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LAGRANGE POLYNOMIAL AND CUBIC SPLINE INTERPOLATIONS AS THE ALTERNATIVE PROCEDURES FOR ESTIMATING SMOOTHING PARAMETER IN THE SINGLE EXPONENTIAL SMOOTHING METHOD

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Abstract: In this paper, we propose the alternative methods to estimate the smoothing parameter which is used on the Exponential Smoothing Methods. The present study is focused on estimating one smoothing parameter in the Single Exponential Smoothing (SES) Forecasting Method. This research provides an algorithm to estimate the smoothing parameter utilizing the 3rd order Lagrange polynomial interpolation and cubic spline interpolation. Furthermore, this study is provided by an example of the use of these methods. The results from these proposed alternative methods for estimating the optimal parameter are then compared to the results obtained from the Excel solver. This research shows that the estimation result of the smoothing parameter in SES with cubic spline interpolation is able to produce a better estimation than the 3rd order Lagrange polynomial interpolation and Excel solver.

Keywords: Lagrange interpolating polynomial; cubic spline interpolation; smoothing parameter; exponential smoothing.

2020 AMS Subject Classification: 41A05, 41A15, 41A10.

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1. INTRODUCTION

The Exponential Smoothing Method is widely used in forecasting in various industries [6], including pharmaceuticals ([17], [20]), healthcare [16], gas cylinders [1], and freight forwarding [11]. This method gives more weight to recent observations than the previous ones ([13], [15]). There are various Exponential Smoothing Methods depending on pattern of the data, i.e., trend, seasonal, or mix of trend - seasonal [6]. The methods are Single, Double, and Triple Exponential Smoothing. The challenge when dealing with the Exponential Smoothing Forecasting Method is the estimation of the parameter. This research focuses on the Single Exponential Smoothing (SES) which is a forecasting method used on data that has no trend and seasonality ([2], [7]). In this problem, there is only one smoothing parameter which is denoted by α [9].

Determining the optimal smoothing parameter α in the Single Exponential Smoothing Method can be done in several ways, among them are trial and error ([8], [9], [14]), a mathematical model by Mu'azu [12], and Golden Section Search ([10], [18]). The trial-and-error method uses a variety of different α values, usually choosing $\alpha = 0.1, 0.2, 0.3, \ldots, 0.9$ and uses α which produces the smallest forecasting error [6]. The disadvantage of this method is that there is a possibility of α values used do not cover the optimal values of α.

Mu'azu [12] tried to propose a heuristic method for estimating the α parameter in the Single Exponential Smoothing Method, resulting in the formula $\alpha = \frac{n-1}{3n}$ where n is the number of observations in data. The weakness of this method, as shown by [15], if n approaches the infinity, then $\alpha = \lim_{n\to\infty} \frac{n-1}{3n} = \frac{1}{3}$. Meanwhile, if $n = 2$ which is the smallest possible value for n, then $\alpha = \frac{1}{6}$. The consequence of aforementioned cases of n is that the range of α that can be yielded by Mu'azu's method are within the interval $\left[\frac{1}{6}, \frac{1}{3}\right)$. Meanwhile, the Single Exponential Smoothing Method allows $\alpha \in (0,1)$ so that Mu'azu's method is not able to produce α values within the interval $\left(0, \frac{1}{6}\right)$ and $\left[\frac{1}{3}, 1\right)$. Hence, there is a possibility that the optimal value of smoothing parameter α lies outside of these possible intervals.

Apart from that, the Golden Section Search Method is a method that solves one-variable Non-Linear Programming (NLP) problem in the form of maximization or minimization of $f(x)$ with constraints $a < x < b$ ([4], [10]). This method reduces the boundary of region x iteratively until a maximum or minimum value is obtained or a stopping criterion is reached such as a stop tolerance value [10]. Therefore, iterative approach of this method takes time and less efficient.

The three methods that have been discussed have their own weaknesses. Therefore, this study suggests an alternative solution in determining the optimal α parameter in the Single Exponential Smoothing Method by using Lagrange polynomial interpolation and cubic spline interpolation. In the present study, the Lagrange polynomial interpolation is focused on the 3rd order so that the degree of polynomial resulting from the interpolation is the same as the degree of polynomial from cubic spline interpolation. The same degree of polynomial used in these two proposed methods allows the fair comparison of the results obtained.

