Available online at http://scik.org
Commun. Math. Biol. Neurosci. 2024, 2024:64
https://doi.org/10.28919/cmbn/8539
ISSN: 2052-2541

SEWON BANTUL WASTEWATER STABILIZATION POND SEDIMENT USING THE DIFFUSION MODEL

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Abstract. The pollutant particles contained on wastewater stabilization pond have the ability to settle at the bottom of the pond. This phenomena is often referred to a sedimentation process. A mathematical model of sedimentation process in wastewater stabilization pond is constructed using a diffusion model. We formulate the model in two different perspective that are one dimensional spread and two-dimensional spread of waste particles. The purpose is to show the distribution of each particle over the entire surface of the pond. The finite difference method is used to gain the analytical solution of the model based on Taylor expansion. The numerical simulation indicates that that the sediment distribution is increasing over the time depend on the number of sediment concentration in the water, and the sedimentation process is moving from initial side to the boundary level of stabilization pond.

Keywords: diffusion model; finite difference; partial differential equation; sediment transport.

2020 AMS Subject Classification: 35Q49, 35Q49.

1. Introduction

Healthy environment is the basic condition for a healthy living. One of the parameters is water quality which can be measure by the water quality index (WQI). WQI determine whether the water is able to be used. Water quality play a significant role to the general purposes such as irrigation, sanitary and water consumption. Therefore, [8] conducted research to examine the WQI

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Received March 12, 2024

which focused on the river. A review for WQI conducted by [1] found the WQI calculation from different country to determine the country water level for specific purposes such as drinking, irrigation use, groundwater and other. One of the objects that can be examined to determine the quality of water is the watershed area of the river. We focused on the river water quality as one of the objects of healthy environment parameter. Currently, water river has been polluted so that it is not proper to be used. Therefore, there are some researches has been carried out to reduce river water pollution such as separation of waste transported by water flow. The waste water treatment process is carried out by channeling river water into a facultative stabilization pond at the Waste Water Treatment Plant Process as stated on [13]. It also examines the economic cost for waste stabilization pond technology. Analysis in waste processing can be done through a mathematical approach. Some of the research discussed about the biological treatment to the wastewater, and some other discussed in the dynamical transport. Research conducted by Facundo in [9] were focused on the dynamical model of biochemical processes to treat the biological pollutant using mathematic approach and analyze it with dynamic sensitivity analysis. In addition, [13] also models the treatment of wastewater in stabilization ponds to carry out a cost analysis in providing treatment. Research on mathematical models to predict the quality of water purification in stabilization ponds has been carried out by Beran in [5]. In this paper can predict the pollutant parameter accurately by increasing the kinetic coefficient. It gains an accurate effluent coefficient faster with adjusting the increment in the implicit Crank-Nicolson method. But the accuracy for a large data is unconfirmed. In [3] also forms a one-dimensional mathematical model to estimate the performance resulting from providing secondary treatment of sludge to the wastewater subsystem 2 by observing the biological factors that occur to determine the location of biochemical reactions in the pond. The results shows that the estimated data produced a slightly error tend to zero. The transport of waste material in the stabilization pond is also a dynamic process which can be analyzed mathematically. Research conducted by Goodarzi in [7] explains the hydraulic performance of waste transportation model in stabilization ponds using partial differential equations combined with Reynold-averaged Navier-Stokes. It gives a minimum impact deflector. He also provides simulations on the model to show the transport processes that occur in the pond. One of the experiments that can be carried out in the stabilization pond is observing the sedimentation process formed from the transport of waste particles. Research conducted by Amin in [2] investigated the coagulation-sedimentation model that occurs in the food waste processing process. This research eliminates COD and TSS factors using three coagulants. The given numerical simulations showed unsignificant differences between the three coagulants. While the statistical analysis did not give a significant error analysis between the generated model with its predicted model. In [16] specifically investigates sediment transport in Bohai Bay with two scenarios to determine the factors that influence sediment transport to the estuary and downstream. It showed that the sediment distributed along the coastal line of Bohay Bay using a numerical simulation. In order to carry out waste processing, it is necessary to aerate the sediment formed in the stabilization pond. As in the experiment carried out by [10] to simulate tank optimization for aeration of the sedimentation process. To achieve optimal values, dynamic modelling is carried out using optimal control. The process of wastewater treatment in the stabilization pond was observed in the process of settling waste particles to form sediments in the stabilization pond. It is observed that the settling process can be described as a dynamic process through the dispersion of the waste particles in all directions in the pond. Thus, it can be seen that the distribution of waste particles spreads out in three-dimensional space. Previously, research had been carried out on the formation of sedimentation in the Yellow River with a three-dimensional mathematical model and was solved using the finite volume method by Sui-Ju in [12]. To determine the model of sediment distribution in three-dimensional space, modelling is carried out in the 1st and 2nd dimensions first. In this paper, we discuss for the first and second dimensional approach. The problem-solving approach is carried out by using the finite element method. [14] construct a model of sedimentation process with the view of the existence of flocculation particle. The model solved with finite difference method. It produced a very small error between the numerical model and the simulation. The finite element method is used to solve dimensional problems in mathematics. There some research examines partial differential equation using finite element method and its modification. In [15] used explicit finite element method for solving advection-diffusion model on the biological transport of pollution in wastewater facultative pond. Furthermore, [11] introduced The Reduced-order finite difference extrapolation to find the the shallow sediment concentration on the two-dimensional model. While [4] investigated the same thing, namely sediment transport with a two-dimensional observation and solved it using the finite volume difference method. The dynamical analysis on the sensitivity of advection-diffusion were conducted by Company in [6].

