Available online at http://scik.org
Commun. Math. Biol. Neurosci. 2024, 2024:53
https://doi.org/10.28919/cmbn/8514
ISSN: 2052-2541

ANALYSIS AND OPTIMAL CONTROL TIME FOR A CONTINUOUS PANIC PROPAGATION IN IN-FLIGHT SCENARIOS

AYOUB ABDELALIM*, NADIA IDRISSI FATMI, YASSINE KHOULDI

LIPIM Laboratory ENSA Khouribga, Sultan Moulay Slimane University, Beni Mellal, Morocco

Abstract. Building on earlier research [1], this paper explores the propagation of panic among airplane passengers using a continuous Susceptible-Infected-Recovered (SIR) model. We apply the Pontryagin Maximum Principle to design and implement effective control strategies with the primary aim of minimizing panicked passengers during flight. The SIR model, divided into three categories - susceptible, panicked, and recovered passengers - was simulated using MATLAB. The results demonstrate the significant effectiveness of the implemented controls, underscoring their vital role in maintaining flight safety. Without these measures, the number of panicked passengers could reach critical levels, posing a considerable risk to flight safety. Furthermore, our simulations identified the optimal timing for the application of these control measures, a factor that significantly enhances their effectiveness.

Keywords: safety; aviation; pontryagine.

2020 AMS Subject Classification: 92C20.

1. Introduction

The use of aeroplanes for both business and leisure purposes has exploded across the globe. Compared to other modes of transportation that are thought to be more time-consuming for connecting large population centres, recent estimates in the United States indicate that 20 to 30 per cent of residents are worried and fearful of flying [2]. Fear of flying can grow into a

*Corresponding author

E-mail address: ayoubabdelalim1981@gmail.com

Received February 29, 2024

severe phobia with many emotional and professional consequences [3]. For certain individuals, piloting an aeroplane becomes a formidable challenge. Others fear lightning bolts and turbulent air.

In most cases, aeroplane anxiety is induced by the loss of both control and experience in various external conditions, particularly when flying at an altitude of 10,000 meters. This leads to the spread of terror on board, especially when the aircraft experiences certain phases such as air turbulence, go-arounds, air pockets, bad weather, and emergency situations [4]. Panic is typically characterized by a generalized dread and the quick dissemination of information to an entire group, resulting in a widespread disorder of chaos [5]. It is often believed that in a panic situation, all human activities will have opposite outcomes, posing a threat to one's own and others' lives [6, 7]. However, panic can also signify confusion and flight, and a crowd may comprise both panicked and non-panicked individuals [8]. Humans are typically terrified and anxious in emergency situations due to distinct external factors that influence everyone's risk management. Due to the contagiousness of emotions, the mass response to danger and tragedy can lead to social breakdown [9].

There is a correlation between panic and excessive fear of flight. Frequently, panic is evaluated retrospectively, particularly in instances where significant loss of life has occurred [10, 11]. However, what may be perceived as excessive or irrational behavior may not be deemed as such by others. In situations of danger, such as a building fire, running for the exits may be considered the most rational course of action. However, excessive panic is characterized by symptoms such as anxiety, vomiting, vertigo, and nausea which may be deemed normal in certain situations [12]. The presence of a dangerous threat can cause any group of individuals to become a mob, with each individual having a highly activated fear instinct. The term ”panic” has been defined as a mass flight caused by frenzied beliefs about a general threat [13, 14]. Additionally, the duration of flight is typically brief and linked to the feeling of having reached a safe distance from the threat [15]. There are two key factors that determine the path of panicked passengers: the perception of familiar exits and the tendency to follow others [8]. In such situations, violence, murder, and shattered social connections may occur in an effort to save one's life [8]. It is important to note that the correlation between panic and excessive fear of flight should be analyzed with caution, as what may be deemed as excessive or irrational behaviour by some may not be considered as such by others. Additionally, it is crucial to consider the potential consequences of panic, such as violence and shattered social connections, when evaluating the severity of the fear of flight.