In term of the validity, this research uses the Mean Squared Error (MSE) as an indicator of the feasibility of forecasting results obtained. Hyndman et al. [9] mentioned that the use of MSE to optimize the value of the smoothing parameter α in the Single Exponential Smoothing Method is customary because MSE is a smooth function of α. The smooth function nature of MSE motivates the estimation of MSE using interpolation. Furthermore, the interpolated MSE is used to determine the optimal α, i.e., $\alpha \in (0, 1)$ which produces the smallest value in the obtained interpolation function. For comparison, the value of α obtained from the method in this study will be compared with the value of α obtained from solver in the Excel software, which is an add-in program that is a versatile optimization modeling system [5].

The purpose of this study is to explore the estimation of parameter α in the Single Exponential Smoothing Method with 3rd order Lagrange polynomial interpolation and cubic spline interpolation and compare the results with Excel solver by paying attention to the MSE obtained. The method initiated in this study is applied to data on the purchase of Pumpitor capsules at RS Dewi Sri Karawang. Pumpitor capsule contains Omeprazole which is a proton pump inhibitor used to treat diseases associated with gastroesophageal reflux disease (GERD), such as ulcers, gastric acid hypersecretion, and helps heal tissue damage and ulcers caused by stomach acid and *Helicobacter pylori* bacterial infection [19].

This article is presented in the following order. In Section 2, the materials and research method are presented, namely the 3rd order Lagrange polynomial interpolation, cubic spline interpolation, and the steps conducted in this research. Furthermore, the identification of data patterns, the algorithm for estimating the α parameter with the 3rd order Lagrange polynomial interpolation and cubic spline are summarized in Section 3. The discussion is completed with a comparison of the smoothing parameter estimation results from the two methods with the Excel solver results. The article is closed with a brief conclusion and some suggestions for future research.

2. MATERIALS AND METHODS

2.1. n^{th} Order Lagrange Polynomial Interpolation

n^{th} order Lagrange polynomial interpolation is given by the following theorem from [3].

Theorem 2.1.1. If $x_0, x_1, x_2, \dots, x_n$ are $n+1$ distinct numbers and f is a function whose values are given at these numbers, then a unique polynomial $P(x)$ of degree at most n exists with $f(x_k) = P(x_k)$, for each $k = 0, 1, 2, \dots, n$. The polynomial $P(x)$ is given by

$$P(x) = f(x_0)L_{n,0}(x) + \cdots + f(x_n)L_{n,n}(x) = \sum_{k=0}^{n} f(x_k)L_{n,k}(x) \tag{1}$$

where for each $k = 0, 1, 2, \dots, n$,

$$L_{n,k}(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_n)}{(x_k-x_0)(x_k-x_1)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_n)} = \prod_{\substack{i=0\\ i\neq k}}^{n} \frac{(x-x_i)}{(x_k-x_i)}. \tag{2}$$

Based from Theorem 2.1.1, the following corollary is obtained by using $n = 3$. The corollary is also known as 3rd order Lagrange polynomial interpolation.

Corollary 2.1.2. If x_0, x_1, x_2, x_3 are 4 distinct numbers and f is a function whose values are given at these numbers, then a unique polynomial $P(x)$ of degree at most 3 exists with $f(x_k) = P(x_k)$, for each $k = 0, 1, 2, 3$. The polynomial $P(x)$ is given by

$$P(x) = f(x_0)L_{3,0}(x) + \cdots + f(x_3)L_{3,3}(x) = \sum_{k=0}^{3} f(x_k)L_{3,k}(x) \tag{3}$$

where for each $k = 0, 1, 2, 3$,

$$L_{3,k}(x) = \frac{(x-x_0)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_3)}{(x_k-x_0)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_3)} = \prod_{\substack{i=0\\ i\neq k}}^{3} \frac{(x-x_i)}{(x_k-x_i)}. \tag{4}$$

2.2. Cubic Spline Interpolation

Cubic spline interpolation is done by creating a cubic spline interpolant by using the following definition from [3].

