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Commun. Math. Biol. Neurosci. 2024, 2024:7

<https://doi.org/10.28919/cmbn/8362>

ISSN: 2052-2541

## IMPACT OF WIND SPEED ON PROFIT MAXIMIZATION IN DUAL-FISHERMEN EXPLOITATION OF TRITROPHIC PREY-PREDATOR FISH POPULATIONS

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**Abstract.** In this study, we examine the effects of wind speed on profit optimization within dual-fishermen exploitation of tritrophic prey-predator fish ecosystems. Recognizing wind speed as an influential external parameter, our bioeconomic model emphasizes its impact on profit maximization for two distinct fishing actors. Grounded in the Nash Equilibrium framework, we posit that optimal outcomes arise when each participant steadfastly adheres to their respective strategies. Utilizing a Python-driven Markov chain methodology, we anticipate future wind states, contingent upon current conditions and inherent transition probabilities. Preliminary findings denote a significant correlation between wind speed variations and the economic viability of fishing ventures. This research not only elucidates the profound influence of wind speed on marine fisheries but also advocates for the integration of advanced computational methodologies, such as Python and Markov chains, in fisheries management.

**Keywords:** Markov chain prediction; profit maximization; external environmental factors; Nash equilibrium.

**2020 AMS Subject Classification:** 91A40, 92D25, 92D40, 91B76, 91A80, 60J10.

### 1. INTRODUCTION

Wind prediction is important for a variety of reasons. For example, it can help individuals and businesses make informed decisions about outdoor activities such as sailing, kite surfing

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Received November 22, 2023

or paragliding. Wind prediction is also an important tool for farmers as it helps them determine when to plant and harvest their crops. Additionally, it is crucial for the operation of wind turbines as it enables energy companies to optimize the production of renewable energy as researched by Tang et al. [14]. Overall, wind prediction plays a critical role in many industries and activities, helping people make better decisions and improve the efficiency of their operations.

There are several mathematical methods used to predict wind. One of the most common methods is numerical weather prediction (NWP), which involves complex mathematical models that simulate the Earth's atmosphere. These models use data from weather stations, satellites, and other sources to predict the behavior of the atmosphere, including wind patterns.

Another common method is statistical modeling, which involves analyzing historical weather data to identify patterns and trends, and then using this information to make predictions about future wind patterns. This method is particularly useful for short-term wind prediction, such as predicting wind gusts during a storm.

Another mathematically based method is machine learning, which involves training algorithms on large amounts of data to identify patterns and make predictions. This method is becoming increasingly popular for wind prediction, as it can be used to analyze complex data sets and make accurate predictions over longer time frames.

Overall, the mathematical methods used to predict wind depend on the complexity of the system being analyzed and the length of time over which the predictions are being made.

This study is designed to investigate the impact of wind speed on the profitability of fishermen within a tritrophic prey-predator system. Here, wind speed is viewed as an external factor that influences the bioeconomic model, which targets the maximization of profits for two fishing actors. The model relies on the concept of the Nash Equilibrium, asserting that each participant in this competitive environment must adhere to their original strategy to realize the desired outcome. This approach is built upon the foundational work by El Foutayeni et Khaladi [5,6]. It's critical to recognize, however, that the system in question does not exist in isolation. It is susceptible to external factors, such as wind speed, that can significantly influence the expected outcomes. The exploration of these external elements and their impacts on the fishing industry is of paramount importance. Recent research by Riahi et al. [10], conducted a comprehensive

study on temperature prediction utilizing the Markov chain method. The research focused on analyzing the temperature data archive of Casablanca, Morocco, specifically from the preceding year. The results of the analysis exhibited an impressive convergence, accurately aligning with the real temperatures observed. Similarly, as wind speed is a significant factor not only for the fishermen but also for fish populations, its influence warrants a detailed study.

Wind can have a significant impact on fishing, both positively and negatively. In general, wind can help to stir up the water and create currents, which can attract fish and make them more active. This can be especially true in areas where there are underwater structures like reefs or drop-offs. On the other hand, strong winds can also make fishing difficult or even dangerous. High winds can create large waves that can make it difficult to navigate a boat or fish from shore. Additionally, strong winds can cause fish to seek shelter in deeper water, making them harder to catch. Overall, the impact of wind on fishing depends on a variety of factors such as wind strength, water temperature, and the specific species of fish being targeted.

The region of Casablanca, Morocco is known for its windy conditions, which are influenced by its coastal location on the Atlantic Ocean. The frequency and intensity of the winds can vary throughout the year, but generally, the summer months are less windy compared to the fall and winter months. During the peak of the windy season, the average wind speed can range from 15 to 25 km/h, with gusts reaching up to 50 km/h or more. The most common wind direction in Casablanca is from the northwest, but it can also come from the northeast or southwest during certain times of the year. These wind conditions can impact various aspects of life in the region, including transportation, agriculture, and outdoor activities.

A Python program for Markov chain wind prediction holds significant utility in the field of meteorology and weather forecasting. By leveraging the power and flexibility of Python, this program can effectively analyze historical wind data and generate probabilistic predictions for future wind patterns. The Markov chain methodology employed in the program allows for the consideration of the current state of the wind system and the transition probabilities to determine the subsequent states. This enables the program to capture the inherent dynamics and dependencies in wind patterns, providing valuable insights for various applications. For instance, in renewable energy planning, accurate wind predictions are essential for optimizing the placement

and operation of wind turbines. Additionally, the program can aid in environmental modeling and pollution dispersion studies by simulating the movement of airborne particles. The ability to predict wind patterns using a Python program based on the Markov chain method offers a valuable tool for industries and researchers to make informed decisions, improve resource management, and enhance risk assessment in fields that rely on accurate wind forecasting.

The current research diverges in perspective by focusing on the implications for fishermen, Differing from earlier works such as those by Takyi et al. [13] and Barman et al. [2]. Their research was primarily concerned with developing a predator-prey model that incorporated wind as a critical abiotic factor influencing predation patterns.

This paper is organized as follows: Section 2 introduces the bioeconomic model aimed at maximizing profits in a tritrophic prey-predator fishery, exploited by two fishermen. Section 3 delves into the influence of wind dynamics on marine life and the application of the Markov Chain. In Section 4, we provide a simulation and analysis based on the proposed model. The paper ends with a conclusion section.

## **2. BIOECONOMIC MODEL FOR PROFIT MAXIMIZATION IN DUAL-FISHERMEN EXPLOITATION OF TRITROPHIC PREY-PREDATOR FISH POPULATIONS**

**2.1. Prey-Predator Biological Model.** In this segment, we analyze a tritrophic prey-predator model that encompasses three interacting populations: the prey, an intermediate predator, and an apex predator. It's postulated that the prey population, denoted as  $D_1(t)$ , proliferates according to a logistic growth model characterized by a specific birth rate constant. Additionally, interactions between the prey and both predator levels are incorporated due to the defensive capacities of the prey.

Similarly, the intermediate predator population, represented as  $D_2(t)$ , also expands following a logistic growth model with its unique birth rate constant. The availability of its favored food source, the prey  $D_1(t)$ , positively impacts the population density of the intermediate predator,  $D_2(t)$ . This model also takes into account the interactions between the intermediate and top predators, considering the defensive abilities of the intermediate predator.

