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THE MATHEMATICAL NIPAH VIRUS MODEL WITH ATANGANA-BALEANU DERIVATIVE

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Abstract. The applicability of the Atangana-Baleanu derivative in modeling and assessing the dynamics of the Nipah virus is investigated in this paper. The Atangana-Baleanu derivative, a fractional derivative operator, is used in the mathematical model of the Nipah virus to add memory effects and non-local behaviour. To do this, we first use fixed point theory to establish the existence and uniqueness of the solutions for the fractional order model. Using various fractional order values, we got a number of numerical simulations emphasizing the significance of the aforementioned derivative. The findings of solving the Nipah virus (NiV) model using the Atangana-Baleanu derivative provide a better understanding of the dynamics and behaviors of the studied systems.

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1. INTRODUCTION

The Nipah virus is a zoonotic pathogen that is a member of the Henipavirus genus and family Paramyxoviridae. It was first identified during an outbreak of encephalitis (inflammation of the brain) in Malaysia and Singapore in 1998-1999. The name ”Nipah” is derived from a village in Malaysia where the outbreak occurred ([31],[27]).

Nipah virus is primarily transmitted to people via direct contact with infected pigs, most notably fruit bats (specifically, some species of flying foxes) or their infected bodily fluids. Transmission from person to person has also been observed, typically through intimate contact with infected people' respiratory fluids or secretions ([27],[10], [20],[1],[23]).

Nipah virus disease can cause moderate to serious symptoms. They commonly include fever, headache, muscle soreness, disorientation, and respiratory problems ([16],[8]). In severe cases, it can progress to encephalitis, characterized by seizures, altered mental status, and coma. Nipah virus disease has a significant fatality rate, ranging from 40% to 75%.

Nipah epidemics have largely occurred in South and Southeast Asia, most notably in Bangladesh and India [10]. Such outbreaks are frequently caused by a mix of transmission from person to person and contact with diseased pigs or their products, such as tainted fruits or raw date sap from palm trees ([23], [32]. [23],[12]).

Pteropus fruit bats are thought to be the Nipah virus's natural reservoir hosts. These bats do not show symptoms of sickness, but the virus is often found in their urine, spit, and stool. In certain outbreaks, intermediate hosts such as pigs have been implicated in multiplying and spreading the virus to people ([15],[15], [20]). Because of its ability to cause severe sickness and epidemics, the Nipah virus remains an important threat to public safety. Ongoing research and surveillance efforts are focused on understanding the virus, its transmission dynamics, and developing effective preventive and therapeutic strategies.

The Atangana-Baleanu derivative is a fractional derivative operator that extends the concept of differentiation to include fractional orders. It was introduced by Dumitru Baleanu and Jean Roger Atangana in 2016 as a generalization of the classical derivative. The Atangana-Baleanu derivative has found applications in various fields, including physics, engineering, and biology. It is particularly useful for modeling phenomena such as viscoelasticity, anomalous diffusion, fractal behavior, and population dynamics, where memory effects and non-local behaviors play a crucial role. In fractional differential equations, researchers use the Atangana-Baleanu derivative to accurately characterize and evaluate systems having memory effects. It is a mathematical instrument for investigating the stability, its current state, uniqueness, and attributes of answers in these systems. The use of the Atangana-Baleanu derivative contributes to a more comprehensive understanding of complex dynamics and facilitates the development of more accurate models for real-world phenomena.

Several writers played a role in the formation of fractional mathematics beginning in 1695, after L'Hospital asked Leibiz what if the order of a derivative is $n = 1/2$. The modeling of biological processes, engineering, physics, finance, and many other fields have shown increased interest in fractional operators [9]. Despite the fact that the classic fractional Riemann Liouville and Caputo derivatives have various advantages for describing reality as more reliable, the singularity that results from the strength of their kernels presents several significant processing challenges [18], and [17]. To overcome these concerns, Caputo and Fabrizio presented the Caputo-Fabrizio (CF) derivative, a novel non-singular fractional derivative that uses an increasing kernel. [17]. Atangana and Baleanu have introduced Atangana-Baleanu (AB) derivatives with Mittag-Leffler kernel function, which are influenced by the concept of CF derivative [2]. Apart from being a derivative, these operators have been viewed as a filter regulator [3]. Atangana and Alkahtani performed a comprehensive examination of the existence and uniqueness of Keller-Segel model solutions including the CF derivative [4]. Baggs and Freedman models with exponential kernels were examined by Atangana and Koca [2]. Singh et al. investigated the epidemiological model for computer viruses with CF derivatives using Banach fixed point theory [13]. Yavuz et al [25] solved fractional partial differential equations with an AB derivative. The traditional model of a contaminated lake system was changed using the notion of fractional differentiation [28]. Uçar [34] used CF and AB variants to investigate a smoking model as it relates to determination and education-related activities.

Therefore, motivated by the applicability of Atangana-Baleanu derivatives, we intend to further explore a Nipah Virus model in the perspective of fractional concept in relation to the effects of the vaccine and condoms. Following is how the remainder of the document is organized: On the following page, we give some introductory information about fractional order derivatives. The method described in Section 2 for the Nipah virus. In Section 3, it is established that our fractional NiV model's solutions exist and are distinct using fixed point theory. A few number findings are presented in Section 4 along with a brief commentary on them. The conclusions are found in section 5 of the research.

1.1. Basic Definitions. In this part, we provide some essential definitions of fractional derivative.

Definition 1.1. The Sobolev space of order one(1) in (k,l) is defined as [19]:

$$W^1(k,l) = \{p \in L^2(k,l) : p' \in L^2(k,l)\} \tag{1.1}$$

Definition 1.2. Let $p \in W^1(k,l), \quad k < l$ be a function and $\alpha \in [0,1]$. The Atangana- Baleanu derivative is Caputo type of order α of p is given by [**?**]:

$$^{ABC}_{k}D^{\alpha}_{t}[p(t)] = \frac{G(\alpha)}{1-\alpha}\int_{k}^{t} p'(x)E_{\alpha}\left[-\alpha\frac{(t-x)^{\alpha}}{1-\alpha}\right]dx \tag{1.2}$$

where $G(\alpha)$ is a normalization function with $G(0) = G(1) = 1$ and E_{α} is the Mittag-Leffler function

Definition 1.3. Let $p \in W^1(k,l), \quad k < l$ be a function and $\alpha \in [0,1]$. The Atangana-Baleanu derivative in Riemann-Liouville type of order α of p is given by [34]:

$$^{ABR}_{k}D^{\alpha}_{t}[p(t)] = \frac{G(\alpha)}{1-\alpha}\frac{d}{dt}\int_{k}^{t} p(x)E_{\alpha}\left[-\alpha\frac{(t-x)^{\alpha}}{1-\alpha}\right]dx \tag{1.3}$$

Definition 1.4. . The fractional integral is defined by [34]:

$$^{AB}_{k}I^{\alpha}_{t}[p(t)] = \frac{1-\alpha}{G(\alpha)}p(t) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\int_{k}^{t} p(\lambda)(t-\lambda)^{\alpha-1}d\lambda \tag{1.4}$$

Theorem 1.5. *For a continuous function p on* $[k,l]$*. The inequality given below holds on* $[k,l]$ [34]*:*

$$||^{ABR}_{0}D^{\alpha}_{t}[p(t)]|| < \frac{G(\alpha)}{1-\alpha}||p(t)|| \tag{1.5}$$

where $||p(t)|| = \underset{k\leq t\leq l}{Max}|p(t)|$

Theorem 1.6. *The Atangana-Baleanu derivative in Caputo and RL type satisfy Lipschitz condition* [34]*:*

$$||{}_0^{ABC}D_t^{\alpha}[p(t)] - {}_0^{ABC}D_t^{\alpha}[q(t)]|| \leq W||P(t)-q(t)|| \tag{1.6}$$

and

$$||{}_0^{ABR}D_t^{\alpha}[p(t)] - {}_0^{ABR}D_t^{\alpha}[q(t)]|| \leq W||P(t)-q(t)| \tag{1.7}$$

2. Model Information

From the humans and pigs population, we have $S_p(t)$: the number of pigs who are not yet infected with the NiV at time t, but may get it if they come into contact with other infected pigs or eat contaminated fruits, $E_p(t)$: the number of pigs that have come into contact with the infectious agent or pathogen that causes the Nipah virus, $I_p(t)$: the number of pigs that are capable of transmitting the virus to others including human, $S(t)$: susceptible with respect to human beings, $S_v(t)$: the susceptible persons who are vaccinated, $S_u(t)$: the susceptible persons who are not yet vaccinated, $S_{vc}(t)$:the susceptible persons who are vaccinated and use condoms, $S_{vn}(t)$: the susceptible persons who are vaccinated and do not use condoms, $S_{uc}(t)$: the susceptible persons who are not yet vaccinated and use condoms, $S_{un}(t)$: the susceptible persons who are not yet vaccinated and do not care for condoms, $E(t)$: people that are in touch with the infectious agent(human and pigs) or pathogen that causes the Nipah virus, $I(t)$: people that are capable of transmitting the virus to others, $C(t)$: they are people who have been infected with the virus but do not develop any symptoms of the disease, and still carry and transmit the virus to others, $I_i(t)$: they are isolated individuals undergoing treatment who are capable of transmitting the virus to others, $I_t(t)$: they are not isolated but undergoing treatment as individuals that are capable of transmitting the virus to others, $R(t)$: they are individuals who have recovered from the Nipah virus and are capable of contacting the virus again, $D(t)$: the bodies of those who died due to the virus.

