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NET PREMIUM ESTIMATION ON TERM-HEALTH INSURANCE BASED ON MARKOV CHAIN

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Abstract: Health insurance is a part of life insurance with life as an object and product that provides some sum assured if the insured is exposed to certain illness or disability. This research aims to estimate net premium based on trend movement on every possible entry age and probability of being exposed to diabetes mellitus, chronic kidney disease, and hemodialysis on every age. Net premium estimation is based on multistate model using Markov chain to determine transition probability on a case study of 5 status, namely healthy, diabetes mellitus, chronic kidney disease, hemodialysis, and death. Specified case study is conducted on a 56-year-old man taking 20 years of insurance protection, paying premium for 5 years with sum insured of one billion IDR on 4 benefit categories. Calculation on every entry age shows that the older someone, the higher annual net premium that must be paid as the transition probability towards high severity disease increases.

Keywords: chronic kidney disease; health insurance; Markov chain; multistate.

2020 AMS Subject Classification: 62P10.

1. INTRODUCTION

Insurance is a form of agreement between two parties, insurer and insured or policyholder, which become a base of premium receive for insurer as a reward for covering the insured's loss or

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expenses. Health insurance is a kind of insurance that diverse health risks from the insured to the insurer under a written contract called policy [1]. In its application, benefit of health insurance is paid upon insured's claim on being sick or any disabilities being covered in the policy.

Health insurance premium calculation could be based under multistate model since its correlation with the transition between states from being healthy until death [2]. Hence, this research will show the need of Markov chain to determine the probability of transitioning from one to another state. Markov chain is a stochastic process which assumes the probability of further transition is only relying on the current state and free from the impact of the previous state. In relation to health insurance premium calculation, the transition probability holds a crucial role to determine the right premium charged upon an insured.

This research is a development from previous research [3] in a form of adding state involved into 5, changing the study case into chronic kidney disease, and display the graphic of net premium charged on any possible entry age. In chronic kidney disease study case, there are 5 states in order from lowest to highest severity: healthy, diabetes mellitus, chronic kidney disease, haemodialysis, and death. It is assumed that someone who is in the state of higher sickness severity cannot reverse to the lower sickness severity. This research aims to determine the net premium from health insurance product with additional of life insurance.

2. METHOD

2.1. LITERATURE REVIEW

2.1.1. Stochastic process. Stochastic process is a set of random variable which depends on time basis changes. This random variable is symbolized as $\boldsymbol{M_t}$ or $\boldsymbol{M(t)}$ in which $\boldsymbol{t}$ represents discrete unit from time to time which moves along a set of time $\boldsymbol{T} = \{\boldsymbol{0}, \boldsymbol{1}, \boldsymbol{2} \dots\}$. A set of possible $\boldsymbol{M_t}$ is defined as state space [4]. Due to its wide applications, a stochastic process $\boldsymbol{M_t}$ could represent various random events, such as probability, frequency of occurrences, and success amount from an experiment. There are two stochastic models, continuous model for continuous $\boldsymbol{T}$ and discrete model for discrete $\boldsymbol{T}$ [5].

Markov process is one of discrete stochastic model with property that given random variable $\boldsymbol{M_s}$ for $\boldsymbol{s > t}$ are not influenced by by $\boldsymbol{X_u}$ in which $\boldsymbol{u < t}$ and only being influenced by the current state $\boldsymbol{M_t}$. In other words, discrete Markov process assumes that the probability of one event occurs in the future is not influenced by its past. Mathematically, Markov process property is defined as follows, with $\boldsymbol{n}$ representing time the transition begins:

$$\Pr\{M_{n+1} = j | M_0 = i_0, \dots, M_{n-1} = i_{n-1}, M_n = o\} \\ = \Pr\{M_{n+1} = j | M_n = o\}. \tag{1}$$

By giving integer notation on every state, transition probability is defined as transition probability $\boldsymbol{M_{n+1}}$ being in a state $\boldsymbol{j}$, which $\boldsymbol{j} \in \{\mathbf{1}, \mathbf{2}, \dots, \boldsymbol{J}\}$ with the prior knowledge that $\boldsymbol{M_n}$ is in state $\boldsymbol{o}$, which $\boldsymbol{o} \in \{\mathbf{1}, \mathbf{2}, \dots, \boldsymbol{O}\}$ and $\boldsymbol{O} = \boldsymbol{J}$. Transition probability is symbolized as follows:

$$P_{ij} = \Pr\{M_{n+1} = j | M_n = o\}. \tag{2}$$

Transition probability matrix is a matrix that collects $P_{o,j} \forall o, j$. Transition probability matrix is generally stated as follows:

