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PREDATOR-PREY MODEL WITH FUZZY PARAMETERS AND FUZZY INITIAL CONDITIONS: A SYSTEMATIC LITERATURE REVIEW

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Abstract: A predator-prey model with fuzzy parameters and fuzzy initial conditions is a model of interaction between species that takes into account uncertainty in both parameters and initial conditions expressed in fuzzy numbers. Over the past years, numerous studies have been conducted on this model. This paper provides a review of research carried out by several previous scholars in developing and studying this model, focusing on the model evolution and the methodologies employed. The review reveals that starting in 2015, researchers began expanding the fuzzy predator-prey models by incorporating additional biological processes, such as considering harvesting factors, disease, prey protection, toxic effects, etc. Various approaches, including the Hukuhara derivative (H-derivative) approach, the generalized Hukuhara derivative (gH-derivative) approach, and the Zadeh extension principle approach, were utilized to study the predator-prey fuzzy model. Based on the findings of this review, there are still opportunities for further predator-prey fuzzy model development by incorporating additional biological factors, such as Allee effects. Furthermore, the limitations associated with the employed methodologies indicate potential for future research

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exploring alternative approaches to fuzzy derivatives, such as generalized derivative (g-derivatives) and granular derivatives (gr-derivatives). Another development that can be carried out for further research is to combine parameters and initial conditions as a fuzzy uncertainty so that it is more realistic and in accordance with real conditions.
Keywords: predator-prey model; fuzzy numbers; fuzzy differential equations; fuzzy parameters; fuzzy initial conditions.

2020 AMS Subject Classification: 34A07, 92-10.

1. INTRODUCTION

The predator-prey model represents a dynamic system of differential equations that characterizes the interaction between two species: predator and prey. The foundational work on this model was introduced by Lotka in 1925 and later expanded by Volterra in 1926, thus earning the name Lotka-Volterra predator-prey model. Throughout its development, the Lotka-Volterra predator-prey model has undergone several modifications, as evidenced by references [1]–[14]. Dawes & Souza [2] and Ma et al. [4], explored the incorporation of functional responses into predator-prey model. Meanwhile, Das et al. [1] and Jana & Kar [8] investigated the impact of diseases on the predator-prey dynamics. Harvesting effect were discussed by Das et al. [9]; Rojas-Palma & González-Olivares [10]; Supriatna & Possingham [3], [14]. Furthermore, the inclusion of Allee effects was addressed by Flores & González-Olivares [7]; Huang et al. [13]; Terry [11]; Xie [5]; Ye et al. [12]; and Zu & Mimura [6]. In existing literature, all parameters and initial values in the predator-prey model are typically assumed to be certain. However, in practice, uncertainty often arises due to unclear, inadequate, or incomplete information regarding variables and parameters resulting from errors in observations, measurements, experiments, and so on. To address this issue, various approaches have been employed, including stochastic, fuzzy, and fuzzy stochastic approaches, to construct models that account for uncertainty.

In recent decades, the application of fuzzy theory has proven to be a valuable tool for mathematical modeling of real-world phenomena. This approach enables the representation of uncertain variables and parameters using intervals and fuzzy numbers. In the study of differential equations in a fuzzy environment, the term fuzzy differential equation is used as a reference to differential equations with fuzzy coefficients, differential equations with fuzzy initial values or fuzzy limit values, or even differential equations that relate to functions in the space of fuzzy intervals [15]–[18]. The stability of fuzzy dynamical systems in population dynamics has also been discussed through fuzzy differential equations and the problem of fuzzy initial values [18]. Various methods are used in solving fuzzy differential equations including through the Hukuhara derivative approach (H-derivative) [15], [16], [19]; differential inclusion [20], [21]; Zadeh extension principle [18], [22]–[27], fuzzification of derivative operators [28], strongly generalized Hukuhara derivative (SGH-derivative) and generalized Hukuhara derivative (gH-derivative) [29]–[32], generalized derivative (g-derivative) [33], interactive derivative [26], and granular derivative (gr-derivative) [34]. Numerical solutions for systems of fuzzy equations have also been introduced in [24], [35], [36].

The fuzzy predator-prey model was initially proposed by da Silva Peixoto et al. [37] in 2008. The advancement of fuzzy differential equations has led to significant advancements in the field of the fuzzy predator-prey model, as evidenced by studies conducted by [38]–[40]. In this particular investigation, the classical deterministic predator-prey model Lotka-Volterra, was reformulated by incorporating fuzzy initial conditions. Moreover, the Lotka-Volterra predator-prey model has undergone further development to incorporate additional biological processes. As a result, research on the fuzzy predator-prey model continues to expand and flourish.

This review employs a systematic literature review (SLR) as an approach to provide a overview of existing studies and emerging trends in the field of the fuzzy predator-prey model.
To support the SLR process, we use the Preferred Reporting Items for Systematic reviews and Meta-Analyses (PRISMA) method. The PRISMA method requires material from database mining articles sourced from various digital libraries. The study incorporates a bibliometric analysis to establish connections between the data presented in the reviewed articles. Through data visualization techniques, the analysis aims to examine the content, patterns, and trends within the collection of documents by measuring the significance of terms and calculating the frequency of keywords or topics that appear simultaneously in the article under review. Additionally, an evolutionary analysis was conducted to identify temporal changes in the topic covered.

