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COMPARISON OF ITERATIVELY REGULARIZED GAUSS-NEWTON METHOD WITH ADAM OPTIMIZATION FOR IMAGE RECONSTRUCTION IN ELECTRICAL IMPEDANCE TOMOGRAPHY

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Abstract. Electrical impedance tomography (EIT) is a non-invasive imaging technique that visualizes the distribution of electrical conductivity within biological tissues. In this paper, we explore the potential of the Adam optimization algorithm as an innovative approach to reconstructing thoracic conductivity from EIT data. Originally developed for machine learning and signal processing tasks, the Adam method offers significant promise for enhancing EIT reconstruction accuracy and spatial resolution. Through comprehensive numerical simulations conducted on thorax models, we compare the Adam method and the traditional iterative Gauss-Newton method. The results demonstrate that the Adam method provides superior performance, improving spatial resolution and accuracy in resolving thoracic conductivity. The method is still under investigation, and further research and validation are needed to fully establish its effectiveness and reliability. Although preliminary findings are promising, additional research and clinical trials are required to identify the degree of its benefits and limits in the context of thoracic imaging. This study contributes to the growing body of research aimed at exploring advanced optimization methods to optimize EIT applications in medical imaging. This will result in better diagnostic capabilities and medical decision making in the field of thoracic health.

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1. INTRODUCTION

Recently, Electrical Impedance Tomography (EIT) has attracted significant attention as a promising non- invasive medical imaging technique. Its unique capability to visualize and monitor electrical conductivity variations within the body's tissues offers valuable insights into physiological processes and pathological conditions. EIT works by injecting small electrical currents through electrodes placed on the body's surface and measuring the resulting voltages at other electrode positions. By analyzing these voltage measurements, EIT can create real-time images of the internal conductivity distribution, providing clinicians with dynamic information without the need for ionizing radiation[1,14,15,16]. The non-invasive and radiation-free nature of EIT makes it particularly attractive for continuous and repeated imaging of patients, especially those who may be sensitive to radiation exposure, such as infants and pregnant women. Furthermore, EIT is relatively cost-effective compared to more complex and expensive imaging modalities like computed tomography (CT) and magnetic resonance imaging (MRI)[2, 3]. Despite these advantages, EIT faces significant challenges, primarily due to the ill-posed and nonlinear nature of the inverse problem it seeks to solve[4, 5]. The process of reconstructing internal conductivity from external voltage measurements is inherently underdetermined, leading to limited spatial resolution and accuracy in the conductivity images. Noise and measurement errors further exacerbate these issues, hampering the widespread clinical application of EIT[6, 7]. To address these challenges, researchers have turned to optimization techniques to enhance the accuracy and quality of EIT conductivity reconstructions[8, 9, 10]. The Adam method, renowned in the fields of machine learning and signal processing, has shown promising results in optimizing complex, non- linear problems. Its adaptability in adjusting learning rates and momentum parameters makes it well-suited to handle the ill-conditioned inverse problems encountered in EIT[11, 12]. This article aims to investigate the effectiveness of the Adam method in reconstructing thoracic conductivity from EIT data. To achieve this, we will conduct numerical simulations using sophisticated thorax models to evaluate and compare the performance of the Adam method against traditional EIT reconstruction methods, such as the Gauss-Newton method[13]. The insights gained from these simulations will shed light on the potential of the Adam method to enhance spatial resolution and accuracy in thoracic electrical imaging, thereby contributing to the advancement of EIT as a valuable medical imaging tool. The primary objective of this study is to offer an extensive comprehension of the applicability of the Adam method within the domain of Electrical Impedance Tomography (EIT) and its potential to enhance the precision and efficiency of EIT imaging. Our investigation is structured as follows: In Section 2, we present a comprehensive overview of the mathematical model of EIT. Moving forward, Section 3 delves into an in-depth discussion concerning the inverse problem and outlines the methodology utilized for the reconstruction of the conductivity distribution. In Section 4, we present the results of numerical simulations conducted to substantiate our assertions. The ensuing analysis and discourse of our findings are encapsulated within Section 5. This study undertakes a comparative examination between the iterative Gauss-Newton method and the Adam optimization algorithm, aiming to underscore the relative advantages and implications of both techniques in the context of EIT.