Definition 2.2.1. Given a function f defined on $[a,b]$ and a set of nodes $a = x_0 < x_1 < \cdots < x_n = b$, a cubic spline interpolant S for f is a function that satisfies the following conditions:

(a) $S(x)$ is a cubic polynomial, denoted $S_j(x)$, on the subinterval $[x_j, x_{j+1}]$ for each $j = 0,1,\dots,n-1$;

(b) $S_j(x_j) = f(x_j)$ and $S_j(x_{j+1}) = f(x_{j+1})$ for each $j = 0, 1, \dots, n-1$;

(c) $S_{j+1}(x_{j+1}) = S_j(x_{j+1})$ for each $j = 0, 1, 2, \dots, n-2$;

(d) $S'_{j+1}(x_{j+1}) = S'_j(x_{j+1})$ for each $j = 0, 1, 2, \dots, n-2$;

(e) $S''_{j+1}(x_{j+1}) = S''_j(x_{j+1})$ for each $j = 0, 1, 2, \dots, n-2$;

(f) One of the following sets of boundary conditions is satisfied:

(i) $S''(x_0) = S''(x_n) = 0$ (natural (or free) boundary);

(ii) $S'(x_0) = f'(x_0)$ and $S'(x_n) = f'(x_n)$ (clamped boundary).

2.3 Research Method

This study is conducted by comparing the performance of estimating the α parameter in the Single Exponential Smoothing Method from the 3rd order Lagrange polynomial interpolation and cubic spline interpolation. The following are the steps taken in this research:

Step 1: Inputting time series data that will be used in this research.

Step 2: Performing stationarity test with Augmented Dickey-Fuller test on the data. If the data is stationary, go to step 4. If the data is not stationary, go to step 3.

Step 3: Performing the difference on the data and return to step 2.

Step 4: Estimating the parameter α in the Single Exponential Smoothing Method with the 3rd order Lagrange polynomial and cubic spline interpolation. From this step, the optimal α is obtained from each estimation.

Step 5: Comparing the MSE generated by using α from step 4 with the MSE generated by the optimal α from the Excel solver.

Step 6: Interpretating the results from the comparison done in step 5.

3. RESULTS AND DISCUSSION

3.1. Data Pattern Identification

The parameter α will be determined by utilizing the MSE value obtained from each different and equidistant use of α so as to obtain a set of points (α, MSE) where these points will be interpolated by the 3rd order Lagrange polynomial interpolation and cubic spline. Next, the critical point that produces the smallest value in the function obtained from each interpolation is determined. This critical point is the optimal α parameter. Forecasting will also be carried out using an Excel solver whose results are compared with the results of the 3rd order Lagrange polynomial interpolation and cubic spline.

The data used in this study are secondary data on monthly purchases of Pumpitor capsules at Dewi Sri Karawang Hospital, Indonesia, from 2019 to 2021. The data used in the Single Exponential Smoothing Method must be stationary. Thus, it is necessary to check the stationarity of the data to be forecast. The p-value result obtained from the augmented Dickey-Fuller test on the data is $0.3703 \geq 0.05$ so it is concluded that this data is not stationary. For this reason, the data needs to be differenced so that it can be applied to the Single Exponential Smoothing Forecasting Method.

After differencing process, the p-value obtained from the augmented Dickey-Fuller test is $0.01102 < 0.05$. It means the differenced data is stationary and can be used in the Single Exponential Smoothing Forecasting Method. The monthly purchase data and differencing results are presented in Table 1.

Table 1. Monthly Purchase Data of Pumpitor Capsule Medicine at Dewi Sri Karawang Hospital, Indonesia, and Differencing Results

t^{th} Time	Purchase (Capsule)	Difference	t^{th} Time	Purchase (Capsule)	Difference
0	4400		18	4600	1260
1	5880	1480	19	2860	-1740
2	4700	-1180	20	2560	-300
3	1640	-3060	21	5400	2840
4	5160	3520	22	3800	-1600
5	4400	-760	23	3220	-580
6	5100	700	24	5380	2160
7	5980	880	25	4260	-1120
8	4280	-1700	26	2780	-1480
9	6800	2520	27	7120	4340
10	4420	-2380	28	5320	-1800
11	6100	1680	29	2380	-2940
12	4440	-1660	30	3800	1420
13	2820	-1620	31	3780	-20
14	3360	540	32	4460	680
15	1500	-1860	33	4800	340
16	3620	2120	34	1800	-3000
17	3340	-280	35	5540	3740

To illustrate the differences, the data and differencing results are presented in graphical form in Figure 1.

