was transferred to a regular inpatient room,

t_{11} : time is taken for the $k-$th patient since arrival at the hospital until it is managed according to basic disease PPK by DPJP room.

Variables that indicate the length of time in each process service are defined as follows.

$v_{t_1,k}$: length of time for the $k-$th patient to wait to be directed to the isolation room,

$v_{t_2,k}$: length of time to go to the isolation room and examine the $k-$th patient,

$v_{t_3,k}$: the length of time the $k-$th patient waits for the examination results to be declared positive for COVID-19,

$v_{t_4,k}$: length of time for the $k-$th patient to wait for the examination results to be declared PDP suspects

$v_{t_5,k}$: the length of time the $k-$th patient was transferred to the COVID-19 positive room for then managed according to the clinical path of COVID-19,

$v_{t_6,k}$: the length of time the $k-$th patient was transferred to the suspect room and waited to do a COVID-19 swab,

$v_{t_7,k}$: length of time for the $k-$th patient swab examination and waiting for it to test positive based on the results of the first COVID-19 swab,

$v_{t_8,k}$: length of time for the $k-$th patient swab examination and waiting for it to tested negative based on the results of the first COVID-19 swab and are waiting to carry out the 2nd COVID-19 swab,

$v_{t_9,k}$: length of patient's $k-$th swab examination and waiting for it tested positive for COVID-19,

$v_{t_{10},k}$: length of patient's $k-$th swab examination and waiting until tested negative for COVID-19,

$v_{t_{11},k}$: the time it takes for the k −th patient to arrive at the room hospitalization for management according to PPK Basic Diseases by the DPJP room.

Based on Figure 2, it can be seen that there are 7 places with a set of places $P = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}$, and there are 11 transitions with a transition set $T = \{T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8, T_9, T_{10}, T_{11}\}$, so that a matrix of order 7×11 can be formed. Weight for each arc is as large as one. The following is a representation of the Petri net in Figure 2 using Incidence matrix.

Backward Incidence Matrix (A_b)

$$\boldsymbol{A_b} = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

Forward Incidence Matrix (A_f)

$$\boldsymbol{A_f} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \end{bmatrix}$$

Incidence Matrix ($\dot{A}$)

$$\dot{A} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

Next, the Coverability Tree is built with the initial conditions $x_0 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$. This state is defined with the root node and shows that place P_1 has not been filled by tokens until transition T_1 fired. Based on Figure 2 and Equation 4, the coverability tree is obtained as follows.

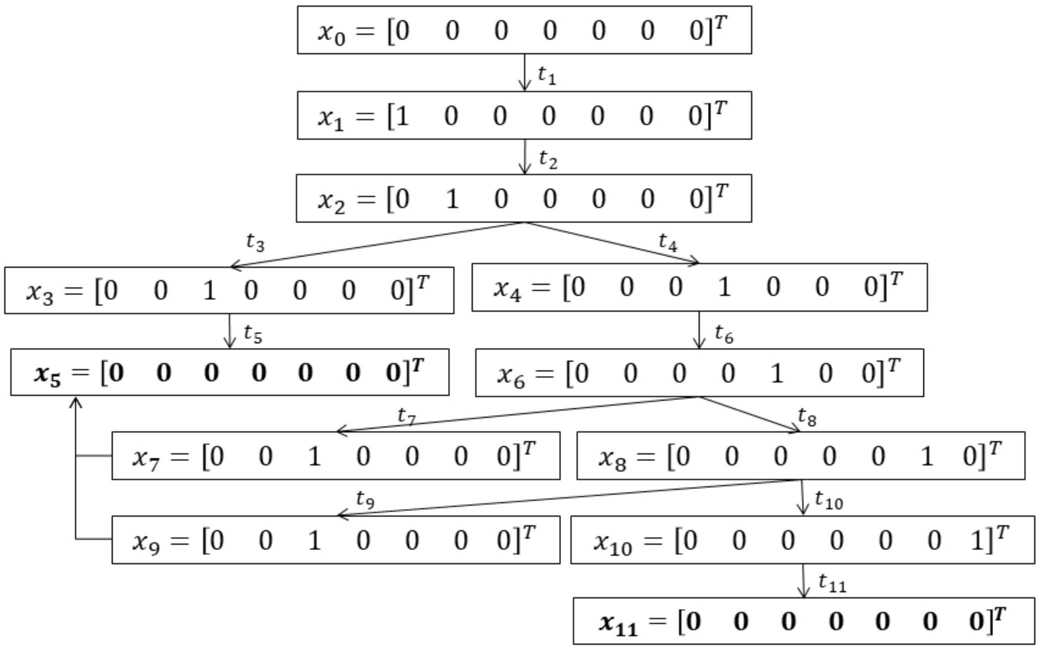

Figure 3. Coverability Tree of the service Petri Net Model COVID-19 patient

Based on Figure 3, it can be seen that the states x_5 and x_{11} appear which makes T_1 re-enable and the process will repeat so on. Furthermore, the max-plus algebra model is built based on the Petri net model and coverability tree that has been obtained.