2. The Theory of EIT

The EIT problem configuration involves a designated region known as Ω, which must be analyzed with tomography. To facilitate this process, a set of Nel electrodes is strategically positioned along the boundary $\partial\Omega$. These electrodes serve a dual role: they apply an excitation current j while measuring the boundary voltage (where n = 1, 2, ..., Nel). The boundary of interest is indicated by a dashed line, outlining the extent of the domain under investigation. This domain holds significance as it relates to the reconstruction of the electrical impedance σ of interest [22]. In a specific excitation measurement mode, the EIT hardware system samples the body's boundary voltages, which are then processed in the EIT software system to extract relevant impedance information. To better comprehend the principle behind this diagram, forward and inverse problems are introduced in the subsequent section. It is critical to note that electrical impedance can also be expressed using complex admittance [17], where:

$$\gamma = \sigma + i\omega\varepsilon \tag{1}$$

Here, ω represents the angular frequency, σ denotes the electrical conductivity, and ε represents the electric permittivity. EIT combines electrical conductivity tomography (ECT) and electrical permittivity tomography (EPT). However, in certain applications of EIT (e.g., lung imaging), the real part or the static state ($\omega = 0$) of the admittance carries more crucial information due to additional pathological insights. To simplify the problem, reconstruction typically focuses on only the real part. Hence, in many literatures, electrical impedance is simplified to electrical conductivity. This paper also adopts this simplification to facilitate the problem description.

3. The Forward Problem

The forward problem in electrical impedance tomography (EIT) involves calculating the boundary voltage U at the measurement electrodes when the spatial distribution of conductivity σ of the objects and boundary conditions are known. In other words, given the electrical properties of the medium being studied and the boundary conditions, we aim to determine the resulting voltage measurements at the electrode positions. This is a crucial step in EIT as it forms the basis for comparing the measured data with the predicted data obtained from mathematical models. Solving the forward problem allows us to simulate the expected electrical responses of the system under different conductivity distributions and conditions, providing valuable insights for image reconstruction and interpretation in EIT applications. The problem at hand involves finding a unique solution to the Laplace equation under the Newman and Dirichlet boundary condition. Specifically, based on Maxwell's equation, Ohm's law, and charge conservation, we obtain the Laplace equation as follows:

$$\nabla \cdot [\sigma(x) \nabla U(x)] = 0, \quad x \in \Omega \tag{2}$$

Where U denotes the electric potential, and Ω represents the domain of interest. The boundary condition on the electrodes is given by:

$$\sigma(x) \frac{\partial U(x)}{\partial n} = j, \quad x \in \partial\Omega \tag{3}$$

Where j is the current density at the electrodes, and $\partial\Omega$ denotes the boundary of the body Ω Additionally, the current conservation equation $\nabla \cdot \mathbf{j} = 0$ needs to be satisfied according to

Gauss's theorem and a zero potential condition must be defined.Due to the irregular shape of the boundary $\partial\Omega$, obtaining an analytical solution to the above differential equation is challenging [22, 23]. In fact, the relationship among excitation, system reaction, and measurement can be expressed simply as follows:

$$U = F(\sigma(x))|j \tag{4}$$

Where σ(x) represents the conductivity distribution, U represents the theoretical boundary voltage; and F is a nonlinear function mapping from the conductivity distribution space to the measurement space. There are four main types of EIT solvers based on electrode modeling: continuum model, gap model, shunt model, and complete model [24, 25]. Regardless of the specific model used, the relationship between the boundary voltage U and the conductivity distribution σ is always represented by a nonlinear function F, which is challenging to express analytically. Consequently, some methods employ Taylor's formula for linearization approximation , resulting in the following expression:

$$\begin{aligned} U &= U_0 + \Delta U \approx F(\sigma_0)|j| + \frac{\partial \sigma}{\partial j} \cdot \Delta\sigma \\ &= F(\sigma_0)|j + J(\sigma(x) - \sigma_0) \end{aligned} \tag{5}$$