The apex predator population,  $D_3(t)$ , sees an increase in its population density with the presence of its favored food sources (prey and the intermediate predator). Therefore, the model can

be expressed mathematically as follows:

$$(2.1) \quad \begin{cases} \dot{D}_1 = r_1 D_1 (1 - D_1) - \alpha D_1 D_2 - \beta D_1 D_3, \\ \dot{D}_2 = r_2 D_2 (1 - D_2) + \bar{\alpha} D_2 D_1 - \delta D_2 D_3, \\ \dot{D}_3 = r_3 D_3 (1 - D_3) + \bar{\beta} D_1 D_3 + \bar{\delta} D_2 D_3 \end{cases}$$

where  $(D_j)_{j=1,2,3}$  represent the densities of the three populations, with each having positive initial conditions  $D_1(0) > 0$ ,  $D_2(0) > 0$ ,  $D_3(0) > 0$ . The variables  $(r_j)_{j=1,2,3}$  correspond to the intrinsic growth rates of the prey, the middle predator, and top predator, respectively. The parameters  $\alpha$ ,  $\beta$ , and  $\delta$  represent the maximum values that the per capita reduction rate of  $D_1$  and  $D_2$  can attain, respectively.  $\bar{\alpha}$  is the conversion rate of prey  $D_1$  into middle predator  $D_2$ , and  $\bar{\beta}$  and  $\bar{\delta}$  are the conversion rate of prey  $D_1$  into top predator  $D_3$  and the conversion rate of middle predator  $D_2$  into top predator  $D_3$ , respectively.

**Proposition 2.1.** *The Persistent Model of a dynamical system of differential equations can be estimated by setting all derivatives to zero.*

The equilibrium points for the system outlined in equations (1) can be determined by finding the solutions to the following equations:

$$(2.2) \quad \begin{cases} \dot{D}_1 = r_1 D_1 (1 - D_1) - \alpha D_1 D_2 - \beta D_1 D_3 = 0, \\ \dot{D}_2 = r_2 D_2 (1 - D_2) + \bar{\alpha} D_2 D_1 - \delta D_2 D_3 = 0, \\ \dot{D}_3 = r_3 D_3 (1 - D_3) + \bar{\beta} D_1 D_3 + \bar{\delta} D_2 D_3 = 0. \end{cases}$$

The solution to the system denoted as (2.2) is represented by  $P(D_1^*, D_2^*, D_3^*)$ , where the values are defined in the equation set (2.3):

$$(2.3) \quad \begin{aligned} D_1^* &= \frac{(r_1 r_2 r_3 + r_1 \delta \bar{\delta} + r_3 \alpha \bar{\alpha} - r_2 r_3 \alpha - r_2 r_3 \beta - r_2 \beta \bar{\delta})}{\Delta}, \\ D_2^* &= \frac{(r_1 r_2 r_3 + r_2 \beta \bar{\beta} - r_3 \beta \bar{\alpha} - r_1 r_3 \delta + r_1 r_3 \bar{\alpha} - r_1 \beta \bar{\delta})}{\Delta}, \\ D_3^* &= \frac{(r_1 r_2 r_3 + r_1 r_2 \bar{\beta} + r_1 r_2 \bar{\delta} + r_1 \bar{\alpha} \bar{\delta} - r_2 \beta \alpha + r_3 \alpha \bar{\alpha})}{\Delta} \end{aligned}$$

where  $\Delta = r_1 r_2 r_3 + r_1 \delta \bar{\delta} + r_2 \beta \bar{\beta} + r_3 \alpha \bar{\alpha} - \alpha \delta \bar{\beta} + \beta \bar{\alpha} \bar{\delta}$ .

The equilibrium point exists if  $r_1 > \max\{\alpha, \beta\}$ ,  $r_2 > \delta$ , and  $r_3 > \max\{\delta, \bar{\beta}\}$ .

**Remark 2.1.** *This system potentially has other solutions that were not considered due to their contradiction with the established hypotheses. For instance, the point  $P(0, 0, 0)$  was excluded*

because the system would lose its ecological meaning if the population densities were to equal zero.

**Proposition 2.2.** *A steady state is stable if it is locally asymptotically stable.*

**Theorem 2.1.** *The point  $P(D_1^*, D_2^*, D_3^*)$  is locally asymptotically stable.*

*Proof.* The Jacobian matrix for system (1) is given by:

$$(2.4) \quad J^* = \begin{bmatrix} -r_1 D_1^* & -\alpha D_1^* & -\beta D_1^* \\ \bar{\alpha} D_2^* & -r_2 D_2^* & -\delta D_2^* \\ \bar{\beta} D_3^* & \bar{\delta} D_3^* & -r_3 D_3^* \end{bmatrix},$$

by formulating the characteristic equation and applying Routh-Hurwitz criterion, we find:

$$(2.5) \quad Q(\lambda) = n_0 \lambda^3 + n_1 \lambda^2 + n_2 \lambda + n_3,$$

where

$$n_0 = 1,$$

$$n_1 = r_1 D_1^* + r_2 D_2^* + r_3 D_3^*,$$

$$n_2 = r_3 D_3^* (r_1 D_1^* + r_2 D_2^*) + D_1^* D_2^* (\alpha \bar{\alpha} + r_1 r_2) + D_1^* D_3^* \beta \bar{\beta} + D_2^* D_3^* \delta \bar{\delta},$$

$$n_3 = D_3^* (r_1 D_1^* + r_2 D_2^*) (D_1^* \beta \bar{\beta} + D_2^* \delta \bar{\delta}) - \bar{\delta} D_2^* D_3^* (\delta r_2 D_2^* - \bar{\alpha} \beta D_1^*) - \bar{\beta} D_1^* D_3^* (\beta r_1 D_1^* + \alpha \delta D_2^*) + r_3 D_1^* D_3^* D_2^* (\alpha \bar{\alpha} + r_1 r_2).$$

as  $n_0, n_1, n_2, n_3$ , and  $(n_1 n_2 - n_0 n_3)$  are all positive. The Routh-Hurwitz conditions are satisfied. Therefore, the point  $P(D_1^*, D_2^*, D_3^*)$  is locally asymptotically stable.  $\square$

**2.2. Bioeconomic Model.** We characterize the bioeconomic model by the following set of equations:

$$(2.6) \quad \begin{cases} \dot{D}_1 = r_1 D_1 (1 - D_1) - \alpha D_1 D_2 - \beta D_1 D_3 - q_1 E_1 D_1, \\ \dot{D}_2 = r_2 D_2 (1 - D_2) + \bar{\alpha} D_2 D_1 - \delta D_2 D_3 - q_2 E_2 D_2, \\ \dot{D}_3 = r_3 D_3 (1 - D_3) + \bar{\beta} D_1 D_3 + \bar{\delta} D_2 D_3 - q_3 E_3 D_3. \end{cases}$$

Catchability, denoted as  $q$ , is a pivotal parameter in the establishment and verification of the fishing simulation model. This constant value represents the efficiency of a fishing operation or how effectively a unit of fishing effort captures fish. The fishing effort itself is a composite

metric representing the degree of fishing activity and fishing power. Each fleet's fishing effort is an aggregate of the individual efforts of its fishing units. Importantly, a unit's fishing power signifies its capacity to harvest fish. Numerous variables influence the fishing effort, including the type of vessel, the duration of fishing, the number of fishing expeditions, the technological level, the fishing equipment, and the size of the crew. However, within the context of this paper, 'effort' denoted  $E$  is interpreted as an encompassing term, encapsulating all these influencing factors.