The Max-Plus Algebra Model for COVID-19 Patient

Based on the Petri net in Figure 2, the max-plus algebra model is obtained as follows with $k = 1,2, \dots$.

$$t_1(k) = v_{t_1,k} \otimes t_1(k-1),$$

$$\begin{aligned} t_2(k) &= v_{t_2,k} \otimes t_1(k) \\ &= v_{t_2,k} \otimes v_{t_1,k} \otimes t_1(k-1), \end{aligned}$$

$$\begin{aligned} t_3(k) &= v_{t_3,k} \otimes t_2(k) \\ &= v_{t_3,k} \otimes v_{t_2,k} \otimes v_{t_1,k} \otimes t_1(k-1), \end{aligned}$$

$$\begin{aligned} t_4(k) &= v_{t_4,k} \otimes t_2(k) \\ &= v_{t_4,k} \otimes v_{t_2,k} \otimes v_{t_1,k} \otimes t_1(k-1), \end{aligned}$$

$$t_6(k) = v_{t_6,k} \otimes t_4(k)$$

$$= v_{t_6,k} \otimes v_{t_4,k} \otimes v_{t_2,k} \otimes v_{t_1,k} \otimes t_1(k-1),$$

$$t_7(k) = v_{t_7,k} \otimes t_6(k)$$

$$= v_{t_7,k} \otimes v_{t_6,k} \otimes v_{t_4,k} \otimes v_{t_2,k} \otimes v_{t_1,k} \otimes t_1(k-1),$$

$$t_8(k) = v_{t_8,k} \otimes t_6(k)$$

$$= v_{t_8,k} \otimes v_{t_6,k} \otimes v_{t_4,k} \otimes v_{t_2,k} \otimes v_{t_1,k} \otimes t_1(k-1),$$

$$t_9(k) = v_{t_9,k} \otimes t_8(k)$$

$$= v_{t_9,k} \otimes v_{t_8,k} \otimes v_{t_6,k} \otimes v_{t_4,k} \otimes v_{t_2,k} \otimes v_{t_1,k} \otimes t_1(k-1),$$

$$t_5(k) = v_{t_5,k} \otimes \{t_3(k) \oplus t_7(k) \oplus t_9(k)\}$$

$$= (v_{t_5,k} \otimes t_3(k)) \oplus (v_{t_5,k} \otimes t_7(k)) \oplus (v_{t_5,k} \otimes t_9(k))$$

$$= (v_{t_5,k} \otimes v_{t_3,k} \otimes v_{t_2,k} \otimes v_{t_1,k} \otimes t_1(k-1)\ ) \oplus (v_{t_5,k} \otimes v_{t_7,k} \otimes v_{t_6,k} \otimes v_{t_4,k} \otimes v_{t_2,k} \otimes v_{t_1,k} \otimes t_1(k-1))\ \oplus (v_{t_5,k} \otimes v_{t_9,k} \otimes v_{t_8,k} \otimes v_{t_6,k} \otimes v_{t_4,k} \otimes v_{t_2,k} \otimes v_{t_1,k} \otimes t_1(k-1)),$$

$$t_{10}(k) = v_{t_{10},k} \otimes t_8(k)$$

$$= v_{t_{10},k} \otimes v_{t_8,k} \otimes v_{t_6,k} \otimes v_{t_4,k} \otimes v_{t_2,k} \otimes v_{t_1,k} \otimes t_1(k-1),$$

$$t_{11}(k) = v_{t_{11},k} \otimes t_{10}(k)$$

$$= v_{t_{11},k} \otimes v_{t_{10},k} \otimes v_{t_8,k} \otimes v_{t_6,k} \otimes v_{t_4,k} \otimes v_{t_2,k} \otimes v_{t_1,k} \otimes t_1(k-1),$$

Based on the Petri net in Figure 2 and the max-plus algebra model that has been obtained above, a particular model can be formed that can be expressed in the following matrix according to the condition of the patient's examination results.

Based on the Petri net in Figure 2 and the max-plus algebra model that has been obtained above, a particular model can be formed that can be expressed in the following matrix according to the condition of the patient's examination results.

i). The Max-Plus Algebra Model in the Covid-19 Positive Referral Patient Service Process

$$\begin{bmatrix} t_1(k) \\ t_2(k) \\ t_3(k) \\ t_5(k) \end{bmatrix} = \begin{bmatrix} v_{t_1,k} & a & a & a \\ b & a & a & a \\ c & a & a & a \\ e & a & a & a \end{bmatrix} \otimes \begin{bmatrix} t_1(k-1) \\ t_2(k-1) \\ t_3(k-1) \\ t_5(k-1) \end{bmatrix},$$

ii). Max-Plus Algebra Model of Referral Patients with Suspected PDP Covid-19 and The Results of the 1st Positive Swab Test for Covid-19