Here, σ_0 represents the background materials or inclusions, and J denotes the sensitivity matrix, often called the Jacobian matrix.The sensitivity matrix maps the conductivity distribution to the measurement distribution. When the electrode positions and the current injection/measurement protocol are determined, the sensitivity matrix J can be computed using the finite element method (FEM). This allows for the accurate reconstruction of the electrical impedance distribution within the object of interest [19, 20, 21].

4. The Inverse Problem

The inverse problem of Electrical Impedance Tomography (EIT) involves determining the internal distribution of electrical conductivity σ(x) within an object based on measurements of boundary voltages Un and known boundary conditions. Based on equation (4), we can formulate the inverse problem as follows:

$$\sigma(x) = F^{-1}(U_n)|j \tag{6}$$

Where U_n represents the measurement boundary voltage, possibly with noise distinct from the forward problem solver U. This equation (6) introduces two significant challenges. Firstly, the mapping F is nonlinear, making it challenging to deduce the inverse mapping F^{-1} analytically. Secondly, the theoretical boundary voltage U, which functions as the forward problem solver, adheres to an elliptic equation with Cauchy data. This condition renders the problem ill-posed, implying that even a minor perturbation (noise) in the measurement data Un can significantly impact the solution σ(x). Recent years, there has been significant interest in developing advanced algorithms to solve the inverse problem in Electrical Impedance Tomography (EIT). As a result, the field has seen the emergence of a variety of advanced techniques [26, 27].

4.1. The classic method: Gauss-Newton Approach.

Electrical Impedance Tomography (EIT) is a challenging inverse problem because it is both nonlinear and ill- posed.To get an approximate solution, this problem is often handled utilizing minimization techniques. The key concept is to minimize an objective function that measures the difference between the measured and predicted voltages. The basic idea is to linearize the relationship between the conductivity distribution and the boundary voltage measurements using the Jacobian matrix. In each iteration, an update to the conductivity distribution is computed based on the linearized model, bringing the predicted voltages closer to the measured ones. This iterative process continues until a convergence criterion is met. The Gauss-Newton method is widely employed in EIT due to its effectiveness in handling nonlinearities and its ability to provide relatively rapid convergence to a solution. However, it's important to note that the method's performance can be sensitive to the initial guess of the conductivity distribution and the presence of noise in the measurements [8, 28]. The primary objective is to determine the optimal solution σ that minimizes the cost function ϕ:

$$\phi(\sigma) = \frac{1}{2}\|F(\sigma(x)) - U_n\|^2 \tag{7}$$

The problem of EIT can be formulated as an optimization problem .

$$\sigma^* = \operatorname{argmin}(\sigma) \tag{8}$$

The Tikhonov regularization technique involves a regularization term added to the cost function. The purpose of this modification is to improve the stability of the optimization solution.

$$\phi(\sigma) = \frac{1}{2}\|F(\sigma(x)) - U_n\|^2 + \lambda R(\sigma) \tag{9}$$

Input: ϕ: Objective function of a least square problem

Input: σ_0: Initial guess

Input: $J(\sigma)$: Jacobian matrix with respect to parameter σ

Input: U_n: Measured voltages

Input: U: Simulated voltages using σ_0

$k \leftarrow 0$ (Initialize iteration);

while *not converged* **do**

  $k \leftarrow k+1$ (Calculate conductivity update);