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Transition probability matrix is a matrix that collects $\boldsymbol{P_{o,j}} \forall \boldsymbol{o}, \boldsymbol{j}$. Transition probability matrix is generally stated as follows:

$$\boldsymbol{P} = \begin{bmatrix} P_{11} & \dots & P_{1J} \\ \dots & \dots & \dots \\ P_{O1} & \dots & P_{OJ} \end{bmatrix}, \tag{3}$$

with $\boldsymbol{P}$ defines square matrix $O \times O = J \times J$. To comply with the probability rules, the following properties must be fulfilled:

$$P_{oj} \geq 0, \forall\, o, j \tag{4}$$

$$\sum_{j=0}^{J} P_{oj} = 1. \tag{5}$$

Equation (4) represents non-negative probability property, while Equation (5) represents the summation of transitional probability from state o until j for all possible state is one.

2.1.2. Modification on transitional probability matrix with age consideration. Under general model, matrix P will be constant along transitions, but in this research transition probability will be dynamic following the transition probability on every age. It is done in order to make the calculation more realistic with the real condition as the transition probability on every sickness state until death from a younger group is different with the older ones. Hence, the modified transitional probability being used in this research is defined as follows, with x representing insurance entry age:

$$\boldsymbol{P_x} = \begin{bmatrix} P_{11}^x & P_{12}^x & P_{13}^x & P_{14}^x & P_{15}^x \\ P_{21}^x & P_{22}^x & P_{23}^x & P_{24}^x & P_{25}^x \\ P_{31}^x & P_{32}^x & P_{33}^x & P_{34}^x & P_{35}^x \\ P_{41}^x & P_{42}^x & P_{43}^x & P_{44}^x & P_{45}^x \\ P_{51}^x & P_{52}^x & P_{53}^x & P_{54}^x & P_{55}^x \end{bmatrix}. \tag{6}$$

Information: State 1 for healthy, State 2 for diabetes mellitus, State 3 for chronic kidney disease, State 4 for haemodialysis, and State 5 for death.

In order to obtain the n-steps transition probability, ${}_n P_x$ matrix will be multiplied as follows:

$${}_n \boldsymbol{P_x} = \prod_{i=0}^{n-1} \boldsymbol{P_{x+i}} = \boldsymbol{P_x P_{x+1}} \dots \boldsymbol{P_{x+n-1}}, \tag{7}$$

so that the elements in the ${}_n \boldsymbol{P_x}$ matrix are as follows:

$${}_n \boldsymbol{P_x} = \begin{bmatrix} {}_nP_{11}^x & {}_nP_{12}^x & {}_nP_{13}^x & {}_nP_{14}^x & {}_nP_{15}^x \\ {}_nP_{21}^x & {}_nP_{22}^x & {}_nP_{23}^x & {}_nP_{24}^x & {}_nP_{25}^x \\ {}_nP_{31}^x & {}_nP_{32}^x & {}_nP_{33}^x & {}_nP_{34}^x & {}_nP_{35}^x \\ {}_nP_{41}^x & {}_nP_{42}^x & {}_nP_{43}^x & {}_nP_{44}^x & {}_nP_{45}^x \\ {}_nP_{51}^x & {}_nP_{52}^x & {}_nP_{53}^x & {}_nP_{54}^x & {}_nP_{55}^x \end{bmatrix}. \tag{8}$$

2.1.3. Actuarial present value for pure endowment health insurance. Actuarial present value is one-time payment premium which is paid by the insured on protection in form of benefit upon death or treatment on certain disease being covered in the insurance policy under coverage agreed between both parties. The n-year term protection pure endowment health insurance will pay benefit at the ending year of death or treatment and by the end of protection term. It is formulated as follows [6]:

$$b_{k+1} = \begin{cases} 1 & ; k \geq 0 \\ 0 & ; k \ lainnya \end{cases} \tag{9}$$

$$v_{k+1} = (1+i)^{-(k+1)} = v^{k+1}; k = 0,1,2 \dots, n-1 \tag{10}$$

$$Z = \begin{cases} v^{k+1} & ; k \geq 0 \\ 0 & ; k \ lainnya. \end{cases} \tag{11}$$

