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Commun. Math. Biol. Neurosci. 2024, 2024:24

<https://doi.org/10.28919/cmbn/8407>

ISSN: 2052-2541

SEIHR MODEL FOR INDIAN COVID-19: TRUSTWORTHINESS OF THE GOVERNMENT REGULATORY PROCEDURE FOR CORONAVIRUS ASPECTS

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Abstract: In this research paper the identification and large implementation of a powerful SARS-CoV-2 vaccine may prevent considerable illnesses and fatalities from COVID-19 while minimizing the negative effects associated with non-pharmaceutical treatments. We used an enhanced, age-structured SEIHR model with social interaction matrices to evaluate age-related vaccination allocation strategies in India. We used specific to the state age patterns and disease transmission ratios calculated from confirmed COVID-19 incident cases between July 1 and August 31, 2020. How vaccination distribution tactics that prioritize distinct age groups reduce mortality and morbidity relative to one another, and how these strategies combine with concurrent non-pharmaceutical treatments [24], were looked into through simulations. Because of the uncertainty surrounding the development of the COVID-19 vaccine, we modified the vaccination features in the modeling simulations. The biggest relative reduction in mortality was obtained by allocating COVID-19 vaccines to older populations (those over 60 years old), regardless of vaccination effectiveness, control methods, rollout velocity, or immunity dynamics. Compared to other evaluated strategies, preferential

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Received December 18, 2023

vaccination of this group typically resulted in bigger total symptomatic infections and more dramatic predictions of peak incidence. The overall plan success was significantly influenced by vaccination efficacy, immunity type, target coverage, and rollout pace. The relative mortality benefit compared to no vaccination was significantly impacted by the time necessary to achieve goal coverage. The results of our study back up worldwide guidelines for deciding on COVID-19 vaccination allocation for those over 65. As the frequency of vaccine distribution increased the relative discrepancies between allocation plans reduced. Vaccine allocation choices which are optimum will be determined according to vaccine characteristics, the strength of ongoing non-pharmaceutical medication, and specific to that area goals. In comparison to not having been vaccinated, there is a benefit.

Keywords: basic reproduction number; stability analysis; disease free and endemic equilibrium; optimum control strategies.

2020 AMS Subject Classification: 92C60.

1. INTRODUCTION

The World Health Organization, or WHO, stated in December 2019 that COVID-19 was a pandemic of worldwide threat after the identification of the disease's fifth human case by China's Wuhan City officials. On June 1, 2022, the virus had caused over 530 million infections and over 6.2 million fatalities globally. The 2nd COVID-19 wave in India has been extremely serious; during the latter week of April 2021, around 30,000 new cases were confirmed daily. According to WHO data, as of December 15, 2022, there were over 6,637,512 confirmed COVID-19 instances and 646,740,524 confirmed cases of the virus. On February 14, 2020, the first COVID-19 case to be reported in Africa took place in Egypt [1].

Since COVID-19 symptoms take at least two to ten days to manifest, it might be challenging to segregate affected people in the early stages of the illness. The main signs and symptoms of COVID-19 are a high fever, respiratory difficulties, and a dry cough. When an infected person coughs or sneezes, respiratory droplets from them may enter the environment and spread the virus. People who have not previously been afflicted by the virus may contract it by breathing in contaminated air or by contacting infected objects. Such human transmission was widespread during the early phases of the COVID-19 outbreak since the general public was ignorant of these

risk factors and sick people were not quarantined, which allowed them to unintentionally spread the virus to other people [4].

- (i) Use soap and water or a hydro alcoholic solution to wash your hands often [5].
- (ii) If you cough or sneeze, cover your mouth and nose with the crease of your elbow or a handkerchief; promptly discard the handkerchief and wash your hands [5].
- (iii) Steer clear of people who are sick with a fever and cough [5].
- (iv) Seek medical attention as soon as you have a fever, cough, or trouble breathing [5].
- (v) Avoid unprotected direct contact with live animals and surfaces that have come into contact with animals in markets located in areas where cases of the new coronavirus are currently occurring [5].
- (vi) It is best to refrain from consuming raw or undercooked meat or poultry. To prevent cross contamination with raw food, raw meat, milk, or organ meats should be handled carefully in accordance with acceptable food safety practices [5].

Novel virus infection outbreaks in humans are always a cause for concern for the public's health, particularly when little is known about the virus's properties, how it travels among individuals, the severity of the resulting infections, and available treatments. One of the most effective ways to address all of these issues is through mathematical modeling. Effective nondrug treatments, like social exclusion and personal protection, will be essential in managing the outbreak in the absence of COVID-19 vaccines or antivirals [5].

State wise in India, Maharashtra, Kerala, Karnataka, Andhra Pradesh and Tamil Nadu are the top five hotspot of COVID-19 virus spreads, In Maharashtra as on 09th December 2023 totally 81,71,942 COVID-19 cases are conformed in that 8023379 are recovered and 148563 are Dead due to COVID-19, In Kerala as on 09th December 2023 totally 69,07,976 COVID-19 cases are conformed in that 6835931 are recovered and 72045 are Dead due to COVID-19, In Karnataka as on 09th December 2023 totally 40,88,956 COVID-19 cases are conformed in that 4048597 are recovered and 40359 are Dead due to COVID-19, In Andhra Pradesh as on 09th December 2023 totally 23,40,677 COVID-19 cases are conformed in that 2325944 are recovered and 14733 are Dead due to COVID-19, In Tamil Nadu as on 09th December 2023 totally 36,10,774 COVID-19

cases are conformed in that 35,72,693 are recovered and 38,081 are Dead due to COVID-19 these data are published in MoHFW [23].

Indian Government imposed various disease control strategy such as Lockdown, Social Distancing, Washing Hands Frequently Wearing Mask, Sanitize hands with alcohol base sanitizer and vaccine administer etc., In these Lock down helps us to make self-isolation by staying home and it helps us to recover from mild infection of COVID-19, While maintaining six feet Social Distancing helps to avoid the disease spread through the tiny droplets in Public places meanwhile wearing mask will prevent the spread of tiny droplets through air.

In light of the global rollout of COVID-19 vaccinations, we develop and examine a COVID-19 model that accounts for treatment options, immunization of susceptible persons, and hospitalized/infected patient care. Several important biological and epidemiological aspects of COVID-19 are included in our suggested model, such as demographic characteristics (recruitment/birth and death). Pontryagin's maximum concept, which is employed in epidemiological models and outlined in, is used to achieve optimal control. A sensitivity analysis utilizing partial rank correlation coefficients is performed in order to determine which model parameters, when vaccination and treatment are adopted, have a bigger influence on the initial disease transmission R_0 . The results are graphically displayed. This identification is essential in order to guide policy decisions about which parameters to prioritize for data gathering or to slow the disease's progress [3].

2. MODEL FORMATIONS

Based on the real situation we are going to form a Mathematical Modelling of Indian Pandemic COVID-19 Equations at any time t [38], we can subdivide the human population into five compartments namely $S(t)$ Susceptible, $E(t)$ Exposed, $I(t)$ infected, $H(t)$ Hospitalized and Removed $R(t)$ respectively [24].

SEIHR MODEL FOR INDIAN COVID-19

Parameters	Description
A	Birth Rate
β	Outflow of the S to E
α	Outflow of the E to I
ϕ	Outflow of the I to H
δ	Outflow of the H to R
γ	COVID-19 induced death rate
ϵ	Outflow of the E to S
λ	Outflow of the E to H
θ	Outflow of the I to R

The parameter M mathematically stands for the Vaccine administered to the public portion from the category $(R(t))$, the parameter M stands for the Vaccine administered to the public should depend on time and should provide the best result in formulating an optimal control problem that we must maintain the infected person at a minimum level, regardless of where $0 \leq M(t) \leq 1$. We have a propensity to draw up the expected COVID-19 scheme in Figure1 followed the above assumptions.

