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MODELING MIXED GEOGRAPHICALLY WEIGHTED NEGATIVE BINOMIAL REGRESSION ON THE NUMBER OF TUBERCULOSIS CASES IN SOUTH SULAWESI

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Abstract: Tuberculosis is an infectious disease caused by bacteria known as *Mycobacterium Tuberculosis*, which is a problem in various regions, one of which is in the province of South Sulawesi which has experienced tuberculosis problems in recent years. Tuberculosis data in South Sulawesi shows overdispersion. This may be caused by the different geographical location of each region, so it is necessary to know the variables that affect tuberculosis cases. The overdispersion problem in the data can be overcome by using the Negative Binomial model. However, this model is only global while tuberculosis cases have different location characteristics. Therefore, a method is needed that can overcome overdispersion and consider the effects of spatial heterogeneity. Mixed Geographically Weighted Negative Binomial Regression (MGWNBR) is a model used for spatially heterogeneous discrete data that can overcome overdispersion in the data. The results of the study using MGWNBR show that the global variable that has a significant effect on the number of tuberculosis cases in all observed locations is the number of medical personnel, while the local variables that have a significant effect on the number of tuberculosis cases in some observed locations are the number of health facilities, population, and population density.

Keywords: tuberculosis; overdispersion; MGWNBR; spatial heterogeneity.

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1. INTRODUCTION

Regression is a method that serves to predict the effect of two or more variables. Regression was first introduced by Francis Galton [15]. According to Galton, regression analysis is a method to examine the relationship between two or more variables. In regression analysis, there are two types of variables, namely predictor variables and response variables. The application of regression analysis is used to analyze response variable data in the form of continuous, but often obtained response variable data in the form of discrete data. One of the regression models that can be used to explain the relationship between response variables in the form of discrete data and predictor variables in the form of discrete, continuous, categorical or mixed data is the Poisson regression model [1]. Poisson regression has assumptions that must be met, namely that the response variable is discrete data and equidispersion. Equidispersion is the average value equal to the variance value. However, often the data count has a variance that is greater than the mean or commonly called the term overdispersion [2], [16]. Violation of this assumption causes the parameters resulting from Poisson regression to be less accurate. The Negative Binomial model is a model that can overcome overdispersion and is more flexible than the Poisson regression model because the mean and variance assumptions do not have to be the same [2], [9].

One of the developments of regression models for count data that takes into account spatial heterogeneity factors with geographic weighting and can overcome overdispersion in data is Geographically Weighted Negative Binomial Regression. GWNBR is also one of the right solutions to form a regression analysis that is local to each observation location. GWNBR was first introduced [3]. Often not all predictor variables in the GWNBR model are locally influential. Sometimes, some predictor variables are globally influential, while other predictors may retain their local or spatial influence. Therefore, the GWNBR model is developed into a Mixed Geographically Weighted Negative Binomial Regression (MGWNBR) model. The MGWNBR model is a combination of the Negative Binomial regression model with the GWNBR model, so that the MGWNBR model will produce parameter estimators that are partly global and partly local according to data observations [11].

The number of tuberculosis cases is one example of count data and overdispersion often occurs in the data due to the geographical conditions of the region which causes differences in the number of tuberculosis cases between one region and another [12]. Tuberculosis is a contagious lower respiratory tract disease caused by *Mycobacterium Tuberculosis* bacteria [4]. South Sulawesi Province has experienced tuberculosis problems in recent years [13]. Based on data from the South

Sulawesi Provincial Health Office, in 2018 there were 18 thousand cases, in 2019 it rose to almost 19 thousand cases, while in 2020 it decreased to 12 thousand cases, then in 2021 there was an increase in cases of 15 thousand cases. This figure is said to be still quite high, if not handled immediately, it can cause every active patient to infect. Tuberculosis data in South Sulawesi is count data that experiences overdispersion due to the geographical location in each region is different. So to overcome the problem of overdispersion and consider the effects of spatial heterogeneity, the Mixed Geographically Weighted Negative Binomial Regression (MGWNBR) model is used.