In essence, this study offers a survey report that presents a systematic examination of the fuzzy predator-prey model. It encompasses various aspects, including the development and modification of the model, the methodologies employed in studying and analyzing the model, as well as an evaluation of the strengths and weaknesses associated with each approach. Finally, it will offer the development of models and approaches that can be used in the studying and analyzing models as novelty for further research.

2. MATERIALS AND METHODS

2.1. Scientific Article Data

The research process for this study involved searching and collecting relevant article data. The initial step was to conduct a systematic search of indexed publications in three databases: Google Scholar, Dimensions, and Scopus. The search was performed using specific keywords to ensure comprehensive coverage of relevant literature. The keywords used in the search included: ("Predator-Prey Model" OR "Predator Prey Model" OR " Prey Predator Model" OR "Prey-Predator Model") AND (“Fuzzy Set” OR “Fuzzy Numbers”) AND ("Fuzzy Differential Equations" OR “Fuzzy Initial Value" OR “Fuzzy Initial Condition" OR " Fuzzy Initial Population" OR "Fuzzy Parameters”). For more details, refer to Table 1 for the specific keywords combinations employed during each stage of the search.

Table 1. Literature search keywords

Code	Keywords
KW-I	("Predator-Prey Model" OR "Predator Prey Model" OR " Prey Predator Model" OR "Prey-Predator Model") AND ("Fuzzy Set” OR “Fuzzy Numbers").
KW-II	("Predator-Prey Model" OR "Predator Prey Model" OR " Prey Predator Model" OR "Prey-Predator Model") AND ("Fuzzy Differential Equations" OR “Fuzzy Initial Value" OR “Fuzzy Initial Condition" OR " Fuzzy Initial Population" OR "Fuzzy Parameters”).
KW-III	KW-I AND KW-II

The article search process was facilitated by utilizing the Publish or Perish software. This software enabled the creation of a database containing articles relevant to the specified keywords. The information within the database encompassed various aspect, including the author's name, article title, publication date, publisher, number of citations, and abstract text. The search was limited to articles written in English and published in peer-reviewed journals between the years 2014 and 2023. By utilizing Publish or Perish, a comprehensive collection of articles meeting the specified criteria was obtained, forming the basis for further analysis and examination in this study.

The initial step involved searching for articles related to the first keyword. The search yielded 313 articles from Google Scholar, 296 articles from Dimension, and 21 articles from Scopus. Furthermore, by using the second keyword results are obtained 245 articles from Google Scholar, 152 articles from Dimension, and 10 articles from Scopus. Finally, by combining the first keyword with the second keyword using the "AND" operator, the search resulted in 183 articles from Google Scholar, 112 articles from Dimension, and 5 articles from Scopus. To facilitate the selection process, the articles obtained from the search results in the three databases were stored in BibTex format. A summary of the search results from the three databases, based on the aforementioned keywords, is provided in Table 2.

Table 2. Search results based on keywords

Code	Keywords	*Google Scholar*	*Dimensions*	*Scopus*	Total
KW-I	("Predator-Prey Model" OR "Predator Prey Model" OR " Prey Predator Model" OR "Prey-Predator Model") AND ("Fuzzy Set" OR "Fuzzy Numbers").	313	296	21	630
KW-II	("Predator-Prey Model" OR "Predator Prey Model" OR " Prey Predator Model" OR "Prey-Predator Model") AND ("Fuzzy Differential Equations" OR "Fuzzy Initial Value" OR "Fuzzy Initial Condition" OR " Fuzzy Initial Population" OR "Fuzzy Parameters").	245	152	10	407
KW-III	KW-I AND KW-II	183	112	5	300

The search results for the next stage comprised at total of 300 articles obtained from the three databases using the third keyword. These articles will be further examined and analyzed in subsequent phase of the research.

2.2 Selection of Literature Database

In this section, the data collection and selection process was conducted using the PRISMA flowchart, as presented in Figure 1. Out of the initial pool of 300 articles obtained from the combined search results in the three databases using the JabRef software, a series of step were followed to refine the selection. Duplicate articles (80), books (24), and conference proceedings (11) were exluded, resulting in a remaining set of 185 articles. These articles were then assessed based on their titles and abstracts, leading to the identification of 29 relevant articles. The full texts of these 29 articles were obtained and further examined, resulting in a final set of 20 relevant articles. Lastly, a manual selection process involving a careful reading of the complete articles led to identification of 18 articles that were deemed relevant to the research being conducted.