The biomass representation as a function of fishing effort at the biological equilibrium is determined by solving the system:

$$(2.7) \quad \begin{cases} r_1(1 - D_1) = \alpha D_2 + \beta D_3 + q_1 E_1, \\ r_2(1 - D_2) = -\bar{\alpha} D_1 + \delta D_3 + q_2 E_2, \\ r_3(1 - D_3) = -\bar{\beta} D_1 - \bar{\delta} D_2 + q_3 E_3. \end{cases}$$

The solution to the aforementioned system (7) is provided as follows:

$$(2.8) \quad \begin{cases} D_1 = n_{11} E_1 + n_{12} E_2 + n_{13} E_3 + D_1^*, \\ D_2 = n_{21} E_1 + n_{22} E_2 + n_{23} E_3 + D_2^*, \\ D_3 = n_{31} E_1 + n_{32} E_2 + n_{33} E_3 + D_3^*. \end{cases}$$

where

$$(2.9) \quad \begin{aligned} n_{11} &= \frac{-(\delta \bar{\delta} q_1 - r_2 r_3 q_1)}{\Delta}, \\ n_{21} &= \frac{(\delta \bar{\beta} q_1 - \bar{\alpha} r_3 q_1)}{\Delta}, \\ n_{31} &= \frac{(-\bar{\alpha} \bar{\beta} q_1 - \bar{\beta} r_2 q_1)}{\Delta}, \\ n_{12} &= \frac{(\beta \bar{\delta} q_2 - \alpha r_3 q_2)}{\Delta}, \\ n_{22} &= \frac{(\beta \bar{\beta} q_2 - r_1 r_3 q_2)}{\Delta}, \\ n_{32} &= \frac{(\alpha \bar{\beta} q_2 - \bar{\delta} r_1 q_2)}{\Delta}, \\ n_{13} &= \frac{(-\delta \alpha q_3 + \beta r_2 q_3)}{\Delta}, \\ n_{23} &= \frac{(\beta \bar{\alpha} q_3 - \delta r_1 q_3)}{\Delta}, \\ n_{33} &= \frac{(-\alpha \bar{\alpha} q_3 - r_1 r_2 q_3)}{\Delta}. \end{aligned}$$

We define the matrix  $D$ , where  $D = -NE + D^*$  where  $N = (-n_{ij})_{1 \leq i, j \leq 3}$  and  $D = (D_1^*, D_2^*, D_3^*)^T$  with  $n_{ii} < 0$  for all  $i = 1, 2, 3$ .

**2.2.1. Expression of the Profit.** The profit accrued by each fisherman is characterized by the ensuing function:

$$(2.10) \quad \Pi_i(E) = (TR)_i - (TC)_i,$$

where  $\Pi_i(E)$  is the profit for each fisherman,  $(TR)_i$  is the total revenue and  $(TC)_i$  is the total cost.

The Total Revenue  $(TR)_i$  is linearly associated with the catch, expressed as  $TR = price \times catches$ .

$$(2.11) \quad (TR)_i = p_i \times H_{ij},$$

where  $H_{ij} = q_j E_{ij} D_j$  represent the catches of species  $j$  by the fisherman  $i$ .

The total catches of species  $j$  by all fishermen is expressed by:

$$(2.12) \quad H_j = \sum_{i=1}^2 H_{ij},$$

The total fishing effort dedicated to species  $j$  by all fishermen can be represented as:

$$(2.13) \quad E_j = \sum_{i=1}^2 E_{ij},$$

We elaborate these notations as follows:

$$(2.14) \quad \begin{aligned} (TR)_i &= p_1 H_{i1} + p_2 H_{i2} + p_3 H_{i3} \\ &= p_1 q_1 E_{i1} (n_{11} E_1 + n_{12} E_2 + n_{13} E_3 + D_1^*) + p_2 q_2 E_{i2} (n_{21} E_1 + n_{22} E_2 + n_{23} E_3 + D_2^*) \\ &\quad + p_3 q_3 E_{i3} (n_{31} E_1 + n_{32} E_2 + n_{33} E_3 + D_3^*) \\ &= p_1 q_1 E_{i1} \left( n_{11} \sum_{i=1}^n E_{i1} + n_{12} \sum_{i=1}^n E_{i2} + n_{13} \sum_{i=1}^n E_{i3} + D_1^* \right) \\ &\quad + p_2 q_2 E_{i2} \left( n_{21} \sum_{i=1}^n E_{i1} + n_{22} \sum_{i=1}^n E_{i2} + n_{23} \sum_{i=1}^n E_{i3} + D_2^* \right) \\ &\quad + p_3 q_3 E_{i3} \left( n_{31} \sum_{i=1}^n E_{i1} + n_{32} \sum_{i=1}^n E_{i2} + n_{33} \sum_{i=1}^n E_{i3} + D_3^* \right). \end{aligned}$$

Then

$$(2.15) \quad (TR)_i = \langle E^i, -pqNE^i \rangle + \left\langle E^i, pqD^* - \sum_{j=1, j \neq i}^n pqNE^j \right\rangle,$$

where  $(p_j)_{j=1,2,3}$  is the price per unit biomass of the species  $j$ . In this work, we take  $p_1, p_2$  and  $p_3$  as constants.



In alignment with numerous established fisheries models, such as those proposed by Clark [4] and Gordon [7], we assume that:

$$(2.16) \quad (TC)_i = \langle c^i, E^i \rangle,$$

where  $(TC)_i$  is the total effort cost of the fisherman  $i$ , and  $c^i$  is the constant cost per unit of harvesting and  $E^i$  is the total effort of the fisherman  $i$ .

Leveraging these notations, we can now formulate the expression of profit.

$$(2.17) \quad \begin{aligned} \Pi_i(E) &= (TR)_i - (TC)_i \\ &= \langle E^i, -pqNE^i \rangle + \left\langle E^i, pqD^* - \sum_{j=1, j \neq i}^n pqNE^j \right\rangle. \end{aligned}$$

As the biological model is meaningful only if the biomass of all the marine species are strictly positive.

$$\text{Then } D = -NE + D^* \geq D_0 > 0.$$

For each fisherman  $i$  we have:

$$(2.18) \quad NE^i \leq - \sum_{j=1, j \neq i}^n NE^j + D^*.$$

### 2.2.2. Profit Maximization.

**Nash Equilibrium.** A Nash equilibrium solution is realized when each involved fisherman maintains their fishing strategy while aiming to optimize their individual profit and establish a certain level of fishing effort.

We can reformulate this issue into an optimization problem:

To maximize the profit of the first fisherman, we need to resolve problem  $(P_1)$ :

$$(2.19) \quad \begin{aligned} \max \Pi_1(E) &= \langle E^1, -pqNE^1 + pqD^* - c - pqNE^2 \rangle, \\ \text{subject to} & \quad NE^1 \leq -NE^2 + D^*, \\ & \quad E^1 \geq 0, E^2 \text{ is given.} \end{aligned}$$

To optimize the second fisherman's profit, we resolve problem  $(P_2)$ :

$$(2.20) \quad \begin{aligned} \max \Pi_2(E) &= \langle E^2, -pqNE^2 + pqD^* - c - pqNE^1 \rangle, \\ \text{subject to} & \quad NE^2 \leq -NE^1 + D^*, \\ & \quad E^2 \geq 0, E^1 \text{ is given.} \end{aligned}$$

The point  $(E^1, E^2)$  constitutes a generalized Nash equilibrium if and only if  $E^1$  solves problem  $(P_1)$  when  $E^2$  is given, and  $E^2$  solves problem  $(P_2)$  when  $E^1$  is given.