2. PRELIMINARIES

1. Multicollinearity

Detection of multicollinearity cases according to [5] can be seen in several ways as follows:

- a. If the Pearson correlation coefficient (r_{ij}) between predictor variables is greater than 0.95 ($r_{ij} > 0.95$) then there is a correlation between these variables.
- b. A VIF value greater than 10 ($VIF > 10$) indicates that there is multicollinearity between the predictor variables or there is a violation of the assumption of non-multicollinearity. The VIF value is expressed as follows:

$$VIF_j = \frac{1}{1-R_j^2} \quad ; j = 1, 2, \dots, p \quad (1)$$

2. Spatial Heterogeneity

Spatial heterogeneity can occur if the same predictor variable gives a different response at other locations in one research area. The existence of spatial heterogeneity will show spatial diversity, resulting in different coefficient and parameter estimates in each research area. One test of spatial heterogeneity can be done using the Breusch-Pagan (BP) statistic. Hypothesis testing and test statistics are as follows:

$$BP = \left(\frac{1}{2}\right) f^T Z (Z^T Z)^{-1} Z^T f \sim X_{(p)}^2 \quad (2)$$

The rejection region is rejected H_0 if value $BP > X_{(p,\alpha)}^2$ or p-value $< \alpha_{(0,05)}$ which means the variance between locations is different.

3. The Spatial Weight Matrix

The spatial weight matrix describes the proximity between regions or the distance between regions from each other [6]. The function used in the formation of the spatial weighting matrix in

this study is the adaptive bi-square kernel weighting. According to [7], another weighting function is to set the weight to zero if it is outside the bandwidth and monotonically decreases to zero if it is within the bandwidth as the distance from the two observation locations increases.

$$w_{ij} = \left\{ \left[-\frac{1}{2} \left(\frac{d_{ij}}{b_i} \right)^2 \right]^2, \text{ jika } d_{ij} < b_i, 0 \text{ other} \right. \quad (3)$$

4. Mixed Geographically Weighted Negative Binomial Regression

MGWNBR is a method developed from Negative Binomial and GWNBR models that are local and global parameters [10], [17]. In the MGWNBR model, the response variable (y) is estimated with predictor variables (x), each of which is the regression coefficient $\beta_k(u_i, v_i)$ depending on geographic location and γ_p is constant, then the MGWNBR model with $Y_i \sim$ Negative Binomial can be expressed as follows:

$$y_i = \exp \left(\sum_{i=0}^{q^*} \beta_k(u_i, v_i) x_{ik} + \sum_{p=q^*+1}^q \gamma_p x_p + \varepsilon_i \right) \quad (4)$$

The Mixed Geographically Weighted Negative Binomial Regression model can be described as follows [8]:

$$\begin{aligned} y_1 &= \exp(\beta_0(u_1, v_1) + \beta_1(u_1, v_1)x_{11} + \beta_{q^*}(u_1, v_1)x_{1q^*} + \gamma_p x_{1p} + \gamma_{p+1}x_{2(p+1)} + \dots + \gamma_q x_q + \varepsilon_1) \\ y_2 &= \exp(\beta_0(u_2, v_2) + \beta_1(u_2, v_2)x_{21} + \beta_{q^*}(u_2, v_2)x_{2q^*} + \gamma_p x_{2p} + \gamma_{p+1}x_{2(p+1)} + \dots + \gamma_q x_{2q} + \varepsilon_2) \\ y_n &= \exp(\beta_0(u_n, v_n) + \beta_1(u_n, v_n)x_{n1} + \beta_{q^*}(u_n, v_n)x_{nq^*} + \gamma_p x_{np} + \gamma_{p+1}x_{2(p+1)} + \dots + \gamma_q x_{nq} + \varepsilon_n) \end{aligned} \quad (5)$$

Testing the significance of the MGWNBR model parameters consists of simultaneous tests and partial tests. Simultaneous significance tests can use the Maximum Likelihood Ratio Test (MLRT).