On the basis of the aforementioned information, it can be concluded that the aeroplane is a favourable site for the spread of panic; consequently, this places high pressure on airline passengers in the event of an emergency requiring an immediate landing at the nearest airport to avoid terrible scenarios[9]. Keeping in mind the necessity of taking precise measures in order to stop the panic's rapid spread efficiently. Extremely high-level panic managing and planning actions may take a few seconds, minutes, or even hours; therefore, time-pressured decision-making exists within these ranges. When there is pressure to make a decision as quickly as feasible, when the decision itself requires the integration of various information sources, or when there are multiple possible responses to choose from, panic levels increase. Ultimately, there must be significant consequences for selecting one option over another [16]. According to (Websters, 1976), the definition an emergency is an unplanned event demanding rapid action. There have been a number of scenarios necessitating time-sensitive judgments during air travel.

In 1990, a British Airways flight from Birmingham, England, to Malaga, Spain, carrying 87 passengers and crew members, was involved in a similar incident. 13 minutes after takeoff, an explosion occurred, causing the fuselage to fill with mist, the cockpit glass to be ripped off by a rush of air, and the crew commander to be practically sucked into the window opening while leaning out of a flying aircraft. The crew began to comfort frightened passengers and requested an emergency landing from ground services as they tried to calm the passengers' fears. Fortunately, the crew members reacted quickly and prevented the captain from falling overboard by grabbing him by the legs. In those conditions, it is evident that no one will live. However, it would not allow the aircraft to depressurize completely, and if they released the pilot, his body would slam into the aircraft's crucial components at full speed. Then everything would have gone horribly wrong, especially considering that the cockpit door was ripped off and fragments crashed over the console. 25 minutes after takeoff, the aircraft touched down in

Southampton. The flight attendant hugged his icy partner the entire time. The crew not only successfully landed the aircraft but also saved the lives of everyone on board [17, 18].

This dreadful scenario was witnessed in the Lockheed L-1011-200 TriStar when it was necessary to contain panic as quickly as possible to avert dire results. When the aircraft was climbing to an altitude of 6,700 metres, both the rear luggage compartment and engine 2 automatic fire alarms were activated. The flight engineer reported the situation to the captain, who chose to return to Riyadh for an emergency landing after receiving the information. Meanwhile, the fire in the cargo hold was rapidly spreading. Concerned passengers in a panic moved to the front cabin out of fear of toxic smoke, while others began to fight for the front seats, resulting in a disruption of the aircraft's alignment. In preparation for an emergency landing, flight attendants attempted to compel passengers to take their seats. However, instead of immediately stopping on the runway, the pilots turned onto a taxiway, where they spent an additional 2 minutes and 39 seconds of valuable time. The flight crew was unable to open any of the emergency evacuation doors for unknown reasons. At this point, the cargo compartment fire had already spread to the passenger compartment. It took the rescuers another 23 minutes to open the main doors. After the personnel opened the main door, there was an instant ignition, and the fire quickly spread throughout the cabin, engulfing the entire aircraft in flames. After five minutes, the fire was put out. Due to the delayed evacuation, all 287 passengers and 14 staff members perished [18, 8].

Statistics indicate that more than seventy-five percent of aviation crashes occur on the runway or landing strip, hence there are virtually no human casualties. Consequently, if the crash begins at a reasonable altitude, passengers must adhere to all of the survival instructions given by the flight attendant. Everyone experiences anxiety and terror during a plane accident. They can compel them to get out of their seats or undo their seatbelts, which will cause real panic and confusion on board, preventing the pilot from attempting to land the out-of-control plane [19]. Due to the aerodynamics of transportation, a competent pilot can attempt to land an unguided plane even if the airliner's engines fail to operate at high altitudes. It enables a big aeroplane to ascend into the sky and descend smoothly, rather than plummeting to the ground with a massive burden. According to research, an aircraft can cover 155 metres by losing 1 metre of altitude, which is of significant assistance to pilots [20]. However, control of the controls can only be maintained if the passengers cease panicking and strictly adhere to the instructions. Only in this manner will pilots be able to keep an uncontrolled aircraft aloft, which can travel more than 100 kilometres to find the safest landing place.

In [21], the evacuation response of airline passengers were modelled as an autonomous agent and multi-agent system. These agents are initially located in seat squares and relocate to an emergency exit upon the occurrence of an aeroplane accident; the autonomous agent reflects the behaviour of panicked air passengers; as the situation unfolds, the agents experience either mental stress or high levels of panic and anxiety; as a result, panic causes the agents to react in an unfavourable manner prior to an evacuation, causing time delays in the evacuation flow to the exit. To control the panic level, three sets of criteria are suggested: the difficulty of reaching an exit, the frequency of waiting, and the remaining period of time.

