Available online at http://scik.org
Commun. Math. Biol. Neurosci. 2024, 2024:5
https://doi.org/10.28919/cmbn/8359
ISSN: 2052-2541

A NEW MATHEMATICAL MODEL OF DRINKING ALCOHOL AMONG DIABETES POPULATION TAKING ANTI-DIABETIC DRUGS: AN OPTIMAL CONTROL APPROACH

IKRAM IMKEN*, NADIA IDRISSI FATMI

LIPIM Laboratory ENSA Khouribga, Sultan Moulay Slimane University, Beni Mellal, Morocco

Abstract. In this work, we present a new nonlinear mathematical model, describing the spread of high-risk alcohol consumption among people living with diabetes Type 2 and drug interactions with some diabetes medications with three control strategies to gain insights into this increasingly concerned about health and social phenomenon. Some properties of the solutions to the model including positivity, boundedness and existence are shown. The optimal control strategies are derived by proposing an objective functional and using Pontryagin's Maximum Principle. Numerical simulations are also conducted in the analytic results.

Keywords: diabetes; anti-diabetic drugs; alcohol; mathematical model; optimal control.

2020 AMS Subject Classification: 92C50.

1. Introduction

Alcohol misuse is the excessive consumption of alcohol. It is the inability to control drinking, even when it negatively affects a person's life. The person consuming alcohol may develop tolerance and experience withdrawal symptoms when trying to cut back.

A national survey published in 2019 reported that 14.1 million adults (5.6%) and 414,000 adolescents aged 12-17 years (1.7%) were experiencing AUD in 2019 in the United States.

*Corresponding author
E-mail address: ikramimken@gmail.com
Received November 21, 2023

Over time, excessive alcohol use can lead to the development of chronic diseases and other serious problems including: High blood pressure, heart disease, stroke, liver disease, and digestive problems [1], Cancer of the breast, mouth, throat, esophagus, voice box, liver, colon, and rectum [1, 2] weakening of the immune system [1, 2], increasing mental health problems, including depression and anxiety[1]. Social problems, including family problems, job-related problems, and unemployment.[1] Alcohol use disorders, or alcohol dependence [3]. The World Health Organization (WHO) estimated there were 283 million people with alcohol use disorders worldwide as of 2016 which need treatment in hospitals and private or public addiction treatment centers [4]. Alcoholism remains one of the most frequent and important topics discussed by the world community due to its dire consequences [5].

Diabetes [6] is a condition that happens when your blood sugar (glucose) is too high. It develops when your pancreas doesn't make enough insulin or any at all, or when your body isn't responding to the effects of insulin properly [7]. Approximately 37.3 million people in the United States have diabetes, which is about 11% of the population. Type 2 diabetes is the most common form, representing 90% to 95% of all diabetes cases [8].

Alcohol And Diabetes 2:

Mixed together, alcohol and diabetes can spell trouble , alcohol can raise the risk of heart problems for people with diabetes, who are already at an increased risk of heart disease, it can impact your blood sugar and make it difficult for you to identify when problems arise. It can be problematic because of how it impacts blood sugar and other conditions that occur in people with type 2 diabetes. According to a study published by the Journal of the American Heart Association [9], people with type 2 diabetes already at high risk for cardiovascular disease who drink alcohol, even moderately, have an increased risk of high blood pressure. Uncontrolled high blood pressure can increase the likelihood of other serious health problems, including diabetic retinopathy, diabetic nephropathy, diabetic neuropathy, cardiovascular disease (CVD) as a macrovascular complication. The liver regulates blood sugar levels and filters alcohol, but it is not good at doing both at the same time. So, when faced with both tasks, the liver will choose to filter alcohol and stop working to control blood sugar levels. This can lead to hypoglycemia [10]. Low blood sugar (hypoglycaemia) it may include symptoms such as hunger, rapid heartbeat, confusion, irritability, sweating, trembling, and weakness. It can even progress to clumsiness, difficulty speaking, disorientation, loss of consciousness, seizures, or death. One of the significant problems with alcohol-induced hypoglycemia is that the symptoms of hypoglycemia and intoxication can be similar.

Alcohol drug interactions with diabetes medications [11]

- Drug interactions with some diabetes medications can be serious or life-threatening. Consuming alcohol with some medications can lead to dangerously low blood sugar because the alcohol interferes with the liver's ability to regulate blood sugar (called hepatic gluconeogenesis). Many type 2 diabetes medications are also available in combination, increasing the risk for multiple drug-alcohol interactions.
- The mix of alcohol with metformin can increase the risk of a rare but dangerous condition called lactic acidosis. Get emergency medical help if you have any of these symptoms of lactic acidosis: fatigue, weakness, increasing sleepiness, slow or irregular heart beat, cold feeling (chills or shivering), muscle pain, shortness of breath, stomach area pain.
- When alcohol is combined with insulin, the glucose lowering effect of insulin may be increased or decreased. Both hypoglycemia (low blood sugar) and hyperglycemia (high blood sugar) may occur, depending on how much and how often you drink.

Alcohol And diabetic Complications [12]

- Peripheral Neuropathy: This is a type of nerve damage common to those with diabetes. It can cause pain, tingling, or numbness in the hands and feet. Drinking alcohol can intensify these symptoms as well as make them more difficult to treat. Diabetes and alcohol consumption are the two most common underlying causes of peripheral neuropathy.
- Diabetic Retinopathy : This is a common complication of diabetes that damages the blood vessels in your retina (the back part of your eye). Drinking alcohol can cause these blood vessels to leak or swell, further damaging your vision. If you are suffering from this diabetic eye disease, it is best to have a comprehensive dilated eye exam at

least once a year. As a common complication of diabetes, diabetic retinopathy is a leading cause of blindness.

- Uncontrollable Blood Pressure: If you have diabetes, your blood pressure is already at an increased risk. Drinking alcohol can make diabetes disease control harder to keep at bay, which can lead to serious health problems such as a stroke.
- Kidney Damages: Alcohol affects the way kidneys regulate fluid and electrolytes in the body. They are less likely to do their job to filter the blood and remove toxins when you drink alcohol. Also, alcohol dries out the body, which can lead to dehydration that results in cells and organs not working properly.
- High Triglycerides: Triglycerides are a type of fat in your blood. When you drink, your liver is less effective at removing triglycerides from the bloodstream. This can increase your risk for heart disease.