With the following hypothesis:

$$H_0: \beta_i(u_i, v_i) = \beta_2(u_i, v_i) = \dots = \beta_p(u_i, v_i) = 0$$

$$H_1: \beta_j(u_i, v_i) \neq 0; j = 1, 2, \dots, p$$

Test statistic:

$$D(\hat{\beta}) = -2 \ln \left(\frac{L(\hat{\omega})}{L(\hat{\Omega})} \right) = 2 (\ln L(\hat{\Omega}) - L(\hat{\omega})) \quad (6)$$

The rejection criterion is reject H_0 if value $D(\hat{\beta}) \geq X_{(p, \alpha)}^2$ which means that there is at least one parameter in the MGWNBR model that has a significant effect on the response variable. After simultaneous testing, partial testing is carried out to find out which parameters have a significant effect on the response variable at each location with the following hypothesis :

$$H_0: \beta_j(u_i, v_i) = 0$$

$$H_1: \beta_j(u_i, v_i) \neq 0; j = 1, 2, \dots, p$$

Test statistic:

$$Z_{hit} = \frac{\hat{\beta}_j(u_i, v_i)}{se(\hat{\beta}_j(u_i, v_i))} \quad (7)$$

The rejection region is reject H_0 if the value of $|Z_{score}| \geq Z_{\alpha/2}$ which means that the parameter has a significant effect on the response variable at each location in the MGWNBR model.

3. MAIN RESULTS

1. Characteristics of the Number of Tuberculosis Cases in South Sulawesi Province in 2021-2022

Geographically, South Sulawesi Province is located at $0^{\circ}12' - 8^{\circ}$ South latitude and $116^{\circ}48' - 122^{\circ}36'$ East Longitude, South Sulawesi consists of 24 districts/cities, including 21 districts and 3 cities. Based on data from the Ministry of Health of the Republic of Indonesia in 2021, South Sulawesi is listed as one of the eight provinces that contribute the largest number of tuberculosis cases in Indonesia. The number of tuberculosis cases in this study is the response variable. The number of tuberculosis cases is the response variable Y. The following is a mapping of the number of tuberculosis cases in South Sulawesi.

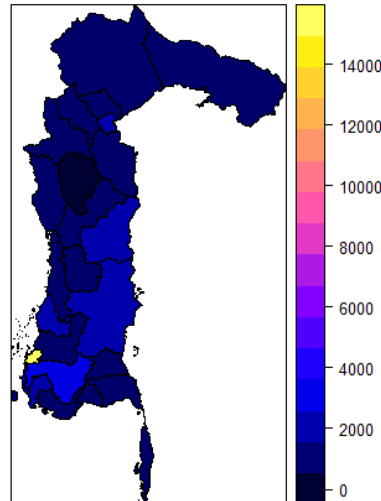


Fig1. Distribution of the Number of TB Cases in South Sulawesi Province in 2021-2022

Based on Figure 1, it can be seen that the brighter the color of the map, the higher the number of tuberculosis cases. Where, the yellow color indicates the highest number of tuberculosis cases in Makassar City with 14,926 cases and the black color indicates the lowest number of tuberculosis cases in Enrekang Regency with 484 cases. The number of tuberculosis cases in South Sulawesi has a large standard deviation of 2843.224 because there is a significant difference between the

number of tuberculosis cases in each district/city in South Sulawesi.

2. Ratio of Health Facilities per District/City in South Sulawesi

Health facilities are places used to organize health efforts, which include hospitals and health centers. The ratio of health facilities is the ratio between the number of tuberculosis cases in each district/city in South Sulawesi and the number of health facilities in each district/city in South Sulawesi. The ratio of health facilities is calculated to determine whether an area needs additional health facilities or not. Number of health facilities is a predictor variable X_1 . The following is a map of the distribution of the ratio of health facilities for each district/city in South Sulawesi.

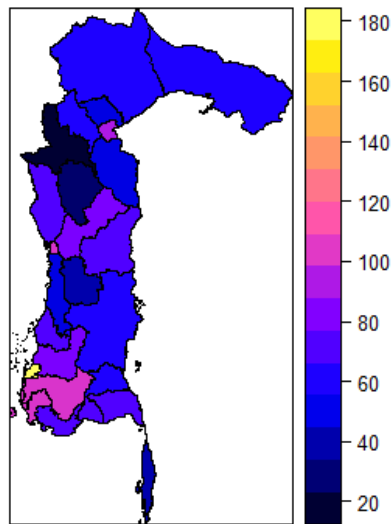


Fig2. Distribution of Health Facilities in South Sulawesi Province

The average ratio of health facilities which includes the number of hospitals and health centers in each district/city in South Sulawesi Province is 70.969. Based on Fig2. it can be seen that the yellow color indicates the area with the highest ratio of health facilities located in Makassar City with a ratio value of 173.558 and the black color indicates the area with the lowest ratio of health facilities located in Tanah Toraja Regency with a ratio value of 22.708.