By employing the Karush-Kuhn-Tucker conditions to problem  $(P_1)$  we obtain vectors  $u_1 \in \mathbb{R}_+^3, v_1 \in \mathbb{R}_+^3$  and  $\lambda^1 \in \mathbb{R}_+^3$  that satisfy the following conditions:

$$(2.21) \quad \begin{cases} 2pqNE^1 + c - pqD^* + pqNE^2 - u^1 + N^T \lambda^1 = 0, \\ NE^1 + v^1 = -NE^2 + D^*, \\ \langle u^1, E^1 \rangle = \langle \lambda^1, v^1 \rangle = 0. \end{cases}$$

Similarly, applying the Karush-Kuhn-Tucker conditions to problem  $(P_2)$ , we have  $u_2 \in \mathbb{R}_+^3, v_2 \in \mathbb{R}_+^3$  and  $\lambda^2 \in \mathbb{R}_+^3$  such that:

$$(2.22) \quad \begin{cases} 2pqNE^2 + c - pqD^* + pqNE^1 - u^2 + N^T \lambda^2 = 0, \\ NE^2 + v^2 = -NE^1 + D^*, \\ \langle u^2, E^2 \rangle = \langle \lambda^2, v^2 \rangle = 0. \end{cases}$$

From (2.21) and (2.22) we have

$$(2.23) \quad \begin{cases} u^1 = 2pqNE^1 + c - pqD^* + pqNE^2 + N^T \lambda^1, \\ u^2 = 2pqNE^2 + c - pqD^* + pqNE^1 + N^T \lambda^2, \\ v^1 = -NE^1 - NE^2 + D^*, \\ v^2 = -NE^1 - NE^2 + D^*, \\ \langle u^i, E^i \rangle = \langle \lambda^i, v^i \rangle = 0 \text{ for all } i = 1, 2, 3, \quad (*1) \\ E^i, u^i, \lambda^i, v^i \geq 0 \text{ for all } i = 1, 2, 3. \quad (*2) \end{cases}$$

From (\*1) and (\*2) we have  $v^1 = v^2$

And as  $D_j > 0$  for all  $j = 1, 2, 3$ ; therefore  $v^1 = v^2 > 0$ .

We also have the scalar product of  $(\lambda^i)_{i=1,2,3} = 0$  and  $(v^i)_{i=1,2,3} = 0$ .

We denote by  $v = v^1 = v^2$ , so we have:

$$(2.24) \quad \begin{cases} u^1 = 2pqNE^1 + pqNE^2 + c - pqD^*, \\ u^2 = 2pqNE^2 + pqNE^1 + c - pqD^*, \\ v = -NE^1 - NE^2 + D^*, \\ \langle u^i, E^i \rangle = 0 \text{ for all } i = 1, 2, 3, \\ E^i, u^i, v^i \geq 0 \text{ for all } i = 1, 2, 3, \end{cases}$$

Thus,

$$(2.25) \quad \begin{pmatrix} u^1 \\ u^2 \\ v \end{pmatrix} = \begin{bmatrix} 2pqN & pqN & N^T \\ pqN & 2pqN & 0 \\ -N & -N & 0 \end{bmatrix} \begin{pmatrix} E^1 \\ E^2 \\ 0 \end{pmatrix} + \begin{pmatrix} c - pqD^* \\ c - pqD^* \\ D^* \end{pmatrix}.$$

Linear Complementarity Problem. We have a linear complementarity problem equivalent to the Nash equilibriumn problem  $LCP(M, b)$  such that  $z, w \in \mathbb{R}^6$  in order that:

$$(2.26) \quad LCP(M, b) \begin{cases} w = Mz + b \geq 0, \\ z, w > 0, \\ z^T w = 0. \end{cases}$$

We verify that the  $LCP(M, b)$  has a unique solution by the next Theorem.

**Theorem 2.2.**  *$LCP(M, b)$  has a unique solution for every  $b$  if and only if  $M$  is a  $P$  – matrix.*

*Proof.* A matrix  $M$  is called  $P$  – matrix if the determinant of every principal submatrix of  $M$  is positive Murty, [9]. □

**Remark 2.2.** *If  $M$  is a  $P$  – matrix the Nash Equilibrium Problem have a unique solution.*

**Theorem 2.3.** *The matrix*

$$(2.27) \quad M = \begin{bmatrix} 2pqN & pqN & N^T \\ pqN & 2pqN & 0 \\ -N & -N & 0 \end{bmatrix},$$

*is a  $P$  – matrix.*

*Proof.* As we have  $n_{ii} < 0$  for all  $i = 1, 2, 3$  and  $\chi > 0$ , we note by  $(M_i)_{i=1, \dots, 9}$  the submatrix of  $M$ , we obtain:

$$(2.28) \quad \begin{aligned} \det(M_1) &= -2p_1q_1n_{11} > 0, \\ \det(M_2) &= 4p_1q_1p_2q_2q_1r_3q_2\Delta > 0, \\ \det(M_3) &= 8p_1q_1p_2q_2p_3q_3q_1q_2\Delta^2 > 0, \\ \det(M_4) &= -12n_{11}p_1^2q_1^2p_2q_2p_3q_3q_1q_2\Delta^2 > 0, \\ \det(M_5) &= 18p_1^2q_1^2p_2^2q_2^2p_3q_3q_1r_3q_2q_3q_1q_2\Delta^3 > 0, \end{aligned}$$

$$\det(M_6) = 27p_1^2q_1^2p_2^2q_2^2p_3^2q_3^2(q_3q_1q_2\Delta^2)^2 > 0,$$

$$\det(M_7) = -9p_1q_1p_2^2q_2^2p_3^2q_3^2n_{11}(q_3q_1q_2\Delta^2)^2 > 0,$$

$$\det(M_8) = 3p_1q_1p_2q_2p_3^2q_3^2q_1r_3q_2\Delta(q_3q_1q_2\Delta^2)^2 > 0,$$

$$\det(M_9) = p_1q_1p_2q_2p_3q_3(q_3q_1q_2\Delta^2)^3 > 0.$$

With all the principal minors  $\det(M_i)_{i=1,\dots,9} > 0$  being greater than zero, it implies that the matrix  $M$  is  $P$ -matrix. Hence, the associated linear complementarity problem, denoted as  $LCP(M, b)$ , is guaranteed to have a unique solution. This unique solution embodies the Nash equilibrium of the problem under consideration.

$$(2.29) \quad \begin{cases} E^1 = \frac{1}{3}N^{-1}(D^* - \frac{c}{pq}), \\ E^2 = \frac{1}{3}N^{-1}(D^* - \frac{c}{pq}). \end{cases}$$

□

**Summary 2.1.** *Ultimately, the fishing effort that optimally maximizes the profit for fishermen exploiting the three species is expressed in equation (29):*