Information:

k ≡ age difference when decrement happens and entry

b_{k+1} ≡ benefit upon death, paid at $k+1$ year after insurance policy applies

i ≡ interest rate

v_{k+1} ≡ discount factor upon benefit payment at $k+1$ year after insurance policy is applies

Z ≡ present value of benefit payment at $k+1$ year after insurance policy is applies

Based on equation (9), it can be identified that k which represents time until insured claim for benefit is a random variable. As a result, the probability of claim on every possible point must be calculated. Therefore, actuarial present value concept which is the expected value of Z is formulated as follows:

$$A_{x:n]} = E(Z) = \sum_{k=0}^{n-1} v^{k+1} P(K = k) + v^{n}{}_{n}\, p_x = \sum_{k=0}^{n-1} v^{k+1}{}_{k} p_x q_{x+k} + v^{n}{}_{n}\, p_x \quad (12)$$

Information:

${}_{k}p_x$ ≡ probability of an insured x-years-old stay alive k years further, or reaching $(x+k)$ years old

q_{x+k} ≡ Probability of an insured $(x+k)$-years-old die in the following year

2.1.4. Multiple decrement model. In insurance practice, multiple decrement model differs the benefit paid for every different claim cause. Developing from Equation (12), actuarial present value which involves multiple decrement symbolized by $d \in \{1,2,\dots,D\}$:

$$bA_{x:n]} = E(Z) = \sum_{d=0}^{D} \sum_{k=0}^{n-1} b_d v^{k+1}{}_{k}\, p_x^{(\tau)} q_{x+k}^{(d)} + v^{n}{}_{n}\, p_x^{(\tau)} \quad (13)$$

Information:

${}_{k}p_x^{(\tau)}$ ≡ probability x-years-old insured stay healthy and alive in the next k years

b_d ≡ insurance benefit paid for cause of claim or decrement d

$q_{x+k}^{(d)}$ ≡ probability of $(x+k)$-years-old insured suffer decrement d in the following year

2.1.5. Term-life annuity. Term-life annuity is a series of periodically payment in a constant amount as long as the insured is alive or do not claim under certain period agreed in advance. Mathematically, term-life annuity is modelled as follows [6]:

$$Y = \begin{cases} \ddot{a}_{k+1]} = \dfrac{1-Z}{d} = \dfrac{1-v^{k+1}}{d} & ; 0 \le k < n \\ \ddot{a}_{n]} = \dfrac{1-Z}{d} = \dfrac{1-v^{n}}{d} & ; k \ge n. \end{cases} \quad (14)$$

Since k is a random variable, the expectation form is defined as follows:

$$\ddot{a}_{x:n]} = E(Y) = \sum_{k=0}^{n-1} \ddot{a}_{k+1]}\ {}_{k}\, p_x q_{x+k} + \ddot{a}_{n]}\ {}_{n}\, p_x. \quad (15)$$

Equation (15) will be simplified into this equation:

$$\ddot{a}_{x:n]} = \sum_{k=0}^{n-1} v^{k}\ {}_{k}\, p_x. \quad (16)$$

2.1.6. Interpreting transition probability matrix into actuarial model. As a step to aim the final goal of this paper on the form of net premium calculation, interpretation is needed to connect stochastic model in form of transition probability matrix of Markov chain into terms being used in actuarial model, especially in the premium calculation model. In order to do this, two transition probability matrix must be formed, which already shown on Equation (8) and matrix as follows:

$$\boldsymbol{P_{x+n}} = \begin{bmatrix} P_{11}^{x+n} & P_{12}^{x+n} & P_{13}^{x+n} & P_{14}^{x+n} & P_{15}^{x+n} \\ P_{21}^{x+n} & P_{22}^{x+n} & P_{23}^{x+n} & P_{24}^{x+n} & P_{25}^{x+n} \\ P_{31}^{x+n} & P_{32}^{x+n} & P_{33}^{x+n} & P_{34}^{x+n} & P_{35}^{x+n} \\ P_{41}^{x+n} & P_{42}^{x+n} & P_{43}^{x+n} & P_{44}^{x+n} & P_{45}^{x+n} \\ P_{51}^{x+n} & P_{52}^{x+n} & P_{53}^{x+n} & P_{54}^{x+n} & P_{55}^{x+n} \end{bmatrix}. \quad (17)$$

This matrix shows transition probability matrix when the insured is reaching age $x+n$.