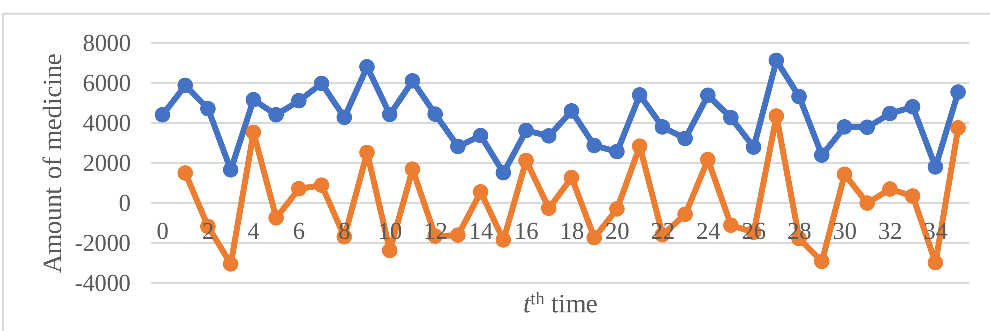


Figure 1. The Purchase Data with the Differencing Results

Next, a set of points (α, MSE) is required to be interpolated. Therefore, forecasting is carried out with the Single Exponential Smoothing Method to obtain MSE with the following steps:

1. Suppose α is the smoothing parameter to be used in the Single Exponential Smoothing Method, X is the secondary data of monthly purchases of Pumpitor capsules at Dewi Sri Karawang Hospital from 2019 to 2021 with X_t is the data from X at time t, n is the amount of data in X, and F_t is the forecasting value at time t.
2. Set $F_1 = X_1$.
3. Obtain F_t with the formula
$$F_{t+1} = F_t + \alpha(X_t - F_t), \forall t = 1, 2, 3, \dots, n-1 \tag{5}$$
4. Calculate the MSE of the forecast with the formula
$$MSE = \frac{1}{n}\sum_{t=1}^{n}(X_t - F_t)^2 \tag{6}$$

In order to see the sensitivity of the estimation to the number of points used in the 3rd order Lagrange polynomial interpolation and cubic spline, several interpolations were carried out by selecting many points as many as $3z + 1, z \in \mathbb{Z}^+$ because the interpolation involved was 3rd order Lagrange polynomial interpolation. In this study, 13, 22, and 103 sets of points were explored. The value of the α parameter used during interpolation includes 0 and 1 despite the Single Exponential Smoothing Method only allows α values in (0,1) such that the interpolation result can represent the overall MSE in the interval (0,1).

3.2. Estimation of α Parameter Using 3rd Order Lagrange Polynomial Interpolation

The mathematical algorithm of α parameter estimation using 3rd order Lagrange polynomial interpolation is as follows:

1. Suppose n is many different values of α to be used for estimation with $n = 3z + 1, z \in \mathbb{Z}^+$. Perform estimation with $n = 13$, $n = 22$, dan $n = 103$.
2. Obtain α_i with the formula
$$\alpha_i = \frac{i-1}{n-1}, \forall i = 1, 2, 3, \dots, n \tag{7}$$
3. Define $f(x)$ as the MSE obtained from forecasting using the Single Exponential Smoothing Method with α equal to x.
4. Obtain $f(\alpha_i)$ for all $i = 1, 2, 3, \dots, n$.

5. Construct the function $p_j(x)$ which is a 3rd order Lagrange polynomial interpolant of the form

$$p_j(x) = \sum_{k=3(j-1)+1}^{3(j-1)+4} f(\alpha_k) L_{3(j-1)+4,k}(x) \tag{8}$$

where

$$L_{3(j-1)+4,k}(x) = \prod_{\substack{i=3(j-1)+1 \\ i\neq k}}^{3(j-1)+4} \frac{(x-\alpha_i)}{(\alpha_k-\alpha_i)} \tag{9}$$

on interval $[\alpha_{3(j-1)+1}, \alpha_{3(j-1)+4}]$ for all $j = 1, 2, 3, \dots, \frac{n-1}{3}$. Lagrange polynomial interpolation of order 3 will produce a polynomial function of degree at most 3, so $p_j(x)$ can be written in the form of