Several mathematicians did a lot of work in order to understand the dynamics and analysis of drinking and reduce its harm on the drinker and society as well as minimizing the number of addicted drinkers. For example, S. H. Ma et al [13] modeled alcoholism as a contagious disease and used an optimal control to study their mathematical model with awareness programs and time delay. S. Sharma et al [14] developed a mathematical model of alcohol abuse and discussed the existence and stability of drinking-free, endemic equilibria and sensitivity analysis of R_0.
B. Benedict [15] used a SIR-type model with a contact rate between susceptibles and alcoholism, he getted alcoholism reproductive number and discussed the existence and stability of two equilibria. H. F. Huo et al [16] proposed a new social epidemic model to depict alcoholism with media coverage which was proven to be an effective way in pushing people to quit drinking. I. k. Adu et al [17] used a non-linear SHT R mathematical model to study the dynamics of drinking epidemic. They divided their population into four classes: non-drinkers (S), heavy drinkers (H), drinkers receiving treatment (T) and recovered drinkers (R). They discussed the existence and stability of drinking-free and endemic equilibrium. Other mathematical models have also been widely used to study this phenomenon (For example, [18],[19],[20]...). Among the first studies of COVID-19 in population living with diabetes is [21] this [21] emphasizes

the impact of covid on diabetic individual which lead the idea of creation a new model of alcohol consumption in a specific population with a chronic disease, namely diabetes to clarify the dangerous of associated of alcohol consumption on with anti-diabetic drugs.

The novelty of this study lies in the selection of a specific population with a chronic disease, namely diabetes, to clarify the direct impact of alcohol on diabetic individuals. More precisely, the main contributions of this paper are summarized as follows:

- Creation of a new compartmental model which D,C,M_D, H_D,C_S, T_C,R_A stand for: potential drinkers living diabetes type 2 without complications whose age is over adolescence and adulthood, people living diabetes with complications using insulin or medication and are susceptible of drinking alcohol, moderate drinkers with diabetes, heavy drinkers taking anti-diabetic medication, drinkers with dangerous complications, the number of diabetic drinkers(moderate, heavy, drinkers with dangerous complications) who join public treatment centers of alcohol addiction, the person quitters of drinking.
- Seeking effective optimal control strategies that minimize the amount of heavy drinkers taking anti diabetic medication and drinkers with dangerous complications and maximizing them for joining center treatment. To satisfy this objective, we introduce optimal control strategies related to three sorts of controls: The awareness program through media and education by raising awareness of the seriousness of the negative impact of moderate diabetic drinkers on diabetic with complications, the effort that prevents the failure of treatment by providing financial and psychological service to diabetic patients at addiction treatment center, can consolidate the number of medical staff in the centers and increase the monitoring of people at risk until they complete successfully their treatment, measure the effort of treatment applied on the heavy diabetic drinkers taking anti-diabetic drugs to be moderate drinkers

This paper is organized as follows: In section 2 we present our mathematical model that describe the classes of drinkers with diabetes, and we give some properties of boundedness, positivity and existence of solution, then we give in section 3 the optimal control problem for the proposed model where we give some results concerning the existence of the optimal controls and we characterize these optimal controls using Pontryagin's Maximum Principle in discrete time. Numerical simulations are given in Section 4, and we concluded the paper in section 5.

2. Mathematical Model

2.1. Description Of The Model. We propose a continuous model $DCM_DH_DC_ST_CR_A$ to describe the interaction between drinkers classes in population living with diabetes, the graphical representation of the proposed model is shown as follows :

The schematic diagram leads to the following system of ordinary differential equations:

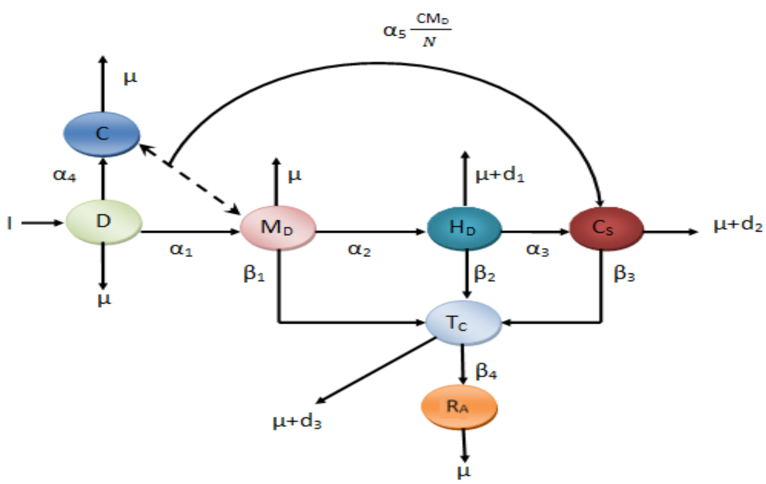


FIGURE 1. compartments Model

$$
(1) \qquad \begin{cases}
\dfrac{dD}{dt} = I - \alpha_1 \dfrac{DM_D}{N} - (\alpha_4 + \mu) D \\
\dfrac{dM_D}{dt} = \alpha_1 \dfrac{DM_D}{N} - (\alpha_2 + \mu + \beta_1) M_D \\
\dfrac{dC}{dt} = \alpha_4 M_D - \alpha_5 \dfrac{CM_D}{N} - \mu C \\
\dfrac{dH_D}{dt} = \alpha_2 M_D - (\beta_2 + \alpha_3 + (\mu + d_1)) H_D \\
\dfrac{dC_S}{dt} = \alpha_3 H_D - (\mu + d_2 - \beta_3) C_S + \alpha 5 \dfrac{CM_D}{N} \\
\dfrac{dT_C}{dt} = \beta_1 M_D + \beta_2 H_D + \beta_3 C_S - (\mu + d_3) T_C - \beta_4 T_C \\
\dfrac{dR_A}{dt} = \beta_4 T_C - \mu R_A
\end{cases}
$$

The model in equation (1) subdivides human population into mutually exclusive compartments:

- D: represents the potential drinkers living diabetes type 2 without complications whose age is over adolescence and adulthood.
- C: refers people living diabetes with complications using insulin treatment like Peripheral Neuropathy, Diabetic Retinopathy, Uncontrollable Blood Pressure, High Triglycerides,....
- M_D: is composed of moderate drinkers with diabetes.
- H_D: contains heavy drinkers who are addicted to alcohol, while managing their diabetes and taking anti-diabetic medication including chlorpropamide (Diabinese), metformin, and troglitazone.....
- C_S: drinkers with dangerous complications as cardiovascular disease, alcohol may increase triglyceride levels and increase blood pressure and also alcohol can cause flushing, nausea, increased heart rate, and slurred speech......
- T_C: represents the number of diabetic drinkers(moderate, heavy, drinkers with dangerous complication) who join public treatment centers of alcohol addiction.
- R_A: represent the person quitters of drinking.