Referring into net premium calculation model for multiple decrement case in Equation (13) and assuming premium is calculated for insured at entry age of x in healthy condition, it can be understood that ${}_n P_{11}^{x}$ from Equation (17) is equal to ${}_n p_x^{(\tau)}$ in the actuarial model. Then, the value of $q_{x+k}^{(d)}$ for any decrement is equal to the following equation:

$$q_{x+n}^{(d)} = P_{1(d+1)}^{x+n}, h = 1, \dots, 4. \quad (18)$$

Information: (1) diabetes mellitus, (2) chronic kidney disease, (3) haemodialysis, (4) death.

Therefore, Equation (13) can be adapted into transition probability matrix notation into following equation:

$$bA_{x:n]}^{|} = E(Z) = \sum_{d=0}^{4} \sum_{k=0}^{n-1} b_d v^{k+1} \; {}_k P_{11}^{x} P_{1(d+1)}^{x+k} + v^n \; {}_n P_{11}^{x}. \quad (19)$$

Then, the same thing applies for life annuity calculation on Equation (16) which will be modified into this equation:

$$\ddot{a}_{x:l]} = \sum_{k=0}^{l-1} v^k \; {}_k P_{11}^{x}. \quad (20)$$

2.1.7. Annual premium. Annual premium is the amount of premium that must be paid as long as insured do not claim during certain payment period. Hence, the formula for annual premium is dividing actuarial present value for multiple decrement case in Equation (13) with life annuity in Equation (16) with n years of protection and l years payment (Note that premium payment and insurance terms might be unequal so that the symbol l is introduced to represents premium payment term):

$$P(A_{x:n]}) = \frac{bA_{x:n]}}{\ddot{a}_{x:l]}} \tag{21}$$

2.1.8. Linear interpolation. Linear interpolation is a way to determine value from unknown point, but in between two known point based on linear equation [7]. In this paper, linear interpolation will be used to estimate probabilities in categorical data shown by data source. Formula being used are as follows with q represents the age that want to be determined the interpolation value and g represents prevalence for disease at age q.

$$\frac{q - q_1}{q_2 - q_1} = \frac{g - g_1}{g_2 - g_1} \tag{22}$$

$$q = \frac{q_2 - q_1}{g_2 - g_1}(g - g_1) + q_1. \tag{23}$$

2.2. DATA SOURCE

Data being used in this paper is secondary data consists of prevalence of diabetes mellitus and chronic kidney disease (from age 0-75) in 2018 provided from Indonesia National Bureau of Statistics. To support our calculation, data regarding death probability on every age is taken from Indonesia Mortality Table IV.

2.3. RESEARCH ALGORITHM

In order to calculate net premium on chronic kidney disease multiple decrement health insurance product, these steps are needed:

1. Doing recap on sample data from Indonesia National Bureau of Statistics on probability of getting diabetes mellitus, chronic kidney disease, and percentage of chronic kidney disease patients taking haemodialysis on every available age.
2. Interpolating the age group data source into discrete age.
3. Assuming the transition probability from healthy to death following Indonesia Mortality Table IV.
4. Doing further research on the increasing probability of getting exposed to chronic kidney disease as a result of diabetes mellitus condition before.
5. Doing further research on the increasing probability of death after having diabetes mellitus, chronic kidney disease, or haemodialysis, compared to probability of death from someone who is healthy.
6. Forming transition probability matrix for every age.
7. Forming n-steps transition probability matrix, with n relies on the insurance coverage

term.

8. Classifying the elements of transition probability matrix into probability that is being used in the actuarial model for premium calculation.
9. Calculating actuarial present value for the study case.
10. Calculating life annuity.
11. Calculating annual net premium that has to be paid by the insured.

3. RESULTS AND DISCUSSION

3.1. Data interpolation for age group type of data. As a beginning step, data from Indonesia National Bureau of Statistics, as shown on Table (1), must be interpolated using Equation (23). The result from data interpolation is shown on Table (2).