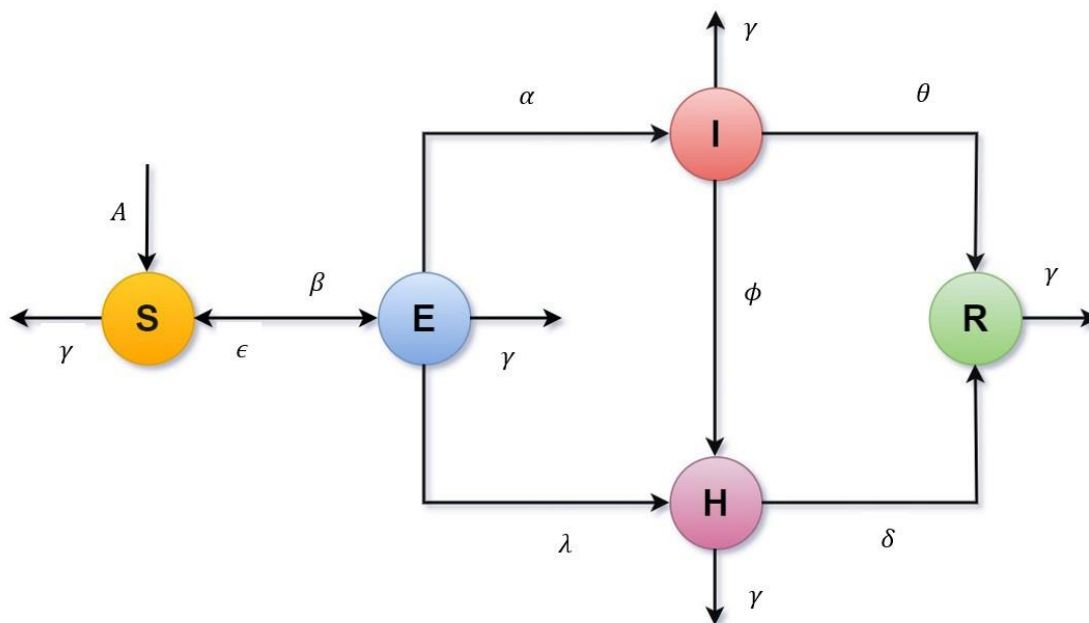


Fig 1: Compartmental Diagram of SEIHR Model for Indian COVID-19 Pandemic

Under this Assumption the Mathematical Model of Indian Pandemic COVID-19

$$\begin{aligned}
 \dot{S} &= A - \beta SE - \gamma S + \epsilon E \\
 \dot{E} &= \beta SE - (\gamma + \alpha + \epsilon + \lambda)E \\
 \dot{I} &= \alpha E - (\gamma + \phi + \theta)I \\
 \dot{H} &= \lambda E + \phi I - (\gamma + \delta)H \\
 \dot{R} &= \theta I + \delta H - \gamma R
 \end{aligned} \tag{1}$$

The initial states of the system (1) are $S(0) \geq 0, I(0) \geq 0, E(0) \geq 0, R(0) \geq 0$.

We assume that control parameter N has a fixed value. In this section we mainly focus the study of uniformly boundedness solutions and the basic reproduction number, Stability Criteria for different equilibria and sensitivity analysis etc.

3. UNIFORMLY BOUNDEDNESS OF INDIAN PANDEMIC COVID-19 EQUATIONS

We verify the boundedness property of the system of nonlinear equations (1). The system of nonlinear equations(1). are uniformly bounded. Based on the assumption that

$$Z = S + E + I + H + R$$

Therefore

$$\begin{aligned}
 \frac{dz}{dt} &= \frac{dS}{dt} + \frac{dE}{dt} + \frac{dI}{dt} + \frac{dH}{dt} + \frac{dR}{dt} \\
 \frac{dz}{dt} &= A - \gamma z \\
 \frac{dz}{dt} + \gamma z &= A
 \end{aligned}$$

Integrating above inequality by applying Birkhoff and Rota [2] theorem of differential equation we get

$$z \leq \frac{A}{\gamma} [1 - e^{-\gamma t}] + z_0 e^{-\gamma t}$$

Now for $t \rightarrow \infty$

$$0 < z \leq \frac{A}{\gamma}$$

Hence all the solutions of System of Nonlinear Equations (1) that are commence in $\{R_+^5\}$ are restricted in the region

$$\{z \in R_+^5: 0 < X(S, E, I, H, R) \leq \frac{A}{\gamma} + \epsilon\}$$

For any $\epsilon > 0$ and for $t \rightarrow \infty$

4. BASIC REPRODUCTION NUMBER OF INDIAN PANDEMIC COVID-19 EQUATIONS

Basic reproduction number (R_0) parameter plays an important role within the epidemic model for determinant the character of disease “The number of Secondary individual infected can caused by a single infected individual within the whole time interval” [5,9,22,44]

$$\dot{E} = \beta SE - (\gamma + \alpha + \epsilon + \lambda)E \quad (2)$$

$$\dot{I} = \alpha E - (\gamma + \phi + \theta)I$$

$$\dot{H} = \lambda E + \phi I - (\gamma + \delta)H$$

We can write the above system as

$$\frac{dy}{dt} = \varphi(y) - \Psi(y)$$

$$\text{Where } y = \begin{pmatrix} E \\ I \\ H \end{pmatrix} \quad \varphi(y) = \begin{pmatrix} \beta SE \\ 0 \\ 0 \end{pmatrix} \quad \Psi(y) = \begin{pmatrix} (\gamma + \epsilon + \alpha + \lambda)E \\ (\gamma + \phi + \theta)I - \alpha E \\ (\gamma + \delta)H - \lambda E - \phi I \end{pmatrix}$$

The system of Nonlinear Equations (1) that's $E_0 \left(\frac{A}{\gamma}, 0, 0, 0 \right)$ a disease free equilibrium.

Currently the Jacobian matrix of φ and Ψ at the disease free equilibrium are respectively given by

$$F = J \left(\frac{\varphi}{E_0} \right) = \begin{pmatrix} \beta S_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$V = J \left(\frac{\Psi}{E_0} \right) = \begin{pmatrix} (\gamma + \epsilon + \alpha + \lambda) & 0 & 0 \\ -\alpha & (\gamma + \phi + \theta) & 0 \\ -\lambda & -\phi & (\gamma + \delta) \end{pmatrix}$$

$$\text{Then } V^{-1} = \begin{pmatrix} \frac{1}{(\gamma + \epsilon + \alpha + \lambda)} & 0 & 0 \\ \frac{\alpha}{(\gamma + \epsilon + \alpha + \lambda)(\gamma + \phi + \theta)} & \frac{1}{(\gamma + \phi + \theta)} & 0 \\ \frac{\lambda}{(\gamma + \epsilon + \alpha + \lambda)(\gamma + \phi + \theta)(\gamma + \delta)} & \frac{\phi}{(\gamma + \phi + \theta)(\gamma + \delta)} & \frac{1}{\gamma + \delta} \end{pmatrix}$$

The matrix's spectral radius is also known as the basic reproductive number (R_0) (FV^{-1}) and is indicated in the current model by

$$\begin{aligned}
 & FV^{-1} \\
 & = \begin{pmatrix} \beta S_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{(\gamma + \epsilon + \alpha + \lambda)} & 0 & 0 \\ \frac{\alpha}{(\gamma + \epsilon + \alpha + \lambda)(\gamma + \phi + \theta)} & \frac{1}{(\gamma + \phi + \theta)} & 0 \\ \frac{\lambda}{(\gamma + \epsilon + \alpha + \lambda)(\gamma + \phi + \theta)(\gamma + \delta)} & \frac{\phi}{(\gamma + \phi + \theta)(\gamma + \delta)} & \frac{1}{\gamma + \delta} \end{pmatrix} \\
 & \quad FV^{-1} = \frac{\beta S_0}{(\gamma + \epsilon + \alpha + \lambda)} \\
 & \quad R_0 = \frac{\beta A}{\gamma(\gamma + \epsilon + \alpha + \lambda)} \tag{3}
 \end{aligned}$$

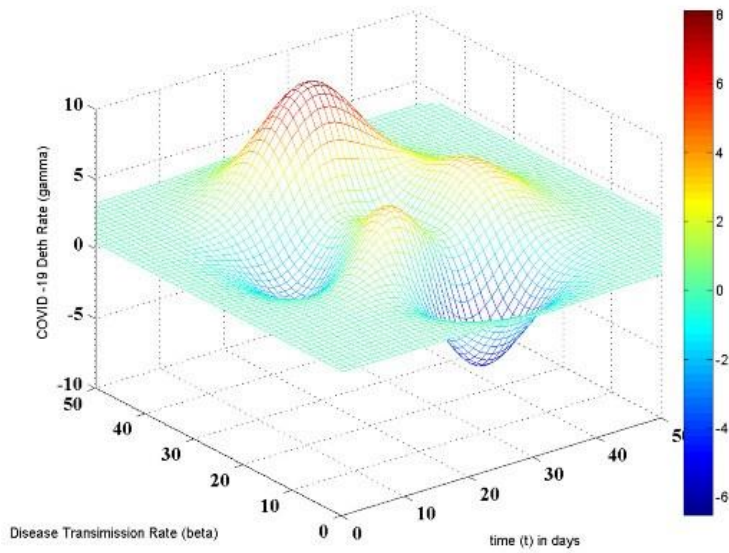


Fig 2: Surface graph of R_0 with respect to Disease Transmission Rate (β) and Covid -19 Death Rate (γ)