2. Materials and Method

2.1. Stabilization Pond Illustration. The stabilization pond system in Wastewater Treatment Process Installation (WWTP) Sewon, Bantul, Yogyakarta is a series of ponds used to improve the quality of river water from waste. The WWTP centrally manages household waste carried by water in all rivers in Bantul, Yogyakarta. The structure is formed to separate river water from the pollutant. One of the ponds called as facultative stabilization pond II. In this pond, River water flows in to undergo a purification process. There is a biological degradation process using BOD, COD and CO; and the process of deposition of waste particles to the bottom of the WWTP pond. However, the biological degradation process is not described in this paper. We focused on the sediment formation in the base of the pond for the solid particle in wastewater. The process can be shown in Figure 1.

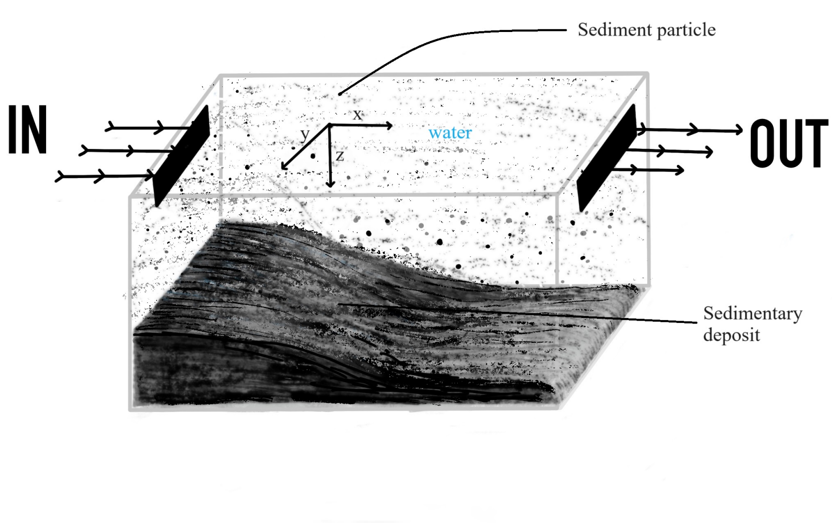

FIGURE 1. Illustration of the process of sediment formation in a stabilization pond by solid waste particles