Table 1. Disease Prevalence on Age Group in Indonesia

(a) Diabetes mellitus

Age Group	<1	1-4	5-14	15-24	25-34	35-44	45-54	55-64	65-74	75+
Percentage	0.01	0.00	0.00	0.05	0.22	1.08	3.88	6.29	6.03	3.32

(b) Chronic kidney disease

Age Group	<1	1-4	5-14	15-24	25-34	35-44	45-54	55-64	65-74	75+
Percentage	0.00	0.00	0.00	0.13	0.23	0.33	0.56	0.72	0.82	0.75

(c) Haemodialysis

Age Group	<1	1-4	5-14	15-24	25-34	35-44	45-54	55-64	65-74	75+
Percentage	0.00	0.00	0.00	24.06	19.29	14.99	18.85	22.91	20.08	12.68

Table 2. Linear Interpolation Result

(d) Diabetes mellitus

Age Group	1	…	15	16	…	31	32	…	74	75
Percentage	0.01	…	0.05	0.07	…	0.74	0.82	…	3.59	3.32

(e) Chronic kidney disease

Age Group	1	…	15	16	…	31	32	…	74	75
Percentage	0.00	…	0.13	0.14	…	0.29	0.30	…	0.76	0.75

(f) Haemodialysis

Age Group	1	…	15	16	…	31	32	…	74	75
Percentage	0.00	…	24.06	23.58	…	16.71	16.28	…	13.42	12.68

3.2. Formulation of transition probability matrix. By using the interpolated data, elements in the transition probability matrix will be determined. The states involved in this research are: $(\mathbf{1})$ healthy, $(\mathbf{2})$ diabetes mellitus, $(\mathbf{3})$ chronic kidney disease, $(\mathbf{4})$ haemodialysis, and $(\mathbf{5})$ death. All possible one step transition between states is visualized in Figure 1.

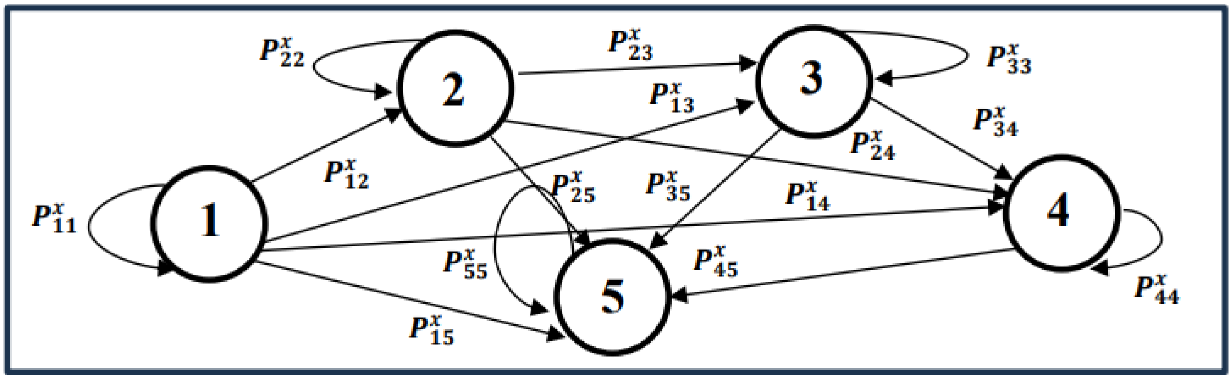

Figure 1: One Step Transition Between States on Insured $\boldsymbol{x}$-years Old

Figure 1 shows the impossibility of transition from higher severity disease into the lower one as assumed in the study case of chronic kidney disease and proven by medical research [8]. Due to data limitation from our sources, some assumptions need to be added in order to complete the transition probability matrix for every state:

1. Transition probability from healthy into death (P_{15}^x) follows Indonesia Mortality Table IV.
2. Transition probability from healthy into chronic kidney disease patient (P_{13}^x) is not directly using data provided from data source, but needs to be deducted by the percentage of patient directly taking haemodialysis. As a result, (P_{13}^x) is formulated as follows:
$$P_{13}^x = (1 - P_{34}^x) \cdot \text{ Prevalence of chronic kidney disease at age } x. \tag{25}$$
3. Transition probability from healthy into haemodialysis patient (P_{14}^x) is the remaining part of the chronic kidney patient categorized as non-haemodialysis patient in Equation (25).
$$P_{14}^x = (P_{34}^x) \cdot \text{ Prevalence of chronic kidney disease at age } x. \tag{26}$$
4. Transition probability from diabetes mellitus patient into chronic kidney patient (P_{23}^x) is 5.1x transition probability from the healthy ones, based on medical research [9].
$$P_{23}^x = 5{,}1(P_{13}^x). \tag{27}$$
5. Transition probability from diabetes mellitus into haemodialysis (P_{24}^x) is using the same assumption with (P_{23}^x).
6. Transition probability from diabetes mellitus patient into death (P_{25}^x) is using assumption that is based on medical research that shows that diabetes mellitus patient is 4x riskier that

the healthy ones [9].