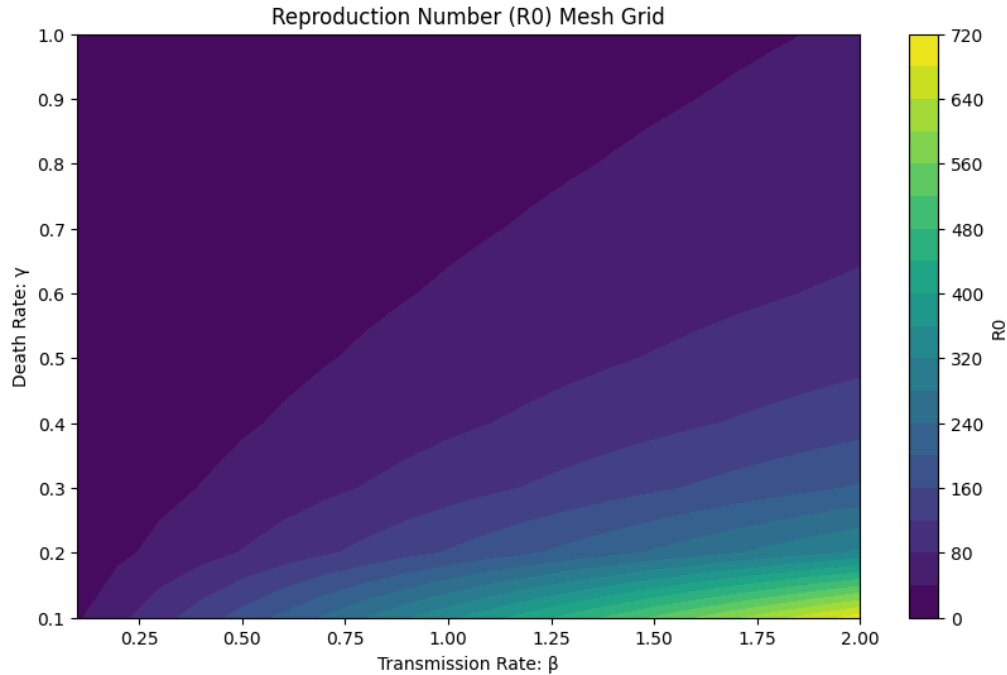


Fig 3: Contour graph of R_0 with respect to Disease Transmission Rate (β) and Covid -19 Death Rate (γ)

5. EQUILIBRIA SOLUTION OF INDIAN PANDEMIC COVID-19

Two possible equilibria's are available for the system of nonlinear equations (1), one is the disease-free equilibrium and the disease disappears in this disease-free equilibrium. And give $E_0(S_0, 0, 0, 0, R_0)$, Where $S_0 = \frac{A}{\gamma}$, $R_0 = \frac{\beta A}{\gamma(\gamma + \epsilon + \alpha + \lambda)}$ and the other one is Endemic equilibrium point in this endemic equilibrium the infections present always and gives

$E_1(S^*, E^*, I^*, H^*, R^*)$

$$S^* = \frac{(\gamma + \epsilon + \lambda + \alpha)}{\beta}$$

$$E^* = \frac{A\beta - \gamma(\gamma + \epsilon + \lambda + \alpha)}{\beta(\gamma + \alpha + \lambda)}$$

$$I^* = \frac{\alpha E^*}{(\gamma + \theta + \phi)}$$

$$H^* = \frac{\lambda E^* + \phi I^*}{(\gamma + \delta)}$$

$$R^* = \frac{\delta H^* + \theta I^*}{\gamma}$$

(4)

6. COVID-19 INDIAN PANDEMIC FOR LOCALLY ASYMPTOTICALLY STABLE

We planned to examine the local asymptotic stability of both infected Free and Endemic Equilibria, If $R_0 < 1$ then the disease free symmetry E_0 is locally asymptotically stable if $R_0 > 1$ then the disease free equilibrium E_0 is locally asymptotically unstable. [12,21,19,44]

The disease free equilibrium of the system of Nonlinear Equation (1) is given by the following Jacobian Matrix

$$J = \begin{pmatrix} -\gamma & 0 & 0 & 0 & 0 \\ -\beta S_0 & -(\gamma + \epsilon + \lambda + \alpha) & \alpha & \lambda & 0 \\ 0 & 0 & -(\gamma + \phi + \theta) & \phi & \theta \\ 0 & 0 & 0 & -(\gamma + \delta) & \delta \\ 0 & 0 & 0 & 0 & -\gamma \end{pmatrix}$$

$$|J - \rho I| = \begin{pmatrix} -\gamma - \rho & 0 & 0 & 0 & 0 \\ -\beta S_0 & -(\gamma + \epsilon + \lambda + \alpha) - \rho & \alpha & \lambda & 0 \\ 0 & 0 & -(\gamma + \phi + \theta) - \rho & \phi & \theta \\ 0 & 0 & 0 & -(\gamma + \delta) - \rho & \delta \\ 0 & 0 & 0 & 0 & -\gamma - \rho \end{pmatrix}$$

The characteristic equation of the system Nonlinear equations (1) at its disease free equilibrium is given by

$$(\rho + \gamma)^2(\rho + (\gamma + \delta))(\rho + (\gamma + \phi + \theta))(\rho + (\gamma + \epsilon + \lambda + \alpha)) = 0 \quad (5)$$

From the above Jacobian matrix if all the eigen value are non-positive only when $R_0 < 1$. Then the system is locally asymptotically stable if $R_0 < 1$ and it is unstable if $R_0 > 1$.

Note 1: If R_0 increases to its value greater than 1 then the disease free equilibrium E_0 losses its stability.

Note 2: When $R_0 = 1$ the system of nonlinear equations (1) permits through a Transcritical bifurcation around its disease free equilibrium. When $R_0 < 1$ the disease free equilibrium happens and locally asymptotically stable if $R_0 > 1$ is the beginning criteria for both Present and asymptotic stability for endemic equilibrium point, At the beginning $R_0 > 1$ then the disease free equilibrium reduces to unstable in nature. Then there is a change of feasibility besides stability occurs at $R_0 = 1$. [12,21,19,44].

7. COVID-19 INDIAN PANDEMIC FOR ENDEMIC EQUILIBRIUM

If $R_0 > 1$ then the Endemic equilibrium E_1 is locally asymptotically stable from the following Jacobian Matrix.

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$$J = \begin{pmatrix} -\gamma - \beta E^* & -\beta S^* + \epsilon & 0 & 0 & 0 \\ \beta E^* & \beta S^* - (\gamma + \epsilon + \lambda + \alpha) & 0 & 0 & 0 \\ 0 & \alpha & -(\gamma + \phi + \theta) & 0 & 0 \\ 0 & \lambda & \phi & -(\gamma + \delta) & 0 \\ 0 & 0 & \theta & \delta & -\gamma \end{pmatrix}$$

$$|J - \rho I|$$

$$= \begin{pmatrix} -\gamma - \beta E^* - \rho & -\beta S^* + \epsilon & 0 & 0 & 0 \\ \beta E^* & \beta S^* - (\gamma + \epsilon + \lambda + \alpha) - \rho & 0 & 0 & 0 \\ 0 & \alpha & -(\gamma + \phi + \theta) - \rho & 0 & 0 \\ 0 & \lambda & \phi & -(\gamma + \delta) - \rho & 0 \\ 0 & 0 & \theta & \delta & -\gamma - \rho \end{pmatrix}$$

The characteristic equation of the system (1) around its endemic equilibrium E_2 is

$$(\rho + \gamma)(\rho + (\gamma + \delta))(\rho + (\gamma + \phi + \theta))$$

$$\left(\rho^2 + \rho \left(\frac{A\beta - \gamma\epsilon}{(\gamma + \alpha + \lambda)} \right) + \frac{(\gamma + \alpha + \lambda)^2 + \gamma(\gamma + \epsilon + \lambda + \alpha)}{(\gamma + \alpha + \lambda)} (R_0 - 1) \right) = 0 \quad (6)$$

Equation (5) shows that the first pair of roots are positive real and rest of the roots are Quadratic polynomial and all the other parametric values are positive. We accomplish that equation (1) is locally asymptotically steady everywhere its endemic equilibrium E_1 in the view of Routh-Hurwitz criterion.