We assume the following: Natural sickness recovery can take place because of powerful antibodies [7]. Casual touching of the dead bodies will expose the individuals to the virus

[5]. There is an interaction between the farmer and the infectious pigs [6]. Since they are continuously watched, medical personnel safeguard themselves against the virus, and infection can happen in a therapy class, isolated individuals do not aid in the spread of NiV [7]. The general public has easy access to and can afford condoms, an infected isolation facility, and vaccinations [6]. After some time, people who have recovered become susceptible to infection once more [35].

TABLE 2.1. Describe the Variables

parameters	**Parameter Description**
Λ_p	The proportion of new pigs introduced
σ	Exposure rate of pigs
ρ	The rate at which infected pigs become exposed
Λ	Human resource recruitment level
χ_1	Rate of vulnerable non-vaccinated peoplee
χ_2	Vaccination coverage among vulnerable populations
η_1	The fraction of unvaccinated vulnerable people who use condoms
η_2	The fraction of unvaccinated vulnerable people who do not use condoms.
τ_1	The fraction of vaccinated people who use condoms
τ_2	The fraction of vaccinated vulnerable people who do not use condoms
Γ_3	Infection force on S_{nc}
Γ_4	Infection force on S_{un}
Γ_1	Infection force on S_{vc}
Γ_2	Infection force on S_{vn}
κ	The rate at which an exposed population becomes infected
θ	The proportion of the exposed population who becomes a NiV carrier
ψ_1	Rate of isolation of infected people having treatment
ψ_2	Treatment rate of infected persons
γ_1	Recovery rate from the disease therapy class
γ_2	Recovery rate from the infectious isolated undergoing treatment class
γ_3	The NiV carrier recovery rate
γ_4	The infectious recovery rate
ε	Rate of susceptibility among recovered persons
δ_1	Illness-related death rate in NiV-Carriers
δ_2	Illness-related death rate in infectious population
δ_3	Illness-related death rate in infectious isolated people undergoing treatment
δ_4	Illness-related death rate in infectious people undergoing treatment
δ_d	Illness-related death rate in infectious pigs
μ_d	The rate at which deceased bodies are disposed of (burial/cremation)
μ_p	Pig mortality rate
μ	Natural death rate

$$ {}_0^{ABC}D_t^{\alpha}[S(t)] = \Lambda - (\chi_1 + \chi_2 + \mu)S + \varepsilon R \tag{2.1} $$

$$ {}_0^{ABC}D_t^{\alpha}[S_v(t)] = \chi_2 S - (\tau_1 + \tau_2 + \mu)S_v \tag{2.2} $$

$$ {}_0^{ABC}D_t^{\alpha}[S_u(t)] = \chi_1 S - (\eta_1 + \eta_2 + \mu)S_u \tag{2.3} $$

$$ {}_0^{ABC}D_t^{\alpha}[S_{vc}(t)] = \tau_1 S_v - (\Gamma_1 + \mu)S_{vc} \tag{2.4} $$

$$ {}_0^{ABC}D_t^{\alpha}[S_{vn}(t)] = \tau_2 S_v - (\Gamma_2 + \mu)S_{vn} \tag{2.5} $$

$$ {}_0^{ABC}D_t^{\alpha}[Suc(t)] = \eta_1 S_u - (\Gamma_3 + \mu)S_{uc} \tag{2.6} $$

$$ {}_0^{ABC}D_t^{\alpha}[S_{un}(t)] = \eta_2 S_u - (\Gamma_4 + \mu)S_{un} \tag{2.7} $$

$$ {}_0^{ABC}D_t^{\alpha}[E(t)] = \Gamma_2 S_{vn} + \Gamma_4 S_{un} + \Gamma_3 S_{uc} + \Gamma_1 S_{vc} - (\mu + \theta + \kappa)E \tag{2.8} $$

$$ {}_0^{ABC}D_t^{\alpha}[C(t)] = \theta E - (\gamma_3 + \mu + \delta_1)C \tag{2.9} $$

$$ {}_0^{ABC}D_t^{\alpha}[I(t)] = \kappa\, E - (\psi_1 + \psi_2 + \mu + \delta_2 + \gamma_4)I \tag{2.10} $$

$$ {}_0^{ABC}D_t^{\alpha}[I_{it}(t)] = \psi_1 I - (\gamma_2 + \mu + \delta_3)I_{it} \tag{2.11} $$

$$ {}_0^{ABC}D_t^{\alpha}[I_t(t)] = \psi_2 I - (\gamma_1 + \mu + \delta_4)I_t \tag{2.12} $$

$$ {}_0^{ABC}D_t^{\alpha}[R(t)] = \gamma_2 I_{it} + \gamma_4 I + \gamma_1 I_t + \gamma_3 C - \mu R - \varepsilon R \tag{2.13} $$

$$ {}_0^{ABC}D_t^{\alpha}[D(t)] = \delta_4 I_t + \delta_3 I_{it} + \delta_1 C + \delta_2 I - \mu_d D \tag{2.14} $$

$$ {}_0^{ABC}D_t^{\alpha}[S_p(t)] = \Lambda_p - (\sigma + \mu_p)S_p \tag{2.15} $$

$$ {}_0^{ABC}D_t^{\alpha}[E_p(t)] = \sigma S_p - (\rho + \mu_p)E_p \tag{2.16} $$

$$ {}_0^{ABC}D_t^{\alpha}[I_p(t)] = \rho E_p - (\mu_p + \delta_p)I_p \tag{2.17} $$

where the force of infections are

$$ \Gamma_1 = \beta_1\left(\frac{a_1 I_p}{N_p} + \frac{a_2 C + a_3 I + a_4 I_t + a_5 D}{N}\right), \quad \Gamma_2 = \beta_2\left(\frac{b_1 I_p}{N_p} + \frac{b_2 C + b_3 I + b_4 I_t + b_5 D}{N}\right) $$

$$ \Gamma_3 = \beta_3\left(\frac{q_1 I_p}{N_p} + \frac{q_2 C + q_3 I + q_4 I_t + q_5 D}{N}\right), \quad \Gamma_4 = \beta_4\left(\frac{z_1 I_p}{N_p} + \frac{z_2 C + z_3 I + z_4 I_t + z_5 D}{N}\right), $$

N is number of human beings, N_p is number of pigs. Therefore a_i, b_i, q_i, z_i, $i = 1,2,3,4,5$ are contact rates, ${}_0^{ABC}D_t^{\alpha}$ is Atangana-Baleanu derivative in Caputo type, and $\alpha \in [0,1]$.

3. Existence and Uniqueness of Niv Model Solution

The solution of nonlinear equations is a complicated matter in differential calculus [?]. The fractional order model under examination is nonlinear, accurate solutions to these kind of problems may be hard to find. Therefor, we analyze the existence and uniqueness of Niv model solution with the use of fixed point theory.

Consider $\mathscr{C}[k,l] \times \mathscr{C}[k,l] \times \mathscr{C}[k,l] \times \mathscr{C}[k,l] \times \mathscr{C}[k,l] \times \mathscr{C}[k,l] \times \mathscr{C}[k,l] \times \mathscr{C}[k,l] \times \mathscr{C}[k,l] \times \mathscr{C}[k,l] \times \mathscr{C}[k,l] \times \mathscr{C}[k,l] \times \mathscr{C}[k,l] \times \mathscr{C}[k,l] \times \mathscr{C}[k,l] \times \mathscr{C}[k,l] \times \mathscr{C}[k,l]$ such that $\mathscr{C}[k,l]$ is a Banach space of continuous $\mathbb{R} \to \mathbb{R}$ valued functions on the interval $[k,l]$ with the norm defined as $||(S,S_u,S_v,S_{uc},S_{un},S_{vc},S_{vn},E,C,I,I_{it},I_t,R,S_p,E_p,I_p)|| = ||S|| + ||S_u|| + |S_v|| + ||S_{uc}|| + ||S_{un}|| + ||S_{vc}|| + ||S_{vn}|| + ||E|| + ||C|| + ||I|| + ||I_{it}|| + ||I_t|| + ||R|| + ||S_p|| + ||E_p|| + ||I_p||$

where $||S|| = sup\{|S(t)| : t \in [k,l]\}$, $|S_u|| = sup\{|S_u(t)| : t \in [k,l]\}$, $||S_v|| = sup\{|S_v(t)| : t \in [k,l]\}$, $||S_{uc}|| = sup\{|S_{uc}(t)| : t \in [k,l]\}$, $||S_{un}|| = sup\{|S_{un}(t)| : t \in [k,l]\}$, $||S_{vc}|| = sup\{|S_{vc}(t)| : t \in [k,l]\}$, $||S_{vn}|| = sup\{|S_{vn}(t)| : t \in [k,l]\}$, $||E|| = sup\{|E(t)| : t \in [k,l]\}, ||C|| = sup\{|C(t)| : t \in [k,l]\}$, $||I|| = sup\{|I(t)| : t \in [k,l]\}$, $||I_{it}|| = sup\{|I_{it}(t)| : t \in [k,l]\}$, $||I_t|| = sup\{|I(t)| : t \in [k,l]\}$, $||R|| = sup\{|R(t)| : t \in [k,l]\}$, $||S_p|| = sup\{|S_p(t)| : t \in [k,l]\}$, $||E_p|| = sup\{|E_p(t)| : t \in [k,l]\}$, $||I_p|| = sup\{|I_p(t)| : t \in [k,l]\}$.