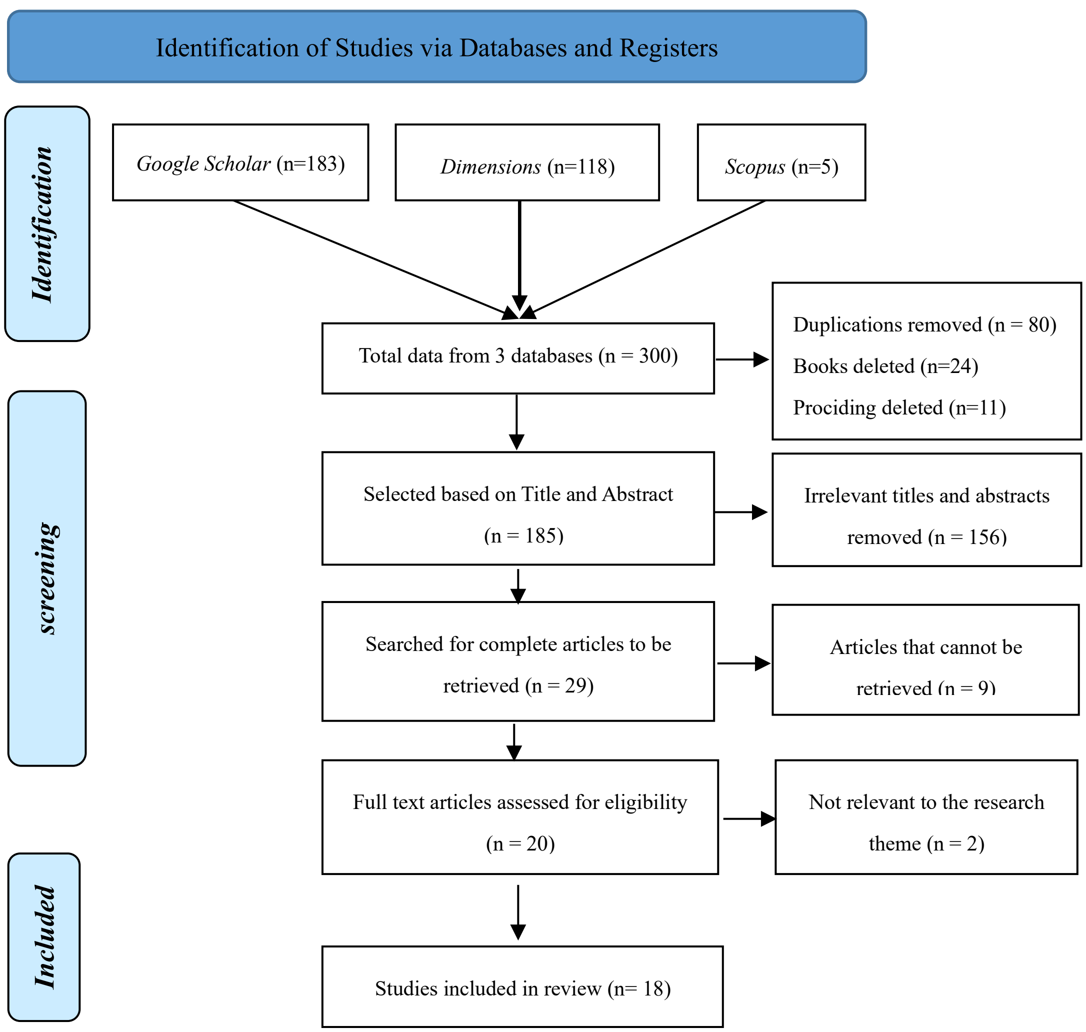

Figure 1. PRISMA flow chart

2.3 Bibliometric Analysis

In this section, we employed the ".RIS" format to store the articles selected from our final selection process. Subsequently, we conducted bibliometric analysis using VOSviewer software, a widely-used tool in library science research. Bibliometric analysis allows us to gain valuable insights and a comprehensive overview of scientific publications by examining their citation patterns. The analysis results were visualized, providing a clear depiction of the content, patterns, and trends present within our document collection. This visualization facilitated the measurement of term strength and the identification of keywords or topics that appeared concurrently across the research documents under study. To further organize the emerging topics, we utilized clustering techniques. The size of each cluster directly corresponded to the number of articles addressing specific keywords related to our research theme. Larger clusters indicated a broader coverage of the corresponding keywords throughout the database, while smaller clusters indicated a more limited discussion on those specific keywords within the articles.

3. RESULTS AND DISCUSSION

The following section describes the results of data analysis obtained from 18 articles. Explanations include article data visualization, development of fuzzy predator-prey models, model analysis, and methods used in model analysis and numerical simulations.

3.1 Article Data Visualization

In this section, the results of the bibliometric analysis conducted using VOSviewer are presented in the form of article data visualization. The analysis focused on identifying topics that appeared at least twice out of the total 827 words analyzed, resulting in 351 mappings that met the threshold. From these mappings, 42 distinct topics were derived and categorized into four clusters, represented by different colors: red (18 topics), green (10 topics), blue (8 topics), and yellow (6 topics). The network visualization analysis of the article data is illustrated in Figure 2.