2. Model Description

In this section, we introduce a model that uses mathematical equations to understand how panic spreads during a flight. The model divides passengers into three groups: those who are at risk of becoming panicked (X), those who are already panicked (Y), and those who have recovered from panic (Z). The model assumes that panic can be transmitted through proximity and is specific to one flight with a certain number of passengers. We use this model to study the dynamics of panic propagation during the flight.

$$
\begin{cases}
\dot{X}(t) = -\beta X(t)Y(t) - \mu X(t) + \theta Z \\
\dot{Y}(t) = \beta X(t)Y(t) - \sigma Y(t) \\
\dot{Z}(t) = \mu X(t) + \sigma Y(t) - \theta Z(t)
\end{cases}
\tag{1}
$$

with $X(0) = X_0 \geq 0$, $Y(0) = Y_0 \geq 0$ and $Z(0) = Z_0 \geq 0$ are the initial conditions, and $t \in [0, T_f]$. This equation describes the changes in the number of susceptible passengers (X), panicked passengers (Y), and recovered passengers (Z) over time. The term $\dot{X}$ represents the change in the number of susceptible passengers, $\dot{Y}$ represents the change in the number of panicked passengers, and $\dot{Z}$ represents the change in the number of recovered passengers. The term β, XY represents the rate at which panic spreads from panicked passengers to susceptible passengers, μ, X represents the rate at which susceptible passengers recover from panic, θ, Z represents the rate at which recovered passengers become susceptible to panic again, σ, Y represents the rate at which panicked passengers recover from panic, and $\mu, X+\sigma Y$ represents the rate at which recovered passengers increase.

3. Model Analysis

3.1. Positivity of solutions.

Theorem 3.1. *if $X(0)\geq 0$, $Y(0)\geq 0$ and $Z(0)\geq 0$, the solutions $X(t), Y(t)$ and $Z(t)$ of System* (1) *are positive for all $t\geq 0$.*

Proof. It follows from the first equation of system (1) that

$$\dot{X}=\frac{dX}{dt}=-\beta XY-\mu X+\theta Z\geq -\beta XY-\mu X, \frac{dX}{dt}+(\beta Y+\mu)\geq 0$$

where

$$F(t)=\beta Y(t)+\mu$$

by multiplying the both side of the last inequality by $\exp(\int_0^t F(s)ds)$ we obtain

$$\frac{dX}{dt}\exp(\int_0^t F(s)ds)+F(t)(\exp(\int_0^t F(s)ds))X(t)\geq 0$$

Then

$$\frac{d}{dt}X(t)\exp(\int_0^t F(s)ds)\geq 0$$

integrating this inequality from 0 to t gives

$$\int_0^t \frac{d}{ds}(X(s)\exp(\int_0^t(\beta Y(s)+\mu)ds)\geq 0$$

then

$$X(t)\geq X(0)\exp(\int_0^t(\beta Y(s)+\mu)ds)$$

hence

$$X(t)\geq 0$$

Similarly, we show the positiveness of $Y(t)$ and $Z(t)$.

□

3.2. Boundedness of solutions.

Theorem 3.2. *the set* $\Omega = \{(X,Y,Z) \in \mathbb{R}^3 / 0 \leq X+Y+Z \leq C\}$ *positively invariant under system* (1) *with initial conditions* $X(0) \geq 0$, $Y(0) \geq 0$ *and* $Z(0) \geq 0$.

Proof. We presume that

$$N(t) = X(t) + Y(t) + Z(t)$$

So $\frac{dN}{dt} = 0$ then $N(t) = C = N(0)$. hence, we will have $0 \leq N(t) \leq C$

it implies the region Ω is positively invariant set for the system. □

3.3. Existence of solutions.

Theorem 3.3. *The system* (1) *satisfies a given initial Condition, then it has a unique solution*

Proof. Let

$$\phi(t) = \begin{pmatrix} X(t) \\ Y(t) \\ Z(t) \end{pmatrix} \text{ and } \phi_t(t) = \begin{pmatrix} \frac{dX}{dt} \\ \frac{dY}{dt} \\ \frac{dZ}{dt} \end{pmatrix},$$