$$P_{25}^{x} = 4(P_{15}^{x}). \tag{28}$$

7. Transition probability from chronic kidney disease patient into death (P_{35}^{x}) is using assumption that is based on medical research that shows that chronic kidney disease patient is 4.5x riskier than the healthy ones [10]:

$$P_{35}^{x} = 4.5(P_{15}^{x}). \tag{29}$$

8. Transition probability from haemodialysis patient into death (P_{45}^{x}) is using the medical research that shows that the probability of death from haemodialysis patient a year after is 0.18 [11].

$$P_{45}^{x} = 0.18. \tag{30}$$

9. Probability of no change in state is following the probability concept property shown in Equation (5).

$$P_{oo}^{x} = 1 - \sum_{j=0}^{5} P_{oj}\,; o \neq j. \tag{31}$$

Using all the assumptions stated, transition probability matrix can be formed. As an example, transition probability matrix in 3 years for insured 56-years-old is shown as follows:

$$\boldsymbol{P_{56}} = \begin{bmatrix} 0.923 & 0.063 & 0.006 & 0.002 & 0.007 \\ 0 & 0.934 & 0.288 & 0.008 & 0.027 \\ 0 & 0 & 0.743 & 0.226 & 0.031 \\ 0 & 0 & 0 & 0.820 & 0.180 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}_{3}\,\boldsymbol{P_{56}} = \prod_{i=0}^{2} \boldsymbol{P_{56+i}} = \boldsymbol{P_{56}}\boldsymbol{P_{57}}\boldsymbol{P_{58}}$$

$${}_{3}\,\boldsymbol{P_{56}} = \begin{bmatrix} 0.786 & 0.161 & 0.169 & 0.009 & 0.027 \\ 0 & 0.812 & 0.062 & 0.0355 & 0.090 \\ 0 & 0 & 0.411 & 0.410 & 0.178 \\ 0 & 0 & 0 & 0.551 & 0.448 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

3.3. Study case. In order to implement the calculation model in which the transition probability matrix as the support has already known for each age, case study will be conducted on the following conditions:

1. Sum assured is IDR1,000,000,000 (one billion Indonesian Rupiah)
2. Premium will be paid annually in the first 5 years
3. Insurance will cover covered losses for 20 years of coverage period

4. Insured is not permitted to do more than one claim
5. Insurance will cover the following disease with benefit for every decrement as follows:

- Category non-critical illness (diabetes mellitus and chronic kidney disease)

  Benefit: $\mathbf{25}\%$ sum assured
- Category critical illness (haemodialysis)

  Benefit: $\mathbf{100}\%$ sum assured + $\mathbf{100}\%$ total premium that has been paid
- Death (for any cause)

  Benefit: $\mathbf{160}\%$ total premium that has been paid
- End of coverage period (endowment)

  Benefit: $\mathbf{160}\%$ total premium that has been paid

Study case will be conducted for insured age $\mathbf{56}$-years-old.

The benefit paid will be varies depends on the amount of premium that has been paid by the insured. Hence, equivalence principle must be applied to Equation $(\mathbf{22})$ and Equation $(\mathbf{20})$ as follows:

$$P(A_{56:20]})\ddot{a}_{56:5]} = A_{56:20]} \tag{32}$$

$$P(A_{56:20]})\ddot{a}_{56:5]} = \sum_{k=0}^{19} b_1 v^{k+1}\ {}_kP_{11}^{56} P_{12}^{56+k} + b_2 v^{k+1}\ {}_kP_{11}^{56} P_{13}^{56+k} + b_3 v^{k+1}\ {}_kP_{11}^{56} P_{14}^{56+k} + b_4 v^{k+1}\ {}_kP_{11}^{56} P_{15}^{56+k} + b_5 v^{20}\ {}_{20}P_{11}^{56} \tag{33}$$

Equation (33) will then be split into partition per decrement so that the actuarial present value can be calculated separately as follows:

$$APV^{(d)} = \sum_{k=0}^{19} b_d v^{k+1}\ {}_kP_{11}^{56} P_{1(d+1)}^{56+k},\ d = 1,2,3,4 \tag{34}$$

$$APV^{(5)} = b_5 v^{20}\ {}_{20}P_{11}^{56} \tag{35}$$

Then, the value of $\boldsymbol{b_d}$ for every decrement will be substituted with the nominal value stated in the study case.