$$(2.30) \quad \begin{aligned} E_{11} &= \frac{1}{3} \left[ \frac{r_1}{q_1} \left( D_1^* - \frac{c_1}{p_1q_1} \right) + \frac{\alpha}{q_1} \left( D_2^* - \frac{c_1}{p_2q_2} \right) + \frac{\beta}{q_1} \left( D_3^* - \frac{c_1}{p_3q_3} \right) \right], \\ E_{12} &= \frac{1}{3} \left[ \frac{r_2}{q_2} \left( D_2^* - \frac{c_1}{p_2q_2} \right) - \frac{\bar{\alpha}}{q_2} \left( D_1^* - \frac{c_1}{p_1q_1} \right) + \frac{\delta}{q_2} \left( D_3^* - \frac{c_1}{p_3q_3} \right) \right], \\ E_{13} &= \frac{1}{3} \left[ \frac{r_3}{q_3} \left( D_3^* - \frac{c_1}{p_3q_3} \right) - \frac{\bar{\beta}}{q_3} \left( D_1^* - \frac{c_1}{p_1q_1} \right) - \frac{\bar{\delta}}{q_3} \left( D_2^* - \frac{c_1}{p_2q_2} \right) \right], \\ E_{21} &= \frac{1}{3} \left[ \frac{r_1}{q_1} \left( D_1^* - \frac{c_2}{p_1q_1} \right) + \frac{\alpha}{q_1} \left( D_2^* - \frac{c_2}{p_2q_2} \right) + \frac{\beta}{q_1} \left( D_3^* - \frac{c_2}{p_3q_3} \right) \right], \\ E_{22} &= \frac{1}{3} \left[ \frac{r_2}{q_2} \left( D_2^* - \frac{c_2}{p_2q_2} \right) - \frac{\bar{\alpha}}{q_2} \left( D_1^* - \frac{c_2}{p_1q_1} \right) + \frac{\delta}{q_2} \left( D_3^* - \frac{c_2}{p_3q_3} \right) \right], \\ E_{23} &= \frac{1}{3} \left[ \frac{r_3}{q_3} \left( D_3^* - \frac{c_2}{p_3q_3} \right) - \frac{\bar{\beta}}{q_3} \left( D_1^* - \frac{c_2}{p_1q_1} \right) - \frac{\bar{\delta}}{q_3} \left( D_2^* - \frac{c_2}{p_2q_2} \right) \right]. \end{aligned}$$

### 3. WIND DYNAMICS: INFLUENCE ON MARINE LIFE AND MARKOV CHAIN PREDICTIVE MODELLING

**3.1. Wind Impact on Prey-Predator Interactions.** The relation that is more affected by wind speed in fishing can vary depending on the specific fishing location, the species of fish present, and the fishing techniques employed. However, in general, wind speed can have a more significant impact on fishing in relation to the behavior and movements of prey and predator fish rather than species in competition.

It's also important to note that the effects of wind might not be the same for all species or even all individuals within a species. Some predators might be better able to cope with or exploit windy conditions than others, for example, due to differences in body size or foraging strategy.

The interaction between wind speed and predator-prey relationships can significantly impact predator-prey system dynamics, either by decreasing or increasing predation rates.

- (1) **Influence on Predation Success:** High wind conditions can make hunting more challenging for predators. This adversity initially triggers an increase in their population density. However, beyond a certain threshold, the predator population begins to diminish due to hunting difficulties, suggesting that extremely windy conditions could potentially lead to species extinction.
- (2) **Wind as a Stabilizer:** Conversely, wind can serve to stabilize the predator-prey ecosystem. In situations where predation follows a Holling Type II functional response, an uptick in wind flow can transition the system into a stable state. By mitigating over-predation, strong winds can foster long-term system stability.
- (3) **Effects on Species Coexistence:** Wind can help maintain a stable coexistence of predator and prey populations when predation aligns with the law of mass action principle. However, under a Holling Type II functional response, wind doesn't impact the system's stability. Importantly, extreme wind strength can lead both predator and prey populations to dwindle to zero, thereby risking species extinction.
- (4) **Impact of Time-Variant Wind Flow:** The incorporation of time-variant wind flow into the model adds a layer of complexity to the system dynamics. The periodic variation in wind flow has a profound influence on predator predation patterns, highlighting an area ripe for further exploration.

Overall, wind speed plays a critical role in shaping predator-prey dynamics within a fishing context. It influences predator hunting success rates, prey vulnerability, and the overall stability of the ecosystem. This understanding is vital for effectively predicting and managing population dynamics amid fluctuating environmental conditions.

**3.2. Fishermen and Wind Speed Interactions.** Fishermen target a variety of species as prey and predators, with the types and techniques varying based on location, regulations, and practices. Notably, wind speed can impact predator-prey dynamics in the fishing ecosystem.

**Wind Speed Influence:** In high wind conditions, the hunting challenges for predators lead to an increase and then a decrease in their population, suggesting potential species extinction with extreme winds. Contrarily, wind can stabilize the predator-prey ecosystem, preventing over-predation and promoting system stability. While wind fosters predator and prey coexistence, extreme wind strength can risk species extinction. The introduction of time-variant wind flow adds complexity to system dynamics.

**Variables Impacting Fishing Trips:** The likelihood of fishing trips is influenced by factors such as wind speed, wave height, expected catch weight, and expected unit price. As wind speed increases, trip likelihood decreases. Conversely, successful fishing or high market prices can incentivize trips even in adverse weather. The need for accurate weather forecasts is emphasized for fishermen's informed and safe decisions.

The study conducted by Sainsbury [11] examines the impact of wind speed on the likelihood of embarking on a trip. The hypothesis suggests that wind speed in a favourable direction would have a negative impact on trip likelihood. The rationale behind this hypothesis is that stronger winds and larger waves increase discomfort, pose operating challenges, and reduce safety, which in turn, may dissuade people from taking trips.

The study made by Sainsbury [11] illustrates the results of two distinct logistic regression models: Conditional Logit and Random Parameter Logit. Both models are used to analyze the relationship between wind speed (an independent variable) and trip decisions (the dependent variable) for fishermen.

Here's the interpretation of the coefficients with respect to wind speed and trip decisions:

- (1) Wind speed: In both models, the coefficient for wind speed is positive, which suggests that an increase in wind speed is associated with an increase in the log-odds of the outcome (presumably, the decision to go on a fishing trip).
- (2) In the Conditional Logit model, the coefficient estimate for wind speed is 0.09344, indicating that for each unit increase in wind speed, the log-odds of the outcome increases

by about 0.09344. The 2.5% and 97.5% confidence intervals (CIs) are 0.04401 and 0.14286 respectively, which gives us an interval where the true coefficient lies with a 95% confidence level.

- (3) In the Random Parameter Logit model, the coefficient estimate for wind speed is a bit larger, at 0.12772. This means that for each unit increase in wind speed, the log-odds of the outcome increases by about 0.12772. The 2.5% and 97.5% CIs are 0.06448 and 0.19096 respectively. This result is also statistically significant at the 99% confidence level.

The attribute levels for wind speed are presented as 10, 20, 30, 40, and 50 mph. This suggests that the analysis is considering varying degrees of wind speed and their impact on trip likelihood.

The Conditional Logit and Random Parameter Logit models indicate a significant positive correlation between wind speed and the probability of choosing to go on a fishing trip. Thus, as wind speed rises, so does the likelihood of initiating a fishing trip. However, in practice, high wind speeds, which pose safety risks and operational challenges, could deter fishermen from setting out. This dichotomy underscores the importance of precise and timely weather forecasting in aiding fishermen to make safe, informed decisions about their trips.

### **3.3. Wind Speed Prediction.**

**3.3.1. Data and Modeling Strategy.** The study utilizes a dataset derived from the Anfa operational meteorological tower located in Casablanca, Morocco. This dataset includes hourly average wind speed data collected over a year (from March 1, 2022, to February 28, 2023), and is publicly accessible via the Windguru website.

The main objective of this research is to construct a model capable of accurately capturing and illustrating seasonal variations, despite the constraints of a limited annual time series dataset. To this end, the dataset is divided into four seasons - winter (December, January, February), spring (March, April, May), summer (June, July, August), and autumn (September, October, November).

To better analyze and utilize this seasonally-divided data, transition matrices were created for each selected month. These matrices were then employed to generate synthetic data, a method that will allow for a more detailed exploration of seasonal wind speed variations.

**3.3.2. Markov Chain Simulation Technique.** In this study, we utilize a first-order Markov chain to model the wind speed. The first-order Markov chain is a type of stochastic process where the state at the next time step depends only on the current state, as outlined by Shamshad et al. [12], rather than on the sequence of events that came before it.