So, the system (1) can be rewritten in the following form

$$\phi_t(t) = A\phi + N(\phi)$$

$$A = \begin{pmatrix} -\mu & 0 & \theta \\ 0 & -\sigma & 0 \\ \mu & \sigma & -\theta \end{pmatrix} \text{ and } N(\phi) = \begin{pmatrix} -\beta XY \\ \beta XY \\ 0 \end{pmatrix}$$

and ϕ_t denotes derivative of ϕ with respect to time. The second term on the right-hand side satisfies

$$|N(\phi_1) - N(\phi_2)| = |N_1(\phi_1) - N_1(\phi_2)| + |N_2(\phi_1) - N_2(\phi_2)|$$

then by applying the Triangular inequality, we have

$$\begin{aligned} |N(\phi_1) - N(\phi_2)| &\leq 2\alpha M(|X_1 - X_2| + |Y_1 - Y_2|) \\ &\leq 2\alpha M|X_1 - X_2| + 2\alpha M|Y_1 - Y_2| \\ &\leq K|\phi_1 - \phi_2| \end{aligned}$$

with $K = 2\alpha M =$. Thus, it follows that the function N is uniformly Lipschitz continuous, therefore the solution of the system exists. □

3.4. The optimal control problem. The following system gives the controlled proposed model related to the system (1)

$$\begin{cases} \dot{X}(t) = -\beta X(t)Y(t) - \mu X(t) + \theta, Z(t) - u(t)X(t) \\ \dot{Y}(t) = \beta X(t)Y(t) - \sigma Y(t) - v(t)Y(t) \\ \dot{Z}(t) = \mu X + \sigma Y - \theta Z(t) + u(t)X(t) + v(t)Y(t) \end{cases} \tag{2}$$

with $X(0) = X_0 \geq 0$, $Y(0) = Y_0 \geq 0$ and $Z(0) = Z_0 \geq 0$ are the initial conditions, and $t \in [0, T_f]$. The problem is to minimize the objective cost

$$J(u, v, T_f) = \int_0^{T_f} [Y(t) - Z(t) + \frac{1}{2}\rho_1 u^2(t) + \frac{1}{2}\rho_2 v^2(t)]dt + \rho_3 T_f^2 \tag{3}$$

where $\rho_i, \quad i = 1, 2, 3$ are weighting positives factors parameters associated to the controls u, v. Our target is to minimize the objective functional (3) by decreasing Panicked Passengers and increasing Recovered ones, by using possible minimal controls variables $(u(t), v(t)) \in U_{ad}$, where U_{ad} is the Control set given by

$$U_{ad} = \{u, v / 0 \leq u(t) \leq u_{max} \leq 1, 0 \leq v(t) \leq v_{max} \leq 1, t \in [0, T_f]\}.$$

3.5. The existence of the optimal solution. In this section, we will prove the existence of an optimal control by using a result of [22, 23, 24, 25]

Theorem 3.4. *There exists control functions u^*, v^* such that*

$$J(u^*, v^*, T_f) = \min_{(u,v) \in U_{ad}} J(u(t), v(t), T_f)$$

Proof. It is simple to confirm that an optimal control exists in order to demonstrate that:

- A_1 The set of controls, and corresponding state variables, is not empty.
- A_2 The admissible set U_{ad} is convex and closed.
- A_3 The right-hand side of the state system is bounded by a linear function in the state and control variables.
- A_4 The integrand L of the objective functional defined in (4) is convex on U_{ad}.

A_5 There exist constants $\delta_1 \geq 0, \delta_2 \geq 0$ and $\eta \geq 1$ such the integrand $L(X,Y,Z,u,v)$ of the objective functional satisfies.

$$L(X,Y,Z,u,v) \geq \delta_2 + \delta_1 (|u^2| + |v^2|)^{\eta/2}$$

The first condition A_1 is verified using Lukes' results [22].

The set U_{ad} is convex and closed by definition, thus the condition A_2.

Our state system is bounded hence the condition A_3.

Note that the integrand of our objective function is convex from where conditions A_4.

To prove the condition A_5, we have $u+v \geq |u|^2 + |v|^2$ since $0 \leq u, v \leq 1$, from which the last condition is derived. The result follows directly from (Fleming and Rishel 1975) [23].