We reorganize the model (2.1)– (2.17) in the simple method shown below.

$$ {}_0^{ABC}D_t^{\alpha}[S(t)] = W_1(t,S) \tag{3.1}$$

$$ {}_0^{ABC}D_t^{\alpha}[S_v(t)] = W_2(t,S_v(t)) \tag{3.2}$$

$$ {}_0^{ABC}D_t^{\alpha}[S_u(t)] = W_3(t,S_u(t)) \tag{3.3}$$

$$ {}_0^{ABC}D_t^{\alpha}[S_{vc}(t)] = W_4(t,S_{vc}(t)) \tag{3.4}$$

$$ {}_0^{ABC}D_t^{\alpha}[S_{vn}(t)] = W_5(t,S_{vn}(t)) \tag{3.5}$$

$$ {}_0^{ABC}D_t^{\alpha}[Suc(t)] = W_6(t,S_{uc}(t)) \tag{3.6}$$

$$ {}_0^{ABC}D_t^{\alpha}[S_{un}(t)] = W_7(t,S_{un}(t)) \tag{3.7}$$

$$ {}_0^{ABC}D_t^{\alpha}[E(t)] = W_8(t,E(t)) \tag{3.8}$$

$$ {}_0^{ABC}D_t^{\alpha}[C(t)] = W_9(t,C(t)) \tag{3.9}$$

$$ {}_{0}^{ABC}D_t^{\alpha}[I(t)] = W_{10}(t, I(t)) \tag{3.10} $$

$$ {}_{0}^{ABC}D_t^{\alpha}[I_{it}(t)] = W_{11}(t, I_{it}(t)) \tag{3.11} $$

$$ {}_{0}^{ABC}D_t^{\alpha}[I_t(t)] = W_{12}(t, I_t(t)) \tag{3.12} $$

$$ {}_{0}^{ABC}D_t^{\alpha}[R(t)] = W_{13}(t, R(t)) \tag{3.13} $$

$$ {}_{0}^{ABC}D_t^{\alpha}[D(t)] = W_{14}(t, D(t)) \tag{3.14} $$

$$ {}_{0}^{ABC}D_t^{\alpha}[S_p(t)] = W_{15}(t, S_p(t)) \tag{3.15} $$

$$ {}_{0}^{ABC}D_t^{\alpha}[E_p(t)] = W_{16}(t, E_p(t)) \tag{3.16} $$

$$ {}_{0}^{ABC}D_t^{\alpha}[I_p(t)] = W_{17}(t, I_p(t)) \tag{3.17} $$

such that $W_i, \quad i = 1, 2, ..., 17$ are the kernels

Applying fractional integral [11] to the equation (2.1)– (2.17), we have

$$ S(t) - S(0) = \frac{1-\alpha}{G(\alpha)} W_1(t, S) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_1(\nu, S) d\nu \tag{3.18} $$

$$ S_v(t) - S_v(0) = \frac{1-\alpha}{G(\alpha)} W_2(t, S_v) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_2(\nu, S_v) d\nu \tag{3.19} $$

$$ S_u(t) - S_u(0) = \frac{1-\alpha}{G(\alpha)} W_3(t, S_u) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_3(\nu, S_u) d\nu \tag{3.20} $$

$$ S_{vc}(t) - S_{vc}(0) = \frac{1-\alpha}{G(\alpha)} W_4(t, S_{vc}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_4(\nu, S_{vc}) d\nu \tag{3.21} $$

$$ S_{vn}(t) - S_{vn}(0) = \frac{1-\alpha}{G(\alpha)} W_5(t, S_{vn}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_5(\nu, S_{vn}) d\nu \tag{3.22} $$

$$ S_{uc}(t) - S_{uc}(0) = \frac{1-\alpha}{G(\alpha)} W_6(t, S_{uc}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_6(\nu, S_{uc}) d\nu \tag{3.23} $$

$$ S_{un}(t) - S_{un}(0) = \frac{1-\alpha}{G(\alpha)} W_7(t, S_{un}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_7(\nu, S_{un}) d\nu \tag{3.24} $$

$$ E(t) - E(0) = \frac{1-\alpha}{G(\alpha)} W_8(t, E) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_8(\nu, E) d\nu \tag{3.25} $$

$$ C(t) - C(0) = \frac{1-\alpha}{G(\alpha)} W_9(t, C) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_9(\nu, C) d\nu \tag{3.26} $$

$$ I(t) - I(0) = \frac{1-\alpha}{G(\alpha)} W_{10}(t, I) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_{10}(\nu, I) d\nu \tag{3.27} $$

$$ I_{it}(t) - I_{it}(0) = \frac{1-\alpha}{G(\alpha)} W_{11}(t, I_{it}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_{11}(\nu, I_{it}) d\nu \tag{3.28} $$

$$I_t(t) - I_t(0) = \frac{1-\alpha}{G(\alpha)} W_{12}(t, I_t) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_{12}(\nu, I_t) d\nu \tag{3.29}$$

$$R(t) - R(0) = \frac{1-\alpha}{G(\alpha)} W_{13}(t, R) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_{13}(\nu, R) d\nu \tag{3.30}$$

$$D(t) - D(0) = \frac{1-\alpha}{G(\alpha)} W_{14}(t, D) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_{14}(\nu, D) d\nu \tag{3.31}$$

$$S_p(t) - S_p(0) = \frac{1-\alpha}{G(\alpha)} W_{15}(t, S_p) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_{15}(\nu, S_p) d\nu \tag{3.32}$$

$$E_p(t) - E_p(0) = \frac{1-\alpha}{G(\alpha)} W_{16}(t, E_p) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_{16}(\nu, E_p) d\nu \tag{3.33}$$

$$I_p(t) - I_p(0) = \frac{1-\alpha}{G(\alpha)} W_{17}(t, I_p) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_{17}(\nu, I_p) d\nu \tag{3.34}$$

Theorem 3.1. *If* $0 \leq \bar{\xi}_i < 1$, *then the kernels* W_i, $i = 1, 2, ..., 17$ *fulfill the Lipschitz condition and contraction such that*

$$||W_1(t,S) - W_1(t,S_*)|| \leq \bar{\xi}_1 ||S(t) - S_*(t)||, ..., ||W_{17}(t,I_p) - W_{17}(t,I_{p*})|| \leq \bar{\xi}_{17} ||I_p(t) - I_{p*}(t)|| \tag{3.35}$$

Proof. Given two functions S and S_*, we have

$$\begin{aligned} ||W_1(t,S) - W_1(t,S_*)|| &= ||\Lambda - (\chi_1 + \chi_2 + \mu)S + \varepsilon R - [\Lambda - (\chi_1 + \chi_2 + \mu)S_* + \varepsilon R]|| \\ &\leq (\chi_1 + \chi_2 + \mu)||S - S_*|| \\ &= \bar{\xi}_1 ||S - S_*|| \end{aligned} \tag{3.36}$$

where $\bar{\xi}_1 = (\chi_1 + \chi_2 + \mu)$. Therefore

$$||W_1(t,S) - W_1(t,S_*)|| \leq \bar{\xi}_1 ||S - S_*||$$

Hence, the Lipschitz condition satisfied for W_1 and $0 \leq \chi_1 + \chi_2 + \mu < 1$ implies W_1 is also contraction.

Similarly, it can be demonstrated that the other kernels satisfy the Lipschitz condition and contraction. However, observe that

$$S(t) + S_u(t) + S_v(t) + S_{uc}(t) + S_{un}(t) + S_{vc}(t) + S_{vn}(t) + E(t) + C(t) + I(t) + I_{it}(t) + I_t(t) + R(t) \leq \frac{\Lambda}{\mu} \Longrightarrow C(t) \leq \frac{\Lambda}{\mu}, I(t) \leq \frac{\Lambda}{\mu}, I_{it}(t) \leq \frac{\Lambda}{\mu}, I_t(t) \leq \frac{\Lambda}{\mu}, \ldots, R(t) \leq \frac{\Lambda}{\mu} \text{ as } t \geq 0.$$

Without loss of generality, $\bar{\xi}_2 = (\tau_1 + \tau_2 + \mu)$, $\bar{\xi}_2 = (\eta_1 + \eta_2 + \mu)$, $\bar{\xi}_4 = \beta_1 \left(\frac{a_1 \Lambda_p}{N_p \mu_p} + \frac{(a_2 + a_3 + a_4 + a_5)\Lambda}{N\mu} \right) + \mu$, $\bar{\xi}_5 = \beta_2 \left(\frac{b_1 \Lambda_p}{N_p \mu_p} + \frac{(b_2 + b_3 + b_4 + b_5)\Lambda}{N\mu} \right) + \mu$, $\bar{\xi}_6 = \beta_3 \left(\frac{q_1 \Lambda_p}{N_p \mu_p} + \frac{(q_2 + q_3 + q_4 + q_5)\Lambda}{N\mu} \right) + \mu$, $\bar{\xi}_7 = \beta_4 \left(\frac{z_1 \Lambda_p}{N_p \mu_p} + \frac{(z_2 + z_3 + z_4 + z_5)\Lambda}{N\mu} \right) + \mu$, $\bar{\xi}_8 = (\mu + \theta + \kappa)$, $\bar{\xi}_9 = (\gamma_3 + \mu + \delta_1)$, $\bar{\xi}_{10} = (\psi_1 + \psi_2 + \mu + \delta_2 + \gamma_4)$, $\bar{\xi}_{11} = \gamma_2 + \mu + \delta_3$, $\bar{\xi}_{12} = \gamma_1 + \mu + \delta_4$, $\bar{\xi}_{13} = \mu + \varepsilon$, $\bar{\xi}_{14} = \mu_d$, $\bar{\xi}_{15} = \sigma + \mu_p$, $\bar{\xi}_{16} = \rho + \mu_p$, $\bar{\xi}_{17} = \delta_p + \mu_p$.