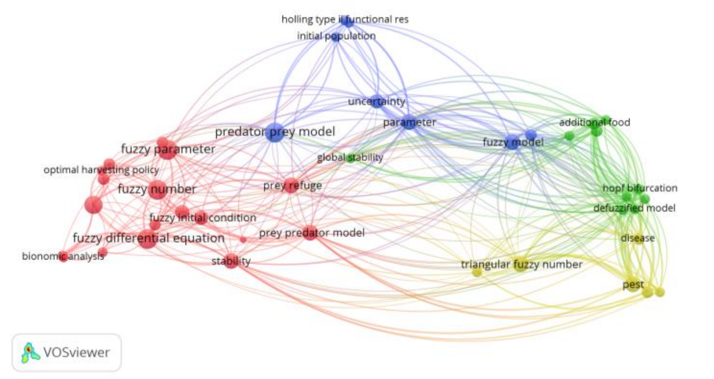

Figure 2. Article data network visualization

The network visualization results reveal that the topic that occurs most frequently is “stability” and “fuzzy model” which appears 27 times each in the analyzed articles. This is closely followed by the topic of “triangular fuzzy numbers” which appears 22 times, and “predator prey model", “fuzzy parameter”, and “fuzzy numbers”, which appears 21 times each. The topic of “stability” and “fuzzy model” demonstrates strong connections to several other topics, particularly predator-prey models, fuzzy parameter, fuzzy differential equations, fuzzy initial condition, and fuzzy numbers.

The overlay visualization map, as depicted in Figure 3, provides insights into the publication timeline of articles related to the analyzed topics. The map utilizes different shades of color, with lighter shades indicating more recent publications and darker shades representing articles published at a later time. Based on the visualization, it is evident that the majority of the topics covered in the articles were published within the period of 2016 to 2022.

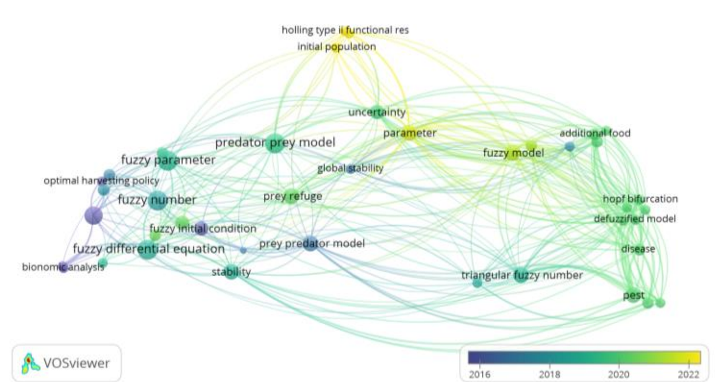

Figure 3. Visualization of article data overlay

The density visualization map, as depicted in Figure 4, provides insights into the publication depth of topics within related research. The map utilizes different shades of color, with lighter shades indicating topics that are widely used and referenced in the research, while darker shades suggest topics that are less prevalent and not extensively explored.

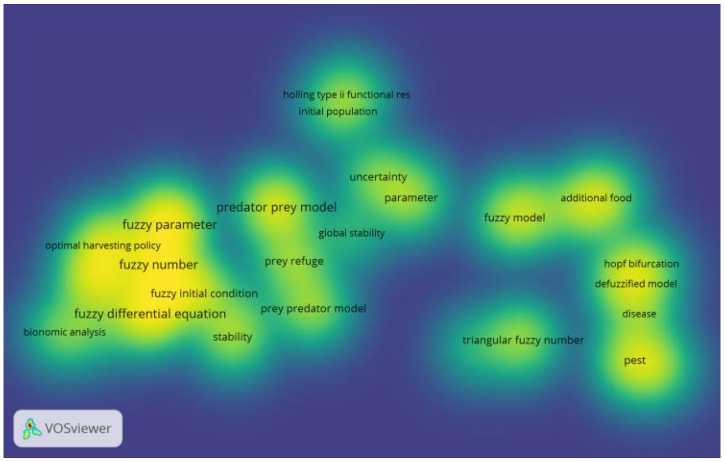

Figure 4. Density visualization map

Based on the density visualization map, it is evident that the topics of "fuzzy initial condition" and "initial population" appear with darker shades, indicating a relatively lower frequency of research conducted on these specific topics.

The development of research on the fuzzy initial condition and initial population is shown in Figure 5 (a)-(b).