The 'order' in this context refers to how many previous time steps, or states, from each season we consider when calculating the probability of transitioning to the next state. In a first-order Markov chain, we only look at the current state to predict the next one.

These transition probabilities are encapsulated within a matrix known as the 'transition probability matrix.' The dimensions of this matrix are determined by the number of distinct, or 'dominant,' states observed in each season.

**Definition 3.1.** *A discrete-time Markov chain is a sequence of random variables  $X_0, X_1, \dots$ , with the Markov property, known as a stochastic process, in which the value of the next variable depends only on the value of the current variable*

$$(3.1) \quad P(X_{n+1} = x | X_1 = x_1, X_2 = x_2, \dots, X_n = x_n) = P(X_{n+1} = x | X_n = x_n).$$

*The probability of  $X_{n+1}$  only depends on the probability of  $X_n$  that precedes it.*

**Definition 3.2.** *A Markov chain is called homogeneous if and only if the transition probabilities are independent of the time, such that :*

$$(3.2) \quad P(X_{n+1} = x | X_n = x_n) = P(X_1 = j | X_0 = i) = p_{ij},$$

where  $j + i = 1$ .

The transition probability matrix can be written as:

$$(3.3) \quad P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}.$$



The elements of  $P$  are nonnegative and, given that the state at time  $n$  is  $i$ , the process must be somewhere at time  $n + 1$  which means that the elements in each row must sum to one.

**3.3.3. Wind Prediction Using Markov Chain Algorithm: A Python-Based Approach.** The wind prediction algorithm based on the Markov chain is a computational method implemented in Python to forecast wind patterns. The algorithm leverages the principles of the Markov chain, which models the probabilistic transition between different states based on historical data.

In this algorithm, historical speed wind data, is used to estimate the transition probabilities between different wind states. By analyzing the patterns and dependencies in the historical data, the algorithm predicts the likelihood of wind transitioning from one state to another over time.

The algorithm involves the following steps:

- (1) Data preprocessing: The historical wind data is collected and prepared for analysis. This may involve cleaning the data, handling missing values, and ensuring data consistency.
- (2) State discretization: The continuous wind data is discretized into a set of states to facilitate the Markov chain modeling. The number and definition of states depend on the specific application and desired granularity.
- (3) Transition probability estimation: The transition probabilities between different wind states are calculated based on the historical data. This involves analyzing the frequency and occurrence of state transitions.
- (4) Markov chain modeling: The transition probabilities are used to construct a transition matrix that represents the Markov chain. Each element of the matrix corresponds to the probability of transitioning from one state to another.
- (5) Wind prediction: Given an initial wind state, the Markov chain algorithm uses the transition matrix to iteratively forecast the wind state at future time steps. The prediction is based on the current state and the calculated transition probabilities.
- (6) Evaluation and validation: The accuracy and performance of the wind predictions are evaluated using appropriate metrics and compared against observed wind data to assess the algorithm's effectiveness.

By employing this algorithm, researchers and practitioners can obtain insights into future wind patterns, which can be valuable for various applications such as renewable energy planning, environmental modeling, and decision-making processes that involve wind-dependent activities.

The Algorithm. `import random`

```

def calculate_transition_matrix(states, transition_probabilities):
    transition_matrix = {}
    for i in range(len(states)):
        current_state = states[i]
        transition_matrix[current_state] = {}
        cumulative_prob = 0.0
        for j in range(len(states)):
            next_state = states[j]
            prob = transition_probabilities[i][j]
            cumulative_prob += prob
            transition_matrix[current_state][next_state] = cumulative_prob
    return transition_matrix

def predict_wind_speed(initial_state, transition_matrix, steps):
    current_state = initial_state
    predicted_speeds = [current_state]
    for _ in range(steps):
        rand = random.random()
        for next_state in transition_matrix[current_state]:
            if rand < transition_matrix[current_state][next_state]:
                current_state = next_state
                break
        predicted_speeds.append(current_state)
    return predicted_speeds

def main():

```

```

# Define wind speed states and transition probabilities
states = [
[84/90, 3/90],
[0, 3/90]
]
transition_probabilities = [
[0.6, 0.4],
[0.2, 0.8]
]
# Calculate the transition matrix
transition_matrix = calculate_transition_matrix(states, transition_probabilities)
# Set initial wind speed state and prediction steps
initial_state = [84/90, 3/90]
prediction_steps = 10
# Predict wind speeds
predicted_speeds = predict_wind_speed(initial_state, transition_matrix, prediction_steps)
# Print the predicted wind speeds
print("Predicted Wind Speeds:", predicted_speeds)

```

**3.3.4. Results.** In the subsequent analysis, we take into account the seasonal variations in wind speed as proposed by Karatepe and Corscadden [8]. represented by Markov graphs and their corresponding stochastic matrices. For each season—Winter, Spring, Summer, and Autumn—we’ve developed a distinct Markov graph that encapsulates the state transitions of wind speed specific to that time of year. Concurrently, we’ve created a stochastic matrix for each season, presenting a quantitative representation of the probabilities associated with each state transition within the Markov graph. Together, the graphs and matrices illustrate the dynamic and complex nature of wind speed as it fluctuates seasonally, serving as a predictive tool that can guide decision-making in fields such as fishing, renewable energy generation, and meteorology.

In our study focusing on wind speed prediction in the region of Casablanca, we identified two distinct states: a favorable and an unfavorable condition for fishing. According to Agmour et al. [1], the favorable state is characterized by wind speeds ranging from 1 to 5 km/h. In these conditions, fishing operations are typically unhindered, and safety levels are optimal.

On the other hand, the unfavorable state is associated with higher wind speeds ranging from 29 to 38 km/h. These conditions present significant challenges to fishing operations, as they are typically accompanied by moderate waves and increased chances of spray. These conditions can pose operational difficulties and safety risks for fishermen.

Seasonal Wind Speed Variations: Autumn (September-November). During Autumn, from September through November, wind speed variations demonstrate distinct patterns. These variations are represented using a Markov graph and a corresponding stochastic matrix.

The Markov graph visually depicts the transitions between different wind speed states. Conversely, the stochastic matrix numerically encapsulates these transition probabilities. Each matrix entry, denoted as  $p_{ij}$ , represents the probability of transitioning from state  $i$  to state  $j$ .

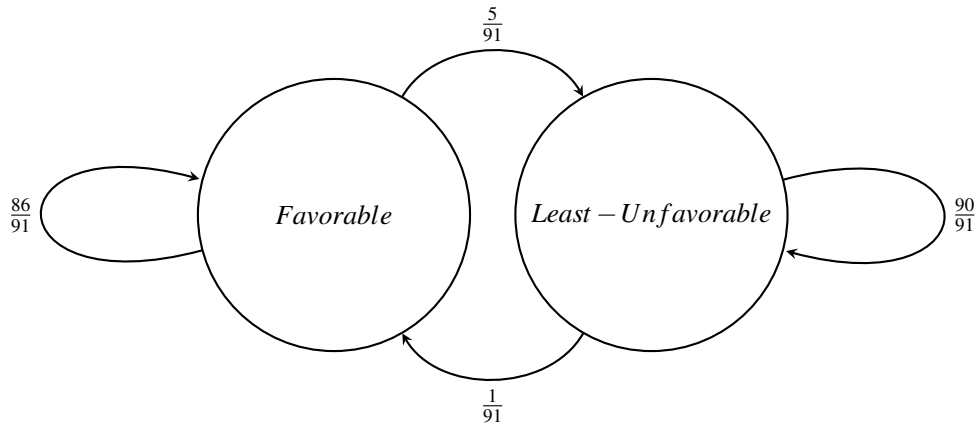


FIGURE 1. Markov graph, from 1 September to 30 November

Here is the Autumn season's stochastic matrix:

$$(3.4) \quad P_1 = \begin{bmatrix} \frac{86}{91} & \frac{5}{91} \\ \frac{1}{91} & \frac{90}{91} \end{bmatrix}.$$

In Figure 1, during September to November, there's a 94.5% chance of staying in the "Favorable" wind state and a 5.5% chance of moving to "Least-Unfavorable". Conversely, the

”Least-Unfavorable” state shows a 98.9% likelihood of remaining and only a 1.1% chance of reverting to ”Favorable”.