Therefore the kernels W_i, $i = 1, 2, ..., 17$ fulfill the Lipschitz condition and $0 \leq \bar{\xi}_i < 1$ implies W_i, $i = 1, 2, ..., 17$ are contraction. □

We define the system (3.18)– (3.34) in the following recursive form:

$$S_{(n)}(t) = \frac{1-\alpha}{G(\alpha)} W_1(t, S_{(n-1)}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_1(\nu, S_{(n-1)}) d\nu \tag{3.37}$$

$$S_{v(n)}(t) = \frac{1-\alpha}{G(\alpha)} W_2(t, S_{v(n-1)}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_2(\nu, S_{v(n-1)}) d\nu \tag{3.38}$$

$$S_{u(n)}(t) = \frac{1-\alpha}{G(\alpha)} W_3(t, S_{u(n-1)}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_3(\nu, S_{u(n-1)}) d\nu \tag{3.39}$$

$$S_{vc(n)}(t) = \frac{1-\alpha}{G(\alpha)} W_4(t, S_{vc(n-1)}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_4(\nu, S_{vc(n-1)}) d\nu \tag{3.40}$$

$$S_{vn(n)}(t) = \frac{1-\alpha}{G(\alpha)} W_5(t, S_{vn(n-1)}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_5(\nu, S_{vn(n-1)}) d\nu \tag{3.41}$$

$$S_{uc(n)}(t) = \frac{1-\alpha}{G(\alpha)} W_6(t, S_{uc(n-1)}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_6(\nu, S_{uc(n-1)}) d\nu \tag{3.42}$$

$$S_{un(n)}(t) = \frac{1-\alpha}{G(\alpha)} W_7(t, S_{un(n-1)}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_7(\nu, S_{un(n-1)}) d\nu \tag{3.43}$$

$$E_{(n)}(t) = \frac{1-\alpha}{G(\alpha)} W_8(t, E_{(n-1)}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_8(\nu, E_{(n-1)}) d\nu \tag{3.44}$$

$$C_{(n)}(t) = \frac{1-\alpha}{G(\alpha)} W_9(t, C_{(n-1)}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_9(\nu, C_{(n-1)}) d\nu \tag{3.45}$$

$$I_{(n)}(t) = \frac{1-\alpha}{G(\alpha)} W_{10}(t, I_{(n-1)}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_{10}(\nu, I_{(n-1)}) d\nu \tag{3.46}$$

$$I_{it(n)}(t) = \frac{1-\alpha}{G(\alpha)} W_{11}(t, I_{it(n-1)}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_{11}(\nu, I_{it(n-1)}) d\nu \tag{3.47}$$

$$I_{t(n)}(t) = \frac{1-\alpha}{G(\alpha)} W_{12}(t, I_{t(n-1)}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_{12}(\nu, I_{t(n-1)}) d\nu \tag{3.48}$$

$$R_{(n)}(t) = \frac{1-\alpha}{G(\alpha)} W_{13}(t, R_{(n-1)}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} W_{13}(\nu, R_{(n-1)}) d\nu \tag{3.49}$$

$$D_{(n)}(t) = \frac{1-\alpha}{G(\alpha)}W_{14}(t,D_{(n-1)}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\int_0^t (t-\nu)^{\alpha-1}W_{14}(\nu,D_{(n-1)})d\nu \tag{3.50}$$

$$S_{p(n)}(t) = \frac{1-\alpha}{G(\alpha)}W_{15}(t,S_{p(n-1)}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\int_0^t (t-\nu)^{\alpha-1}W_{15}(\nu,S_{p(n-1)})d\nu \tag{3.51}$$

$$E_{p(n)}(t) = \frac{1-\alpha}{G(\alpha)}W_{16}(t,E_{p(n-1)}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\int_0^t (t-\nu)^{\alpha-1}W_{16}(\nu,E_{p(n-1)})d\nu \tag{3.52}$$

$$I_{p(n)}(t) = \frac{1-\alpha}{G(\alpha)}W_{17}(t,I_{p(n-1)}) + \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\int_0^t (t-\nu)^{\alpha-1}W_{17}(\nu,I_{p(n-1)})d\nu \tag{3.53}$$

with the initial conditions:

$S_0 = S(0)$, $S_{u0} = S_u(0)$, $S_{v0} = S_v(0)$, $S_{uc0} = S_{uc}(0)$, $S_{un0} = S_{un}(0)$, $S_{vc0} = S_{vc}(0)$, $S_{vn0} = S_{vn}(0)$, $E_0 = E(0)$, $C_0 = C(0)$, $I_0 = I(0)$, $I_{it0} = I_{it}(0)$, $I_{t0} = I_t(0)$, $R_0 = R(0)$, $S_{p0} = S_p(0)$, $E_{p0} = E_p(0)$, $I_{p0} = I_p(0)$

Next, we look at the difference in the successive terms as follows:

$$\Psi_{1n}(t) = S_n(t) - S_{(n-1)}(t) = \frac{1-\alpha}{G(\alpha)}[W_1(t,S_{(n-1)}) - W_1(t,S_{(n-2)})]$$

$$+ \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\int_0^t (t-\nu)^{\alpha-1}[W_1(\nu,S_{(n-1)}) - W_1(\nu,S_{(n-2)})]d\nu \tag{3.54}$$

$$\Psi_{2n}(t) = S_{v(n)}(t) - S_{v(n-1)}(t) = \frac{1-\alpha}{G(\alpha)}[W_2(t,S_{v(n-1)}) - W_2(t,S_{v(n-2)})] \tag{3.55}$$

$$+ \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\int_0^t (t-\nu)^{\alpha-1}[W_2(\nu,S_{v(n-1)}) - W_2(\nu,S_{v(n-2)})]d\nu \tag{3.56}$$

$$\Psi_{3n}(t) = S_{u(n)}(t) - S_{u(n-1)}(t) = \frac{1-\alpha}{G(\alpha)}[W_3(t,S_{u(n-1)}) - W_3(t,S_{u(n-2)})] \tag{3.57}$$

$$+ \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\int_0^t (t-\nu)^{\alpha-1}[W_3(\nu,S_{u(n-1)}) - W_3(\nu,S_{u(n-2)})]d\nu \tag{3.58}$$

$$\Psi_{4n}(t) = S_{vc(n)}(t) - S_{vc(n-1)}(t) = \frac{1-\alpha}{G(\alpha)}[W_4(t,S_{vc(n-1)}) - W_4(t,S_{vc(n-2)})] \tag{3.59}$$

$$+ \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\int_0^t (t-\nu)^{\alpha-1}[W_4(\nu,S_{vc(n-1)}) - W_4(\nu,S_{vc(n-1)})]d\nu \tag{3.60}$$

$$\Psi_{5n}(t) = S_{vn(n)}(t) - S_{vn(n-1)}(t) = \frac{1-\alpha}{G(\alpha)}[W_5(t,S_{vn(n-1)}) - W_5(t,S_{vn(n-2)})] \tag{3.61}$$

$$+ \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\int_0^t (t-\nu)^{\alpha-1}[W_5(\nu,S_{vn(n-1)}) - W_5(\nu,S_{vn(n-2)})]d\nu \tag{3.62}$$

$$\Psi_{6n}(t) = S_{uc(n)}(t) - S_{uc(n-1)}(t) = \frac{1-\alpha}{G(\alpha)}[W_6(t,S_{uc(n-1)}) - W_6(t,S_{uc(n-2)})] \tag{3.63}$$

$$+ \quad \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\int_0^t (t-\nu)^{\alpha-1}[W_6(\nu,S_{uc(n-1)})-W_6(\nu,S_{uc(n-2)})]d\nu \tag{3.64}$$

$$\Psi_{7n}(t) = S_{un(n)}(t)-S_{un(n-1)}(t)=\frac{1-\alpha}{G(\alpha)}[W_7(t,S_{un(n-1)})-W_7(t,S_{un(n-2)})] \tag{3.65}$$

$$+ \quad \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\int_0^t (t-\nu)^{\alpha-1}[W_7(\nu,S_{un(n-1)})-W_7(\nu,S_{un(n-2)})]d\nu \tag{3.66}$$

$$\Psi_{8n}(t) = E_{(n)}(t)-E_{(n-1)}(t)=\frac{1-\alpha}{G(\alpha)}[W_8(t,E_{(n-1)})-W_8(t,E_{(n-2)})] \tag{3.67}$$

$$+ \quad \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\int_0^t (t-\nu)^{\alpha-1}[W_8(\nu,E_{(n-1)})-W_8(\nu,E_{(n-2)})]d\nu \tag{3.68}$$

$$\Psi_{9n}(t) = C_{(n)}(t)-C_{(n-1)}(t)=\frac{1-\alpha}{G(\alpha)}[W_9(t,C_{(n-1)})-W_9(t,C_{(n-2)})] \tag{3.69}$$