8. SENSITIVITY ANALYSIS

In this component, it is established whether modifying values for parameters impact the functional value of the reproduction number. It is vital to find the critical parameter that may act as a critical threshold for disease management. The sensitivity index for R_0 to $\beta, \gamma, \epsilon, \alpha$ be as follows

$$\left. \begin{aligned} \frac{\partial R_0}{\partial \beta} &= \frac{A}{\gamma(\gamma + \lambda + \alpha + \epsilon)} \\ \frac{\partial R_0}{\partial \epsilon} &= \frac{-\beta A \gamma}{(\gamma(\gamma + \lambda + \alpha + \epsilon))^2} \\ \frac{\partial R_0}{\partial \gamma} &= \frac{-\beta A (2\gamma + \lambda + \alpha + \epsilon)}{(\gamma(\gamma + \lambda + \alpha + \epsilon))^2} \\ \frac{\partial R_0}{\partial \alpha} &= \frac{-\beta A \gamma}{(\gamma(\gamma + \lambda + \alpha + \epsilon))^2} \end{aligned} \right\} \quad (7)$$

$$\frac{\partial R_0}{\partial \lambda} = \frac{-\beta A \gamma}{(\gamma(\gamma + \lambda + \alpha + \epsilon))^2}$$

All partial derivatives are positive, therefore increasing any of the aforementioned factors enhances the basic reproductive number R_0 . The proportionate reaction to proportional stimulation can be utilized to assess elasticity.

$$\begin{aligned} E_\beta &= \frac{\beta}{R_0} \left(\frac{\partial R_0}{\partial \beta} \right) = \frac{\beta}{R_0} \left(\frac{A}{\gamma(\gamma + \lambda + \alpha + \epsilon)} \right) = 1 \text{ (Positive)} \\ E_\epsilon &= \frac{\epsilon}{R_0} \left(\frac{\partial R_0}{\partial \epsilon} \right) = \frac{\epsilon}{R_0} \left(\frac{-\beta A \gamma}{(\gamma(\gamma + \lambda + \alpha + \epsilon))^2} \right) = -0.13 \text{ (Negative)} \\ E_\gamma &= \frac{\theta_1}{R_0} \left(\frac{\partial R_0}{\partial \gamma} \right) = \frac{\gamma}{R_0} \left(\frac{-\beta A (2\gamma + \lambda + \alpha + \epsilon)}{(\gamma(\gamma + \lambda + \alpha + \epsilon))^2} \right) = -1.43 \text{ (Negative)} \\ E_\alpha &= \frac{\alpha}{R_0} \left(\frac{\partial R_0}{\partial \alpha} \right) = \frac{\alpha}{R_0} \left(\frac{-\beta A \gamma}{(\gamma(\gamma + \lambda + \alpha + \epsilon))^2} \right) = -0.33 \text{ (Negative)} \\ E_\lambda &= \frac{\lambda}{R_0} \left(\frac{\partial R_0}{\partial \lambda} \right) = \frac{\lambda}{R_0} \left(\frac{-\beta A \gamma}{(\gamma(\gamma + \lambda + \alpha + \epsilon))^2} \right) = -0.11 \text{ (Negative)} \end{aligned} \quad (8)$$

From the above system of equations E_β is positive, while $E_\gamma, E_\epsilon, E_\theta, E_\alpha$ are all negative, it is all negative. This clearly shows that increasing the value of β will increase the value of R_0 while increasing the value of $\epsilon, \gamma, \theta, \alpha$ will decrease the value of R_0 . A highly sensitive parameter should be carefully analysed since tiny changes in the system might result in big numerical changes R_0 .

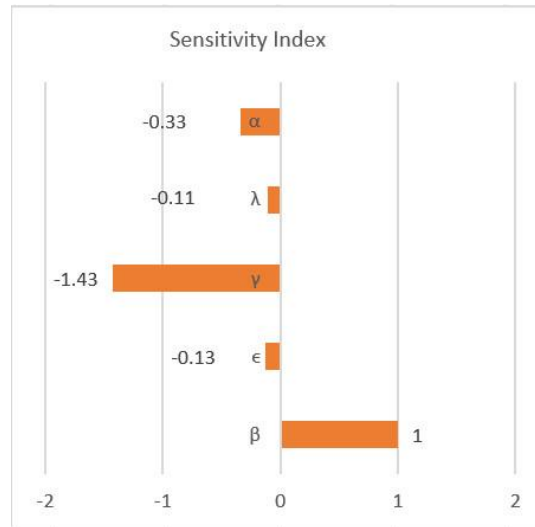


Fig 4: Sensitivity Index R_0 with respect to $(\beta), (\gamma), (\epsilon), (\lambda), (\alpha)$

9. OPTIMAL CONTROL FOR COVID-19 INDIAN PANDEMIC

Here $M(t)$ stands for the Vaccine administered to the public it is one of the time varying control strategy implemented by Indian government, which gave the awareness of Vaccine administered to the public. Recent days it is very important strategy implemented by Indian Government which minimizes the virus spread between the peoples. In order to determine such a strategy, we can use the optimal control theory to minimise the spread of the virus between people, the main objective of the optimal control problem. We may construct the goal in the following way: [27,31]

$$\left. \begin{aligned} \dot{S} &= A - \beta SE - \gamma S + \epsilon E - pMS \\ \dot{E} &= \beta SE - (\gamma + \alpha + \epsilon + \lambda)E \\ \dot{I} &= \alpha E - (\gamma + \phi + \theta)I \\ \dot{H} &= \lambda E + \phi I - (\gamma + \delta)H \\ \dot{R} &= \theta I + \delta H - \gamma R + pMS \end{aligned} \right\} \quad (9)$$

$$J = \int_0^{t_\tau} [c_1 I(t) + c_2 M(t)] dt \quad (10)$$

Subject to the model proposed (1). Where c_1 is defined for the infected population and c_2 is restricted to the control. The above functional aim is linear and regulated in bounded states.

We can also use standard results to ensure optimum control and appropriate optimum states [8].

The aim is to determine the optimal control value $N(t)$ in such a way such that

$$J(M^*(t)) = \min_{M \in \Omega} J(M)$$

$$\Omega = \{M(t) : 0 \leq a \leq M(t) \leq b < 1, 0 \leq t \leq t_\tau, M(t) \text{ is a Lebesgue Measurable}\}$$

By Using Pontryagin's Maximum Principal [8,28,37,44] to develop the required conditions for our optimal control and corresponding states. Then the Lagrangian is given by

$$L(I, M) = c_1 I(t) + c_2 M(t) \quad (11)$$

The Hamiltonian is defined as follows

$$V(I, N, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = L(I, N) + \lambda_1 \left(\frac{dS}{dt} \right) + \lambda_2 \left(\frac{dE}{dt} \right) + \lambda_3 \left(\frac{dI}{dt} \right) + \lambda_4 \left(\frac{dH}{dt} \right) + \lambda_5 \left(\frac{dR}{dt} \right) \quad (12)$$

A vice parameter of the Optimal Control $M(t)$ exists according to the states S, E, I, H and R :

$$\lambda'_1(t) = -\frac{\partial V}{\partial S} = \lambda_1(t)[\beta E + \gamma + pM] - \lambda_2(t)[\beta E] - \lambda_5(t)[pM]$$

$$\begin{aligned}\lambda'_2(t) &= -\frac{\partial V}{\partial E} = \lambda_1(t)[\beta S - \epsilon] - \lambda_2(t)[\beta S - (\gamma + \epsilon + \alpha + \lambda)] + \alpha\lambda_3(t) + \lambda\lambda_4(t) \\ \lambda'_3(t) &= -\frac{\partial V}{\partial I} = \lambda_3(t)[(\phi + \gamma + \theta)] - \phi\lambda_4(t) - \theta\lambda_5(t) \\ \lambda'_4(t) &= -\frac{\partial V}{\partial H} = \lambda_4(t)[\gamma + \delta] - \delta\lambda_5(t) \\ \lambda'_5(t) &= -\frac{\partial V}{\partial H} = \gamma\lambda_5(t)\end{aligned}$$

The transversally conditions were satisfying by the adjoin variables

$$\lambda_1(t_\tau) = 0, \lambda_2(t_\tau) = 0, \lambda_3(t_\tau) = 0, \lambda_4(t_\tau) = 0, \lambda_5(t) = 0 \quad (13)$$

We try to minimize the Hamiltonian using the control variable $M(t)$, More over the Hamiltonian is linear in the control parameter, if we consider the optimal control is singular. Then the switching function as

$$\varphi(t) = \frac{\partial V}{\partial M} = c_2 + (\lambda_5 - \lambda_1)pS \quad (14)$$

If switching function vanishes on non-trivial interval of time then the singular control occurs, and then the optimal control is to take its upper bound or its lower bound according as

$\frac{\partial V}{\partial M} < 0$ or > 0 to verify the singular case, we may assume that $\frac{\partial V}{\partial M} = 0$.