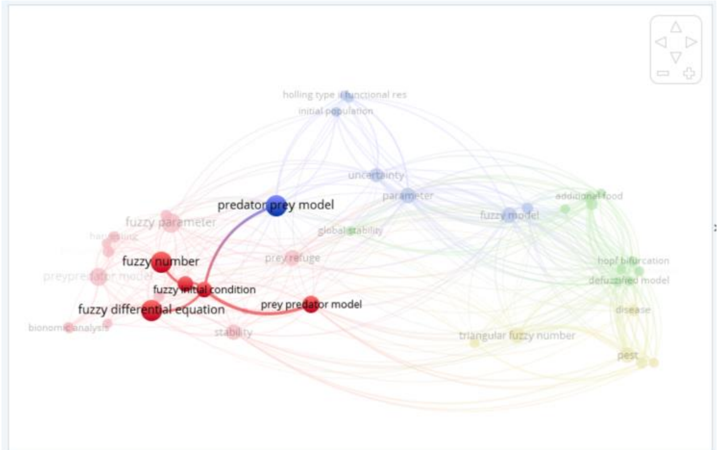

Figure 5.a. Overlay visualization: The topic of fuzzy initial condition

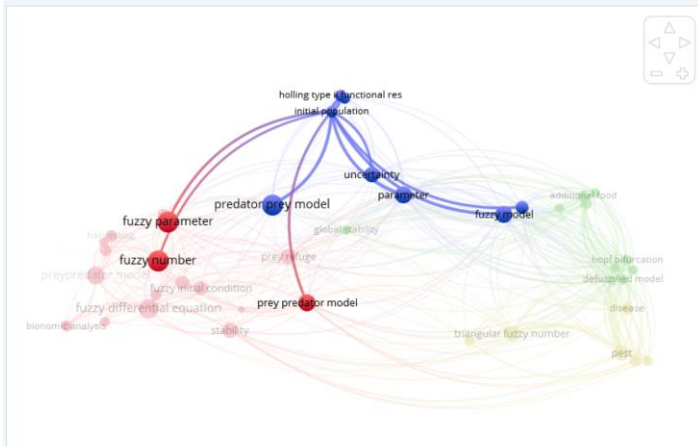

Figure 5.b. Overlay visualization: The topic of initial population

Based on the density visualization map, it is evident that limited research has been conducted on the predator-prey model with fuzzy parameter and fuzzy initial population. This is indicated by the infrequent occurrence of the term "fuzzy initial condition” appearing only five times. Similarly, the problem of the initial population is mentioned merely ten times. It is noteworthy that predator-prey model with fuzzy parameters and fuzzy initial population are frequently linked to various concepts, particularly triangular fuzzy numbers, equilibrium points, and stability.

3.2 Overview of Models from Each Article and Potential to Develop for Future Research

The fuzzy predator-prey model was initially introduced in 2008 by Peixoto et al. Following its introduction, publications related to this research have gradually emerged. Table 3 shows the summary of developments in the fuzzy predator-prey model in 2014 to 2023.

Table 3. Summary of developments in the fuzzy predator-prey model in 2014 to 2023.

No	Author, Years, and Citation Number	Basic Model	Functional Response	Biological Factors	Purpose and Objectives	Focus	Citation
1	(Pandit and Singh, 2014) [41]	Lotka-Volterra	-	-	Overcoming the shortcomings of the Hukuhara derivative approach for Predator-Prey models with fuzzy initial values using Taylor Expansion.	The stability of the equilibrium point	11
2	(Barzinji et al., 2014) [42]	Lotka-Volterra	-	-	Demonstrates the existence of a solution for the fuzzy delay predator-prey system.	Existence of the solution and oscillation property	15
3	(Omar and Ahmed, 2015) [43]	Leslie-Gower	Holling type II	-	Studying a predator-prey model with ratio-dependent functional responses.	Local and global stability Hopf bifurcation Optimum harvest	3
4	(Pal et al., 2015) [44]	Lotka-Volterra	-	Harvesting	Study optimal harvesting policies and obtain optimal solutions with imprecise biological parameters.	The stability of the equilibrium point	36
5	(Pal et al., 2016) [45]	Lotka-Volterra	-	harvesting with toxicity effect	Studying the dynamic behavior and harvesting of a fuzzy predator-prey system with toxic effects.	Local and global stability Optimum harvest	22
6	(Barzinji, 2017) [46]	Lotka-Volterra	-	-	Developing an efficient and accurate fuzzification scheme for fuzzy delay predator-prey systems.	Numerical solution	2
7	(Pal et al., 2018) [47]	Leslie-Gower	Holling type II	harvesting	- Developed a fuzzy prey-predator system with time delays. - Study the effect of pregnancy delay on system stability in a fuzzy environment.	Local and global stability Hopf bifurcation	15
8	(Yuan and Zhang, 2018) [48]	Lotka-Volterra	-	Harvesting and protection	Proposed a fuzzy predator-prey harvesting model that combines the effects of prey protection and predator mutual interference	Stability Optimal harvesting	17
9	(Rizwan and Mohamed, 2019) [49]	Lotka-Volterra	-	-	Studying predator-prey population models under uncertainty the environment that constitutes the initial population of predators and prey is given in interval-valued fuzzy numbers.	Numerical solution	0