Each population represents a proportion of the total population. The total population at time t, denoted by $N(t)$, is given by $N(t) = D(t) + C(t) + M_D(t) + H_D(t) + C_S(t) + T_C(t) + R_A(t)$. (Figure 1)provides a schematic representation of model interactions between the compartment of the epidemiological model. The arrows represent transition between compartments.
The evolution of the model depends on several parameters. All the parameters are non negative constants, and they are described as follows:

- α_1: the rate of effective contact with the moderate drinkers.
- α_2: the rate of moderate diabetic drinkers becomes heavy drinkers.
- α_3: the rate of heavy drinkers taking anti-diabetic medicine who have been with dangerous complications according Reference [22].
- α_4: the rate of diabetic people without complications becomes diabetic with complications.

- α_5: the rate of negative effect of moderate consumption on diabetic people with complications according reference [23].
- β_1: the rate of moderate drinkers which join the treatment center for listing and guiding.
- β_2: the rate of heavy drinkers with precautions who join the treatment centers.
- β_3: the rate of diabetic drinkers people whit dangerous complications who join the treatment centers.
- β_4: the rate of people who quitting alcohol.
- d_1: hypoglycemic coma [24].
- d_2: mortality rate due dangerous complication specially cardiovascular disease [9].
- d_3: the rate of mortality in the public center of treatment due to a lack of medical staff.
- μ: natural mortality.
- I: denote the incidence of diabetes-alcohol.

2.2. Some Proprieties Of The Model.

2.2.1. *Positivity Of Solution Of The Model.* It is necessary to prove that all solutions of system (1) with positive initial data will remain positive for all times $t > 0$. This will be established by the following theorem.

Theorem 1. *If* $D(0) \geq 0, M_D(0) \geq 0, H_D(0), T_C(0) \geq 0, C(0) \geq 0$ *and* $C_S \geq 0, R_A(0) \geq 0$*, then the solution of system* (1) $D(t), M_D(t), H_D(t), C(t), T_C(t), C_S(t), R_A(t)$ *are positive for all* $t \geq 0$.

Proof. From the first equation of system (1) we have

$$\frac{dD(t)}{dt} = I - \big(\alpha_1 \frac{M_D(t)}{N} + \mu + \alpha_4\big)D(t) \tag{2}$$

Let

$$B(t) = \alpha_1 \frac{M_D(t)}{N} + \mu + \alpha_4$$

We multiply equation (2) by $\exp\big(\int_0^t B(s)ds\big)$ to obtain

$$\frac{dD(t)}{dt} \times \exp\big(\int_0^t B(s)ds\big) = \big[I - B(t)D(t)\big] \times \exp\big(\int_0^t B(s)ds\big)$$

which implies that

$$\frac{dD(t)}{dt}\times\exp\big(\int_0^t B(s)ds\big)+B(t)D(t)\times\exp\big(\int_0^t B(s)ds\big)=I\times\exp\big(\int_0^t B(s)ds\big)$$

Therefore

$$\frac{d}{dt}\big[D(t)\times\exp\big(\int_0^t B(s)ds\big)\big]=I\times\exp\big(\int_0^t B(s)ds\big)$$

Taking integral with respect to s from 0 to t , we obtain

$$D(t)\times\exp\big(\int_0^t B(s)ds\big)-D(0)=I\times\int_0^t\big(\exp\big(\int_0^w B(s)ds\big)dw$$

Multiplying equation by $\exp\big(\int_0^t B(s)ds\big)$ we obtain

$$D(t)-D(0)\times\exp\big(-\int_0^t B(s)ds\big)=I\times\exp\big(-\int_0^t B(s)ds\big)\times\bigg(\int_0^t\exp\big(\int_0^w B(s)ds\bigg)dw$$

Then, we can obtain

$$D(t)=D(0)\times\exp\big(-\int_0^t B(s)ds\big)+I\times\exp\big(-\int_0^t B(s)ds\big)\times\int_0^t\exp\big(\int_0^w B(s)ds\big)dw\geq 0$$

So the solution $D(t)$ is positive.

Similarly from the second equation of system (1) we have:

$$M_D(t)\geq M_D(0)\exp\big(-\int_0^t B_1(s)ds\bigg)\geq 0$$

where

$$B_1(t)=\alpha_1\frac{D(t)}{N}-\big(\alpha_1+\mu+\beta_1\big)$$

Similarly, from the third fourth and fifth equation of system (1), we have

$$H_D(t)\geq H_D(0)\exp\big(-\big(\beta_2+\alpha_3+\mu+d_1\big)t\big)\geq 0$$

$$C_S(t)\geq C_S(0)\exp\big(-\big(\beta_3+\mu+d_2\big)t\big)\geq 0$$

$$C(t)\geq C(0)exp\big(-\int_0^t B_2(s)ds\big)\geq 0$$

where

$$B_2(s)=-\alpha_5\frac{M_D(t)}{N}-\mu$$

$$T_C(t)\geq T_C(0)\exp\big(-\big(\beta_4+\mu+d_3\big)t\big)$$

$$R_A(t)\geq R_A(0)\exp\big(-\mu t\big)\geq 0$$

Therefore, we can see that $D(t) \geq 0, M_D(t) \geq 0, H_D(t) \geq 0, C(t) \geq 0, C_S(t) \geq 0, T_C(t) \geq 0$ and $R_A(t) \geq 0, \forall t \geq 0$ and this complete the proof. □

2.2.2. *Boundedeness Of Trajectories.* The model under consideration monitors population as such, we assume that all the variables and parameters of the model are positive for all $t \geq 0$. In order to show that the solution of model system (1) is bounded it is needed to prove that the total population size $N(t)$ is bounded.

Lemma 1. All feasible solution $D(t), M_D(t), H_D(t), C(t), C_S(t), T_C(t), R_A(t)$ of system equation (1) are bounded by the region.

$$\Omega = \left\{ \left(D, M_D, H_D, C, C_S, T_C, R_A\right) \in \mathrm{IR}^7_+ ; D + M_D + H_D + C + C_S + T_C + R_A \leq \frac{I}{\mu} \right\}$$

Proof. from system equation (1).

$$\frac{dN(t)}{dt} = \frac{dD(t)}{dt} + \frac{dM_D}{dt} + \frac{dC_D}{dt} + \frac{dH_D}{dt} + \frac{dC_S}{dt} + \frac{dT_C}{dt} + \frac{dR_A}{dt}$$

$$\frac{dN(t)}{dt} = I - \mu N(t) - \left(d_1 H_D(t) + d_2 C_S(t) + d_3 T_C(t)\right)$$

implies that :

$$\frac{dN(t)}{dt} \leq I - \mu N(t)$$

and it follows that

$$N(t) \leq \frac{I}{\mu} + N(0) e^{-\mu t}$$

where $N(0)$ is the initial value of total number of people;

thus ,

$$\limsup_{t \to \infty} N(t) \leq \frac{I}{\mu}$$

then

$$D(t) + M_D(t) + H_D(t) + C(t) + C_S(t) + T_C(t) + R_A(t) \leq \frac{I}{\mu}$$

Hence, fo analysis of model (1), we get the region which is given by the set

$$\Omega = \left\{ \left(D, M_D, H_D, C, C_S, T_C, R_A\right) \in \mathrm{IR}^7_+ ; D + M_D + H_D + C + C_S + T_C + R_A \leq \frac{I}{\mu} \right\}$$