$$APV^{(d)} = 25\% UP \sum_{k=0}^{19} v^{k+1}\ {}_kP_{11}^{56} P_{1(d+1)}^{56+k}, d = 1,2 \tag{36}$$

$$APV^{(3)} = 100\% UP \sum_{k=0}^{19} v^{k+1}\ {}_kP_{11}^{56} P_{14}^{56+k} + 100\% P\left(A_{56:20]}\right) \sum_{k=0}^{19} k v^{k+1}\ {}_kP_{11}^{56} P_{14}^{56+k} \tag{37}$$

$$APV^{(4)} = 160\% P\left(A_{56:20]}\right) \sum_{k=0}^{19} k v^{k+1}\ {}_kP_{11}^{56} P_{15}^{56+k} \tag{38}$$

$$APV^{(5)} = 160\% \cdot 5P(A_{56:20]}) v^{20}\ {}_{20}P_{11}^{56} \tag{39}$$

To simplify the equation, summation components will be symbolized as follows:

$$A^{(d)} = \sum_{k=0}^{19} v^{k+1} \; {}_{k}P_{11}^{56} P_{1(d+1)}^{56+k}, d = 1,2,3 \tag{40}$$

$$B^{(d)} = \sum_{k=0}^{19} k v^{k+1} \; {}_{k}P_{11}^{56} P_{1(d+1)}^{56+k}, d = 3,4 \tag{41}$$

$$C = v^{20} {}_{20} P_{11}^{56} \tag{42}$$

Substituting this symbol to Equation (33) and the partitions will result in the following equation:

$$P\left(A_{56:20]}\right)\ddot{a}_{56:5]} = 25\%SA \cdot A^{(1)} + 25\%SA \cdot A^{(2)} + 100\%SA \cdot A^{(3)} + 100\%P\left(A_{56:20]}\right) \cdot B^{(3)} + 160\%P\left(A_{56:20]}\right) \cdot B^{(4)} + 160\% \cdot 5P\left(A_{56:20]}\right)C. \tag{43}$$

By inputting the amount of sum assured, annual net premium that must be paid by the insured is formulated as follows:

$$P\left(A_{56:20]}\right) = \frac{250M \cdot A^{(1)} + 250M \cdot A^{(1)} + 1.000M \cdot A^{(3)}}{\ddot{a}_{56:5]} - B^{(3)} - 160\%B^{(4)} - 160\% \cdot 5C}. \tag{44}$$

Calculation on study case is done using R Studio will results summarized in Table 3. Interest rate is assumed to be constant 5%.

Table 3: Study Case Calculation Result

Component	$A^{(1)}$	$A^{(2)}$	$A^{(3)}$	$B^{(3)}$	$B^{(4)}$	C	$\ddot{a}_{56:5]}$	$\boldsymbol{P(A_{56:20]})}$
Result	0.428	0.045	0.012	0.054	0.353	0.089	3.928	**IDR40.953M**

The annual net premium for 56-years-old insured is IDR40.953M.

This paper also shows how annual net premium increases along with the entry age (Figure 2) as a result of decreasing probability of staying healthy and increasing probability of getting any disease being covered in the study case policy (Figure 3). Hence, actuarial present value increases while life annuity decreases, resulting in higher net annual premium.

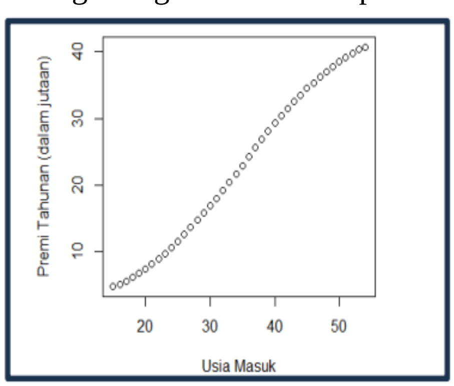

Figure 2: Annual Net Premium for Any Entry Age

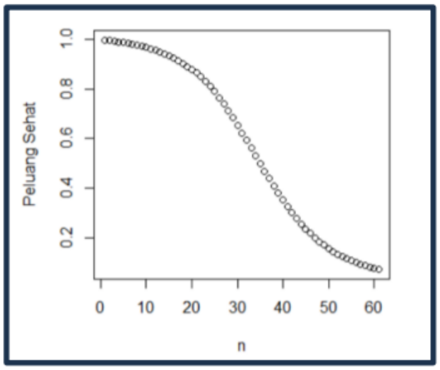

Figure 3: Probability of Staying Healthy

From Figure 4, it can be seen that a 15-years-old diabetes mellitus patient is predicted to have a very low chance to stay alive by the age of 75. Then, a more extreme case is on the chronic kidney disease in which probability of death by the age of 50-55 nearly one, while the most extreme one is on the haemodialysis patient with probability of death age of 40-45 nearly one, meaning almost no chance to live.