$$p_j(x) = a_j + b_j x + c_j x^2 + d_j x^3 \tag{10}$$

6. Derive $p_j(x)$ from Equation 10, we get

$$p_j'(x) = b_j + 2c_j x + 3d_j x^2 \tag{11}$$

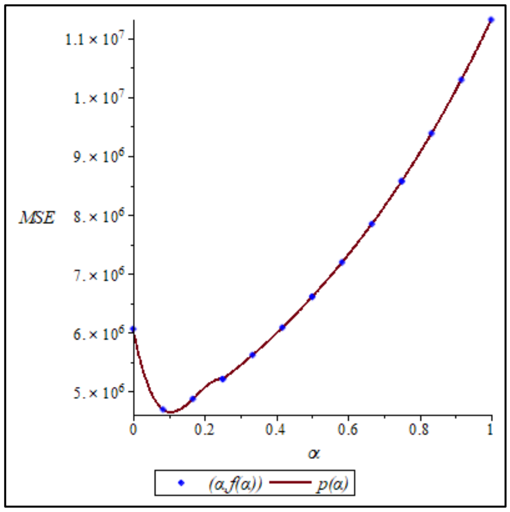
$p_j'(x)$ is a polynomial function with degree up to 2, therefore this function has at most 2 real roots. Since there are $\frac{n-1}{3}$ functions obtained from the fifth step, we will obtain at most $\frac{2(n-1)}{3}$ real roots.

7. Obtain real roots x_r from $p_j'(x)$ for all $j = 1, 2, 3, \dots, \frac{n-1}{3}$ with $r = 1, 2, 3, \dots, s$ where $s \leq \frac{2(n-1)}{3}$. x_r alongside α_a for all $a = 1, 4, 7, 10, \dots, n$ (α_a are the endpoints for each interpolant $p_j(x)$) are critical points from $p_j(x)$ for all $j = 1, 2, 3, \dots, \frac{n-1}{3}$.

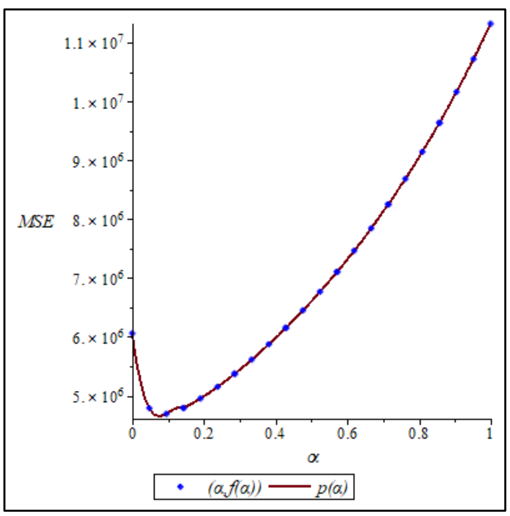
8. Pick x_{optimal} that conforms

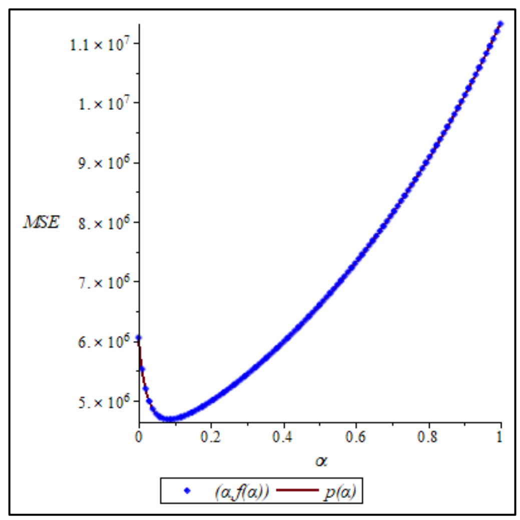
$$f(x_{\text{optimal}}) = \min\{f(\alpha_a) | a = 1, 4, 7, \dots, n\} \cup \{f(x_r)\}_{r=1}^{s} \tag{12}$$

If $x_{\text{optimal}} = 0$, then the optimal α is $\lim_{\alpha \to 0^+} \alpha$. If $x_{\text{optimal}} = 1$, then the optimal α is $\lim_{\alpha \to 1^-} \alpha$. If $x_{\text{optimal}} \neq 0$ and $x_{\text{optimal}} \neq 1$, then x_{optimal} is the optimal α. Stop algorithm.



(a) $n = 13$





(b) $n = 22$ (c) $n = 103$

Figure 2. Plot of the 3rd Order Lagrange Polynomials with n points

The interpolation results of the three sets of point (α, MSE) by applying the aforementioned algorithm are shown in Figure 2. From those figures, it can be observed that if the more points are used, the closer the interpolant is to the overall MSE value. The optimal parameter estimation results by using the 3rd order Lagrange polynomial interpolation can be seen in Table 2.

Table 2. Parameter Estimation Results by Using the 3rd Order Lagrange Polynomial Interpolation and Its MSE

n	α	**MSE**
13	0.106	4703376.466
22	0.075	4689771.731
103	0.084	4685699.423

The algorithm for α parameter estimation using 3rd order Lagrange polynomial interpolation can be represented as a flowchart shown in Figure 3.