□

2.3. Existance of Solution. let

$$X(t)=\big(D(t),M_D(t),C(t),H_D(t),C_S(t),T_C(t),R_A(t)\big)^T$$

The system can be writing as follows

$$\frac{dX}{dt}=AX+F(X)=G(X)$$

$$A=\begin{pmatrix} a_{11} & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & a_{22} & 0 & 0 & 0 & 0 & 0\\ 0 & a_{32} & a_{33} & 0 & 0 & 0 & 0\\ 0 & a_{42} & 0 & a_{44} & 0 & 0 & 0\\ 0 & 0 & 0 & a_{54} & a_{55} & 0 & 0\\ 0 & a_{62} & 0 & a_{64} & a_{65} & a_{66} & 0\\ 0 & 0 & 0 & 0 & 0 & a_{76} & a_{77} \end{pmatrix}$$

Where

$$\begin{array}{lllll} a_{11}=-\big(\alpha_4+\mu\big) & a_{22}=-\big(\alpha_4+\mu+\beta_1\big) & a_{32}=\alpha_4 & a_{33}=-\mu & a_{42}=\alpha_2\\ a_{44}=-\big(\beta_2+\alpha_3+\mu+d_1\big) & a_{54}=\alpha_3 & a_{55}=-\big(\beta_3+\mu+d_2\big) & a_{62}=\beta_1 & a_{64}=\beta_2\\ a_{65}=\beta_3 & a_{66}=-\big(\mu+d_3+\beta_4\big) & a_{76}=\beta_4 & a_{77}=-\mu & \end{array}$$

$$F(X)=\begin{pmatrix} I-\alpha_1\frac{DM_D}{N}\\ \alpha_1\frac{DM_D}{N}\\ -\alpha_5\frac{CM_D}{N}\\ 0\\ \alpha_5\frac{CM_D}{N}\\ 0\\ 0 \end{pmatrix}$$

The function F satisfies

$$|F(X_1)-F(X_2)|\leq M_1|X_1-X_2|$$

where M_1 is positive constant, independent of state variable and

$|X_1 - X_2|$

$= |D_1 - D_2| + |M_{D_1} - M_{D_2}| + |C_1 - C_2| + |H_{D_1} - H_{D_2}| + |C_{S_1} - C_{S_2}| + |T_{C_1} - T_{C_2}| + |R_{A_1} - R_{A_2}|$

Moreover, one has

$$|G(X_1) - G(X_2)| \leq M|X_1 - X_2| where M = max\big(M_1 + ||A||\big) < \infty$$

This it follows that the function G is uniformly Lipschitz continuous and the restriction of the state variable, we conclude that there exist a solution of the system (1).

3. Optimal Control Problem

In this section, we present our optimal control problem discuss the existence of the optimal control and then give a characterization of control terms .

Note the state variable R_A does not appear in the first equation of system (1), so the other variable do not depend on R_A, and we can limit our study to the following system.

$$(3)\qquad \begin{cases} \dfrac{dD}{dt} = I - \alpha_1 \dfrac{DM_D}{N} - \big(\alpha_4 + \mu\big)D \\ \dfrac{dM_D}{dt} = \alpha_1 \dfrac{DM_D}{N} - \alpha_2\big(1 - v_3\big)M_D - \big(\mu + \beta_1\big)M_D \\ \dfrac{dC}{dt} = \alpha_4 D - \alpha_5\big(1 - v_1\big)\dfrac{CM_D}{N} - \mu C \\ \dfrac{dH_D}{dt} = \alpha_2\big(1 - v_3\big)M_D - \beta_2 H_D - \alpha_3 H_D - \big(\mu + d_1)\big)H_D \\ \dfrac{dC_S}{dt} = \alpha_3 H_D - \beta_3\big(1 + v_2\big)C_S - \big(\mu + d_2\big)C_S + \alpha_5\big(1 - v_1\big)\dfrac{CM_D}{N} \\ \dfrac{dT_C}{dt} = \beta_1 M_D + \beta_2 H_D + \beta_3\big(1 + v_2\big)C_S - \big(\mu + d_3 + \beta_4\big)T_C \end{cases}$$

where:

v_1: The awareness program through media and education by raising awareness of the seriousness of the negative impact of moderate consumption on diabetic with complications using insulin treatment according to the reference [11].

v_2: The effort that prevents the failure of treatment by providing financial and psychological service to diabetic patients at addiction treatment center, can consolidate the number of medical staff in the centers and increase the monitoring of people at risk until they complete successfully their treatment [25].

v_3: Measure the effort of treatment applied on the heavy diabetic drinkers to be moderate drinkers [26].

The control strategy aims at minimizing the number of heavy drinkers H_D and the drinkers with dangerous complications and maximize the number who join the center of treatment as well as minimizing the cast of this strategy mathematically.

For a fixed terminal time t_f the problem is to minimize the objective functional

$$J\big(v_1,v_2,v_3\big) = H_D(t_f)+C_S(t_f)-T_C(t_f)+\int_0^{t_f}\left\{H_D(t)+C_S(t)-T_C(t)+\sum_{i=1}^{3}\frac{A_i}{2}v_i^2(t)\right\}dt$$

where $A_i \geq 0$ $\big($ for $i=1,2,3\big)$ denote weights that blance the size of terms. In others words, we seek the optimal values v_1^*, v_2^* and v_3^* of the controls v_1, v_2 and v_3 such that

$$J\big(v_1^*,v_2^*,v_3^*\big) = \min\left\{J\big(v_1,v_2,v_3\big)/\big(v_1,v_2,v_3\big)\in U\right\}$$

with U is the set of admissible controls defined by

$$U=\left\{\Big(v_1(.),v_2(.),v_3(.)\Big)\in\Big(L^\infty\big(0,t_f\big)^3\Big)/0\leq v_1(t),v_2(t),v_3(t)\leq 1,\forall t\in\big[0,t_f\big]\right\}$$

Remark 1. *In following, to avoid some mathematical complexities, we consider that the total population N remains constant during the control period.*

In order to derive the necessary conditions for the optimal control, the Pontryagin's Maximum Principale given in [27] is used. The principal converts the problem (3) into a problem of minimizing a Hamiltonian H, defined by .