Seasonal Wind Speed Variations: Winter (December-February). During Winter, from December through February, wind speed variations exhibit unique patterns. These variations are illustrated using a Markov graph and a corresponding stochastic matrix.

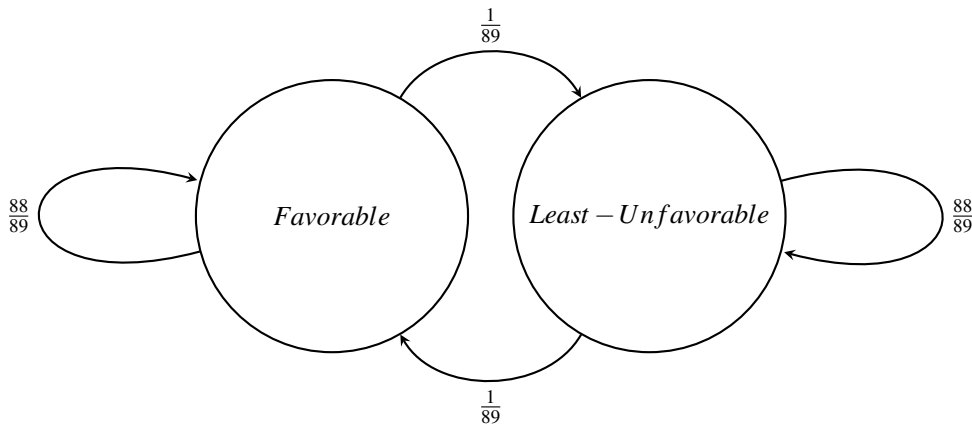


FIGURE 2. Markov graph, from 1 December to 28 February

Here is the Winter season’s stochastic matrix:

$$(3.5) \quad P_2 = \begin{bmatrix} \frac{88}{89} & \frac{1}{89} \\ \frac{1}{89} & \frac{88}{89} \end{bmatrix} .$$

For the December to February period, as illustrated in Figure 2, the ”Favorable” wind state has a high stability with a 88/89 probability of remaining unchanged and only a 1/89 chance of shifting to the ”Least-Unfavorable” state. The ”Least-Unfavorable” state mirrors this stability, with an 88/89 likelihood of maintaining its status and a 1/89 probability of converting to the ”Favorable” state.

Seasonal Wind Speed Variations: Spring (March-May). During Spring, from March through May, wind speed variations exhibit unique patterns. These variations are illustrated using a Markov graph and a corresponding stochastic matrix.

Here is the Spring season’s stochastic matrix:

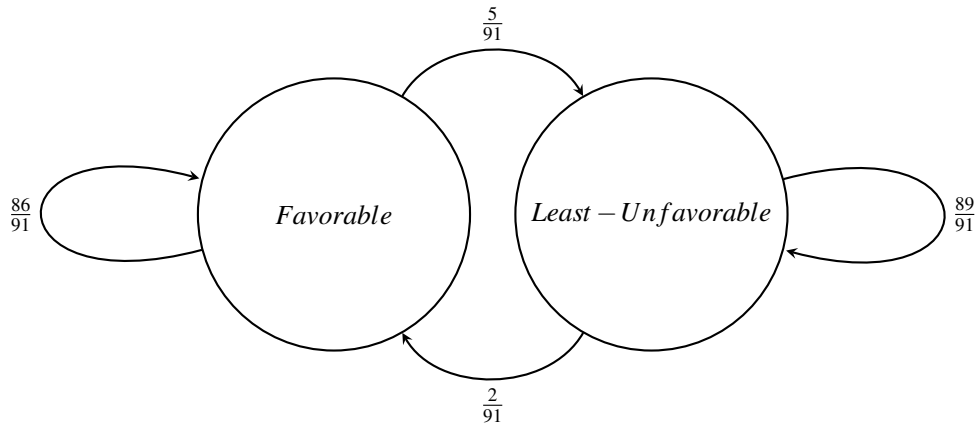


FIGURE 3. Markov graph, from 1 March to 31 May

$$(3.6) \quad P_3 = \begin{bmatrix} \frac{86}{91} & \frac{5}{91} \\ \frac{2}{91} & \frac{89}{91} \end{bmatrix}.$$

In Figure 3, for March to May, the "Favorable" wind state has an  $\frac{86}{91}$  probability of persisting and a  $\frac{5}{91}$  chance of transitioning to "Least-Unfavorable". The "Least-Unfavorable" state is more stable, with an  $\frac{89}{91}$  probability of maintaining itself, while there's a  $\frac{2}{91}$  chance it will switch to the "Favorable" state.

Seasonal Wind Speed Variations: Summer (June-August). During Summer, from June through August, wind speed variations exhibit distinct patterns. These variations are illustrated using a Markov graph and a corresponding stochastic matrix.

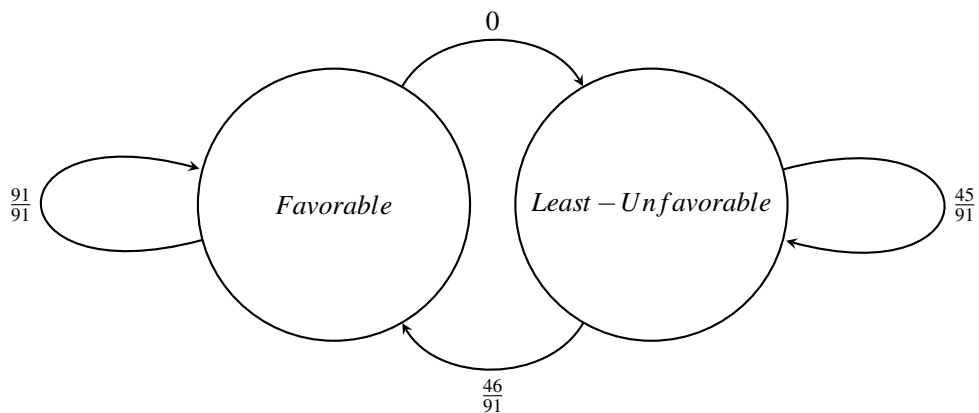


FIGURE 4. Markov graph, from 1 June to 31 August

Here is the Summer season’s stochastic matrix:

$$(3.7) \quad P_4 = \begin{bmatrix} \frac{91}{91} & \frac{0}{91} \\ \frac{46}{91} & \frac{45}{91} \end{bmatrix} .$$

During the summer months of June to August, as depicted in Figure 4, the "Favorable" wind state showcases a complete stability, with a probability of 91/91, indicating it remains consistently in the "Favorable" state without any transition to the "Least-Unfavorable" state. Conversely, the "Least-Unfavorable" state has a higher transition rate, with a 46/91 chance of switching to the "Favorable" state and a 45/91 probability of remaining in its current state.