  $\sigma_{k+1} \leftarrow \sigma_k + (J^T J + \lambda L^T L)^{-1} J^T (U_n - U(\sigma_k))$;

end

Output: return σ_k (Final conductivity estimate)

Algorithm 1: Gauss-Newton Algorithm

The presented Gauss-Newton algorithm aims to solve a least-squares problem, specifically within the context of Electrical Impedance Tomography (EIT). In this algorithm, the initial conductivity distribution σ_0 is iteratively updated to approach the optimal solution. During each iteration, the Jacobian matrix J(σ) is employed to model the relationship between conductivity variations and differences between simulated measurements U and actual measurements U_n. Tikhonov regularization, parameterized by λ , is applied to stabilize the estimation. The iterative process involves solving a system of equations to compute the conductivity update $\Delta\sigma$ based on differences between simulated and actual measurements. This update is then added to the current conductivity σ_k . The algorithm continues these steps until a convergence criterion is met, indicating that the conductivity estimation has converged to a stable solution. Ultimately, the algorithm provides the final estimation of the conductivity distribution σ_k, which is the optimal approximation based on electrical measurements and successive iterations. This Gauss-Newton algorithm plays a crucial role in addressing reconstruction challenges in Electrical Impedance Tomography, enabling a realistic and accurate estimation of material conductivity from observed electrical data.

4.2. The proposed method: Adam optimizer.

The Adaptive Moment Estimation (Adam) optimizer is a widely used optimization algorithm in the field of machine learning and deep neural networks. Adam combines the advantages of both the Adagrad and RMSProp optimizers to dynamically adjust learning rates for each parameter during training. It maintains a running average of both the gradient and the squared gradient, adapting the learning rate based on the historical information. This approach allows Adam to converge quickly and efficiently, making it well-suited for tasks with large datasets and complex models. The algorithm's adaptive nature makes it particularly effective for handling sparse gradients and varying magnitudes of parameters. By automatically adjusting learning rates, Adam optimizes the training process and helps in achieving faster convergence while reducing the need for manual tuning of hyperparameters.

This algorithm involves hyperparameters such as decay rates for the exponential moving averages of gradients (β_1 and β_2), a learning rate α and a small constant ε for numerical stability. The main objective is to iteratively refine the model parameters σ until the loss function ϕ reaches convergence. During each iteration, the algorithm computes the gradient of the loss function with respect to the current parameters σ_k. The exponential moving averages of the gradient M_k and squared gradient V_k are updated using decay rates (β_1 and β_2). Additionally, bias correction is applied to the moving averages using factors $1-\beta_1$ and $1-\beta_2$, yielding V_k and M_k. The model parameters σ_k are then updated using the moving averages and the learning rate α) while a small constant ε is introduced to ensure numerical stability. To ensure the effective optimization of the objective function, the selection of an appropriate learning rate is crucial. In a preliminary investigation, we evaluated the performance of different optimizers by varying the learning rate within the range [0; 0.5]. Our findings indicated that selecting a learning rate

Input: α: Stepsize

Input: $b_1, b_2 \in [0,1)$: Exponential decay rates for moment estimates

Input: $\phi(\sigma)$: Objective function with parameter σ

Input: σ_0: Initial guess

$m_0 \leftarrow 0$ (Initialize 1st moment vector);

$v_0 \leftarrow 0$ (Initialize 2nd moment vector);

$k \leftarrow 0$ (Initialize step);

while ϕ *not converged* **do**

$k \leftarrow k+1$ (Increment step);

$g_k \leftarrow \nabla\phi_\sigma(\sigma_{k-1})$ (Get gradients);

$m_k \leftarrow b_1 \cdot m_{k-1} + (1-b_1)\cdot g_k$ (Update biased first moment estimate);

$v_k \leftarrow b_2 \cdot v_{k-1} + (1-b_2)\cdot g_k^2$ (Update biased second raw moment estimate);

$\hat{m} \leftarrow \frac{m_k}{1-b_1^k}$ (Compute bias-corrected first moment estimate);