$$\begin{bmatrix} t_1(k) \\ t_2(k) \\ t_4(k) \\ t_6(k) \\ t_7(k) \\ t_5(k) \end{bmatrix} = \begin{bmatrix} v_{t_1,k} & a & a & a & a & a \\ b & a & a & a & a & a \\ d & a & a & a & a & a \\ f & a & a & a & a & a \\ g & a & a & a & a & a \\ e & a & a & a & a & a \end{bmatrix} \otimes \begin{bmatrix} t_1(k-1) \\ t_2(k-1) \\ t_4(k-1) \\ t_6(k-1) \\ t_7(k-1) \\ t_5(k-1) \end{bmatrix},$$

iii). Max-Plus Algebra Model of Referral Patients with Conjecture PDP Covid-19 where The Results of the 1st Swab Examination were Negative and The Results of The 2nd Swab Examination were Positive for Covid-19

$$\begin{bmatrix} t_1(k) \\ t_2(k) \\ t_4(k) \\ t_6(k) \\ t_8(k) \\ t_9(k) \\ t_5(k) \end{bmatrix} = \begin{bmatrix} v_{t_1,k} & a & a & a & a & a & a \\ b & a & a & a & a & a & a \\ d & a & a & a & a & a & a \\ f & a & a & a & a & a & a \\ h & a & a & a & a & a & a \\ i & a & a & a & a & a & a \\ e & a & a & a & a & a & a \end{bmatrix} \otimes \begin{bmatrix} t_1(k-1) \\ t_2(k-1) \\ t_4(k-1) \\ t_6(k-1) \\ t_8(k-1) \\ t_9(k-1) \\ t_5(k-1) \end{bmatrix},$$

iv). The Max-Plus Algebra Model in the Service Process for Referral Patients with Negative Covid-19

$$\begin{bmatrix} t_1(k) \\ t_2(k) \\ t_4(k) \\ t_6(k) \\ t_8(k) \\ t_{10}(k) \\ t_{11}(k) \end{bmatrix} = \begin{bmatrix} v_{t_1,k} & a & a & a & a & a & a \\ b & a & a & a & a & a & a \\ d & a & a & a & a & a & a \\ f & a & a & a & a & a & a \\ h & a & a & a & a & a & a \\ j & a & a & a & a & a & a \\ \dot{k} & a & a & a & a & a & a \end{bmatrix} \otimes \begin{bmatrix} t_1(k-1) \\ t_2(k-1) \\ t_4(k-1) \\ t_6(k-1) \\ t_8(k-1) \\ t_{10}(k-1) \\ t_{11}(k-1) \end{bmatrix},$$

with $k = 1, 2, \ldots,$ and

$a = \varepsilon,$

$b = v_{t_2,k} \otimes v_{t_1,k},$

$c = v_{t_3,k} \otimes v_{t_2,k} \otimes v_{t_1,k}$,

$d = v_{t_4,k} \otimes v_{t_2,k} \otimes v_{t_1,k}$,

$f = v_{t_6,k} \otimes v_{t_4,k} \otimes v_{t_2,k} \otimes v_{t_1,k}$,

$g = v_{t_7,k} \otimes v_{t_6,k} \otimes v_{t_4,k} \otimes v_{t_2,k} \otimes v_{t_1,k}$,

$h = v_{t_8,k} \otimes v_{t_6,k} \otimes v_{t_4,k} \otimes v_{t_2,k} \otimes v_{t_1,k}$,

$i = v_{t_9,k} \otimes v_{t_8,k} \otimes v_{t_6,k} \otimes v_{t_4,k} \otimes v_{t_2,k} \otimes v_{t_1,k}$,

$j = v_{t_{10},k} \otimes v_{t_8,k} \otimes v_{t_6,k} \otimes v_{t_4,k} \otimes v_{t_2,k} \otimes v_{t_1,k}$,

$\dot{k} = v_{t_{11},k} \otimes v_{t_{10},k} \otimes v_{t_8,k} \otimes v_{t_6,k} \otimes v_{t_4,k} \otimes v_{t_2,k} \otimes v_{t_1,k}$,

$e = (v_{t_5,k} \otimes v_{t_3,k} \otimes v_{t_2,k} \otimes v_{t_1,k}) \oplus (v_{t_5,k} \otimes v_{t_7,k} \otimes v_{t_{6,k}} \otimes v_{t_4,k} \otimes v_{t_2,k} \otimes v_{t_1,k}) \oplus (v_{t_5,k} \otimes$
$v_{t_9,k} \otimes v_{t_8,k} \otimes v_{t_6,k} \otimes v_{t_4,k} \otimes v_{t_2,k} \otimes v_{t_1,k})$.

To achieve Lyapunov stability, we need to find a vector Φ that satisfies the condition that a Petri net is considered stable if there exists a strictly positive vector Φ such that $e^T A^T \Phi \leq 0$.. Since e^T is a non-negative vector, it is sufficient to show that $A^T \Phi \leq 0$. Proposition 1 states that the vector Φ given in equation (25) should be strictly positive. However, it has been shown that the system modeled by the timed Petri net is unstable. The system can be stabilized by solving the homogeneous linear equation Ae=0, which yields $\Phi = [0, 0, 0, 0, 0, 0, 0]^T$. By solving this equation, we also obtain a non-zero vector $e = [a, a, b, c, b, c, d, e, f, g, g]$ which allows us to conclude that the system can be stabilized.

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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