3. Ratio of Medical Personnel per District/City in South Sulawesi

Adequate health services are the foundation of public health, the presence of medical personnel is needed to support the reduction of the number of tuberculosis cases. The number of medical personnel in this study includes the number of doctors, specialists, nurses in each district/city. The ratio of medical personnel is a comparison between the number of tuberculosis cases in each regency/city in South Sulawesi and the number of medical personnel in each regency/city in South Sulawesi. The number of medical personnel is a predictor variable X_2 . The following map shows the distribution of the ratio of medical personnel for each regency/city in South Sulawesi in Fig3.

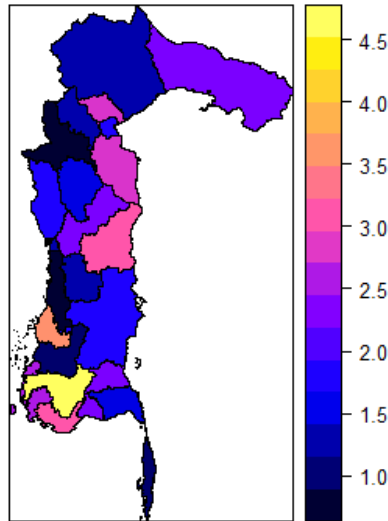


Fig3. Distribution of Medical Personnel in South Sulawesi Province

The average ratio of medical personnel consisting of general practitioners, specialists and nurses in each district/city in South Sulawesi Province is 2.006. Based on Fig3, it can be seen that the yellow color indicates the area with the highest ratio of medical personnel located in Gowa Regency with a ratio value of 4.514 and the black color indicates the area with the lowest ratio of medical personnel located in Tanah Toraja and Barru Districts with a ratio value of 0.9.

4. Population Ratio for each Regency/City in South Sulawesi

Population is the number of people who reside in an area or region and have a permanent livelihood in the area and are legally recorded based on the regulations that apply in the area. Population ratio is the ratio between the number of tuberculosis cases in each regency/city in South Sulawesi and the total population of each regency/city in South Sulawesi. Total population is predictor variable X_3 The following distribution of population ratio for each district/city in South Sulawesi is presented in Fig4.

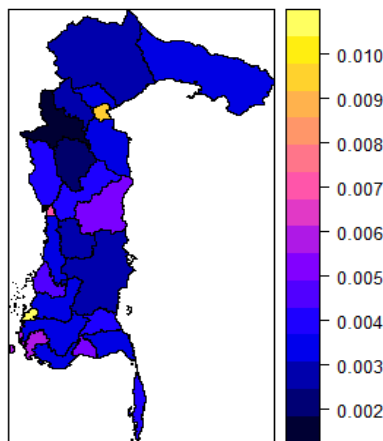


Fig4. Distribution of Population in South Sulawesi Province in 2022

The average population ratio in each regency / city in South Sulawesi Province is 0.004. Based on Fig4. it can be seen that the yellow color indicates the area with the highest population ratio located in Makassar City with a ratio value of 0.0104 and the black color indicates the area with a population ratio located in Tanah Toraja Regency with a ratio value of 0.0018.

5. Population Density

Population density is the ratio between the total population and the area inhabited. High population density results in an imbalance with the area, which will increase the area that is slum, which can trigger health problems, especially tuberculosis, which is a direct infectious disease. Population density is a predictor variable X_4 . The following map of the distribution of population density of each Regency / City in South Sulawesi is presented in Fig5.