$$\begin{aligned}H &= H_D+C_S-T_C+\sum_{i=1}^{3}A_iv_i^2+\lambda_1\big(I-\alpha_1\frac{DM_D}{N}-\big(\alpha_4+\mu\big)D\big)+\lambda_2\big(\alpha_2\frac{DM_D}{N}-\alpha_2\big(1-v_3\big)M_D\\&-\big(\mu+\beta_1\big)M_D\big)+\lambda_3\big(\alpha_4M_D-\alpha_5\big(1-v_1\big)\frac{CM_D}{N}-\mu C\big)+\lambda_4\big(\alpha_2\big(1-v_3\big)M_D-\beta_2H_D-\alpha_3H_D\\&-\big(\mu+d_1\big)H_D\big)+\lambda_5\big(\alpha_3H_D-\beta_3\big(1+v_2\big)C_S+\alpha_5\big(1-v_1\big)\frac{CM_D}{N}\big)+\lambda_6\big(\beta_3M_D-\beta_2H_D\\&+\beta_3\big(1+v_2\big)C_S-\big(\mu+d_3+\beta_4\big)T_C\big)\end{aligned}$$

By applying Pontrygain's Maximum Principal and the existence result for optimal control and corresponding optimal states form the study [26], we obtain the following theorem.

Theorem 2. *Consider the optimal problem* (3)*, there exists an optimal control* $\left(v_1^*, v_2^*, v_3^*\right) \subset U$ *and corresponding solution* $D^*, M_D^*, C^*, H_D^*, C_S^*, T_C^*$ *such that*

$$J\left(v_1^*, v_2^*, v_3^*\right) = \min_{\left(v_1, v_2, v_3\right) \in U} J\left(v_1, v_2, v_3\right)$$

Furthermore, there exists adjoint functions $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6$ *such that*

$$\begin{aligned}
\lambda_1' &= \lambda_1\left(\alpha_1 \frac{M_D}{N} + \alpha_4 + \mu\right) - \lambda_2 \alpha_1 \frac{M_D}{N} - \lambda_3 \alpha_4 \\
\lambda_2' &= \lambda_1 \alpha_1 \frac{D}{N} - \lambda_2 \frac{D}{N} \alpha_1 + \lambda_2\left(\mu + \beta_1 + \alpha_2\left(1 - v_3\right)\right) + \lambda_3 \alpha_5\left(1 - v_1\right) \frac{C}{N} - \lambda_4\left(\alpha_2\left(1 - v_3\right)\right) \\
&\quad - \lambda_5 \alpha_5 \left(1 - v\right) \frac{C}{N} - \lambda_6 \beta_1 \\
\lambda_3' &= \lambda_3 \lambda_5\left(1 - v_1\right) \frac{M_D}{N} + \mu \lambda_3 - \lambda_5 \alpha_5\left(1 - v_1\right) \frac{M_D}{N} \\
\lambda_4' &= -1 + \lambda_4\left(\beta_2 + \alpha_3 + \mu + d_1\right) - \lambda_5 \lambda_3 - \beta_2 \lambda_6 \\
\lambda_5' &= -1 + \lambda_5\left(\mu + d_2 + \beta_3\left(1 + v_2\right)\right) - \lambda_6\left(\beta_3\left(1 + v_2\right)\right) \\
\lambda_6' &= 1 + \lambda_6\left(\mu + d_3 + \beta_4\right)
\end{aligned}$$

With the transversality conditions

$$\lambda_1(t_f) = 0, \qquad \lambda_2(t_f) = 0, \quad \lambda_3(t_f) = 0, \qquad \lambda_4(t_f) = 1, \quad \lambda_5(t_f) = 1, \qquad \lambda_6(t_f) = -1$$

Moreover, the optimal controls v_i^* $\left(\textit{for } i = 1, 2, 3\right)$ *given by*

$$\begin{aligned}
v_1^*(t) &= \min\left(1, \max\left(0, \frac{\alpha_5 C M_D\left(\lambda_5 - \lambda_3\right)}{N A_1}\right)\right) \\
v_2^*(t) &= \min\left(1, \max\left(0, \frac{\beta_3 C_s\left(\lambda_6 - \lambda_5\right)}{A_2}\right)\right) \\
v_3^*(t) &= \min\left(1, \max\left(0, \frac{\alpha_2 M_D\left(\lambda_4 - \lambda_2\right)}{A_3}\right)\right)
\end{aligned}$$

Proof. The existence of the optimal control is obtained from Fleming and Rishel [28] $\big($see detailed proof (I.Imken and I.D.Fatmi [21]).

The adjoint equations and transversality condition can be obtained by using Pontraygin's Maximum Principal in the state and control variables [28] such that

$$\lambda_1^{'} = -\frac{\partial H(t)}{\partial D(t)}$$
$$\lambda_2^{'} = -\frac{\partial H(t)}{\partial M_D(t)}$$
$$\lambda_3^{'} = -\frac{\partial H(t)}{\partial C(t)}$$
$$\lambda_4^{'} = -\frac{\partial H(t)}{\partial H_D(t)}$$
$$\lambda_5^{'} = -\frac{\partial H(t)}{\partial C_S(t)}$$
$$\lambda_3^{'} = -\frac{\partial H(t)}{\partial T_C(t)}$$

$$\lambda_1(t_f) = 0$$
$$\lambda_2(t_f) = 0$$
$$\lambda_3(t_f) = 0$$
$$\lambda_4(t_f) = 1$$
$$\lambda_5(t_f) = 1$$
$$\lambda_6(t_f) = -1$$

The optimal controls v_i^* $\big($ for $i = 1,2,3\big)$ can be solved from the optimality conditions

$$\tfrac{\partial H}{\partial v_i} = 0 \text{ (for } i = 1,2;3 \text{)}$$

with further simplification and special attention on the bounds of controls as defined in U. □

4. Numerical Simulation

In this section, we shall solve numerically the optimal control problem for our model. Here, we obtain the optimality system from the state and adjoint equations. The proposed optimal control strategy is obtained by solving the optimal system which consists of five differencial equations and boundary conditions. The optimality system can be solved by using an iterative method. Using an initial guess for the control variables, $v_1(t)$,$v_2(t)$ and $v_3(t)$, the state variables are solved forward and the adjoint variables λ_i for $i = 1;2;3;4;5$ are solved backwards at times step $i = t_0$ and $i = t_f$. If the new values of the state and adjoint variables differ from the previous values, the new values are used to update v_1, v_2 and v_3, and the process is repeated until the system converges.