$\hat{v} \leftarrow \frac{v_k}{1-b_2^k}$ (Compute bias-corrected second raw moment estimate);

$\sigma_k \leftarrow \sigma_{k-1} - \alpha \cdot \frac{\hat{m}}{\sqrt{\hat{v}}+\varepsilon}$ (Update parameters);

end

Output: return σ_k (Resulting parameters)

Algorithm 2: ADAM Optimizer

beyond[0.001,0.1] leads to slow convergence or non-convergence during minimization. In our simulation, we uniformly sampled learning rates from the range [0.001, 0.1], adhering to the recommended hyperparameter values for momentum (β_1 = 0.9) and (β_2 = 0.999) as specified in the existing literature. This description captures the essential steps and considerations when utilizing the Adam optimization algorithm for efficient gradient descent. [11, 12]

5. Numerical Experiments

In our simulation study, we aimed to reconstruct the internal conductivity distribution of a target object using both the Gauss-Newton method and the Adam optimization algorithm.

This allowed us to assess the strengths and limitations of each approach in the context of solving the EIT inverse problem. During this study, we developed a sophisticated thoracic model for electrical impedance tomography to accurately characterize the distribution of electrical properties within the human thorax. The model incorporates crucial elements of anatomical structure, including the lungs, heart, and background domain, aiming to faith- fully represent physiological reality. The utilization of such a complex model was motivated by the need to enhance precision and spatial resolution in images derived from electrical impedance tomography techniques. We carefully considered the specifics of the thoracic region, incorporating details of major tissues like the lungs and heart. To achieve this, we employed an experimental configuration with 16 electrodes strategically placed around the thoracic surface. In order to optimize current and voltage measurements while covering a broad area of the thoracic region, this electrode arrangement was chosen. Starting with the Gauss-Newton method, we observed that its iterative nature led to a gradual refinement of the estimated conductivity distribution. As the iterations progressed, the differences between observed and predicted boundary voltages diminished, indicating an improving fit between the model and the actual data. This iterative refinement approach provided valuable insights into the spatial distribution of conductivity variations within the object, contributing to enhanced imaging capabilities. To generate simulated data, we used the Pyeit Python library, which is an open-source software for solving the EIT forward and inverse problem[29]. The mesh of the thorax is created and customized with anomalies to simulate variations in electrical conductivity. The forward simulation is performed to generate simulated data for subsequent reconstruction.