$$+ \quad \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\int_0^t (t-\nu)^{\alpha-1}[W_9(\nu,C_{(n-1)})-W_9(\nu,C_{(n-2)})]d\nu \tag{3.70}$$

$$\Psi_{10n}(t) = I_{(n)}(t)-I_{(n-1)}(t)=\frac{1-\alpha}{G(\alpha)}[W_{10}(t,I_{(n-1)})-W_{10}(t,I_{(n-2)})] \tag{3.71}$$

$$+ \quad \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\int_0^t (t-\nu)^{\alpha-1}[W_{10}(\nu,I_{(n-1)})-W_{10}(\nu,I_{(n-2)})d\nu \tag{3.72}$$

$$\Psi_{11n}(t) = I_{it(n)}(t)-I_{it(n-1)}(t)=\frac{1-\alpha}{G(\alpha)}[W_{11}(t,I_{it(n-1)})-W_{11}(t,I_{it(n-2)})] \tag{3.73}$$

$$+ \quad \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\int_0^t (t-\nu)^{\alpha-1}[W_{11}(\nu,I_{it(n-1)})-W_{11}(\nu,I_{it(n-2)})]d\nu \tag{3.74}$$

$$\Psi_{12n}(t) = I_{t(n)}(t)-I_{t(n-1)}(t)=\frac{1-\alpha}{G(\alpha)}[W_{12}(t,I_{t(n-1)})-W_{12}(t,I_{t(n-2)})] \tag{3.75}$$

$$+ \quad \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\int_0^t (t-\nu)^{\alpha-1}[W_{12}(\nu,I_{t(n-1)})-W_{12}(\nu,I_{t(n-2)})]d\nu \tag{3.76}$$

$$\Psi_{13n}(t) = R_{(n)}(t)-R_{(n-1)}(t)=\frac{1-\alpha}{G(\alpha)}[W_{13}(t,R_{(n-1)})-W_{13}(t,R_{(n-2)})] \tag{3.77}$$

$$+ \quad \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\int_0^t (t-\nu)^{\alpha-1}[W_{13}(\nu,R_{(n-1)})-W_{13}(\nu,R_{(n-2)})]d\nu \tag{3.78}$$

$$\Psi_{14n}(t) = D_{(n)}(t)-D_{(n-1)}(t)=\frac{1-\alpha}{G(\alpha)}[W_{14}(t,D_{(n-1)})-W_{14}(t,D_{(n-2)})] \tag{3.79}$$

$$+ \quad \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\int_0^t (t-\nu)^{\alpha-1}[W_{14}(\nu,D_{(n-1)})-W_{14}(\nu,D_{(n-2)})]d\nu \tag{3.80}$$

$$\Psi_{15n}(t) = S_{p(n)}(t)-S_{p(n-1)}(t)=\frac{1-\alpha}{G(\alpha)}[W_{15}(t,S_{p(n-1)})-W_{15}(t,S_{p(n-2)})] \tag{3.81}$$

$$+ \quad \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\int_0^t (t-\nu)^{\alpha-1}[W_{15}(\nu,S_{p(n-1)})-W_{15}(\nu,S_{p(n-2)})]d\nu \tag{3.82}$$

$$\Psi_{16n}(t) = E_{p(n)}(t) - E_{p(n-1)}(t) = \frac{1-\alpha}{G(\alpha)}[W_{16}(t, E_{p(n-1)}) - W_{16}(t, E_{p(n-2)})] \tag{3.83}$$

$$+ \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1}[W_{16}(\nu, E_{p(n-1)}) - W_{16}(\nu, E_{p(n-2)})]d\nu \tag{3.84}$$

$$\Psi_{17n}(t) = I_{p(n)}(t) - I_{p(n-1)}(t) = \frac{1-\alpha}{G(\alpha)}[W_{17}(t, I_{p(n-1)}) - W_{17}(t, I_{p(n-2)})] \tag{3.85}$$

$$+ \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1}[W_{17}(\nu, I_{p(n-1)}) - W_{17}(\nu, I_{p(n-1)})]d\nu \tag{3.86}$$

So that,

$S_{(n)}(t) = \sum_{k=1}^n \Psi_{1k}$, $S_{v(n)}(t) = \sum_{k=1}^n \Psi_{2k}$, $S_{u(n)}(t) = \sum_{k=1}^n \Psi_{3k}$, $S_{vc(n)}(t) = \sum_{k=1}^n \Psi_{4k}$, $S_{vn(n)}(t) = \sum_{k=1}^n \Psi_{5k}$, $S_{uc(n)}(t) = \sum_{k=1}^n \Psi_{6k}$, $S_{un(n)}(t) = \sum_{k=1}^n \Psi_{7k}$

$E_{(n)}(t) = \sum_{k=1}^n \Psi_{8k}$, $C_{(n)}(t) = \sum_{k=1}^n \Psi_{9k}$, $I_{(n)}(t) = \sum_{k=1}^n \Psi_{10k}$, $I_{it(n)}(t) = \sum_{k=1}^n \Psi_{11k}$, $I_{t(n)}(t) = \sum_{k=1}^n \Psi_{12k}$, $R_{(n)}(t) = \sum_{k=1}^n \Psi_{13k}$, $D_{(n)}(t) = \sum_{k=1}^n \Psi_{14k}$, $S_{p(n)}(t) = \sum_{k=1}^n \Psi_{15k}$, $E_{p(n)}(t) = \sum_{k=1}^n \Psi_{16k}$, $I_{p(n)}(t) = \sum_{k=1}^n \Psi_{17k}$,

Taking norm of both sides of equations (3.54)–(3.86), applying triangular inequality and Lipschitz condition, we have

$$\begin{aligned}
\| \Psi_{1n}(t) \| &= \| S_n(t) - S_{(n-1)}(t) \| \\
&\leq \frac{1-\alpha}{G(\alpha)} \| [W_1(t, S_{(n-1)}) - W_1(t, S_{(n-2)})] \| \\
&+ \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \| \int_0^t (t-\nu)^{\alpha-1}[W_1(\nu, S_{(n-1)}) - W_1(\nu, S_{(n-2)})]d\nu \| &(3.88)\\
&\leq \frac{1-\alpha}{G(\alpha)}\bar{\xi}_1 \| S_{(n-1)} - S_{(n-2)} \| \\
&+ \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_1 \int_0^t (t-\nu)^{\alpha-1} \| S_{(n-1)}) - S_{(n-2)} \| d\nu &(3.89)\\
&= \frac{1-\alpha}{G(\alpha)}\bar{\xi}_1 \| \Psi_{1(n-1)}(t) \| \\
&+ \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_1 \int_0^t (t-\nu)^{\alpha-1} \| \Psi_{1(n-1)}(t) \| d\nu &(3.90)\\
&= \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_1 + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_1\right] \| \Psi_{1(n-1)}(t) \| &(3.91)
\end{aligned} \tag{3.87}$$

$$\begin{aligned}
\| \Psi_{2n}(t) \| &= \| S_{v(n)}(t) - S_{v(n-1)}(t) \| \\
&\leq \frac{1-\alpha}{G(\alpha)} \| [W_2(t, S_{v(n-1)}) - W_2(t, S_{v(n-2)})] \|
\end{aligned} \tag{3.92}$$

$$+ \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \parallel \int_0^t (t-\nu)^{\alpha-1}[W_2(\nu,S_{\nu(n-1)})-W_2(\nu,S_{\nu(n-2)})]d\nu \parallel \tag{3.93}$$

$$\leq \frac{1-\alpha}{G(\alpha)}\bar{\xi}_2 \parallel S_{\nu(n-1)}-S_{\nu(n-2)} \parallel$$

$$+ \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_2 \int_0^t (t-\nu)^{\alpha-1} \parallel S_{\nu(n-1)})-S_{\nu(n-2)} \parallel d\nu \tag{3.94}$$

$$= \frac{1-\alpha}{G(\alpha)}\bar{\xi}_2 \parallel \Psi_{2(n-1)}(t) \parallel$$

$$+ \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_2 \int_0^t (t-\nu)^{\alpha-1} \parallel \Psi_{2(n-1)}(t) \parallel d\nu \tag{3.95}$$

$$= \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_2 + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_2\right] \parallel \Psi_{2(n-1)}(t) \parallel \tag{3.96}$$

Finally, we have

$$\parallel \Psi_{1n}(t) \parallel \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_1 + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_1\right] \parallel \Psi_{1(n-1)}(t) \parallel \tag{3.97}$$

$$\parallel \Psi_{2n}(t) \parallel \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_2 + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_2\right] \parallel \Psi_{2(n-1)}(t) \parallel \tag{3.98}$$