We present some numerical simulations in order to illustrate our theoretical results, we consider system (3) with the following parameter values

Paramter	Description	Value in d^{-1}
α_1	The rate of effective contact with th moderate drinkers	0.77
α_2	The rate of moderate drinkers becomes heavy drinkers taking anti-diabetic drugs	0.15
α_3	The rate of heavy drinkers taking anti-diabetic drugs who have been with dangerous complications	0.1
α_4	The rate of diabetic people without complication becomes diabetic with complications	0.02
α_5	The rate of negative effect of moderate consumption on diabetic people with complications	0.22
β_1	The rate of moderate drinkers which join the treatment center for listing and guiding	0.02
β_2	The rate of heavy drinkers who join the treatment centers	0.009
β_3	The rate of drinkers people with dangerous complications who join the treatment centers	0.03
β_4	The rate of drinkers who quitting alcohol	0.02
d_1	Mortality rate caused by hypoglycemia	0.003
d_2	Mortality rate due dangerous complications specially cardiovascular diseases	0.002
d_3	Mortality rate in the public center of treatment due to a lack of medical staff	0.003
μ	The rate of natural mortality	0.056
I	Incidence	56

TABLE 1. Parameters values

Population	Description	Value	source
$D(0)$	Potential diabetic without complications drinkers	300	estimated
$M_D(0)$	Moderate drinkers	220	estimated
$C(0)$	Diabetic with complications	200	estimated
$H_D(0)$	Heavy drinkers taking anti-diabetic drugs	100	estimated
$C_S(0)$	Drinkers with dangerous complications	88	estimated
$T_C(0)$	Drinkers joins center of treatment	70	estimated

TABLE 2. Population values

The proposed control strategies in this work help to achieve several objectives:

First Strategy: Considering preventing the people living diabetes with complications taking insulin treatment about the dangerous of moderate consumption alcohol

There are several specific complications related to diabetes for which it is recommended to limit or avoid alcohol consumption [29]

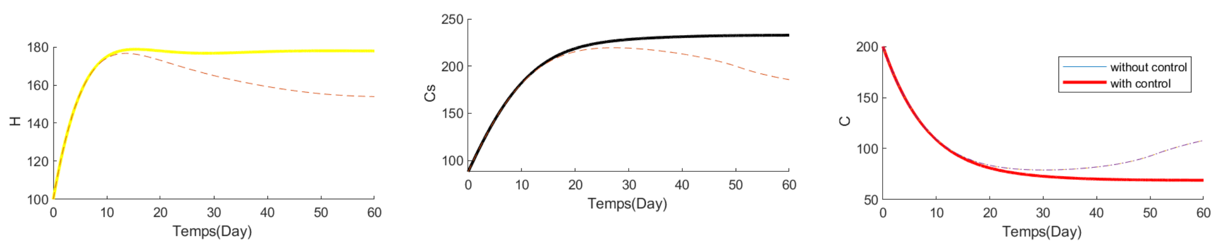
- Diabetic neuropathy: Diabetic neuropathy is a complication that affects the nerves, leading to symptoms such as pain, numbness, tingling, or decreased sensitivity in the limbs. Alcohol consumption can worsen these symptoms and increase the risk of nerve damage.
- Diabetic nephropathy: Diabetic nephropathy is a complication that affects the kidneys, causing progressive damage and impairment of their function. Alcohol consumption can increase blood pressure and worsen kidney damage in people with diabetic nephropathy.
- Diabetic retinopathy: Diabetic retinopathy is a complication that affects the blood vessels of the retina, causing vision problems, and even blindness. Excessive alcohol consumption can worsen blood circulation problems in the retinal vessels and accelerate the progression of diabetic retinopathy.
- Liver issues: Individuals with diabetes are already more likely to develop liver problems, such as non-alcoholic fatty liver disease (NAFLD) or non-alcoholic steatohepatitis (NASH). Alcohol consumption can worsen these liver problems and lead to more significant liver damage.

To realize this strategy, we apply only the control v_1 i.e. the implementation of awareness, and educational programs on diabetic with complications to make them know the risks of this phenomenon on health and society.

(Figure 2A) shows that the number of heavy drinkers with precaution decreased from 177.89 (without control) to 153.9 (withcontrol) at the end of the proposed control. It indicates the effectiveness of optimal control in controlling the growth of heavy diabetic drinkers population with precautions.

(Figure 2B) shows that optimal control affects the number of the poeple with dangerous complications. Before the control is executed, the number of cases with dangerous complications increases gradually until 232.72 at the end of the proposed control. However, when the control is applied, the number of the drinkers population with dangerous complications increases gradually until 185.63 at the end of the proposed control. These changes indicates the effectiveness of optimal control in controlling the growth of drinkers population with dangerous complications.

(Figure 2C) shows that optimal control affects the number of persons living diabetes with complications. Before being controlled, the number of those persons continuously until 69.03 at the end of the proposed control. Whereas, when the control is applied, the number this population decreases and it is will at maximum value 107.83 at the end of the proposed control. It indicates the effectiveness of optimal control in controlling the growth of population with diabtes with complications.



(A) heavy drinkers

(B) drinkers with dangerous compli

(C) diabetic with complications

FIGURE 2. Number of H_D, C_S and C with control v_1

Second Strategy: Providing financial and psychological service to diabetic patients at addiction treatment center.

Providing financial and psychological services to diabetic patients at a public center for alcohol addiction can be beneficial in addressing their specific needs. Here are some potential services that could be offered:

– Financial assistance: Diabetic patients may require financial support to afford essential medications, regular check-ups, and necessary medical supplies. Providing assistance with healthcare expenses or connecting them with relevant financial aid programs can help alleviate the financial burden they may face.

– Counseling and therapy: Managing both diabetes and alcohol addiction can be challenging from a psychological perspective. Offering counseling and therapy services can provide diabetic patients with the necessary support to address their emotional well-being, develop coping mechanisms, and establish healthier habits.

– Education and awareness: It is crucial to provide educational resources and workshops that specifically address the relationship between diabetes and alcohol addiction. This can include raising awareness about the risks of alcohol consumption for diabetic patients, promoting healthier lifestyle choices, and providing guidance on managing both conditions simultaneously.

– Support groups: Facilitating support groups specifically tailored for diabetic patients dealing with alcohol addiction can foster a sense of community and provide a safe space for sharing experiences, challenges, and successes. Support groups can offer encouragement, accountability, and the opportunity to learn from others who are navigating similar situations.

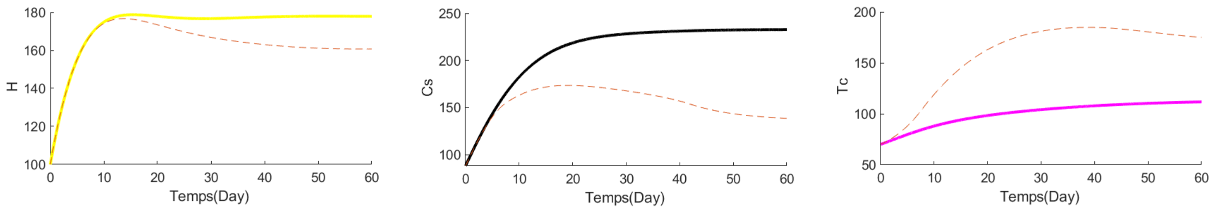
– Collaboration with healthcare professionals: Establishing collaborations between the public center for alcohol addiction and healthcare professionals specializing in diabetes can ensure a comprehensive approach to care. This can involve coordinated treatment plans, regular health assessments, and communication between the healthcare team and the center to ensure holistic support for the patients.