For some non-trivial interval. we calculate $\frac{d}{dt}\left(\frac{\partial V}{\partial M}\right) = 0$ After some simplifications of the time derivative of $\frac{\partial V}{\partial M}$ we obtain

$$0 = \frac{d}{dt}\left(\frac{\partial V}{\partial M}\right) = \frac{d}{dt}[c_2 + (\lambda_5 - \lambda_1)pS] = -\lambda'_1 pS - \lambda_1 pS' + \lambda_5 pS' + \lambda'_5 pS \quad (15)$$

Using the system of equations (2) and (5) we obtain

$$0 = \frac{d}{dt}\left(\frac{\partial V}{\partial M}\right) = -pS[(\lambda_2 - \lambda_1)\beta E - \gamma\lambda_1] + p(\lambda_5 - \lambda_1)(K - \gamma pS) \quad (16)$$

$$\text{Here } K = A - \beta SE - \gamma S + \epsilon E$$

The control parameter \mathbf{M} will not exist for the fore said equations, hence calculation of second order derivative with respect to time is essential.

$$\begin{aligned}0 &= \frac{d^2}{dt^2}\left(\frac{\partial V}{\partial M}\right) = \{-\beta E(\lambda'_2 - \lambda'_1) - \beta(\lambda_2 - \lambda_1)E' + \gamma\lambda'_1\} pS + pS'[\gamma\lambda - (\lambda_2 - \lambda_1)\beta E] + \\ &(\lambda'_5 - \lambda'_1)Kp + (A - \beta(SE' + S'E) - \gamma S' + \epsilon E')(\lambda_5 - \lambda_1)p - \\ &p\gamma S'\lambda_5 - p\gamma S\lambda'_5\end{aligned} \quad (17)$$

$$\begin{aligned}
&= (\lambda_2 - \lambda_1)[\beta\gamma E - (2\beta^2 s^2 pE + \beta^2 E^2 pS)] + (\lambda_5 - \lambda_1)[pM\gamma - \beta p^2 SEM] \\
&+ (\gamma + \lambda + \alpha + \epsilon)\beta pSE(1 + \lambda_2) + \beta pSE(\gamma - \epsilon + \alpha\lambda_3 - \lambda\lambda_4) - \gamma^2\lambda_1 \\
&+ (\lambda_2 - \lambda_1)[\beta^2 SE^2 + \beta\gamma SE - \beta\epsilon E^2 - A\beta E] + \beta SE[pM - \gamma\lambda] \\
&+ (A - \epsilon E + pSM)\gamma\lambda - \gamma^2\lambda S + \{p(\lambda_5 - \lambda_1)[A - (K - pSM)(\beta\epsilon + \gamma) \\
&(\beta S - \epsilon)[(\gamma + \lambda + \alpha + \epsilon)E - \beta SE]\} - (\lambda_2 - \lambda_1)\beta Kp - (\lambda_5 - \lambda_1)[Kp\gamma - Kp^2 M] \\
&+ \lambda_5\gamma p[S\gamma + K - pSM]
\end{aligned} \tag{18}$$

The above equation can be written in the form

$$\frac{d^2}{dt^2} \left(\frac{\partial V}{\partial M} \right) = \varphi_1(t)M(t) + \varphi_2(t) = 0$$

And we solve the singular control as $M_{Singular}(t) = -\frac{\varphi_2(t)}{\varphi_1(t)}$

Provided $\varphi_1(t) \neq 0$ and $a \leq -\frac{\varphi_2(t)}{\varphi_1(t)} \leq b$, where

$$\begin{aligned}
\varphi_1(t) &= (\lambda_5 - \lambda_1)[p\gamma - \beta p^2 SE] + \beta pSE + \gamma\lambda pS + (\lambda_5 - \lambda_1)p^2 S(\beta\epsilon + \gamma) - p^2 K(\lambda_5 - \lambda_1) \\
&+ \lambda_5 p^2 \gamma S
\end{aligned}$$

And

$$\begin{aligned}
\varphi_2(t) &= (\lambda_2 - \lambda_1)[\beta\gamma E - (2\beta^2 s^2 pE + \beta^2 E^2 pS)] \\
&+ (\gamma + \lambda + \alpha + \epsilon)\beta pSE(1 + \lambda_2) + \beta pSE(\gamma - \epsilon + \alpha\lambda_3 - \lambda\lambda_4) - \gamma^2\lambda_1 \\
&+ (\lambda_2 - \lambda_1)[\beta^2 SE^2 + \beta\gamma SE - \beta\epsilon E^2 - A\beta E] + \beta SE[-\gamma\lambda] \\
&+ (A - \epsilon E)\gamma\lambda - \gamma^2\lambda S + \{p(\lambda_5 - \lambda_1)[A - (K)(\beta\epsilon + \gamma) \\
&(\beta S - \epsilon)[(\gamma + \lambda + \alpha + \epsilon)E - \beta SE]\} - (\lambda_2 - \lambda_1)\beta Kp - (\lambda_5 - \lambda_1)[Kp\gamma] \\
&+ \lambda_5\gamma p[S\gamma + K]
\end{aligned} \tag{19}$$

Moreover, it should obey the Generalized Legendre clebsch criteria which is expressed as

$$\frac{d}{dM} \frac{d^2}{dt^2} \left(\frac{\partial V}{\partial M} \right) = \varphi_1(t)$$

To be negative [25], the conclusion part of control profile is defined by

$$\text{If } \frac{\partial V}{\partial M} < 0, \text{ then } M^*(t) = b$$

$$\text{If } \frac{\partial V}{\partial M} > 0, \text{ then } M^*(t) = a$$

$$\text{If } \frac{\partial V}{\partial M} = 0, \text{ then } M_{Singular}(t) = -\frac{\varphi_2(t)}{\varphi_1(t)}$$

Hence the Control optimal provided $\varphi_1(t) < 0$ and $a \leq -\frac{\varphi_2(t)}{\varphi_1(t)} \leq b$.

Govt Control Strategy of Vaccine administered to the Public at 0% compliances

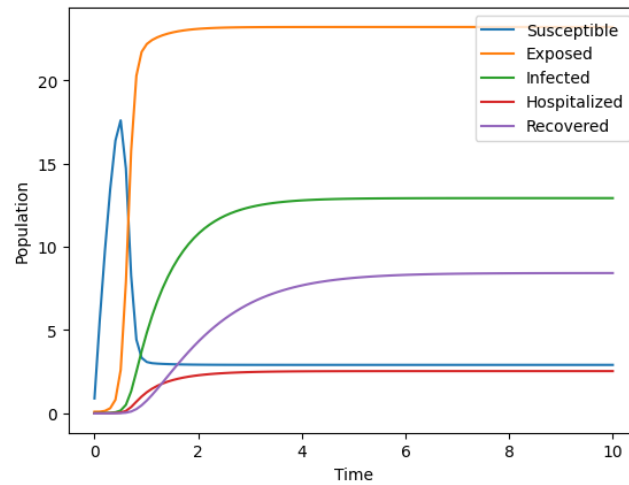


Fig 5: Government control strategy of Vaccine administered to the Public at 0%

Govt Control Strategy of Vaccine administered to the Public at 100% compliances

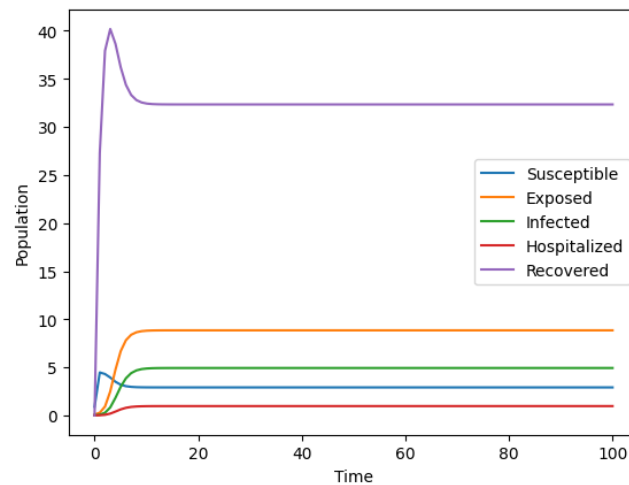


Fig 6: Government control strategy of Vaccine administered to the Public at 100%

10. NUMERICAL ANALYSIS

We look at numerical outcomes in two scenarios: first, when the control is fixed, and second, when the control is applied optimally. For numerical simulations, we first consider the values of the parameters in Table 2. Since δ is the disease-induced rate of mortality and d is the natural death rate, we can conclude that $\delta > d$. Using these characteristics as well as the basic conditions as a

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starting point $S(0) = 60, I(0) = 1, E(0) = 1, R(0) = 190$ We solve our proposed model (1) numerically. When $R_0 < 1$ and the solutions of model (1) converge to the DFE, as shown in Fig. 2, the numerical result is verified.