10	(Narayanamoorthy et al., 2019) [50]	Lotka-Volterra	-	-	Analyzing fractional order predator-prey models with uncertainty, due to their wide application in a variety of ecological systems.	The stability of the equilibrium point	16
11	(Das et al., 2020) [1]	Leslie-Gower	Holling Type-I and Holling Type-II	disease	Studying the dynamics of disease transmission through intermediate pests susceptible prey and infected prey.	Local and global stability Hopf bifurcation	7
12	(Meng and Wu, 2020) [51]	Leslie-Gower	Holling Type-II	harvesting and protection	Studying the influence of fuzzy parameters on the interior balance and bionomic balance of the system.	Local and global stability Hopf bifurcation Optimum harvest	25
13	(Wang et al., 2021) [52]	Leslie-Gower	Holling type-I	Harvesting, and protection	- Develop a predator-prey model by combining prey protection with fuzzy parameters. - Study optimal harvesting strategies with inflation and discount rates as fuzzy numbers	Local and global stability Hopf bifurcation	4
14	(Das et al., 2021) [53]	Leslie-Gower	Holling Type-II	Desease and protection	Studying prey-predator disease control models that incorporate protection on vulnerable prey with fuzzy parameters	The stability of the equilibrium point	10
15	(Mallak et al., 2021) [54]	Lotka-Volterra	Arctan	-	Learning a fuzzy predator-prey model with arctan(ax) functional response.	Local and global stability Optimum harvest	4
16	(Abuasbeh et al., 2022) [55]	Lotka-Volterra	-	-	Demonstrates the existence of a solution for the fuzzy delay predator-prey system with the Caputo derivative.	Existence of a solution	1
17	(Acharya, 2022) [56]	Lotka-Volterra	-	-	Studying the Predator-prey model with a single delay by considering several parameters as fuzzy numbers.	Numerical solutions	1
18	(Sukarsih et al., 2023) [57]	Leslie-Gower	Holling type-II	Harvesting	Develop a numerical scheme to solve the predator-prey model with the Holling type II functional response considering fuzzy parameters and fuzzy initial population.	Numerical solutions	1

From Table 3 it can be seen the limited number of articles published on the fuzzy predator-prey model within the 2014-2023 timeframe. On average, only approximately 1 to 3 articles per year were published, accompanied by a range of 0 to 39 citations per year. Notably, the highest number of articles occurred in 2021 with three articles, receiving a total of 18 citations. This indicates that research on the fuzzy predator-prey model remains relatively scarce, presenting significant opportunities for further exploration and development. Figure 6 provides an overview of the number of articles published and cited between the years 2014 to 2023.

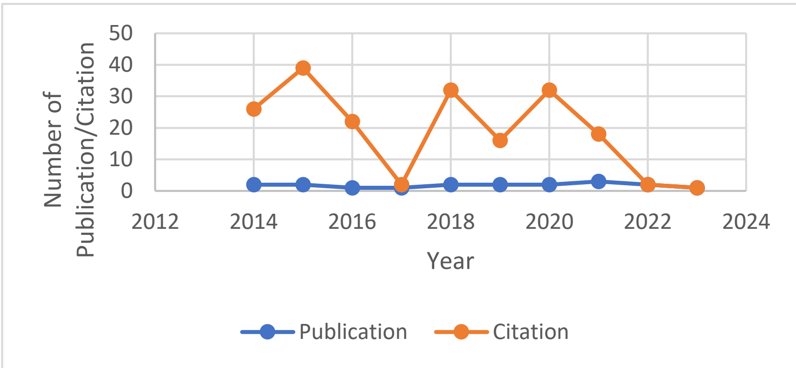

Figure 6. Number of publications and citations for the period 2014 – 2023

The most cited articles are articles [44] by title " Stability and bionomic analysis of fuzzy parameters based prey–predator harvesting model using UFM” with 36 citations. The second position is an article [51] with the title "Dynamical analysis of a fuzzy phytoplankton–zooplankton model with refuge, fishery protection and harvesting" with 25 citations. This shows that the topic of this research is considered interesting enough to be discussed and has a major contribution to further research. In Table 3 it can be seen that, of the 10 articles with the top citations, 5 articles namely [44], [45], [47], [48], [51] discusses the fuzzy predator-prey model with harvesting. This indicates that the fuzzy predator-prey model with harvesting has been extensively studied and remains an intriguing topic for future research endeavors.

From Table 3 it can also be seen that the fuzzy predator-prey model extends the traditional predator-prey model by incorporating uncertainty in parameters and/or initial population expressed as fuzzy numbers. The initial research in this field focused on developing the Lotka-Volterra predator-prey model with fuzzy initial conditions, as evident from studies conducted in 2014 [41]. Subsequently, from 2015 onwards, research on the fuzzy predator-prey model expanded to include various additional biological processes. The development of a fuzzy predator-prey model has also been carried out by adding biological factors, including by considering harvesting as in [44], [45], [47], [48], [51], [52], [57]. Furthermore, another biological factor that has been discussed in the model is the presence of disease factors [1], [53]; protection factor in prey [48], [51]–[53]; and toxic effects [45]. Figure 7 shows the development of a fuzzy predator-prey model based on the addition of various biological factors.

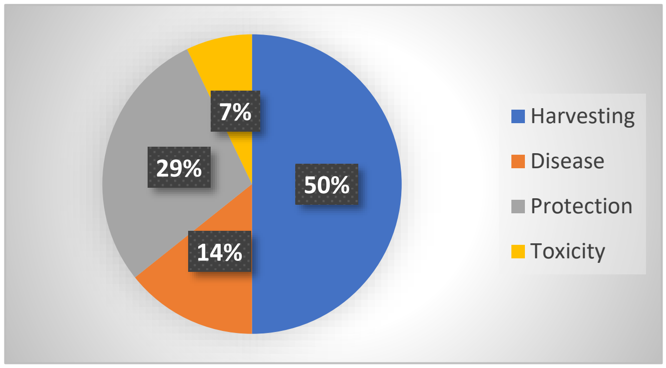

Figure 7. Development of a fuzzy predator-prey model based on biological factors.