To achieve this strategy, we only use the control v_2 i.e. providing financial psychological service.

In (Figure 3A), it is observed that there is a significant decrease in the number of the heavy drinkers with control compared to a situation when there is no control where the decrease is from 177.89 to 126.23 at the end of the proposed control strategy.

(Figure 3B) shows that the number of drinkers with dangerous complications decreased from 232.46 (without control)to 160.13(with control) at the end of the proposed control.

(Figure 3C) shows that the number of person who join the addiction treatment center increases from 111.65 to 186.60.



(A) heavy drinkers (B) drinkers with dangerous compli (C) join the treatment center

FIGURE 3. Number of H_D,C_S and T_C with control v_2

Third strategy: Considering the treatment of heavy drinkers taking anti-diabetic drugs.

Excessive alcohol consumption can have various negative effects on diabetic individuals, leading to complications and worsening their condition. Some of the potential complications include [30] :

– Hypoglycemia: Alcohol can cause a drop in blood sugar levels (hypoglycemia), especially if consumed on an empty stomach or in combination with diabetes medications like insulin or sulfonylureas. Hypoglycemia can be dangerous and lead to symptoms such as dizziness, confusion, sweating, and, in severe cases, loss of consciousness.

– Hyperglycemia: While alcohol can lower blood sugar levels, it can also contribute to hyperglycemia (high blood sugar levels) in some cases. Some alcoholic beverages, especially those with added sugars or high-carbohydrate content, can raise blood sugar levels.

– Increased Risk of Heart Disease: Both diabetes and excessive alcohol consumption independently increase the risk of heart disease. When combined, they can have a synergistic effect, leading to a higher risk of cardiovascular problems such as heart attacks and strokes.

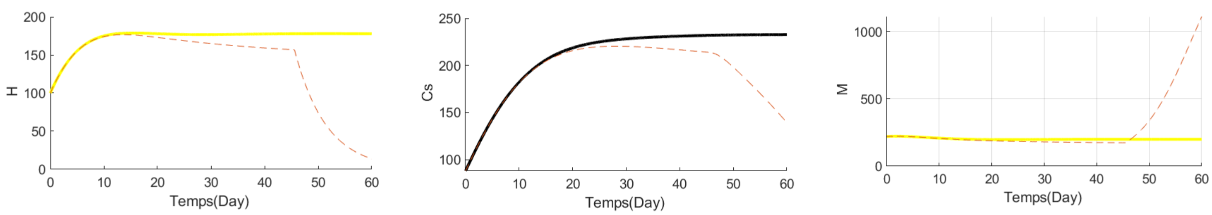
– Liver Damage: Alcohol is processed in the liver, and excessive alcohol intake can lead to liver damage. If a person with diabetes has underlying liver issues or fatty liver disease (common in individuals with type 2 diabetes), alcohol can worsen the condition. – Weight Gain: Alcoholic beverages can be high in calories and often contain added sugars. Excessive alcohol consumption can contribute to weight gain, which can further worsen diabetes management. -Interactions with Diabetes Medications: Some diabetes medications may interact with alcohol, leading to adverse effects or reduced effectiveness. For example, alcohol can increase the risk of lactic acidosis when combined with certain medications like metformin.

To meet this strategy, we use the controls v_3 i.e. effort of treatment applied on on the heavy diabetic drinkers to be moderate drinkers.

(Figure 4A) shows that the number of the heavy drinkers with precaution decreases from 177.9 (without controls) to 14.33 (with controls) at the end of the proposed control.

(Figure 4B)shows that the number of the drinkers with dangerous complications decreases from 580:69 (without controls) to 79:25 (with controls) at the end of the proposed control.

Also, (Figure 4C) depicts clearly an increase in the number of the moderate drinkers from 199.16 (without controls) to 1054.01 (with controls): As a result, the strategy set before has been achieved.



(A) heavy drinkers (B) drinkers with dangerous compli (C) moderate drinkers

FIGURE 4. Number of H_D, C_S and M_D with control v_3

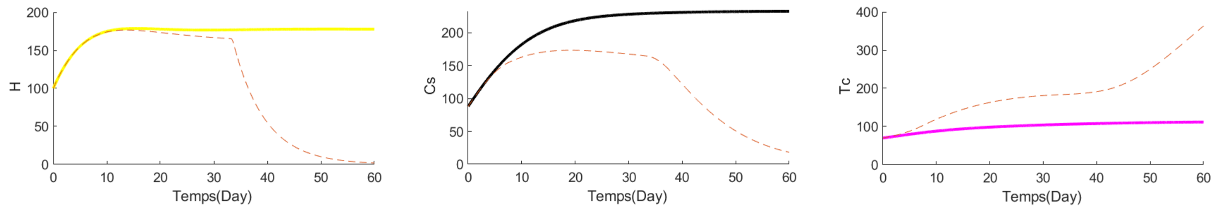
Forth Strategy:Combining the three strategies.

To meet this strategy, we use the controls v_1 and v_2 and v_3

(Figure 5A)shows that the number of the heavy drinkers decreases from the value 177.99 (without controls) to 2.3 (with controls) at the end of the proposed control.

(Figure 5B) shows that the number of the drinkers with dangerous complications decreases from 232.69 (without controls) to 20.34 (with controls) at the end of the proposed control.

Also, (Figure 5C) shows that the number of the person join the treatement centers increases from 111.65 (without controls) to 360.62 (with controls) at the end of the proposed control.



(A) heavy drinkers (B) drinkers with dangerous compli (C) join the treatement center

FIGURE 5. Number of H_D, C_S and T_C with control v_1 and v_2, v_3

5. Conclusion

Alcohol consumption poses significant risks for individuals with diabetes, both those with and without diabetes-related complications. While moderate alcohol intake may be permissible for some people with well-controlled diabetes, excessive alcohol consumption can have adverse effects on blood sugar levels, diabetes management, and overall health.

For diabetics without complications, excessive alcohol intake can lead to hypoglycemia or hyperglycemia, impaired diabetes management, and increased risk of cardiovascular issues. Additionally, it may contribute to weight gain and potential interactions with diabetes medications, further complicating diabetes control.