Table II: Parameter meanings for model

Parameters	Description	Values / Range	Reference
A	Birth Rate	50	Assumed
β	Outflow of the S to E	0.8	[4]
α	Outflow of the E to I	0.79	[5]
ϕ	Outflow of the I to H	0.2	Assumed
δ	Outflow of the H to R	[0.5, 2.3]	Assumed
γ	COVID-19 induced death rate	(0,1)	Assumed
ϵ	Outflow of the E to S	0.4	[4]
λ	Outflow of the E to H	0.35	[4]
θ	Outflow of the I to R	0.3	[4]

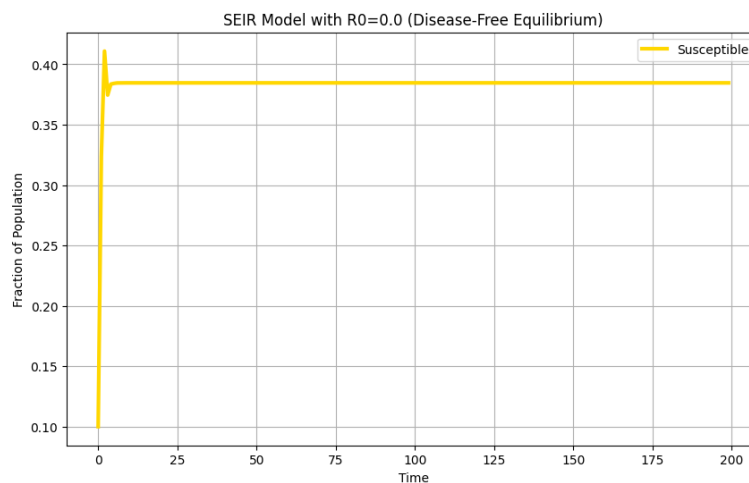


Fig 7 (a): Plot for Change in Susceptible Rate at disease free equilibrium point

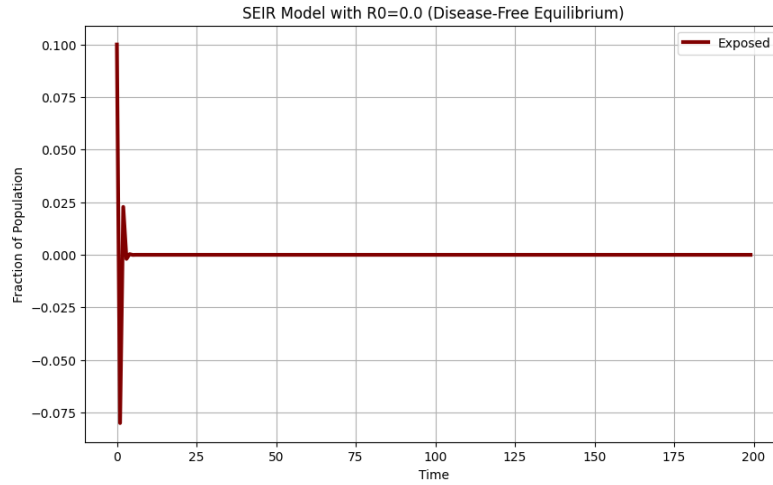


Fig 7 (b): Plot for Change in Exposed Rate at disease free equilibrium point

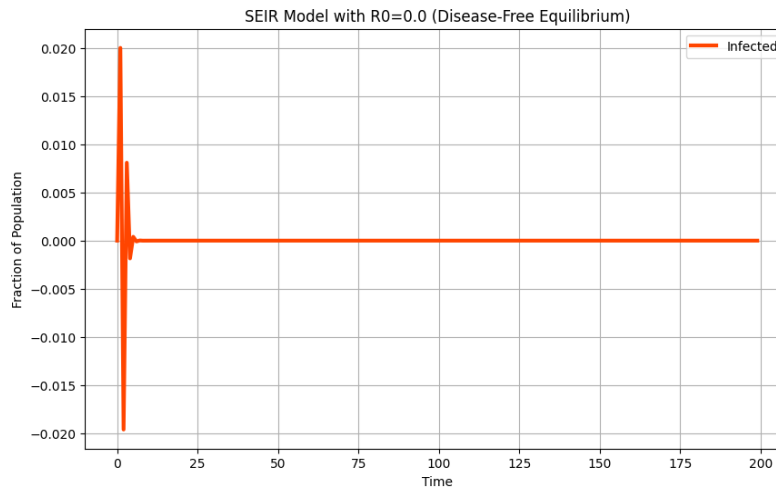


Fig 7 (c): Plot for Change in Infected Rate at disease free equilibrium point

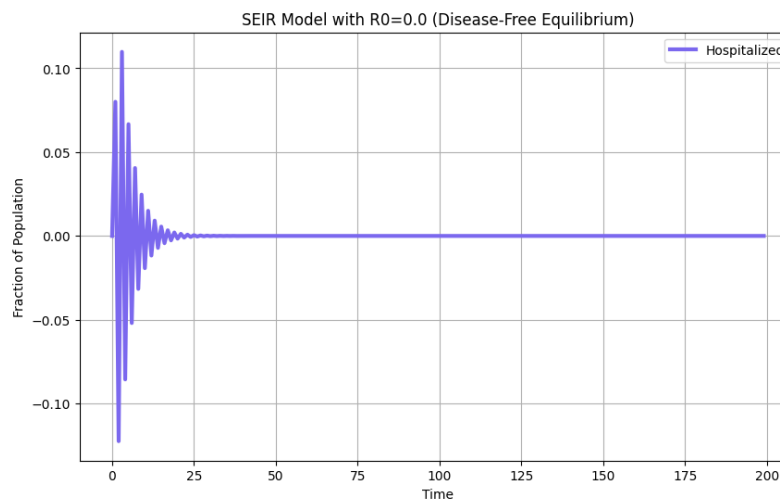


Fig 7 (d): Plot for Change in Hospitalized Rate at disease free equilibrium point

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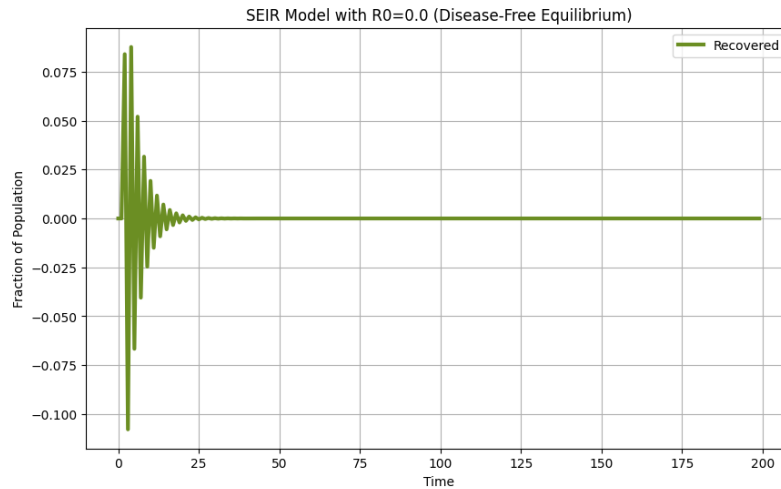


Fig 7 (e): Plot for Change in Recovered Rate at disease free equilibrium point

Fig. 7(a), 7(b), 7(c), 7(d), 7(e) The solution curves for the model (Fig. 1) at disease free equilibrium point with.

Figure 7(a), 7(b), 7(c), 7(d), 7(e) shows the numerical results of the system (Fig. 1) when $R_0 < 1$. These figures show that all numerical solutions for the problem converged to the disease-free equilibrium, $E_0 = (50, 0, 0, 17)$ for the case $R_0 < 1$.