The Figure 1 illustrated that River water carrying waste and pollutants is brought into the stabilization pond. It can be seen that the solid particles carried by the water settle to the bottom of the pond. In a long term, sediment particles accumulate to form mounds that become sediment. As a result, the concentration of solid pollutants in the water is reduced due to this sedimentation distribution process. In Figure 1, we can see that the particles on the fluid are slowly moving down. It is assumed that there is no influent at the beginning of the observation. Thus, the first layer of the pond is the ground layer. Through the particles that begin to settle, a layer of sediment begins to form. Thus, the amount of sediment concentration will increase significantly when the number of pollutant particles is large enough and the velocity of the river water flow is small. The settlement of particles tends to increase throughout summer since the amount of water is small enough compared to the amount of waste that flows into the river.

TABLE 1. Particle settlement data at the WWTP Sewon in 2018

Month	**Average** (10^3 m^3)	**Total** (10^3 m^3)
1	5.8	180.4
2	7.8	227.3
3	8.6	250.5
4	8.7	235.4
5	5.6	163.5
6	0.1	2
7	1.8	32.2
8	1.9	27
9	1.7	20.4
10	0.02	0.3
11	1.7	22.6
12	13.2	53.6

Particle settlement data in Table 1 is taken from the WWTP Sewon in 2018 on a monthly basis. It is clearly stated that the number of sediments is significantly increasing while a dry season. The high rate of water evaporation is a factor influences to the increases of sediment.

3. SEDIMENTATION MODEL

In this research, we construct a mathematical model to describe the concentration distribution of sediment formed in facultative stabilization pond II at WWTP Sewon Bantul. We give two observation schemes for one-dimensional and two-dimensional distribution models utilizing the diffusion model [11]. The sediment transfer equation takes into account for the inflow, outflow, reaction and the displacement process. This process can be described as a geometrical representation. The details of the mathematical formulation given on the description below. Given a one-dimensional sediment transport on Figure 2.

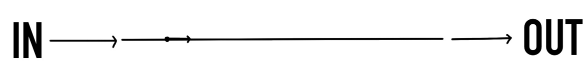

FIGURE 2. Geometric representation of one-dimensional sediment transport

In Figure 2, river water flow into the pond with the flow velocity u containing sediment particle S in straight direction as x-axis direction. We assumed that the pond length is L. Thus, the mathematical formulation is given as

$$\frac{\partial S}{\partial t} = k\frac{\partial (uS)}{\partial x} + A_x\frac{\partial^2 S}{\partial x^2} \tag{1}$$

where

$k =$ displacement coefficient

$A_x =$ sedimentation viscosity coefficient for x axis

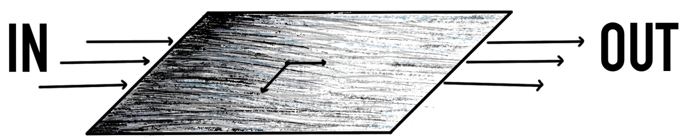

FIGURE 3. Geometric representation of two-dimensional sediment transport

In contrast, the domain presented in Figure 3 shows that the sediment particle contained on the wastewater move along x-axis and y-axis. The stabilization pond dimension is LM where L present as the length pond and M present as the wide pond. Therefore, the two-dimension formula can be written as follows.

$$\frac{\partial S}{\partial t} = k\frac{\partial (uS)}{\partial x} + A_x\frac{\partial^2 S}{\partial x^2} + A_y\frac{\partial^2 S}{\partial y^2}. \tag{2}$$

where

$k =$ displacement coefficient

$A_x =$ sedimentation viscosity coefficient for x axis

$A_y =$ sedimentation viscosity coefficient for y axis

4. RESULTS

In case of solving diffusion equation for Equation (1) and (2) we use the finite difference method to gain the geometrical solution. Take a note to the scheme in Figure 4 below.