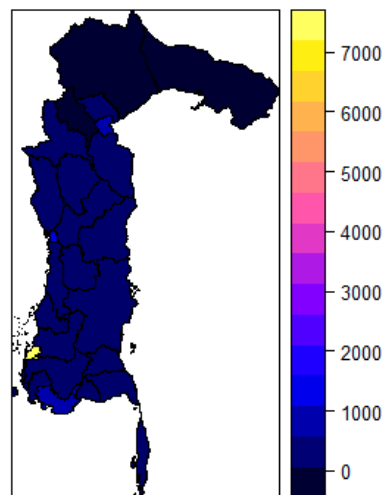


Fig5. Population Density Distribution Map in South Sulawesi Province

Based on Fig5. it can be seen that there is one region that has a value that is too large compared to other regions, the region is Makassar City which is marked in yellow, with a very high population density which is located at 7,188 (soul/km²) and very low population density marked in black are in North Luwu and East Luwu located at 44 (soul/km²).

6. Multicollinearity Detection

Before analyzing Mixed Geographically Weighted Negative Binomial Regression (MGWNBR), multicollinearity testing is carried out on the data used to determine whether the predictor variables do not have a high correlation, because if multicollinearity occurs it can result in the estimated regression parameters obtained will have a large standard error so that it will affect the conclusions obtained. There are several ways to detect the presence of multicollinearity cases, namely by looking at the Pearson correlation coefficient (r_{ij}) and the VIF (Variance Inflation Factor) value.

The following is the correlation coefficient between the predictor variables.

TABLE1. VIF Value of Predictor Variables

Variable	VIF
X ₁	1.70
X ₂	2.68
X ₃	5.21

Based on Table1, shows that the VIF value of each predictor variable has a value of less than 10, which can be concluded that there are no predictor variables that correlate with other predictor variables or there is no multicollinearity, so all of the following predictor variables can be included in the model.

7. Spatial Heterogeneity

Spatial heterogeneity testing was conducted to determine whether there are differences in characteristics between districts/cities after an indication of spatial linkage. This test was conducted with the Breusch Pagan test. The following are the results of the spatial heterogeneity test.

TABLE2. Breusch-Pagan

BP	DF	p-value
0.2641	4	0.0992

The BP value is 0.2641 with a p-value of 0.0992, hence $BP > \chi^2_{0.05;4}$, so the decision is to reject H_0 and it is concluded that with a significance of 5% there is spatial heterogeneity which means there is an indication of diversity between regions and the parameters generated in each region can be different. Spatial heterogeneity conditions can be overcome by using MGWNBR.

8. Mixed Model Geographically Weighted Negative Binomial Regression

Modeling the number of tuberculosis cases in South Sulawesi with the MGWNBR model using the adaptive bisquare kernel function weighting was carried out two tests, namely simultan testing and partial testing. Simultan testing based on the results of calculations on R software that the deviancy value of the MGWNBR model is 45.06704 with $a= 5\%$ obtained $\chi^2_{0.05;4}$ of 9.487. This shows that the deviance value is greater than $\chi^2_{0.05;4}$, So it can be concluded that rejecting H_0 means that there is at least one predictor variable that has a significant effect on the number of tuberculosis cases and simultaneous the predictor variables affect the MGWNBR model. So proceed with partial testing.

Partial testing results in different parameters for each district / city, to find out which variables are significant for each District/City, a comparison of the values is made Z_{score} dengan Z_{table} using $\alpha = 5\%$ obtained $Z_{(0,05/2)} = 1.96$. Based on partial parameter testing, for example, the formation of the MGWNBR model for the observation location in the 10th District / City (u_{10}, v_{10}), namely Makassar City with parameter estimates as in Table 3:

TABLE3. Parameter Estimation of MGWNBR Model in Makassar City

Parameter	Coefficient	Zvalue
θ	1.094937	1.079727
$\hat{\beta}_0$	6.543788	1.112995
$\hat{\beta}_1$	-3.798682	-1.888104
$\hat{\beta}_2$	-1.461783	-6.419152
$\hat{\beta}_3$	2.089954	1.07785
$\hat{\beta}_4$	1.764082	6.539502

Based on the Z_{score} value that can be seen in Table3 above, it shows that the variables that have a significant effect on the MGWNBR model in Makassar City are X_2 and X_4 because the value of the Z_{score} is not significant $|Z| > Z_{0,05/2} = 1.96$. The MGWNBR model formed is the following equation:

$$y_{10} = \exp(6.543788 - 3.798682x_1 - 1.46178x_2 + 2.089954x_3 + 1.764082x_4)$$