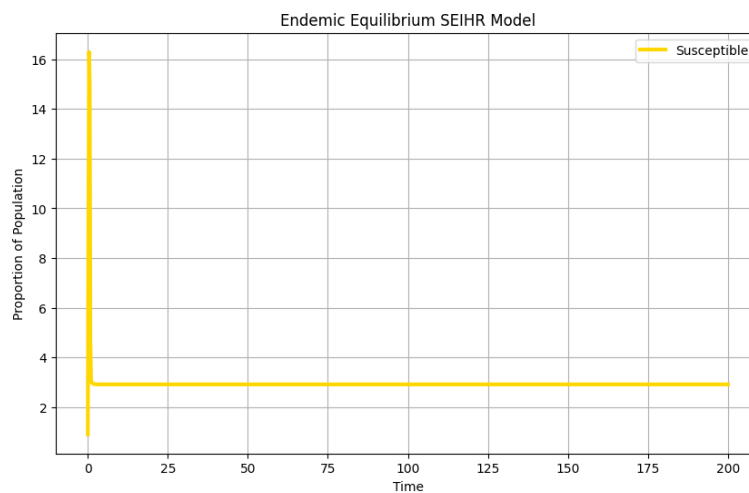


Fig 8 (a) Plot for Change in Susceptible Rate at endemic equilibrium point

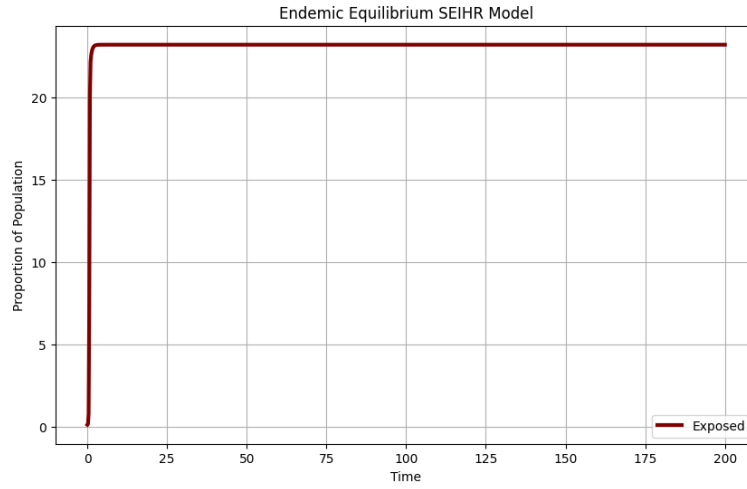


Fig 8 (b) Plot for Change in Exposed Rate at endemic equilibrium point

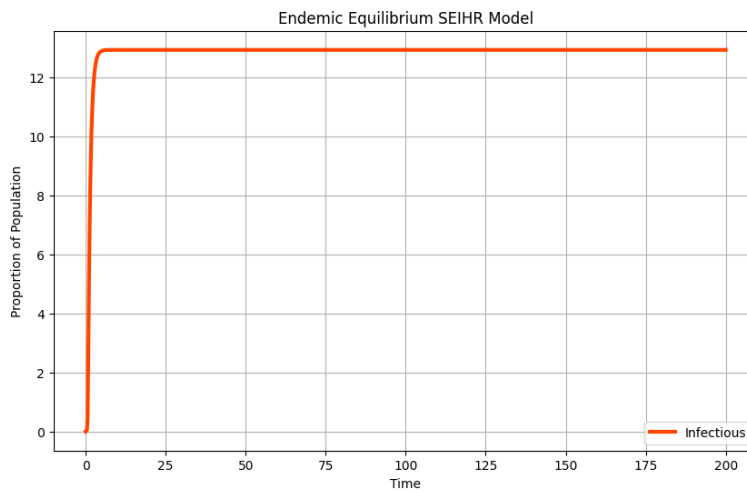


Fig 8 (c) Plot for Change in Infected Rate at endemic equilibrium point

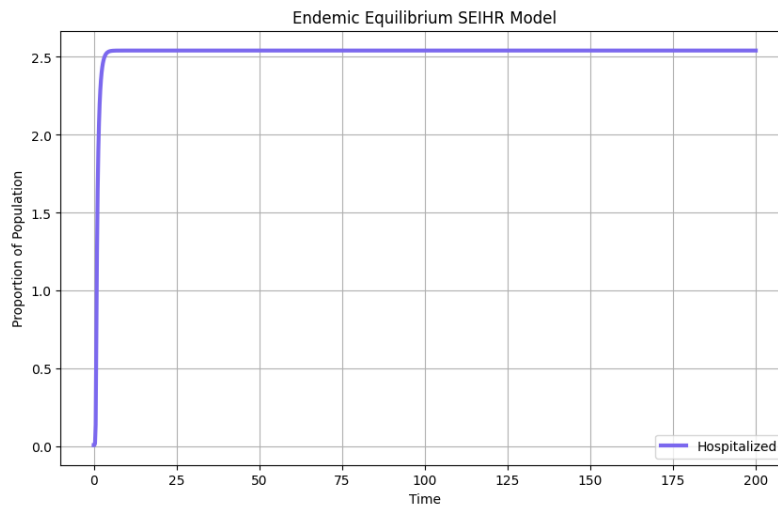


Fig 8 (d) Plot for Change in Hospitalized Rate at endemic equilibrium point

SEIHR MODEL FOR INDIAN COVID-19

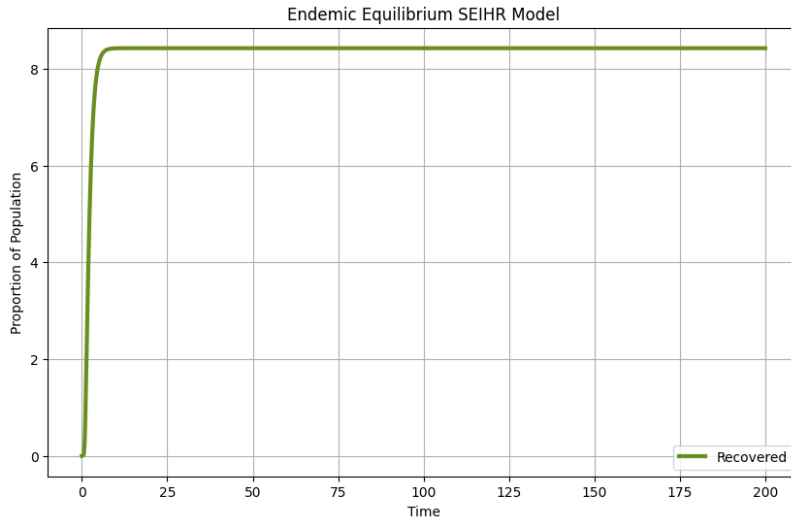


Fig 8 (e) Plot for Change in Recovered Rate at endemic equilibrium point

Fig. 8(a), 8(b), 8(c), 8(d), 8(e) The solution curves for the model (Fig. 1) at disease free equilibrium point with.

Figure 8(a), 8(b), 8(c), 8(d), 8(e) shows the numerical results of the system (Fig. 1) When $R_0 > 1$ the numerical results of the system (Fig. 1) are depicted in Fig.4. These figures show that all numerical solutions converged to the Endemic equilibrium.

$$E^* = (2.91, 23.19, 12.92, 2.54, 8.42) \quad \text{for the case } R_0 > 1 .$$

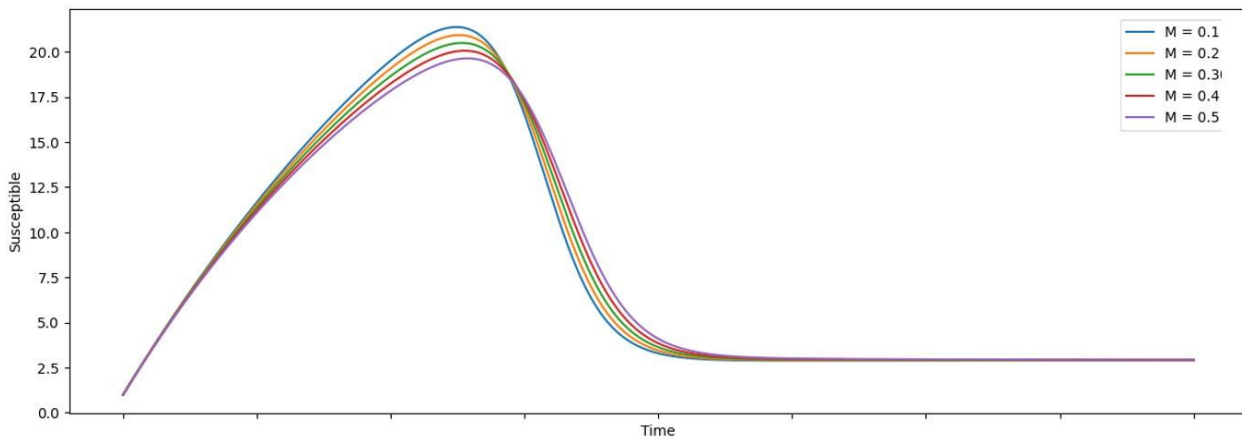


Fig 9 (a) Graph for Change in optimal control parameter $M=(0.1,0.2,0.3,0.4,0.5)$ at Susceptible Rate