From Figure 7 it can be seen that of the 18 articles reviewed, the fuzzy predator-prey model that has been discussed generally has added harvesting (50%) and protection (29%) factors. Biological factors that are still not widely discussed are disease factors (14%) and toxic factors (7%). This opens up opportunities to develop fuzzy predator-prey models by including disease and poison factors for further research. Another biological factor that has not been discussed in the research that has been done in the predator-prey fuzzy model is the alee effect. *Allee effect* broadly defined as a decrease in the fitness of individuals at low population sizes or densities, which may result in a critical population threshold below population extinction [58]. So for some endangered species, the allee effect is more likely. Allee effect is very important for conservation management of endangered species, population development and utilization of natural resources. Therefore, the allee effect in the predator-prey fuzzy model is very important to study and do further research.

Apart from developments in terms of models, purpose and objectives, as well as a research focus, a review of 18 articles was also carried out on discussion material in terms of the concept of fuzzy theory used, including the uncertainty environment (parameters/initial conditions), the use of fuzzy numbers, the methods used, as well as numerical simulations carried out. Table 4 summarizes of the fuzzy theory concepts used in research related to the fuzz predator-prey model based on the literature review.

Table 4. Fuzzy theory concepts used in research related to the fuzzy predator-prey model

No	Author	Title	Uncertainty Environments		Fuzzy Numbers		Methods	Numerical Simulation
			Parameters	Initial conditions	Triangular	Trapezoidal		
1	(Pandit and Singh, 2014) [41]	Prey Predator Model with Fuzzy Initial Conditions	×	√	√	×	Hukuhara derivative	Euler's method
2	(Barzinji et al., 2014) [42]	Fuzzy delay predator-prey system: Existence theorem and oscillation property of solution	×	√	×	×	Hukuhara derivative	-
3	(Omar and Ahmed, 2015) [43]	The Fuzzy Ratio Prey-Predator Model	×	√	√	√	Zadeh extension principle	4th order Runge-Kutta method
4	(Pal et al., 2015) [44]	Stability and bionomic analysis of fuzzy parameters based prey–predator harvesting model using UFM	√	×	√	×	Hukuhara derivative And Utility Function Method	√
5	(Pal et al., 2016) [45]	Stability and Bionomic Analysis of Fuzzy Prey–Predator Harvesting Model in Presence of Toxicity	√	×	√	×	Hukuhara derivative And Utility Function Method	√
6	(Barzinji, 2017) [46]	Numerical solution of fuzzy delay predator-prey system	√	×	√	×	Hukuhara derivative	4th order Runge-Kutta
7	(Pal et al., 2018) [47]	A Study of Bifurcation of Prey-Predator Model with Time Delay and Harvesting Using Fuzzy Parameters	√	×	√	×	Hukuhara derivative	√
8	(Yuan and Zhang, 2018) [48]	About the optimal harvesting of a fuzzy predator–prey system	√	×	√	×	Hukuhara derivative	√
9	(Rizwan and Mohamed, 2019) [49]	Fuzzy Prey-Predator Model Using Taylor's Algorithm	×	√	√	×	Hukuhara derivative	Taylor's Series expansion
10	(Narayanamoorthy et al., 2019) [50]	Analysis for fractional-order predator–prey models with uncertainty	×	√	√	×	Caputo fractional derivatives	fractional Euler
11	(Das et al., 2020) [1]	A Prey Predator Model in Case of	√	×	√	×	Defuzzifi-cation	√

No	Author	Title	Uncertainty Environments		Fuzzy Numbers		Methods	Numerical Simulation
			Parameters	Initial conditions	Triangular	Trapezoidal		
		Disease Transmission via Pest in Uncertain Environment						
12	(Meng and Wu, 2020) [51]	Dynamical analysis of a fuzzy phytoplankton–zooplankton model with refuge, fishery protection and harvesting	√	×	√	×	Hukuhara derivative And Utility Function Method	√
13	(Wang et al., 2021) [52]	Stability and optimal harvesting of a predator-prey system combining prey refuge with fuzzy biological parameters	√	×	√	×	Hukuhara derivative	√
14	(Das et al., 2021) [53]	Disease control prey–predator model incorporating prey refuge under fuzzy uncertainty	×	√	√	×	Defuzzifi-cation	√
15	(Mallak et al., 2021) [54]	Numerical investigation of fuzzy predator-prey model with a functional response of the form arctan(ax)	√	√	√	×	Hukuhara derivative	4th order Runge-Kutta
16	(Abuasbeh et al., 2022) [55]	Oscillatory behavior of solution for fractional order fuzzy neutral predator-prey system	×	√	×	×	Extensi Zadeh dan Caputo derivative	-
17	(Acharya, 2022) [56]	Numerical Solution of Fuzzy Delayed Prey-Predator Model : Hukuhara Derivative Approach	√	×	√	×	generalized Hukuhara (gH) derivative	2th order Runge-Kutta
18	(Sukarsih et al., 2023) [57]	A Runge-Kutta numerical scheme applied in solving predator-prey fuzzy model with Holling type II functional response	√	√	√	×	Zadeh extension principle	4th order Runge-Kutta