**Remark 3.1.** *In this study, the original matrices were transformed into stochastic matrices for use in Markov chain models. The transformation ensured all entries were nonnegative and each row’s sum was 1, aligning with key properties of stochastic matrices. Adjustments were made carefully, maintaining data integrity while ensuring suitability for modeling transition probabilities.*

#### 4. SIMULATION

In this section, we aim to simulate the previously mentioned model that estimates the profits of two fishermen exploiting a Tritrophic Prey-Predator system, with special attention given to the impact of wind speed. To validate the local asymptotic stability and existence of the three fish populations, we use the parameters of system (2.1) as detailed in Table 1.

The economic parameters are obtained from system (2.30) and can be found in Table 2. Profits are then calculated and tabulated in Table 3. This simulation allows us to explore the potential effects of wind speed on the profitability of fishing within a Tritrophic Prey-Predator system, providing valuable insights for fishermen, fisheries management, and policymakers.

TABLE 1. Biological parameters for the Tritrophic system

Prey	Middle predator	Top predator
$r_1 = 5$	$r_2 = 4$	$r_3 = 3$
$\alpha = 9.10^{-6}$	$\bar{\alpha} = 8.10^{-6}$	$\bar{\beta} = 2.10^{-6}$
$\beta = 7.10^{-6}$	$\delta = 6.10^{-6}$	$\bar{\delta} = 10^{-6}$

TABLE 2. Economic parameters for the Tritrophic system

Prey	Middle predator	Top predator
$p_1 = 1$	$p_2 = 3$	$p_3 = 5$
$q_1 = 0.005$	$q_2 = 0.02$	$q_3 = 0.01$
$c_1 = 0.01$	$c_1 = 0.01$	$c_1 = 0.01$
$c_2 = 0.015$	$c_2 = 0.015$	$c_2 = 0.015$

TABLE 3. The Total profit estimation considering the wind speed

$p_1$	$p_2$	$p_3$	$\Pi_1$	$\Pi_2$
20	30	60	96.223	95.221
220	400	510	612.420	611.418
2200	3000	5100	6145.254	6144.25
20000	30000	6000	71496.944	71495.942

where  $(p_1, p_2, p_3)$  are the price of three fish populations and their corresponding fishing profit  $(\Pi_1, \Pi_2)$ .

The table shows how the price points of three fish populations  $(p_1, p_2, p_3)$  affect the corresponding fishing profits  $(\Pi_1, \Pi_2)$ . As the fish prices increase, the fishing profits rise correspondingly. However, our study findings emphasize that external factors like wind speed can significantly influence this relationship, potentially altering the expected profits. Thus, our bioeconomic model integrates these dynamics to predict and optimize outcomes, providing valuable insights for resource management in marine fisheries.

## 5. DISCUSSION

Our study presents a comprehensive exploration of the impact of wind speed on the profitability of dual-fishermen exploitation of a tritrophic prey-predator fish population. Using a bioeconomic model incorporating wind speed as a significant factor, we carried out simulations based on the predicted wind speed in the region of Casablanca. The following plot illustrates the profitability for two fishermen under two conditions: one considering the effect of wind



(marked as 'with Wind Effect') and the other without any wind effect. The x-axis represents different fishing populations, and the y-axis indicates the profit obtained.

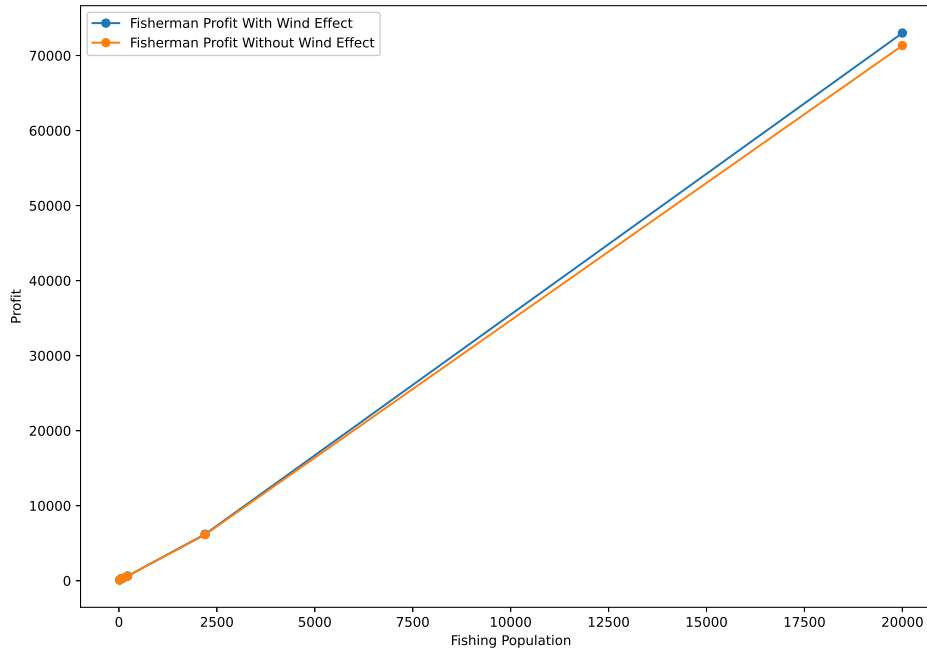


Figure 5: Fisherman profit comparison: with and without wind effect

In Figure 5, we observe a slight deviation in our study's results compared to those obtained by Bentounsi et al. [3]. In our investigation, we incorporated wind speed as a factor and applied the simulation to the predicted wind speed in the region of Casablanca. The wind speed, which was stable, appeared to positively impact profits. Additionally, we found that price also played a significant role in profit maximization. From these observations, we can infer the positive effect of wind speed on profit. However, if additional significant factors are considered, the results may vary. This variability underlines the importance of investigating the impact of other parameters on fishermen's profit.

Our study has clearly shown that wind speed significantly affects the profitability of dual-fishermen exploitation of a tritrophic prey-predator fish population, emphasizing the importance of incorporating environmental factors in bioeconomic modeling.

However, given the regional specificity of wind patterns, validation of the Markov chain model across different geographical areas is necessary. Future studies should aim to explore

the interplay of various environmental factors and extend the application of computational and predictive tools for enhanced accuracy and sustainability in fisheries management.

In summary, our findings underscore the pivotal role of wind speed in fisheries profitability, implying the need for a more environmentally comprehensive approach in fisheries management

## **6. CONCLUSION**

This study offers valuable insights into the seasonal wind speed variations and their potential impacts on fishing activities. Using the Markov chain approach, the study could model and predict wind speed across different seasons, providing a data-driven framework to better understand wind speed variations and their influence on ecosystem dynamics and fishing practices.

The research indicated that wind speed has a profound effect on fishing trip decisions and profitability, reinforcing the need for accurate and timely weather forecasting in aiding fishermen to make safe, informed decisions. Additionally, the study demonstrated the efficacy of employing Markov chains for wind speed prediction, contributing a new perspective to the existing body of literature in this field.

While the study has successfully addressed its research objectives, there are potential avenues for future research. This includes exploring the impact of other weather variables on fishing activities, integrating more comprehensive data sets, and utilizing advanced data analytics techniques for prediction.

Overall, understanding the dynamics of wind speed variations and their influences on various sectors, such as fishing, holds paramount importance in climate change research, fisheries management, and renewable energy sectors.

## **7. DATA AVAILABILITY**

The datasets analysed during the current study are available publicly in the websites; [www.onp.ma] and [www.windguru.cz].

## **CONFLICT OF INTERESTS**

The authors declare that there is no conflict of interests.

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