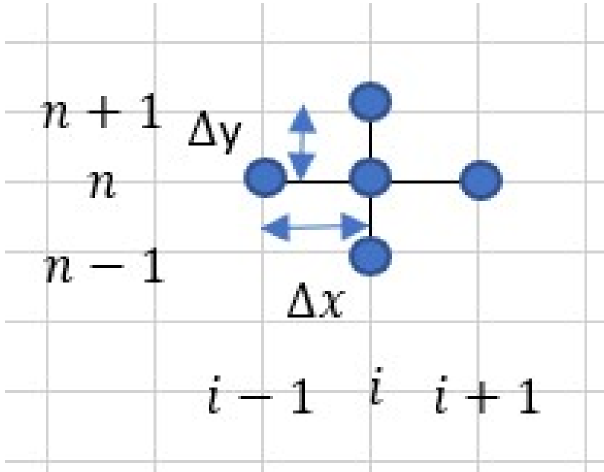

FIGURE 4. Finite difference scheme

In the finite difference method, the initial function is discretized to replace its partial derivative. This discretization is utilizing the past and the future state. Then, the approximation for sediments is written as follows.

$$\frac{\partial S}{\partial t} = S_t = \frac{S_i^{t+1} - S_i^t}{2\Delta t} \tag{3}$$

$$\frac{\partial S}{\partial x} = S_x = \frac{S_{i+1}^t - S_i^t}{2\Delta x} \tag{4}$$

$$\frac{\partial^2 S}{\partial x^2} = S_{xx} = \frac{S_{i+1,j}^t - 2S_{i,j}^t + S_{i-1,j}^t}{\Delta x} \tag{5}$$

$$\frac{\partial^2 S}{\partial y^2} = S_{yy} = \frac{S_{i,j+1}^t - 2S_{i,j}^t + S_{i,j-1}^t}{\Delta y}. \tag{6}$$

Thus, the sediment transport equation on Equation (1) and Equation (2) can be discretized using Equation (3)-(6) as follows.

The one-dimesional

$$S_i^{t+1} = (ku + 2A_x)\frac{\Delta t}{\Delta x} S_{i+1}^t + (2 - ku - 4A_x)\frac{\Delta t}{\Delta x} S_i^t + 2A_x \frac{\Delta t}{\Delta x} S_{i-1}^t \tag{7}$$

The two-dimensional

$$S_{i,j}^{t+1} = (ku + A_x)\frac{\Delta t}{\Delta x} S_{i+1,j}^t + \left(1 - ku\frac{\Delta t}{\Delta x} - 2A_x\frac{\Delta t}{\Delta x} - 2A_y\frac{\Delta t}{\Delta x}\right) S_{i,j}^t + A_x\frac{\Delta t}{\Delta x} S_{i-1,j}^t + A_y\frac{\Delta t}{\Delta x}\left(S_{i,j+1}^t - S_{i,j-1}^t\right). \tag{8}$$

The distribution of sediment particles contained on the river water can be calculated using Equation (7) and Equation (8) at a given grid point ant time $t-1, t$ and $t+1$. Numerically, the solution of Equation (7) and Equation (8) can be solved using MATLAB software. In this paper, we determine the diffusion coefficients using the data collected from WWTP Sewon Bantul at 2018. In this paper, we show the simulation results for one dimensional model.

The numerical simulations with several test values are given in the Table 2.

TABLE 2. Parameter Simulation

δt	δx	u	Ax
1	0.07	3.45	8.23×10^{-2}
1	0.07	4.1	8.23×10^{-2}
1	0.1	3.45	6×10^{-3}
1	0.1	4.1	6×10^{-3}

In the stabilization pond is assumed to have 10 meters length. The simulation uses the grid time equals to 1. Meanwhile the dimension grid point of the pond is given different grid, as for the displacement and sediment viscosity coefficients are obtained from the previous observation data.