From Table 4 it can be seen that the environmental uncertainty discussed in the articles reviewed is mostly only in parameters, namely in [1], [44]–[48], [51], [52], [54], [56] , [57] or in initial conditions only, namely in [41]–[43], [49], [50], [53]–[55], [57]. There are only two articles that discuss uncertainty in both (parameters and initial conditions), namely in [54], [57], but fuzzy parameters and fuzzy initial conditions are discussed separately. This is an opportunity for future research by combining fuzzy parameters and fuzzy initial conditions so that they are more realistic and in accordance with the conditions in real problems. Meanwhile, most studies carry out simulations using triangular fuzzy numbers, there is only one article that discusses trapezoidal fuzzy numbers, namely in [43], but the discussion is only briefly discussed in theory, but in carrying out simulations they still use triangular fuzzy numbers. Further research development can be carried out using trapezoidal fuzzy numbers or others.

The method used in studying the fuzzy predator-prey model generally uses the Hukuhara derivative (H-derivative) [41], [42], [44]–[49], [51], [52], [54]. The advantage of the H-derivative approach is able to show the existence and uniqueness of the system solution. Nevertheless, this approach exhibits a limitation whereby the diameter of the fuzzy function under investigation must always remain constant or increase, thus restricting its applicability in cases where the diameter may decrease. As a result of this limitation, the solutions obtained in numerous cases may deviate from those that would be expected intuitively based on the nature of the system. Besides using the H-derivative approach, another approach is gH-derivative [56]. The gH-derivative was defined based on generalized Hukuhara difference (gH-difference) which is a more general concept than H-difference. Although the existence of gH-differences comes with more limitations little compared to the H-difference, there is a possibility that the gH-difference of the two fuzzy numbers does not exist. Therefore, the existence of the gH derivative of the fuzzy function cannot be guaranteed. This limitation opens opportunities for further research with other approaches to fuzzy derivatives, such as generalized derivative (g-derivative) [32], [33], or granular derivative (gr-derivative) [59]. g-derivative and gr-derivative can overcome the shortcomings of H-derivative and gH-derivative, where these two approaches do not require that the diameter of the fuzzy function under study must always not decrease and the existence of the solution is guaranteed. In addition to the several approaches above, the caputo derivative approach has also been used for the fractional order predator-prey model [50], [55].

Another method implemented in the studied article is through the Zadeh extension principle [44], [57]. This approach does not use fuzzy differential concepts, but can overcome the weaknesses of H-derivatives and gH-derivatives. Zadeh's extension principle approach is easier to understand and apply, but the extension principle can only be applied to systems derived from deterministic systems. This poses a challenge since obtaining a deterministic solution in reality is not always straightforward. However, the extension principle approach offers a convenient means of finding numerical solutions for fuzzy systems by integrating it with various existing numerical methods, thus enabling the development of new numerical techniques specifically designed for fuzzy systems, such as the Adams-Bashforth and Adams-Moulton methods.

4. CONCLUSIONS

This study has presented the results of a literature review on the topic of research on the fuzzy predator-prey model, and the results indicate that there has not been much research related to this matter. The development of fuzzy predator-prey model research, both from the model developed and from the method used, is relatively small. The results of the review show that in 2014 they the classic fuzzy predator-prey model (Lotka-Volterra) is still the focus of research. Starting in 2015, the studied fuzzy predator-prey model has been developed and incorporates various additional biological processes, including by considering harvesting factors, disease factors, protection of prey, and toxic effect.

Based on the results of the above review, model development is still open to be carried out by adding various biological factors, such as Allee effects. From the results of the review, no authors have discussed the fuzzy predator-prey model by considering the allee effect. Allee effect is very important for conservation management of endangered species, population development and utilization of natural resources. Therefore, the allee effect in the predator-prey fuzzy model is very important to study and do further research.

The method used in studying the fuzzy predator-prey model generally uses the Hukuhara derivative (H-derivative). Another method implemented in the studied article is through the generalized Hukuhara derivative (gH-derivative) approach, and the Zadeh extension principle approach. The weakness of the H-derivative approach is that it requires that the diameter of the fuzzy function under study must always not decrease, while the gH-derivative approach has the disadvantage that the existence of the gH derivative of the fuzzy function cannot be guaranteed. Meanwhile, the weakness of Zadeh's extension principle approach is that it can only be applied to systems derived from deterministic systems. This limitation opens opportunities for further research with other approaches to fuzzy derivatives, such as g-derivatives, or granular derivatives. Both of these approaches can overcome the shortcomings of H-derivative and gH-derivative, where these two approaches do not require that the diameter of the fuzzy function under study must always not decrease and the existence of the solution is guaranteed. In addition, research on the fuzzy predator-prey model of the articles reviewed limit the uncertainty environment to parameters only or initial conditions only. Further research development can be carried out by combining parameters and initial conditions as fuzzy uncertainties so that they are more realistic and in accordance with the conditions in real problems.

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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