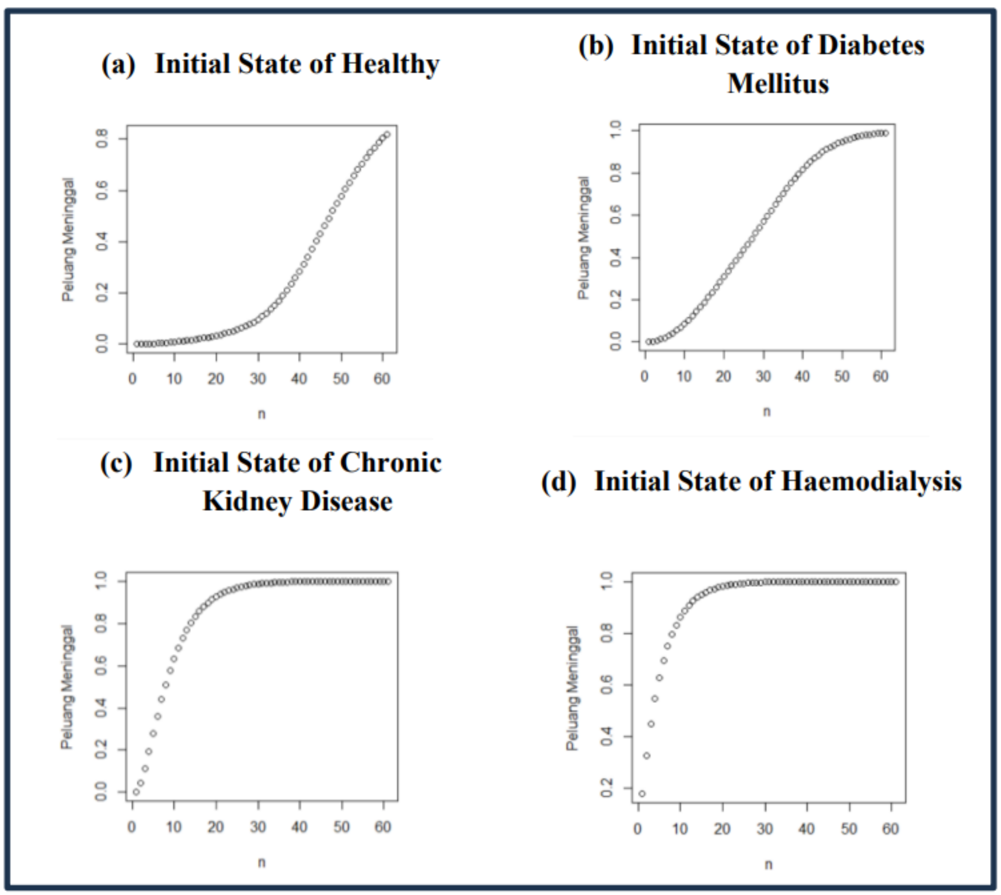

Figure 4: Trend Visualization on Probability of Death ${}_n q_{15}^{(4)}$

4. CONCLUSION

Considering that the decrements being covered in a health insurance product is commonly in a form of state that can move one another in a transition, Markov chain is a model that can represent the reality of this condition into actuarial model of premium calculation. Applying Markov chain in determining insurance premium for health insurance product will help insurance to justify the premium calculation better and also see the probability trend of every disease along the age in order to identify the risks being faced by company during underwriting process. Based on the net annual premium calculation on health insurance endowment product of which the probabilities involved being estimated using Markov chain, it can be summarized that annual net premium will increase along with the entry age of insurance. It can be justified by the increasing probability of getting any disease or death along with the increase of entry age as shown in the visualizations. It is also important to note that background checking for newly insured will be crucial for insurance company as different state of health at the enrolment stage will impact the premium calculation significantly.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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