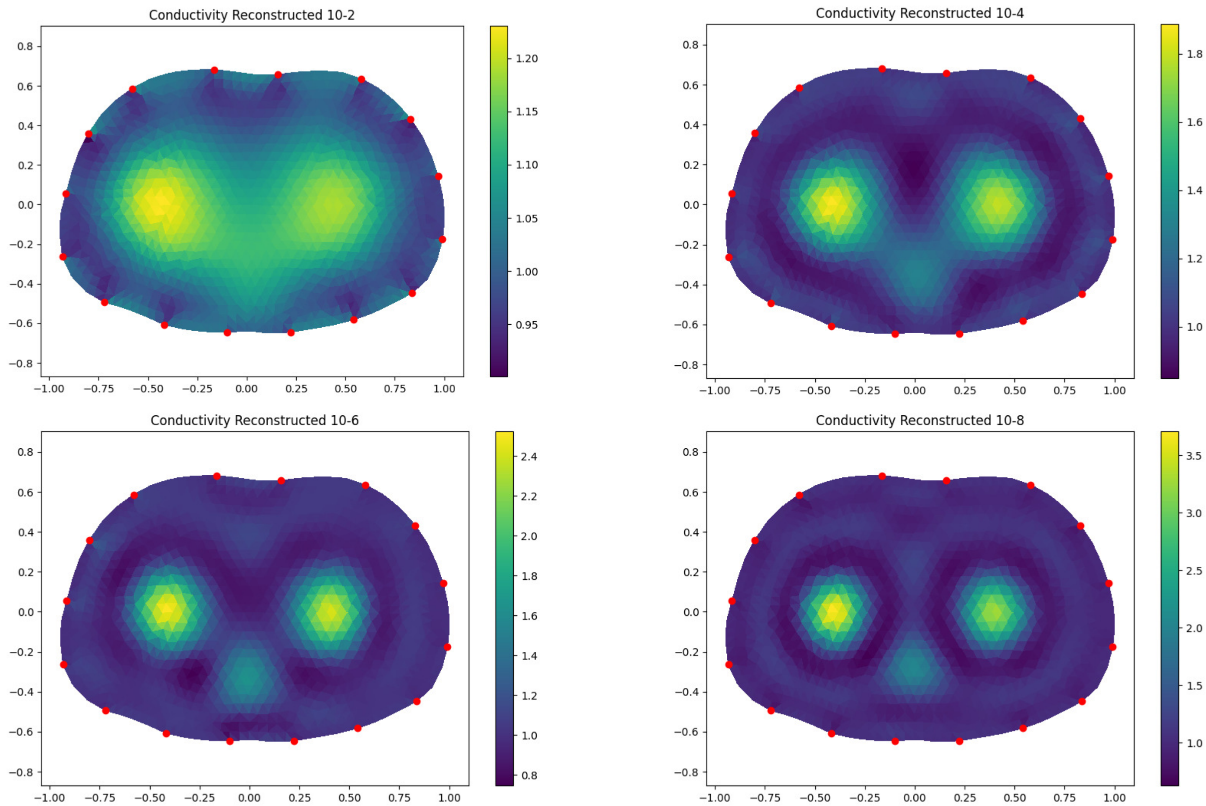

FIGURE 1. The reconstruction of the conductivity distribution within the thoracic model using the Gauss-Newton method

The choice of the regularization parameter λ in the Gauss-Newton method is a pivotal step that can significantly influence the performance and quality of results obtained. In our study, we explored numerous parameters to find the optimal value for λ. Our observations demonstrate that smaller values of λ yielded superior out- comes in our simulations. Specifically, when setting λ to 10^{-6} and 10^{-8}, the reconstructed conductivity images exhibited a marked reduction in artifacts and revealed enhanced resolution of anomalies. This trend suggests that employing a diminished value for the regularization parameter promotes a more effective reconstruction of electrical conductivity anomalies within our experimental setup. The selection of optimization techniques plays a pivotal role in improving the efficacy of electrical impedance tomography (EIT). In this context, adopting the Adam optimization method emerges as a strategic choice with compelling advantages. Adam, an adaptive learning-rate optimization algorithm, is able to adjust learning rates dynamically for each parameter. This adaptability proves particularly beneficial for EIT, given the inherently complex and non-linear nature of the inverse problem. This allows for faster convergence and accommodates varia- tions in parameter scales. Furthermore, its incorporation of both first-moment (mean) and second-moment (uncentered variance) gradient information enhances the algorithm's robustness and stability. As a result, the Adam optimization method is capable of addressing the unique intricacies of EIT, resulting in improved convergence rates and more accurate conductivity reconstructions. Turning our attention to the Adam optimization algorithm, we found that its adaptive learning rate and momentum mechanisms significantly contributed to faster convergence and heightened stability, presenting a notable departure from the conventional Gauss-Newton method. This distinctive efficiency was particularly pronounced when confronted with challenges such as noisy or incomplete boundary voltage measurements. The adaptability inherent in the Adam algorithm, enabling it to dynamically adjust the learning rate for each parameter, facilitated swifter exploration of the parameter space. This attribute proves particularly advantageous as it holds the potential to aid the optimization process in escaping local minima, consequently enabling the algorithm to converge towards more precise solutions. It is important, however, to acknowledge that the performance of the Adam algorithm can be influenced by its hyperparameters, such as the learning rate and momentum coefficients. These hyperparameters necessitate meticulous tuning to harness the full potential of the algorithm, and suboptimal selections might lead to subpar convergence or instability. As a corollary, while the Adam optimization method exhibits remarkable potential for accelerating convergence and enhancing stability in electrical impedance tomography, its effective imple- mentation demands a nuanced understanding of its hyperparameters and their interplay to achieve the desired results. It is also crucial to select the learning rate correctly when solving the minimization problem to obtain accurate and reliable results. Simulations indicate that a learning rate of 0.01 is adequate for good convergence and stable reconstruction of the electrical conductivity distribution. Inversely, larger values of the learning rate, such as 0.1, caused oscillations and instability in the reconstructed images. For this reason, it is important to carefully choose the learning rate and other optimization parameters.