□

3.6. characterization of optimal control. To characterize optimal control, we first define the Lagrangian for the optimal control problem by

$$L(X,Y,Z,u,v,T_f) = Y(t) - Z(t) + \frac{1}{2}\rho_1 u^2(t) + \frac{1}{2}\rho_2 v^2(t) + \rho_3 T_f^2 \tag{4}$$

The related Hamiltonian is given as follows

$$H(X,Y,Z,u,v,\lambda_i,T_f) = L(X,Y,Z,u,v,T_f) + \sum_1^3 \lambda_i f_i \tag{5}$$

with $f_1 = -\beta XY - \mu X + \theta Z,\ f_2 = \beta XY - \sigma Y$ *and* $f_3 = \mu X + \sigma Y - \theta Z$ and where $\lambda_i, i = 1;2;3$ are the adjoint functions to be determined suitably.

Next by applying Pontryagin Maximum Principle to the Hamiltonian H, we get the following theorem[25, 26, 27].

Theorem 3.5. *Given the optimal controls (u^*, v^*), the optimal Time T^* and the solutions X^*, Y^*, Z^* of the corresponding state there exists adjoint variables $\lambda_1, \lambda_2, \lambda_3$ satisfying.*

$$\dot{\lambda_1} = -(1 + \lambda_1(-\beta X - \mu - u) + \lambda_2 \beta X + \lambda_3(-\mu + u)),$$

$$\dot{\lambda_2} = -(-1 - \lambda_1 \beta X + \lambda_2(\beta X - \sigma - v) + \lambda_3(\sigma + v)),$$

$$\dot{\lambda_3} = -(\lambda 1\theta - \lambda 3\theta + 1).$$

with the transversality $\lambda_i(T_f) = 0$ *for* $i = 1,2,3$. *Furthermore, the optimal control*(u^*, v^*) *is given by*

$$u(t) = min\left\{max\left\{0, \frac{(\lambda_1 - \lambda_3)X(t)}{\rho_1}\right\}, m_{max}\right\}, \tag{6}$$

$$v(t) = min\left\{max\left\{0, \frac{(\lambda_2 - \lambda_3)Y(t)}{\rho_2}\right\}, v_{max}\right\}. \tag{7}$$

and the optimal Time is given by

$$T_f^* = \frac{Z(T_f^*) - Y(T_f^*) - \frac{1}{2}\rho_1 u^2(T_f^*) - \frac{1}{2}\rho_2 v^2(T_f^*)}{2\rho_3} \tag{8}$$

Proof. Using The Pontryagin Maximum Principle in the state we obtain the adjoint equations of the problem by direct computation

$$\dot{\lambda_1} = -\frac{\partial H}{\partial X}, \lambda_1(T_f) = 0$$
$$\dot{\lambda_2} = -\frac{\partial H}{\partial Y}, \lambda_2(T_f) = 0$$
$$\dot{\lambda_3} = -\frac{\partial H}{\partial Z}, \lambda_3(T_f) = 0$$

and the controls are obtained by calculating the following equations

$$\frac{\partial H}{\partial u} = 0, \ and \ \frac{\partial H}{\partial v} = 0$$

by the bounds in U_{ad} of the controls, it is easy to obtain u^* and v^* are given in the form (6), (7) the transversality conditions for T_f^* to be the optimal time can be stated as

$$H(T_f^*, X(T_f^*), Y(T_f^*), u(T_f^*), v(T_f^*)) = -\frac{\partial \psi(T_f^*, X(T_f))}{\partial t}$$

where $\psi(T, X(T)) = \rho_3 T^2$, then

$$T_f^* = \frac{Z(T_f^*) - Y(T_f^*) - \frac{1}{2}\rho_1 u^2(T_f^*) - \frac{1}{2}\rho_2 v^2(T_f^*)}{2\rho_3}$$

□

4. Numerical Simulation and Discussion

A numerical simulation is a valuable tool for studying the spread of panic in an airplane. Using mathematical models and computer algorithms, we can simulate the dynamics of passengers and crew and study how factors such as seat layout, plane size, and the presence of trained cabin crew can influence the spread of panic. In this section, we will present the results of our numerical simulations and discuss their significance in terms of in-flight safety.