Therefore, the MGWNBR model formed in Makassar City can be known that an increase in the number of health facilities by 1% will reduce the number of tuberculosis cases in Makassar City by $\exp(-3.798682) = 0.02240$ times, assuming other variables are constant. An increase in the number of medical personnel by 1% will reduce the number of TB cases in Makassar City by $\exp(-1.461783) = 0.23182$ times assuming other variables are constant. A 1% increase in population will increase the number of TB cases in Makassar City by $\exp(2.089954) = 8.08454$ times assuming other variables are constant. If population density increases by 1%, it will increase the number of TB cases in Makassar City by $\exp(1.764082) = 5.83621$ times, assuming other variables are constant. Based on this, the following is a table of district / city groupings in South Sulawesi based on significant variables in each region.

TABLE4. District/City grouping based on significant variables with MGWNBR model

Kelompok	Variabel	Kabupaten/Kota
1	X ₁ , X ₂ , X ₃ , X ₄	Barru, Selayar
2	X ₂ , X ₃ , X ₄	Bone, Pare-Pare, Pinrang, Sidrap, Wajo, Bantareng
3	X ₁ , X ₂ , X ₃	Luwu Timur, Luwu Utara, Luwu, Palopo, Toraja Utara, Toraja, Soppeng
4	X ₁ , X ₂ , X ₄	Bulukumba, Enrekang, Maros, Sinjai, Takalar
5	X ₁ , X ₂	Makassar, Pangkep, Gowa, Jeneponto

Based on Table4, from 24 regencies/cities in South Sulawesi, 5 regional groups or model combinations are formed based on variables that have a significant effect on tuberculosis cases, the distribution map of regency/city groupings in South Sulawesi based on significant variables as follows:

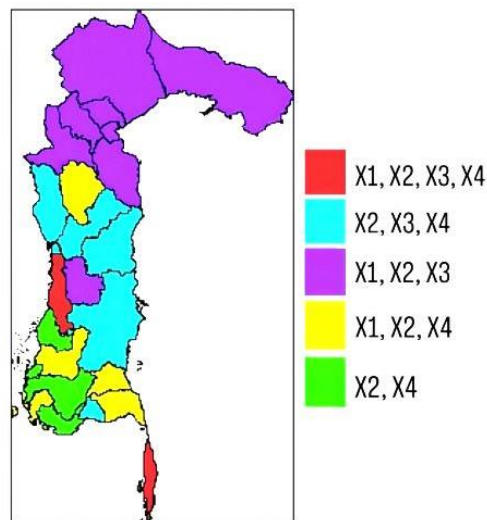


Fig6. Distribution map of Regency/City groupings based on significant variables. Based on the MGWNBR

Fig6. shows the grouping of regencies/cities in South Sulawesi based on significant variables. Where, the red color indicates a group where all variables significantly affect the number of tuberculosis cases in the region, namely the number of health facilities, the number of medical personnel, population and population density. Blue color is a group where the significant variables are X₂, X₃, and X₄ consisting of the number of medical personnel, population and population

density, which means that there are three variables that have a significant effect on the number of tuberculosis cases in the region. Purple color is a group of significant variables X_1 , X_2 , and X_3 consisting of the number of health facilities, number of medical personnel and population so that these three variables have a significant effect on the number of tuberculosis cases in the region. Yellow color is a group of significant variables X_1 , X_2 , X_4 consisting of number of health facilities, number of medical personnel and population density, which means that these three variables have a significant effect on the number of tuberculosis cases in the region. Green color is a group of significant variables X_1 and X_2 consisting of the number of health facilities and the number of medical personnel, which means that both variables have a significant effect on the number of tuberculosis cases in the region.

4. CONCLUSION

The number of tuberculosis cases in South Sulawesi Province from January 2021 to December 2022 caused differences in the number of tuberculosis cases due to regional conditions. There are also several factors that trigger the development of tuberculosis cases, namely population, number of health workers, and number of health facilities. Based on the Mixed Geographically Weighted Negative Binomial model, five groups are obtained, where the globally influential variable is the number of medical personnel, while the locally influential variables are the number of health facilities, the number of medical personnel and population density.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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