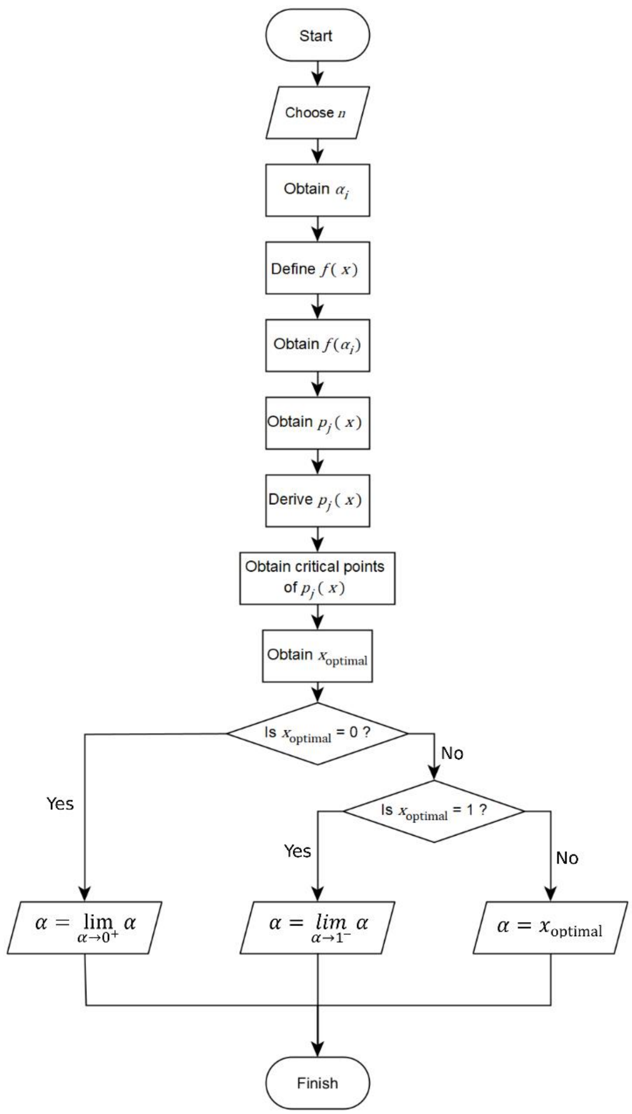


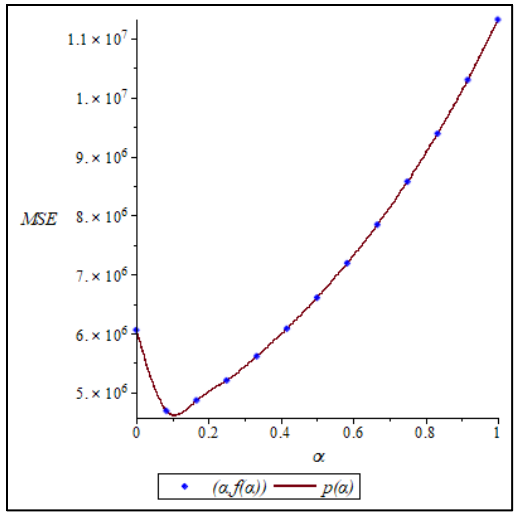
Figure 3. Flowchart of Estimation Using 3rd Order Lagrange Polynomial Interpolation

3.3. Estimation of α Parameter using Cubic Spline Interpolation

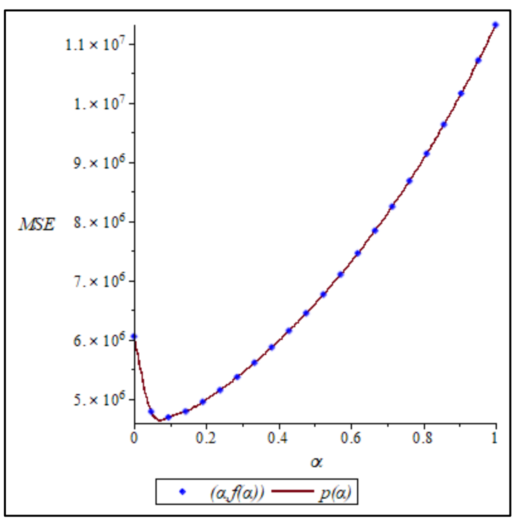
The mathematical algorithm of α parameter estimation using cubic spline interpolation is as follows:

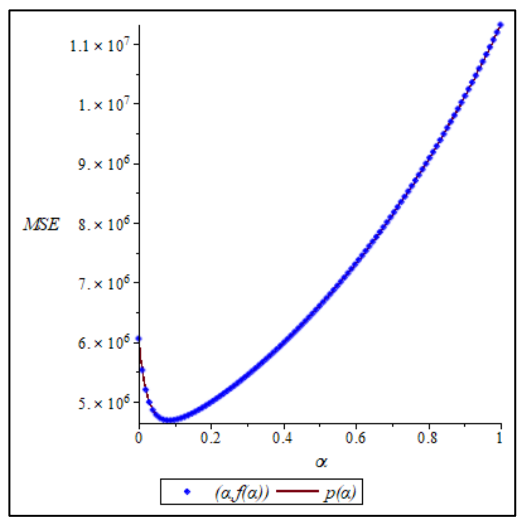
1. Suppose n is many different values of α to be used for estimation with $n = 3z + 1, z \in \mathbb{Z}^+$. Perform estimation with $n = 13$, $n = 22$, dan $n = 103$.
2. Obtain α_i with the formula
$$\alpha_i = \frac{i-1}{n-1}, \forall i = 1, 2, 3, \dots, n \tag{13}$$
3. Define $f(x)$ as the MSE obtained from forecasting using the Single Exponential Smoothing Method with α equal to x.
4. Obtain $f(\alpha_i)$ for all $i = 1, 2, 3, \dots, n$.
5. Construct a cubic spline interpolant S for f by determining the cubic polynomial
$$S_j(x) = a_j + b_j(x - \alpha_j) + c_j(x - \alpha_j)^2 + d_j(x - \alpha_j)^3 \tag{14}$$
on interval $[\alpha_j, \alpha_{j+1}]$ for every $j = 1, 2, 3, \dots, n-1$ that satisfies the conditions mentioned in Definition 2.2.1. Since $f'(x_0)$ and $f'(x_n)$ is unknown, we use the natural boundary variant of cubic spline interpolation. S is a piecewise function that consists of $S_j(x)$ with $j = 1, 2, 3, \dots, n-1$.
6. Derive $S_j(x)$ from Equation 14 for all $j = 1, 2, 3, \dots, n-1$, we get
$$S_j'(x) = b_j + 2c_j(x - \alpha_j) + 3d_j(x - \alpha_j)^2 \tag{15}$$
on interval $[\alpha_j, \alpha_{j+1}]$ for all $j = 1, 2, 3, \dots, n-1$. $S'(x)$ is a piecewise function that consists of $S_j'(x)$ with $j = 1, 2, 3, \dots, n-1$. Since $S'(x)$ contains at most $n-1$ quadratic polynomials, $S'(x)$ has $2(n-1)$ real roots.
7. Obtain real roots x_k from $S'(x)$ with $k = 1, 2, 3, \dots, s$ where $s \leq 2(n-1)$. x_k alongside 0 and 1 are the critical points of $S(x)$.
8. Pick x_{optimal} that conforms
$$f(x_{\text{optimal}}) = \min\{f(0), f(1)\} \cup \{f(x_r)\}_{r=1}^{s} \tag{16}$$

If $x_{\text{optimal}} = 0$, then the optimal α is $\lim_{\alpha \to 0^+} \alpha$. If $x_{\text{optimal}} = 1$, then the optimal α is $\lim_{\alpha \to 1^-} \alpha$. If $x_{\text{optimal}} \neq 0$ and $x_{\text{optimal}} \neq 1$, then x_{optimal} is the optimal α. Stop algorithm.



$(a)\ n = 13$





(b) $n = 22$ (c) $n = 103$

Figure 4. Plot of Cubic Spline Interpolant With n points

As with the 3rd order Lagrange polynomial interpolation, Figure 4 shows that the more points used, the closer the interpolant is to the overall MSE value. The optimal parameter estimation results of cubic spline interpolation can be seen in Table 3.

Table 3. Parameter Estimation Results by Using the Cubic Spline Interpolation and Its MSE

n	α	MSE
13	0.106	4704468.611
22	0.072	4693735.900
103	0.084	4685699.417

The algorithm for α parameter estimation using cubic spline interpolation can be represented as a flowchart shown in Figure 5.