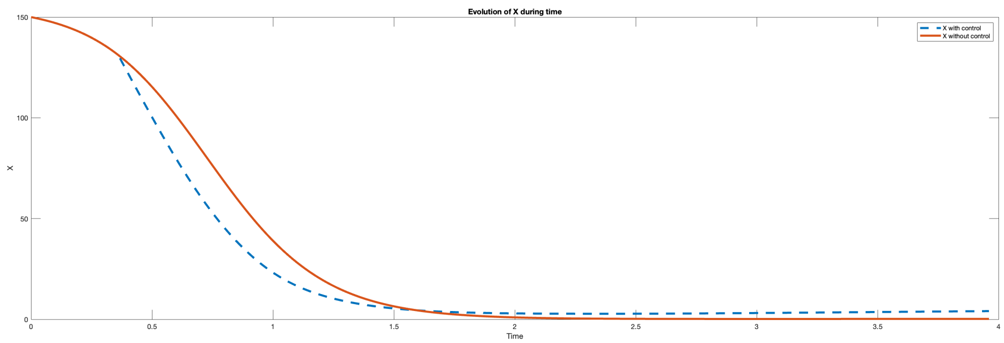

FIGURE 1. The number of susceptible passengers

The graph shows the evolution of the number of susceptible passengers with and without control. It can be seen that when there is control in place, the number of susceptible passengers decreases more quickly than when there is no control. This suggests that control has a positive impact on reducing the susceptibility of passengers. It is important to continue implementing control measures to reduce the number of susceptible individuals and prevent the spread of panic.

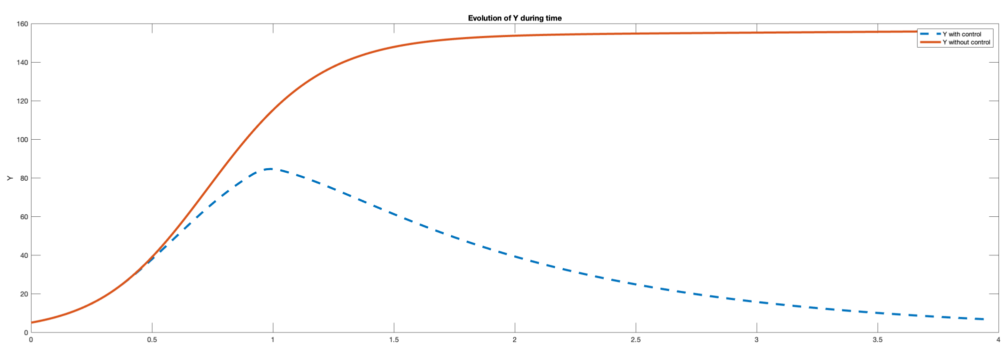

FIGURE 2. The number of Panicked passengers

Figure 2 shows the evolution of the number of panicked passengers with and without control. It can be seen that when control measures are in place, the number of panicked passengers decreases more quickly than when there is no control. This suggests that control has a positive impact on reducing panic among passengers. It is important to continue implementing control measures to prevent the spread of panic and ensure a safe and calm environment for all passengers.

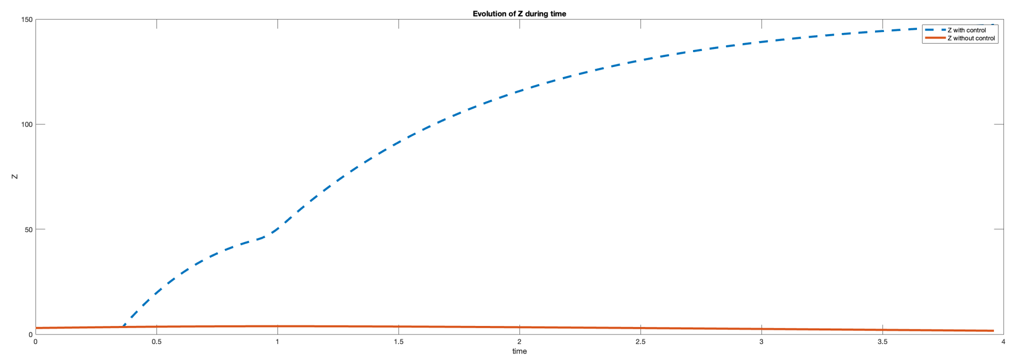

FIGURE 3. The number of recovered passengers with and without control

The graph in Figure 3 presents the evolution of the number of passengers recovering from panic both with and without control measures in place. As can be observed, the presence of control measures appears to expedite recovery. This evidence reinforces the notion that such control measures are beneficial in managing panic levels among passengers. Continuous implementation of these controls is crucial for ensuring efficient recovery processes for those affected by panic, thereby maintaining a safe and calm environment for all passengers.