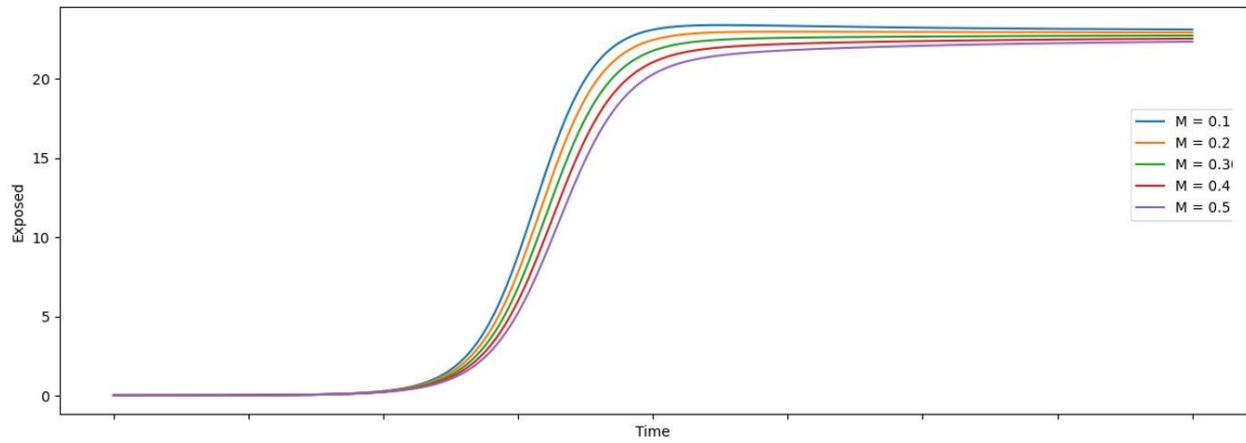


Fig 9 (b) Graph for Change in optimal control parameter $M=(0.1,0.2,0.3,0.4,0.5)$ at Exposed Rate

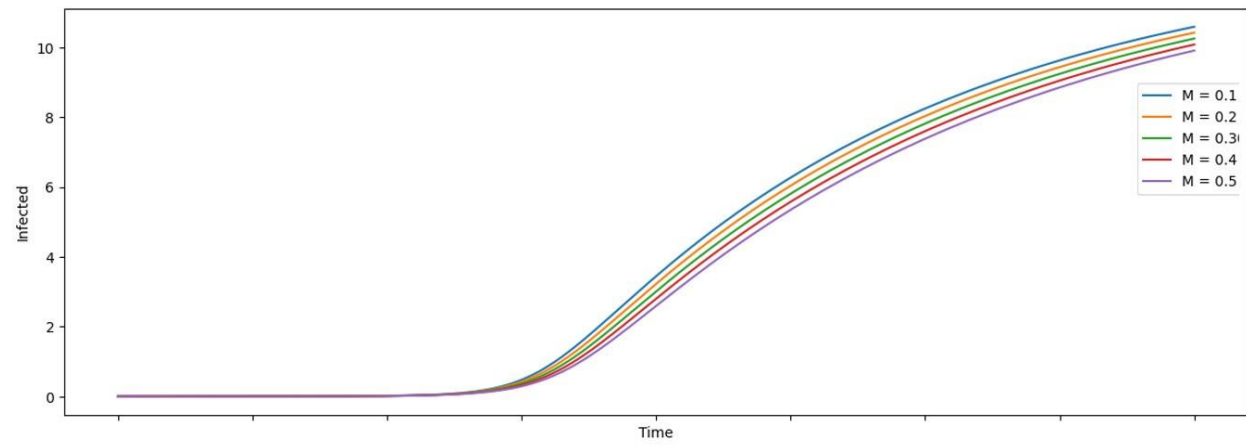


Fig 9 (c) Graph for Change in optimal control parameter $M=(0.1,0.2,0.3,0.4,0.5)$ at Infected Rate

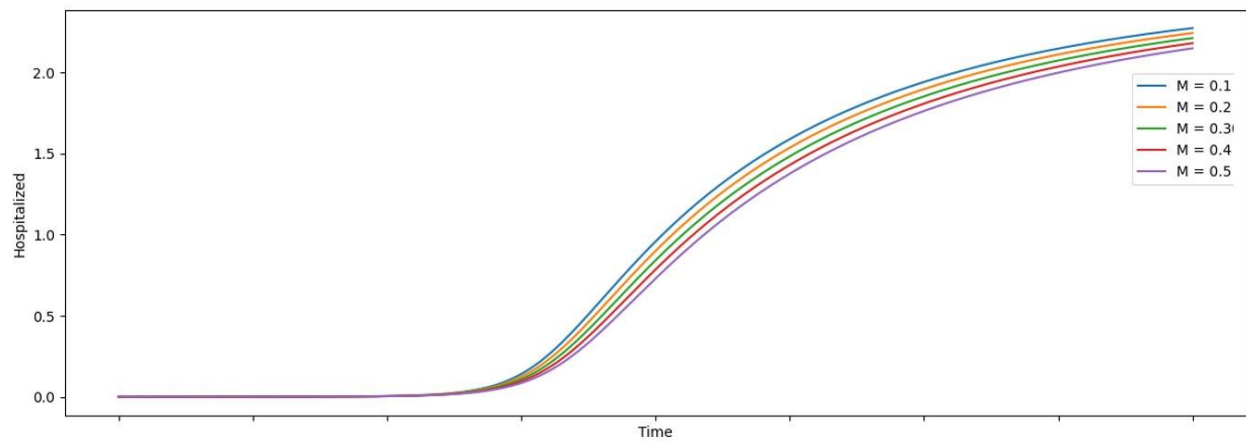


Fig 9 (d) Graph for Change in optimal control parameter $M=(0.1,0.2,0.3,0.4,0.5)$ at Hospitalized Rate

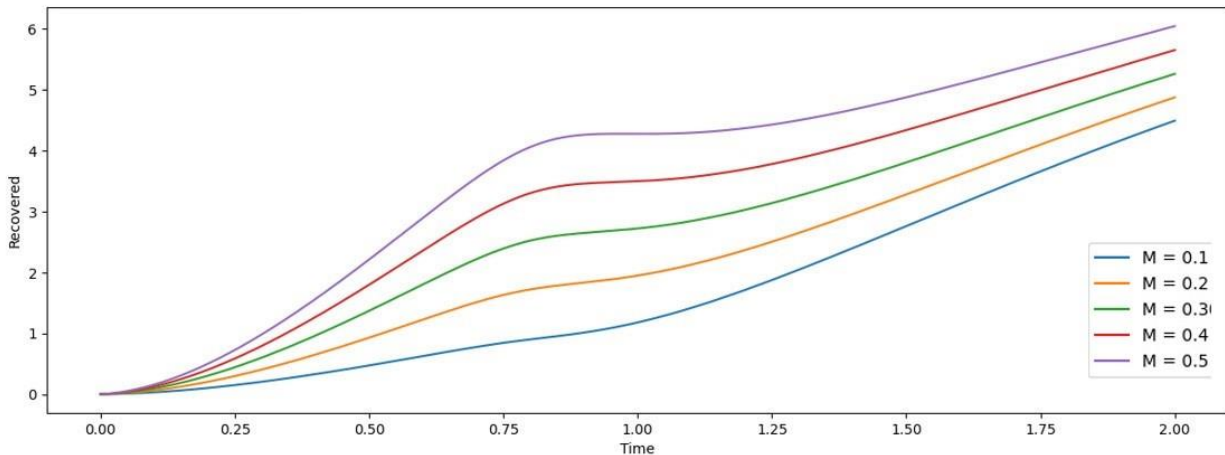


Fig 9 (b) Graph for Change in optimal control parameter $M=(0.1,0.2,0.3,0.4,0.5)$ at Recovered Rate

Fig. 9(a), 9(b), 9(c), 9(d), 9(e). Control parameter M 's control approach is varied at $M=(0.1,0.2,0.3,0.4,0.5)$

11. CONCLUSION

In this work, we examined an optimum control model connected with a deterministic mathematical model of the COVID-19 vaccination epidemic. First, we determined the practical area in which the model is mathematically and epidemiologically well-posed. By applying the next-generation matrix approach, the fundamental reproduction number with respect to the disease-free equilibrium point is calculated. The sensitivity analysis of the model has been studied. Next, using the basic reproduction number as a basis, we examined the disease free equilibrium point's local and global stability, vaccination imposed 100% validated under mathematical model, are some of the parameters analysed by the Indian government policies to determine the optimal control approximation. The model was extended to address the issue of administering vaccines for optimal control. Utilizing cost-effectiveness analysis and the Pontryagin's maximum principle, the optimal control model Applying the incremental cost effectiveness ratio, ascertain the lowest possible cost. The numerical simulation's result showed that each control factor that was used accounted for in the simulation assisted in lowering COVID-19 infections Significant measures can be taken to reduce the number of people who are susceptible to infection by

restricting their contact with susceptible individuals, wearing masks, social distancing, frequently hand wash by soap and quarantining sick individuals. The number of affected people will decline as the inhibitory values rise. The co-administration of immunization and therapy for those who are afflicted is the most effective way to decrease the COVID-19 infection, based on the findings of simulation with economical and optimal control examination

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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