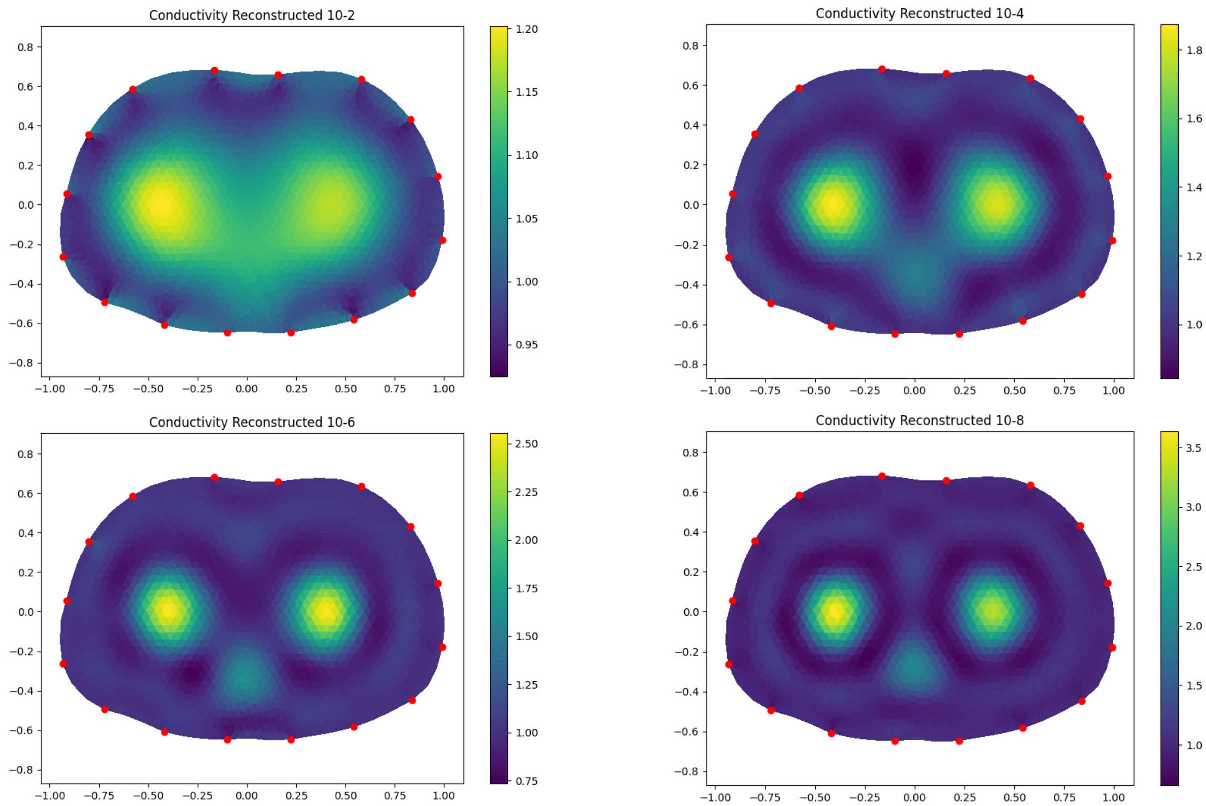

FIGURE 2. The reconstruction of the conductivity distribution within the thoracic model using the Adam optimization method