With the same procedure from (3.88)–(3.96), we reduced the remaining expressions to the form:

$$\parallel \Psi_{3n}(t) \parallel \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_3 + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_3\right] \parallel \Psi_{3(n-1)}(t) \parallel \tag{3.99}$$

$$\parallel \Psi_{4n}(t) \parallel \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_4 + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_4\right] \parallel \Psi_{4(n-1)}(t) \parallel \tag{3.100}$$

$$\parallel \Psi_{5n}(t) \parallel \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_5 + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_5\right] \parallel \Psi_{5(n-1)}(t) \parallel \tag{3.101}$$

$$\parallel \Psi_{6n}(t) \parallel \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_6 + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_6\right] \parallel \Psi_{6(n-1)}(t) \parallel \tag{3.102}$$

$$\parallel \Psi_{7n}(t) \parallel \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_7 + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_7\right] \parallel \Psi_{7(n-1)}(t) \parallel \tag{3.103}$$

$$\parallel \Psi_{8n}(t) \parallel \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_8 + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_8\right] \parallel \Psi_{8(n-1)}(t) \parallel \tag{3.104}$$

$$\parallel \Psi_{9n}(t) \parallel \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_9 + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_9\right] \parallel \Psi_{9(n-1)}(t) \parallel \tag{3.105}$$

$$\| \Psi_{10n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_{10} + \frac{t^{\alpha}}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{10}\right] \| \Psi_{10(n-1)}(t) \| \tag{3.106}$$

$$\| \Psi_{11n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_{11} + \frac{t^{\alpha}}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{11}\right] \| \Psi_{11(n-1)}(t) \| \tag{3.107}$$

$$\| \Psi_{12n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_{12} + \frac{t^{\alpha}}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{12}\right] \| \Psi_{12(n-1)}(t) \| \tag{3.108}$$

$$\| \Psi_{13n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_{13} + \frac{t^{\alpha}}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{13}\right] \| \Psi_{13(n-1)}(t) \| \tag{3.109}$$

$$\| \Psi_{14n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_{14} + \frac{t^{\alpha}}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{14}\right] \| \Psi_{14(n-1)}(t) \| \tag{3.110}$$

$$\| \Psi_{15n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_{15} + \frac{t^{\alpha}}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{15}\right] \| \Psi_{15(n-1)}(t) \| \tag{3.111}$$

$$\| \Psi_{16n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_{16} + \frac{t^{\alpha}}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{16}\right] \| \Psi_{16(n-1)}(t) \| \tag{3.112}$$

$$\| \Psi_{17n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_{17} + \frac{t^{\alpha}}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{17}\right] \| \Psi_{17(n-1)}(t) \| \tag{3.113}$$

Theorem 3.2. *If we can determine t_0 that satisfies the equation*

$$\frac{1-\alpha}{G(\alpha)}\bar{\xi}_i + \frac{t_0^{\alpha}}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_i < 1 \tag{3.114}$$

for $i = 1,2,3,...,17$, the fractional model provided as (2.1)–(2.17) *has a unique solution.*

Proof. Given t_0 satisfying $\frac{1-\alpha}{G(\alpha)}\bar{\xi}_i + \frac{t_0^{\alpha}}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_i < 1$, we show that the fractional model (2.1)–(2.17) has solution and the solution is unique. It is obvious (Theorem 3.1) that

$S, S_v, S_u, S_{vc}, S_{vn}, S_{uc}, S_{un}, E, C, I, I_{it}, I_t, R, D, S_p, E_p, I_p$ satisfied Lipschitz condition and they are also bounded functions. Therefor, from equation (3.99)–(3.113) we have the successive relations [34]:

$$\| \Psi_{1n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_{1} + \frac{t^{\alpha}}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{1}\right]^n \| S_{(n)}(0) \| \tag{3.115}$$

$$\| \Psi_{2n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_{2} + \frac{t^{\alpha}}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{2}\right]^n \| S_{v(n)}(0) \| \tag{3.116}$$

$$\| \Psi_{3n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_{3} + \frac{t^{\alpha}}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{3}\right]^n \| S_{u(n)}(0) \| \tag{3.117}$$

$$\| \Psi_{4n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_{4} + \frac{t^{\alpha}}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{4}\right]^n \| S_{vc(n)}(0) \| \tag{3.118}$$

$$\| \Psi_{5n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_5 + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_5\right]^n \| S_{vn(n)}(0) \| \tag{3.119}$$

$$\| \Psi_{6n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_6 + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_6\right]^n \| S_{uc(n)}(0) \| \tag{3.120}$$

$$\| \Psi_{7n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_7 + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_7\right]^n \| S_{un(n)}(0) \| \tag{3.121}$$

$$\| \Psi_{8n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_8 + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_8\right]^n \| E_{(n)}(0) \| \tag{3.122}$$

$$\| \Psi_{9n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_9 + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_9\right]^n \| C_{(n)}(0) \| \tag{3.123}$$

$$\| \Psi_{10n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_{10} + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{10}\right]^n \| I_{(n)}(0) \| \tag{3.124}$$

$$\| \Psi_{11n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_{11} + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{11}\right]^n \| I_{it(n)}(0) \| \tag{3.125}$$

$$\| \Psi_{12n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_{12} + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{12}\right]^n \| I_{t(n)}(0) \| \tag{3.126}$$

$$\| \Psi_{13n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_{13} + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{13}\right]^n \| R_{(n)}(0) \| \tag{3.127}$$

$$\| \Psi_{14n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_{14} + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{14}\right]^n \| D_{(n)}(0)) \| \tag{3.128}$$

$$\| \Psi_{15n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_{15} + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{15}\right]^n \| S_{p(n)}(0) \| \tag{3.129}$$

$$\| \Psi_{16n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_{16} + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{16}\right]^n \| E_{p(n)}(0) \| \tag{3.130}$$

$$\| \Psi_{17n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_{17} + \frac{t^\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{17}\right]^n \| I_{p(n)}(0) \| \tag{3.131}$$

This establishes the Existence and continuity of the aforementioned solutions.In order to demonstrate that the functions are solutions of equation (2.1)–(2.17), let's say that

$$S(t) - S(0) = S_{(n)}(t) - \bar{\vartheta}_{1n}(t) \tag{3.132}$$

$$S_v(t) - S_v(0) = S_{v(n)}(t) - \bar{\vartheta}_{2n}(t) \tag{3.133}$$

$$S_u(t) - S_u(0) = S_{u(n)}(t) - \bar{\vartheta}_{3n}(t) \tag{3.134}$$

$$S_{vc}(t) - S_{vc}(0) = S_{vc(n)}(t) - \bar{\vartheta}_{4n}(t) \tag{3.135}$$

$$S_{vn}(t) - S_{vn}(0) = S_{vn(n)}(t) - \bar{\vartheta}_{5n}(t) \tag{3.136}$$

$$S_{uc}(t) - S_{uc}(0) = S_{uc(n)}(t) - \bar{\vartheta}_{6n}(t) \tag{3.137}$$

$$S_{un}(t) - S_{un}(0) = S_{un(n)}(t) - \bar{\vartheta}_{7n}(t) \tag{3.138}$$

$$E(t) - E(0) = E_{(n)}(t) - \bar{\vartheta}_{8n}(t) \tag{3.139}$$

$$C(t) - C(0) = C_{(n)}(t) - \bar{\vartheta}_{9n}(t) \tag{3.140}$$

$$I(t) - I(0) = I_{(n)}(t) - \bar{\vartheta}_{10n}(t) \tag{3.141}$$

$$I_{it}(t) - I_{it}(0) = I_{it(n)}(t) - \bar{\vartheta}_{11n}(t) \tag{3.142}$$

$$I_{t}(t) - I_{t}(0) = I_{t(n)}(t) - \bar{\vartheta}_{12n}(t) \tag{3.143}$$

$$R(t) - R(0) = R_{(n)}(t) - \bar{\vartheta}_{13n}(t) \tag{3.144}$$

$$D(t) - D(0) = D_{(n)}(t) - \bar{\vartheta}_{14n}(t) \tag{3.145}$$

$$S_{p}(t) - S_{p}(0) = S_{p(n)}(t) - \bar{\vartheta}_{15n}(t) \tag{3.146}$$

$$E_{p}(t) - E_{p}(0) = E_{p(n)}(t) - \bar{\vartheta}_{16n}(t) \tag{3.147}$$

$$I_{p}(t) - I_{p}(0) = I_{p(n)}(t) - \bar{\vartheta}_{17n}(t) \tag{3.148}$$

At random, observe that

$$\begin{aligned} \| \bar{\vartheta}_{8n}(t) \| &= \| \frac{1-\alpha}{G(\alpha)}[W_1(t,E) - W_1(t,E_{(n-1)})] \\ &+ \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1}[W_1(\nu,E) - W_1(\nu,E_{(n-1)})]d\nu \| \end{aligned} \tag{3.149}$$

$$\begin{aligned} &\leq \frac{1-\alpha}{G(\alpha)} \| [W_1(t,E) - W_1(t,E_{(n-1)})] \| \\ &+ \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \int_0^t (t-\nu)^{\alpha-1} \| [W_1(\nu,E) - W_1(\nu,E_{(n-1)})]d\nu \| \end{aligned} \tag{3.150}$$

$$\begin{aligned} &\leq \frac{1-\alpha}{G(\alpha)}\bar{\xi}_8 \| E - E_{(n-1)} \| \\ &+ \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_8 \int_0^t (t-\nu)^{\alpha-1} \| E - E_{(n-1)} \| \, d\nu \end{aligned} \tag{3.151}$$

$$= \left[\frac{1-\alpha}{G(\alpha)}\bar{\xi}_8 + \frac{t^{\alpha}}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_8\right] \| E - E_{(n-1)} \| \tag{3.152}$$

when we continue recursively at t_0, we have

$$\| \bar{\vartheta}_{8n}(t) \| \leq \left[\frac{1-\alpha}{G(\alpha)} + \frac{t_0^{\alpha}}{G(\alpha)\Gamma(\alpha)}\right]^{n+1} \bar{\xi}_8^{n+1} \frac{\Lambda}{\mu} \tag{3.153}$$