For diabetics with existing complications, such as neuropathy, nephropathy, and retinopathy, excessive alcohol consumption can exacerbate these conditions, leading to further damage to nerves, kidneys, and eyes. Moreover, it increases the risk of cardiovascular problems, liver damage, and delayed wound healing.

Regardless of diabetes status, alcohol consumption should be approached with caution and moderation. Diabetic individuals should consult their healthcare providers to determine if moderate alcohol intake is safe for their specific health condition and to receive personalized guidance on alcohol consumption and diabetes management.

Ultimately, prioritizing a healthy lifestyle, proper diabetes management, and responsible alcohol use can help mitigate the risks associated with alcohol consumption for both diabetics with and without complications, leading to improved overall health and well-being.

ACKNOWLEDGMENTS

The authors are thankful to the anonymous reviewers for their constructive comments and suggestions which helped in significant improvement of this work.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

REFERENCES

[1] V. Poznyak, D. Rekve, Global status report on alcohol and health 2018, World health organization, Geneva, 2018.

[2] H.R. Alpert, M.E. Slater, Y.H. Yoon, et al. Alcohol Consumption and 15 Causes of Fatal Injuries: A Systematic Review and Meta-Analysis, Amer. J. Prevent. Med. 63 (2022), 286–300. https://doi.org/10.1016/j.amepre.2022.03.025.

[3] M.B. Esser, S.L. Hedden, D. Kanny, et al. Prevalence of alcohol dependence among US adult drinkers, 2009-2011, Prev. Chronic Dis. 11 (2014), 140329. https://doi.org/10.5888/pcd11.140329.

[4] National Institute on Alcohol Abuse and Alcoholism, Treatment for alcohol problems: finding and getting help, (2021). https://www.niaaa.nih.gov/sites/default/files/publications/NIAAA_Treatment_Alcohol_Problems_Booklet.pdf.

[5] National library Of Medicine, Alcoholism MeSH descriptor data 2023, (2023). https://meshb.nlm.nih.gov/record/ui?ui=D000437.

[6] Cleveland Clinic, Diabetes, (2023). https://my.clevelandclinic.org/health/diseases/7104-diabetes.

[7] Beyond Type 1, Forms of diabetes. https://beyondtype1.org/other-forms-of-diabetes.

[8] Centers for Disease Control and Prevention, Diabetes, (2023). https://www.cdc.gov/diabetes/index.html.

[9] J.J. Mayl, C.A. German, A.G. Bertoni, et al. Association of alcohol intake with hypertension in type 2 diabetes mellitus: The ACCORD trial, J. Amer. Heart Assoc. 9 (2020), e017334. https://doi.org/10.1161/jaha.120.017334.

[10] National Institute of Diabetes and Digestive and Kidney Diseases, Low blood glucose (hypoglycemia), (2021). https://www.niddk.nih.gov/health-information/diabetes/overview/preventing-problems/low-blood-glucose-hypoglycemia.

[11] Drugs.com, Diabetes medications and alcohol interactions, (2022). https://www.drugs.com/article/diabetes-medications-alcohol.html.

[12] La Hacienda, Alcohol and diabetes danger, (2022). https://www.lahacienda.com/blog/alcohol-and-diabetes-danger.

[13] S.H. Ma, H.F. Huo, X.Y. Meng, Modelling alcoholism as a contagious disease: a mathematical model with awareness programs and time delay, Discr. Dyn. Nat. Soc. 2015 (2015), 260195. https://doi.org/10.1155/2015/260195.

[14] S. Sharma, G.P. Samanta, Drinking as an epidemic: a mathematical model with dynamic behaviour, J. Appl. Math. Inform. 31 (2013), 1–25. https://doi.org/10.14317/JAMI.2013.001.

[15] B. Benedict, Modeling alcoholism as a contagious disease: how ”infected” drinking buddies spread problem drinking, SIAM News. 40 (2007), 1–3.

[16] H.F. Huo, S.R. Huang, X.Y. Wang, et al. Optimal control of a social epidemic model with media coverage, J. Biol. Dyn. 11 (2017), 226–243. https://doi.org/10.1080/17513758.2017.1321792.

[17] I. Adu, M. Osman, C. Yang, Mathematical model of drinking epidemic, Br. J. Math. Computer Sci. 22 (2017), BJMCS.33659. https://doi.org/10.9734/bjmcs/2017/33659.

[18] K.B. Carey, Heavy drinking contexts and indices of problem drinking among college students., J. Stud. Alcohol. Drugs. 56 (1995), 287–292. https://doi.org/10.15288/jsa.1995.56.287.

[19] H.F. Huo, Y.P. Liu, The analysis of the SIRS alcoholism models with relapse on weighted networks, SpringerPlus. 5 (2016), 722. https://doi.org/10.1186/s40064-016-2308-0.

[20] H.F. Huo, N.N. Song, Global stability for a binge drinking model with two stages, Discr. Dyn. Nat. Soc. 2012 (2012), 829386. https://doi.org/10.1155/2012/829386.

[21] I. Imken, N.I. Fatmi, S. Elamari, A new model of the spread of COVID-19 among diabetes population: a mathematical analysis and optimal control approach for intervention strategies, Commun. Math. Biol. Neurosci. 2023 (2023), 123. https://doi.org/10.28919/cmbn/8264.

[22] Drinkaware, Alcohol and diabetes, (2023). https://www.drinkaware.co.uk/facts/health-effects-of-alcohol/alcohol-related-diseases-and-illnesses/alcohol-and-diabetes.

[23] A. Santos-Longhurst, Diabetes, alcohol, and social drinking, (2017). https://www.healthline.com/health/type-2-diabetes/facts-diabetes-alcohol.

[24] M. Sinai, Diabetes and alcohol, (2023). https://mountsinai.org/health-library/selfcare-instructions/diabetes-and-alcohol.

[25] Vertava Health, Alcohol and diabetes, (2023). https://vertavahealth.com/alcohol/diabetes.

[26] J. Dhillon, A complete guide to diabetes and alcohol, (2021). https://360care.ca/blog/health-and-wellness/a-complete-guide-to-diabetes-and-alcohol.

[27] L. Göllmann, D. Kern, H. Maurer, Optimal control problems with delays in state and control variables subject to mixed control–state constraints, Optim. Control Appl. Methods. 30 (2008), 341–365. https://doi.org/10.1002/oca.843.

[28] W. Fleming, R. Rishel, Deterministic and stochastic optimal control, Springer, New York, 1975.

[29] Medical News Today, Diabetes and alcohol, (2021). https://www.medicalnewstoday.com/articles/type-2-diabetes-and-alcohol#diabetes-and-alcohol.

[30] Nutrition and Food Services U.S Department Of Veterans Affairs, Diabetes and alcohol, (2020). https://www.nutrition.va.gov/docs/UpdatedPatientEd/DiabetesandAlcoholUpdated2020.pdf.