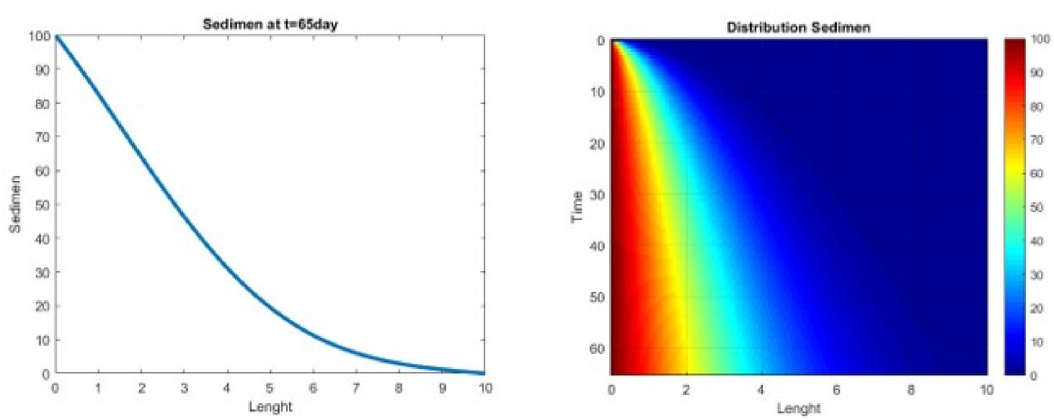

FIGURE 5. Sediment distribution for the first simulation

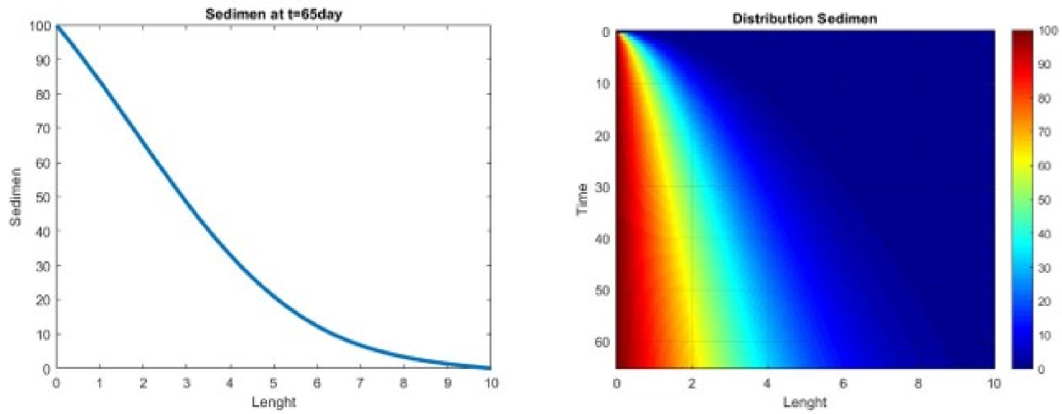

FIGURE 6. Sediment distribution for the second simulation

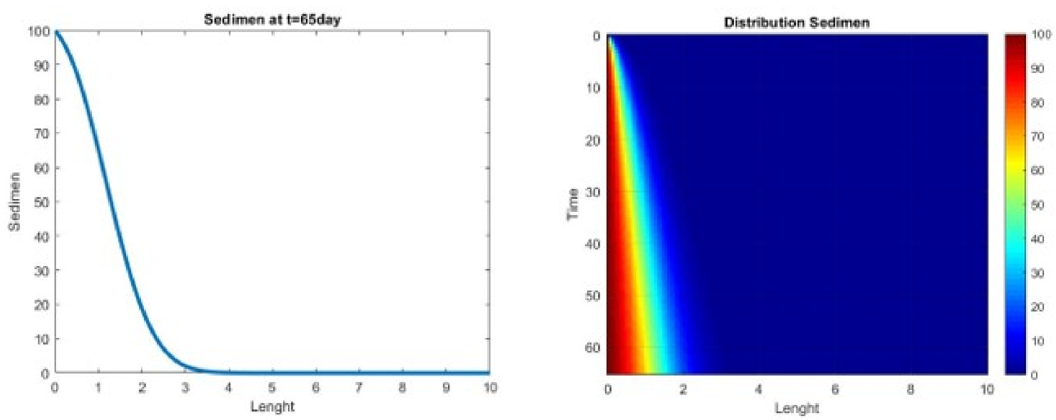

FIGURE 7. Sediment distribution for the third simulation

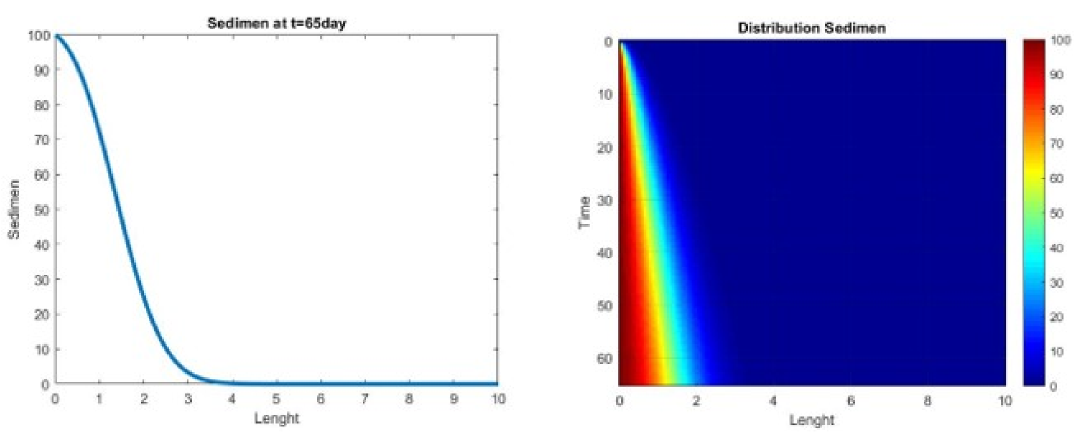

FIGURE 8. Sediment distribution for the forth simulation

In the first simulation, the displacement of sediment set by 3.45 m^2/h with the viscosity coefficient as 8.23×10^{-2} m^2/h in x direction yield a quite thick sediment on the pond. While the second simulation produce a similar sediment thickness although the displacement coefficient is set to be greater than the first simulation. This shows that the thickness depends on the viscosity/ diffusion coefficient. The similar results also show by the third and the fourth simulation where the viscosity coefficient influence the sediment distribution. However, there were quite significant results shown from trials with a diffusion coefficient of 8.23×10^{-2} m^2/h compared to a coefficient of 6×10^{-3} m^2/h. From Figure 5, we can conclude that Sediment thickness is strongly influenced by the diffusion coefficient. A large number of diffusion coefficient results in the formation of sediments that are quite fast at the bottom of the pond along horizon times t.

5. CONCLUSIONS

The process of sediment transport in stabilization pond at WWTP Sewon Bantul can be described using a mathematical formula using the diffusion model. The solution and the interpretation of the model can be achieved while solving the diffusion model of transport sediment using a finite difference method by choosing the grid point for x and y direction along the time t. The simulation showed that sediment formation is influenced by the concentration of sediment particles carried by river water. A large number of concentrations result in relatively faster sediment formation compared to the smaller concentrations. In the future work, we will develop a three-dimensional diffusion model to construct a full sediment formation in the pond which including the dept of the pond. We also validate the constructed model with the field data. Some mathematical tools can be used to optimize the sedimentation process in order to minimize the shallow.

ACKNOWLEDGEMENTS

Thanks to the Leaders and staff of WWTP Sewon Bantul, Public Works Departmen, Housing and ESDM Local Government Yogyakarta who has given permission to conduct research and collect data at WWTP Sewon Bantul. Thanks for the supporting fund from the Ministry of Education, Culture, Research and Technology (Kemendikbudristek) through the fundamental research grant No. 449A-23/UN7.D2/PP/VI/2023.

Conflict of Interests

The authors declare that there is no conflict of interests.

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