Observe that $N \leq \dfrac{\Lambda}{\mu}$ and $N_p \leq \dfrac{\Lambda_p}{\mu_p}$. Therefore without loss of generality, we have

$$\begin{aligned}
\| \bar{\vartheta}_{1n}(t) \| &\leq \left[\frac{1-\alpha}{G(\alpha)} + \frac{t_0^{\alpha}}{G(\alpha)\Gamma(\alpha)}\right]^{n+1} \bar{\xi}_1^{n+1} \frac{\Lambda}{\mu} \\
&\vdots \\
\| \bar{\vartheta}_{14n}(t) \| &\leq \left[\frac{1-\alpha}{G(\alpha)} + \frac{t_0^{\alpha}}{G(\alpha)\Gamma(\alpha)}\right]^{n+1} \bar{\xi}_{15}^{n+1} \frac{\Lambda}{\mu} \\
\| \bar{\vartheta}_{15n}(t) \| &\leq \left[\frac{1-\alpha}{G(\alpha)} + \frac{t_0^{\alpha}}{G(\alpha)\Gamma(\alpha)}\right]^{n+1} \bar{\xi}_{15}^{n+1} \frac{\Lambda_p}{\mu_p} \\
&\vdots \\
\| \bar{\vartheta}_{17n}(t) \| &\leq \left[\frac{1-\alpha}{G(\alpha)} + \frac{t_0^{\alpha}}{G(\alpha)\Gamma(\alpha)}\right]^{n+1} \bar{\xi}_{17}^{n+1} \frac{\Lambda_p}{\mu_p}
\end{aligned}$$

Taking the limit as $n \to \infty$, we have $\| \bar{\vartheta}_{in} \| \to 0, \quad i = 1,2,...,17$ which implies that the fractional model (2.1)–(2.17) has solution

Next, we show that the solution is unique: suppose there is another solution to the model say $S_*, S_{v*}, S_{u*}, S_{uc*}, S_{un*}, S_{vc*}, S_{vn*}, E_*, C_*, I_*, I_{it*}, I_{t*}, R_*, D_*, S_{p*}, E_{p*}, I_{p*}$, then we have

$$\begin{aligned}
D(t) - D_*(t) &= \frac{1-\alpha}{G(\alpha)}[W_{14}(t,D) - W_{14}(t,D_*)] \\
&+ \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \times \int_0^t (t-\nu)^{\alpha-1}[W_{14}(\nu,D) - W_{14}(\nu,D_*)]d\nu
\end{aligned} \tag{3.154}$$

Since all the kernel satisfied Lipschitz condition, applying norm on the both side we have

$$\begin{aligned}
\| D(t) - D_*(t) \| &\leq \frac{1-\alpha}{G(\alpha)} \bar{\xi}_{14} \| D - D_* \| \\
&+ \frac{\alpha}{G(\alpha)\Gamma(\alpha)} \bar{\xi}_{14} \| D - D_* \|
\end{aligned} \tag{3.155}$$

and

$$\| D(t) - D_*(t) \| \left[1 - \frac{1-\alpha}{G(\alpha)}\bar{\xi}_{14} - \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{14}\right] \leq 0$$

Therefor $\left[1 - \frac{1-\alpha}{G(\alpha)}\bar{\xi}_{14} - \frac{\alpha}{G(\alpha)\Gamma(\alpha)}\bar{\xi}_{14}\right] > 0$ since $\alpha \in [0,1]$ and $\bar{\xi}_i \in [0,1)$ for $i = 1,2,\ldots,17$. This implies

$$\| D(t) - D_*(t) \| = 0$$

so that

$$D(t) = D_*(t)$$

Adopting the same approach, we obtain the following; $S = S_*, S_v = S_{v*}, S_u = S_{u*}, S_{uc} = S_{uc*}, S_{un} = S_{un*}, S_{vc} = S_{vc*}, S_{vn} = S_{vn*}, E = E_*, C = C_*, I = I_*, I_{it} = I_{it*}, I_t = I_{t*}, R = R_*, S_p = S_{p*}, E_p = E_{p*}, I_p = I_{p*}$ □

4. Discussion and Simulations of the Model

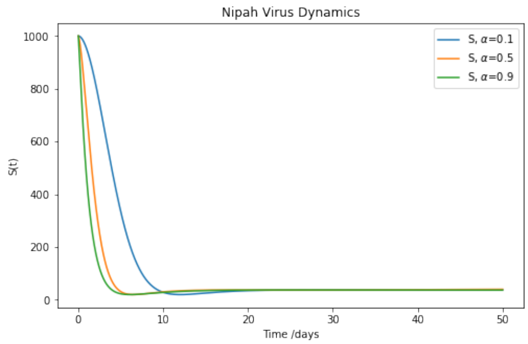
In this part, numerical simulations of model (2.1)–(2.17) for the data presented in Table 4.1 are performed. We run numerical simulations on our model (2.1)–(2.17) using Python software to see the influence of fractional order α changes in the model (2.1)–(2.17) with the initial values and parameters ([6], [22]).

Parameter	Value	Source
χ_1	0.33	Estimated
χ_2	0.62	Estimated
θ	0.486	Estimated
κ	0.715	Estimated
τ_1	0.008	Estimated
τ_2	0.019	Estimated
η_1	0.45	Estimated
η_2	0.39	Estimated
ψ_1	0.825	Estimated
ψ_2	0.342	Estimated
γ_1	0.8	Inferred from [22]
γ_2	0.5	Inferred from [22]
γ_3	0.09	Inferred from [22]
γ_4	0.1	[22]
β_1	0.1134	Inferred from [22]
β_2	0.3969	Inferred from [22]
β_3	0.4455	Inferred from [22]
β_4	0.7209	Inferred from [22]
δ_1	0.02	Inferred from [22]
ρ	0.56	Estimated
σ	0.75	Estimated

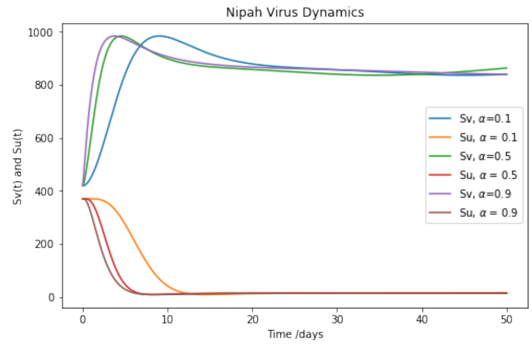
TABLE 4.1. Description of the parameter

Parameter	Value	Source
δ_2	0.15	[22]
δ_3	0.0171	Inferred from [22]
δ_4	0.2	Inferred from [22]
a_1	0.58	Inferred from [22]
a_2	0.513	Inferred from [22]
a_3	0.486	Inferred from [22]
a_4	0.513	Inferred from [22]
a_5	0.000288	Inferred from [22]
b_1	0.69	Inferred from [22]
b_2	0.522	Inferred from [22]
b_3	0.513	Inferred from [22]
b_4	0.504	Inferred from [22]
b_5	0.000324	Inferred from [22]
q_1	0.75	Inferred from [22]
q_2	0.4617	Inferred from [22]
q_3	0.531	Inferred from [22]
q_4	0.513	Inferred from [22]
q_5	0.000648	Inferred from [22]
ε	0.03	Estimated
z_2	0.4374	Inferred from [22]
z_3	0.504	Inferred from [22]
z_4	0.513	Inferred from [22]
z_5	0.000648	Inferred from [22]
μ_p	0.00081	Estimated
μ	0.0003421	[5]

TABLE 4.2. Description of the parameter



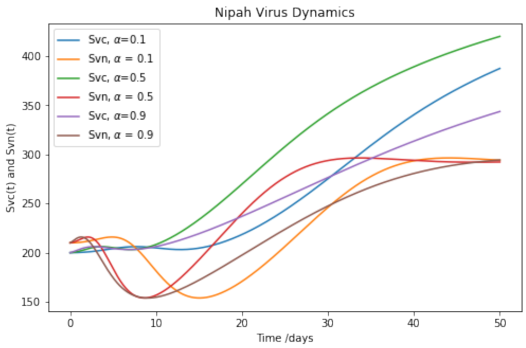
(A) Susceptible individuals S



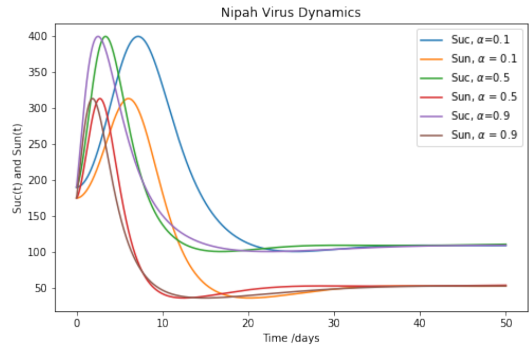
(B) Susceptible vaccinated S_v and unvaccinated S_u

FIGURE 4.1. The Susceptible Population

For different fractional orders α, Figure (4.1) shows how susceptible individuals behave over time. We see that the number of susceptible people declines with time for all values of fractional order α, but it never reaches zero because of population recruitment. Yet when the value of the fractional order α rises, the proportion of susceptible people falls off more quickly over time. The susceptible population, both vaccinated and unvaccinated, is shown in Figure (4.1b). Due to the introduction of the vaccine, the susceptible population was marginally altered as increased but remained steady while responding to the virus. On the other hand, because to the lack of vaccine administration, the susceptible unvaccinated population dramatically shrank and eventually disappeared. Hence, the immune system was compromised. The number of susceptible people who have received vaccinations, on the other hand, rises over time as the value of fractional order α rises, whereas the number of susceptible people who have not received vaccinations falls with time. This suggests that the fractional order derivatives of the dynamical variables are more useful for estimating the proportion of susceptible, susceptible vaccinated, and susceptible unvaccinated people.