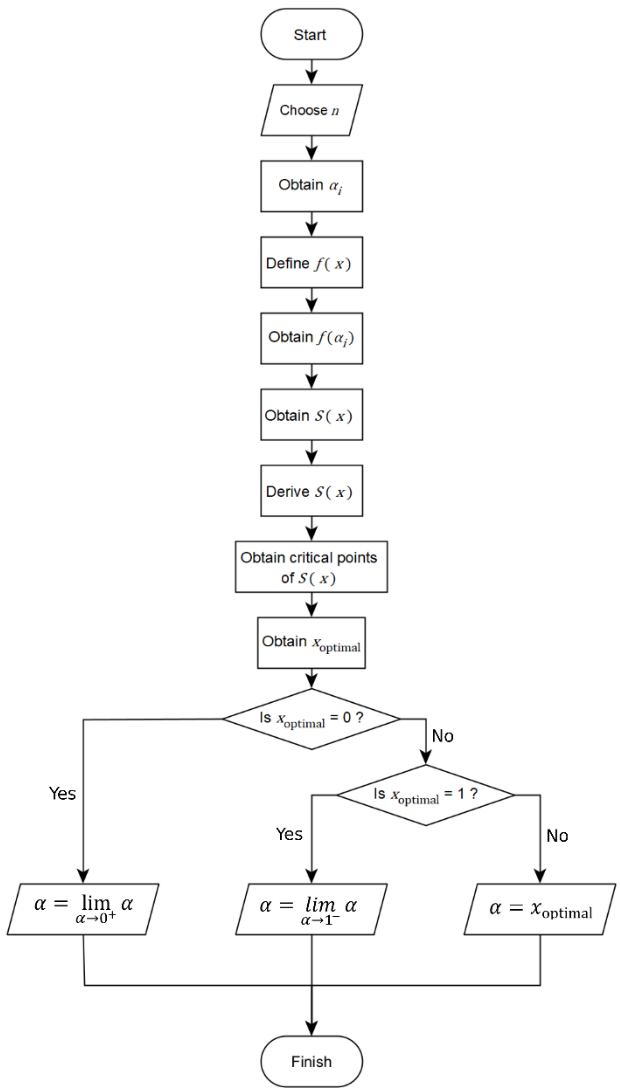


Figure 5. Flowchart of Estimation Using Cubic Spline Interpolation

3.4. Comparison of α Values Obtained

The MSE obtained from each method is used as a benchmark for measuring the effectiveness of each method in estimating α. The following is the value of α along with the MSE of each method that has been done which is summarized in Table 4.

Table 4. Estimation Results of α and MSE from Each Method Along with Excel Solver

n	Method	α	MSE
13	3rd Order Lagrange Polynomial Interpolation	0.106	4703376.446
	Cubic Spline Interpolation	0.106	4704468.611
22	3rd Order Lagrange Polynomial Interpolation	0.075	4689771.731
	Cubic Spline Interpolation	0.072	4693735.900
103	3rd Order Lagrange Polynomial Interpolation	0.084	4685699.423
	Cubic Spline Interpolation	0.084	4685699.417
-	Excel Solver	0.084	4685699.423

Table 4 shows that the more points used, both in the 3rd order Lagrange polynomial interpolation and cubic spline interpolation, the smaller the MSE value. Interestingly, the α parameter estimation results from cubic spline interpolation using 103 points are better than the α parameter estimation results from both Excel Solver and 3rd Order Lagrange Polynomial Interpolation with an MSE difference of 0.006. The α values presented in Table 4 are rounded to 4 digits hence the α values from cubic spline interpolation using 103 points and the Excel solver look the same. In detail, the α values obtained from cubic spline interpolation using 103 points and from the Excel solver are 0.0843224147 and 0.084317840359577, respectively.

4. Conclusions and Future Research

The 3rd order Lagrange polynomial interpolation and cubic spline can be used to estimate the smoothing parameter α in the Single Exponential Smoothing Method. The α parameter estimation results from the 3rd order Lagrange polynomial interpolation and cubic spline are sensitive to the number of points used. The 3rd order Lagrange polynomial interpolation produces an optimal α estimate similar to the Excel solver when a sufficient number of points are used. However, with the same number of points, cubic spline interpolation is able to produce better optimal α estimates than the Excel solver.

To explore the findings of this research, it is necessary to study the minimum number of points to estimate α with cubic spline interpolation so that it can be generally applicable to the case of forecasting with the Single Exponential Smoothing Method and the use of multivariate interpolation to estimate the smoothing parameters in Double Exponential Smoothing and Triple Exponential Smoothing.

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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