Figure 4 illustrates the optimal time for applying control measures to manage the panic situation among passengers. The curve demonstrates a specific point in time where the implementation of controls can have the most significant impact on panic reduction and passenger recovery. The precise timing of these interventions is key to their success; if applied too early or too late, their effectiveness can be diminished. Understanding this optimal timing could drastically improve in-flight safety protocols, ensuring not only a reduction in the overall number of panicked passengers but also an expedited recovery for those affected. Further investigations should explore how this optimal timing might be affected by variables such as flight duration, passenger capacity, and airplane layout.

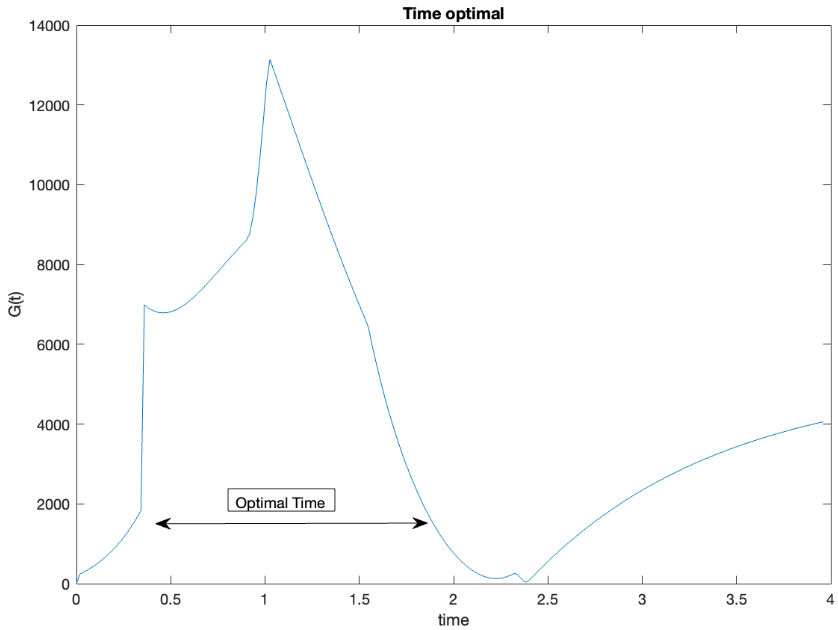

FIGURE 4. Optimal Time

5. CONCLUSION

In conclusion, our numerical simulations strongly demonstrate the positive influence of control measures on mitigating panic among airplane passengers. These measures not only diminish susceptibility and the total count of panicked individuals but also expedite recovery. Therefore, the regular implementation of these controls is essential to guarantee a secure and panic-free in-flight experience. Future research could concentrate on refining these control strategies, potentially customizing them to cater to distinct flight scenarios or passenger demographics. The broader implications of this work extend to various fields where crowd dynamics play a crucial role, such as public transportation, event management, and emergency evacuation planning. By understanding and applying optimal control measures, we can substantially enhance safety and efficiency in these contexts.

ACKNOWLEDGEMENT

Parts of this article were funded by research grant.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

REFERENCES

[1] M. Lafif, A. Ghazaoui, M. Rachik, et al. Study of an optimal control model for the panic propagation in a flying aircraft, Commun. Math. Biol. Neurosci. 2022 (2022), 91. https://doi.org/10.28919/cmbn/7553.

[2] R. Dean, K. Whitaker, Fear offlying: impact on the us air travel industry, Boeing Company Document, Tech. Rep. BCS-00009-RO/OM, (1980).

[3] F.H. Wilhelm, W.T. Roth, Clinical characteristics of flight phobia, J. Anxiety Disord. 11 (1997), 241–261. https://doi.org/10.1016/s0887-6185(97)00009-1.

[4] P. Smith, Ask the pilot: everything you need to know about air travel, Riverhead Books, New York, 2004.

[5] F. Durupınar, U. Güdükbay, A. Aman, et al. Psychological parameters for crowd simulation: from audiences to mobs, IEEE Trans. Visual. Comput. Graph. 22 (2016), 2145–2159. https://doi.org/10.1109/tvcg.2015.2501801.