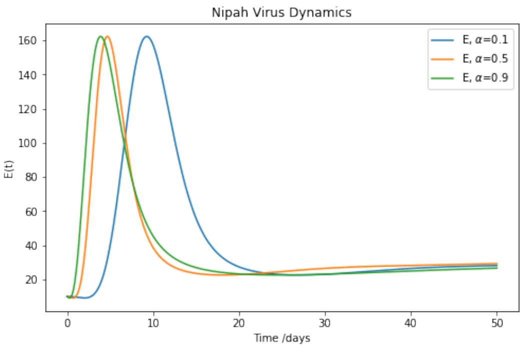
(A) Susceptible vaccinated: condoms S_{vc} and without condoms S_{vn}



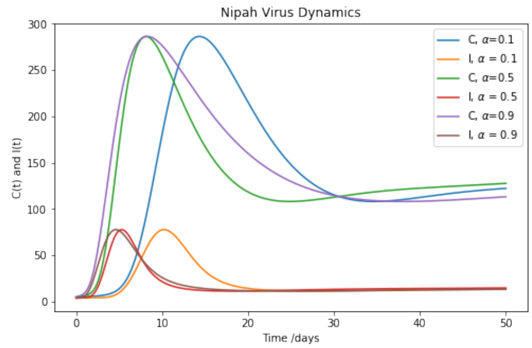
(B) Susceptible unvaccinated: condoms S_{uc} and without condoms S_{un}

FIGURE 4.2. **Susceptible vaccination and unvaccination with and without condom Population**

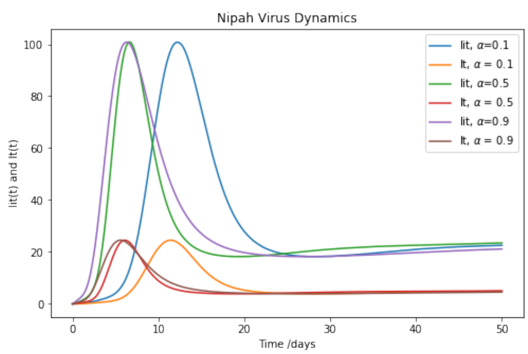
The relationship between susceptible vaccine individuals using condoms and susceptible vaccine individual not using condoms, as well as time for various fractional orders, is shown in Figure (4.2a). The graph showed how vaccination with a condom outperformed vaccination without a condom, which fluctuates a lot. When the fraction order is average, we saw both rises and normalized after a while. This suggests that even though both are immunized, sticking to condom use will be a helpful technique to stop the Nipah virus from spreading. The behavior of susceptible unvaccinated individuals using condom and susceptible unvaccinated individuals not using condoms over time for various fractional orders α is shown in Figure (4.2b). When the virus was first discovered, they surged, but as more individuals became exposed to it, they began to decline quickly. The number of susceptible unvaccinated individuals using condoms did not really decline at the same pace as the susceptible unvaccinated individuals not using condoms, according to our Figure(4.2b). Also, it implies that condom usage is necessary whether or not you have had a vaccination. As the fractional order goes up, both go down.



(A) Exposed individuals E



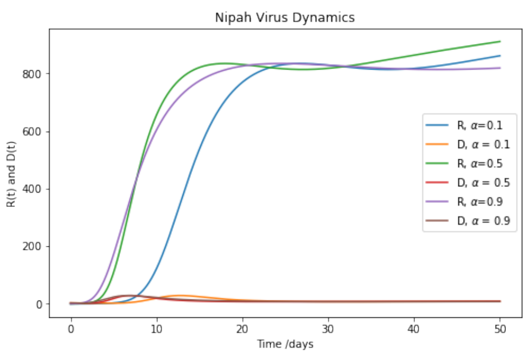
(B) Nipah virus carries and Infectious individual



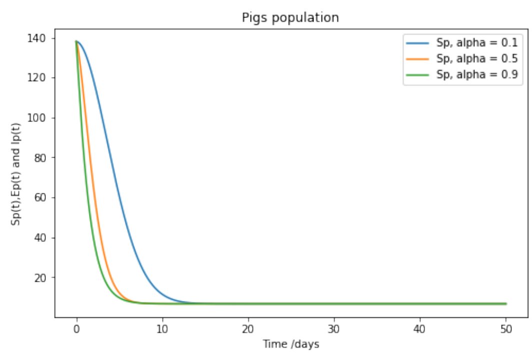
(C) infectious undergoing treatmentI_t and infectious isolated undergoing treatment I_{it}

FIGURE 4.3. Exposed individuals,Nipah Virus Carriers(Asymptomatic) and the Infectious Population

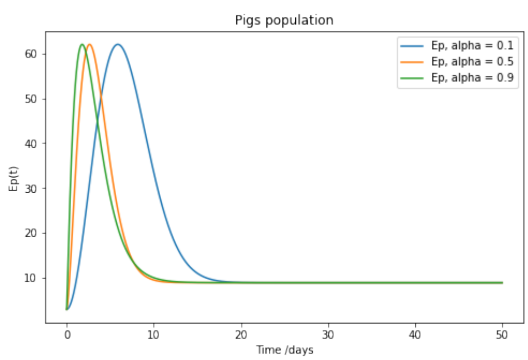
Figure (4.3) depicts the temporal behavior of the fractional orders α of the number of exposed persons, carriers of the Nipah virus, and infected individuals. When humans were exposed to the virus quickly at first, the exposed population rapidly climbed, peaked, and then gradually decreased. The infectious I, infectious undergoing treatmentI_t, infectious isolated undergoing treatment I_{it} and Nipah virus caries C populations all grew before they reduced, but Nipah virus caries population increased the most since it exhibits no symptoms and infects more members of the community as a result. The infectious undergoing treatment's population also increased and later decreased with time but the infectious isolated-treated population increased the most because more infectious people are going for isolation with treatment and few infectious people are going for hospitals for treatment. We observed that when the value of the fractional order α is small, the proportion of exposed individuals, Nipah virus carriers,infectious, infectious undergoing treatment,and infectious isolated undergoing treatment individuals falls slowly over time. This demonstrates that the Nipah virus carrier has to be taken seriously and that the asymptomatic population needs to be tested more often in order to identify and isolate them from the population.



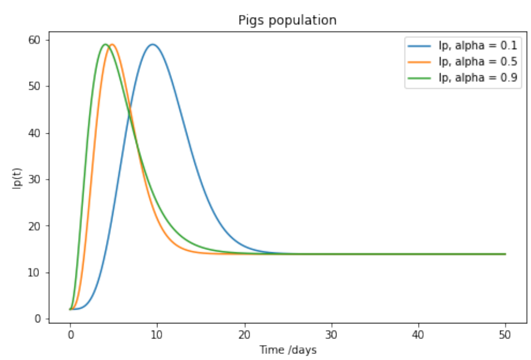
(A) Recovered individuals and the dead bodies compartment



(B) Susceptible pigs



(C) Exposed pigs



(D) Infectious pigs

FIGURE 4.4. Recovered Population, Dead Bodies Population and Pigs Population

Figure (4.4a) illustrates the behavior of recovered people and the population of deceased bodies over time. The graph clearly shows that for all values of α, the number of recovered persons rises with time and eventually stays constant. It may also be inferred that the effect of vaccinations causes an increase in the number of recovered people. After some time, the population that had recovered reached its apex, and the population of dead bodies had almost completely disappeared. Figure (4.4b)-(4.4d) depicts the behavior of the pig population over time at various fractional order levels. The number of susceptible pigs declines as more pigs become exposed and sick. First growing and then declining but never reaching zero was the number of infected and exposed pigs. The exposed and infected pigs grow slowly and eventually diminish slowly while the susceptible pigs decrease slowly when the fractional order is tiny.

5. CONCLUSION

A derivative of AB made up of a Mittag-Leffler kernel has been described by Atangana and Baleanu. We introduce the fractional with vaccination and condoms linked with the model for the first time by the idea of Atangana and Baleanu derivatives in order to see further applications of these fractional derivatives and better investigate Nipah virus dynamics. As with the fixed point approach, our goal is to provide the prerequisites for the models' existence and uniqueness as solutions. To comprehend the efficacy of the fractional order α as well as vaccine and condoms, numerical calculations for these fractional models have been carried out. These simulations show that, depending on the various fractional orders, increasing vaccination and condom use result in a reduction in the Nipah virus's ability to propagate. The description of the Nipah virus's mechanics in light of vaccines and condoms is where we believe the current research will be most helpful.

Finally, the utilization of the Atangana-Baleanu derivative in solving differential equations with memory effects and non-local behaviors offers significant advantages in understanding and modeling complex systems. By incorporating this fractional derivative operator, we can capture the long-term memory effects and non-local behaviors exhibited by various real-world phenomena more accurately. Through the application of the Atangana-Baleanu derivative, we have gained deeper insights into the dynamics and behaviors of systems that cannot be adequately described by classical differential equations. The inclusion of memory effects in the modeling process has allowed us to achieve a more comprehensive and accurate representation of the underlying dynamics.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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