In conducting a comprehensive comparative analysis between the Gauss-Newton and Adam optimization methods, several significant insights come to light. Primarily, it is observed that the Gauss-Newton method often necessitates meticulous parameter adjustments and a finely calibrated initial estimate to achieve accurate reconstructions. This reliance on precise parameterization arises due to the method's sensitivity to the initial state and the intricate balance required for parameter tuning. While the Gauss-Newton method exhibits potential in furnishing high-fidelity reconstructions under optimal conditions, its susceptibility to variations in input conditions can pose challenges in accommodating deviations from the optimal settings. On a divergent trajectory, the Adam algorithm presents a distinct set of attributes that distinguish it from its Gauss-Newton counterpart. Most notably, the Adam algorithm demonstrates a notable degree of resilience to- wards variations in parameter configurations and initial estimations. This intrinsic robustness implies that Adam can potentially yield consistent and reliable out- comes even when parameter calibration is less meticulous. This adaptive nature of the algorithm, underpinned by its utilization of an adaptive learning rate and momentum- based optimization strategy, enables it to traverse parameter spaces more flexibly. It is imperative to acknowledge, however, that the Adam algorithm tends to require a larger number of iterations compared to the Gauss-Newton method. This requirement for an increased number of iterations is a trade-off for its robustness and adaptability. While the Gauss- Newton method may deliver quicker convergence under well-tuned conditions, the Adam algorithm's capacity to provide reasonably accurate reconstructions even in the face of parameter uncertainties and suboptimal initial estimates positions it as a favorable choice for scenarios marked by complex datasets or limited prior information. In summary, the comparative examination between the Gauss-Newton and Adam methods reveals a balancing act: the Gauss-Newton method demands precise tuning for optimal outcomes, while the Adam algorithm show- cases greater flexibility in the presence of parameter un- certainties. The choice between the two methods should factor in the intricacies of the problem, the availability of preliminary information, and the computational resources, alongside an understanding of the trade-offs associated with the number of iterations.

6. Conclusions and Future Work

In conclusion, our comparative exploration of the Gauss- Newton and Adam optimization methods provides valuable insights into their respective strengths and weaknesses in the realm of electrical impedance tomography. The careful parameter tuning demanded by the Gauss-Newton method underscores its potential for high- precision reconstructions under well-optimized conditions, albeit with sensitivity to variations in initial estimates. On the other hand, the adaptive nature and robustness of the Adam algorithm hold promise, especially in scenarios characterized by uncertain data or less refined initial information. Looking ahead, the field of electrical impedance tomography presents intriguing opportunities for further advancements. One avenue of research lies in addressing the challenge of random conductivity distributions. Exploring how optimization methods can cope with such stochastic variations could yield valuable insights into enhancing the stability and accuracy of reconstructions, especially in real-world, noisy environments. Furthermore, the intersection of electrical impedance tomography with neural networks and machine learning algorithms, including variants like Nadam, Adamax, and AMSGrad, holds substantial potential. Integrating these modern optimization techniques with deep learning frameworks could open up new horizons for accelerating the reconstruction process, potentially mitigating the dependency on meticulous parameter calibration. This could further expand the application scope of electrical impedance tomography to scenarios with complex geome- tries and substantial data sets. As we chart the course for future endeavors, it is evident that the optimization landscape in electrical impedance tomography is rich with possibilities. By exploring the interplay between advanced optimization algorithms, machine learning techniques, and the challenges posed by random conductivity distributions, we can anticipate a progressive evolution in the accuracy, efficiency, and adaptability of electrical impedance tomography, ultimately contributing to advancements in medical imaging, industrial monitoring, and beyond.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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