[6] T. Bosse, R. Duell, Z.A. Memon, et al. A multi-agent model for emotion contagion spirals integrated within a supporting ambient agent model, in: J.J. Yang, M. Yokoo, T. Ito, Z. Jin, P. Scerri (Eds.), Principles of Practice in Multi-Agent Systems, Springer Berlin Heidelberg, Berlin, Heidelberg, 2009: pp. 48–67. https://doi.org/10.1007/978-3-642-11161-7_4.

[7] C. Cocking, J. Drury, S. Reicher, The psychology of crowd behaviour in emergency evacuations: Results from two interview studies and implications for the Fire and Rescue Services, Irish J. Psychol. 30 (2009), 59–73. https://doi.org/10.1080/03033910.2009.10446298.

[8] P. Fu, B. Jing, T. Chen, et al. Propagation model of panic buying under the sudden epidemic, Front. Public Health 9 (2021), 675687. https://doi.org/10.3389/fpubh.2021.675687.

[9] A.J. Cook, G. Tanner, European airline delay cost reference values, EUROCONTROL Performance Review Unit, (2011).

[10] A.R. Mawson, Understanding mass panic and other collective responses to threat and disaster, Psychiatry: Interpers. Biol. Processes. 68 (2005), 95–113. https://doi.org/10.1521/psyc.2005.68.2.95.

[11] A.R. Mawson, Mass panic and social attachment: the dynamics of human behavior, Routledge, 2017. https://doi.org/10.4324/9781351153201.

[12] R.S. Lazarus, Psychological stress and the coping process, McGraw-Hill, (1966).

[13] N.J. Smelser, W.T. Smelser, eds., Personality and social systems., John Wiley & Sons, Inc., Hoboken, 1963. https://doi.org/10.1037/11302-000.

[14] B.E. Aguirre, Emergency evacuations, panic, and social psychology, Psychiatry: Interpers. Biol. Processes. 68 (2005), 121–129. https://doi.org/10.1521/psyc.2005.68.2.121.

[15] E.L. Quarantelli, The behavior of panic participants, Sociol. Soc. Res. 41 (1957), 187–194.

[16] A.L. Franklin, E. Hunt, An emergency situation simulator for examining time-pressured decision making, Behav. Res. Methods Instrum. Comput. 25 (1993), 143–147. https://doi.org/10.3758/bf03204487.

[17] S. Stewart, Emergency: crisis on the flight deck, Crowood, 2012.

[18] N. Denton, N. Dennis, Airline franchising in Europe: benefits and disbenefits to airlines and consumers, J. Air Transport Manage. 6 (2000), 179–190. https://doi.org/10.1016/s0969-6997(00)00010-7.

[19] L. Moretti, G. Cantisani, S. Caro, Airport veer-off risk assessment: An Italian case study, ARPN J. Eng. Appl. Sci. 12 (2017), 900–912.

[20] E. Torenbeek, H. Wittenberg, Flight physics: essentials of aeronautical disciplines and technology, with historical notes, Springer, 2009.

[21] T. Miyoshi, H. Nakayasu, Y. Ueno, et al. An emergency aircraft evacuation simulation considering passenger emotions, Computers Ind. Eng. 62 (2012), 746–754. https://doi.org/10.1016/j.cie.2011.11.012.

[22] D.L. Lukes, Differential equations: classical to controlled, Academic Press, New York, 1982.

[23] W.H. Fleming, R.W. Rishel, Deterministic and stochastic optimal control, Springer, 2012.

[24] I. Khaloufi, M. Lafif, Y. Benfatah, et al. A continuous SIR mathematical model of the spread of infectious illnesses that takes human immunity into account, Math. Model. Comput. 10 (2023), 53–65.

[25] O.M. Ogunlaran, S.C. Oukouomi Noutchie, Mathematical model for an effective management of HIV infection, BioMed Res. Int. 2016 (2016), 4217548. https://doi.org/10.1155/2016/4217548.

[26] I. Khaloufi, M. Karim, S.B. Rhila, et al. A mathematical model describing the correlation between smokers and tuberculosis patients, Math. Eng. Sci. Aerospace. 14 (2023), 347–361.

[27] M. Biswas, S. Samad, Optimal control strategy to reduce the infection of the pandemic HIV associated with tuberculosis in Bangladesh, J. Sci. Technol. Res. 3 (2022), 103–114. https://doi.org/10.3329/jscitr.v3i1.62812.