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MODIFIED LEE-CARTER MODEL USING EXTREME VALUE THEORY FOR FORECASTING MORTALITY RATES AMIDST EXTREME EVENTS DURING COVID-19 ERA IN INDONESIA

FEVI NOVKANIZA*, M. SANDY ATHALLA SYACH, MILA NOVITA

Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Indonesia, Depok, 16424, Indonesia

Abstract. In recent years, the world has been grappling with the threat of pandemics, as well as conflicts and wars between countries. Events such as pandemics and conflicts are extreme conditions that can arise unexpectedly, causing significant casualties. Consequently, there is a need for modeling that can account for mortality resulting from these extreme events. The Lee-Carter model utilizes mortality rate data from observed age groups over time. To address extreme mortality rates, the Lee-Carter model has been adapted using Extreme Value Theory (EVT), resulting in the modified Lee-Carter EVT Model. The EVT approach employed is the Peak Over Threshold (POT) approach with Generalized Pareto Distribution (GPD). This model was applied to Indonesian mortality rate data from 1998 to 2020 to forecast mortality rates for the Covid-19 pandemic period in 2021 and 2022. In GPD modeling, a threshold value of 0.02 is determined. Values exceeding the threshold are modeled with GPD, while values below the threshold are modeled with normal and empirical distribution. The results, assessed using Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE), indicate that the Extreme Value Theory Modified Lee-Carter empirical distribution model yields the smallest MAPE value at 12.156%. In comparison, the Extreme Value Theory Modified Lee-Carter normal distribution model has a MAPE value of 13.175%, and the regular Lee-Carter model is at 13.343%. These findings pertain to predicting Indonesia's mortality rate in age groups experiencing extreme events.

*Corresponding author
E-mail address: fevi.novkaniza@sci.ui.ac.id
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1. Introduction

The mortality rate over a period of time is an important indicator for both the government and the private sector in making decisions and planning future policies. The death rate, or mortality, is defined as a measure of the number of deaths in a certain population scaled by the size of that population per unit time [1]. For the government, the mortality rate and life expectancy of the population can serve as references and indicators for determining policies. Decisions in the health sector and the provision of facilities and workforce therein are also influenced by the mortality rate of a region. In the private sector, the mortality rate is an essential indicator of business continuity, especially for insurance companies. Insurance companies require information about population mortality rates to carry out their operations. The population mortality rate is an indicator in calculations to determine premiums and profits obtained by insurance companies. By knowing the mortality rate, insurance companies can plan policies related to the premiums set by the company and achieve optimal company profits. Research related to modeling population mortality rates has been carried out using different approaches. Starting from DeMoivre (1725), Gompertz (1825), and Weibull (1939), who modeled the survival function of a population, stochastic models were then developed related to modeling mortality rates. Stochastic modeling related to mortality rates has been developed, among others, by McNown and Rogers (1989, 1992), Wilmoth (1990), Bell and Monsell (1991), and Lee and Carter (1992).
Lee and Carter proposed a model known as the Lee-Carter model, which was used to model the mortality rate of the United States population in 1992 [2]. They modeled the mortality rate using mortality data from age groups observed over time. This makes the Lee-Carter model not only use age group elements but also consider change factors over time. Age group elements and their changes over time are represented in the three parameters of the Lee-Carter model. The Lee-Carter model has two limitations or constraints to be able to estimate these parameters. The limit is in the form of the sum of the parameters to the index, which is zero or one. The first parameter has an index for age groups, stating the pattern of changes in mortality rates for each age group. This parameter is estimated using the least squares method with Lee-Carter

constraints. The second parameter has an index to the age group, which has a limit of one, and the third parameter has an index to time, which has a limit of zero multiplied by each other. These two parameters are used to see how age group mortality patterns change over time. They are estimated using the Singular Value Decomposition (SVD) method with Lee-Carter constraints. Next, the mortality rate of an age group at a certain point in time is determined as the exponential of the sum of the first parameter multiplied by the product of the second and third parameters. The third parameter in the Lee-Carter model is known as the Lee-Carter mortality index. This parameter receives special attention because it is the only parameter that has an index for time. This parameter shows the change in the mortality rate at that point in time, regardless of which age group the mortality rate is in. The Lee-Carter mortality index differentiates this model from other mortality models. Therefore, the Lee-Carter model has become one of the stochastic models that is popularly used and developed by many researchers worldwide. The Lee-Carter model has been applied to modeling mortality rates in various countries such as the United States [2], Sweden [3], Hungary [4], Argentina [5], Malaysia [6], India [7], Italy [8], French [9], and Spanish [9].

Mortality rates tend to be stable and decrease over time. However, if certain events or conditions occur in one year, the mortality rate can increase or decrease drastically in that year. In the last few years, the world has been facing the dangers of pandemics as well as wars and conflicts between countries. The COVID-19 pandemic is a pandemic that occurred due to the coronavirus, lasting from 2020 to 2022 (kemkes.go.id). The pandemic caused a spike in the death rate in the world's human population, including the Indonesian population. According to data from Indonesia's official Covid-19 handling website, the total number of deaths due to the COVID-19 virus in Indonesia during the period 2020 to 2022 was 160,788 people (covid19.go.id). Apart from that, conflict between countries such as Russia and Ukraine could trigger a war which could cause many casualties and cause the death rate to explode. Cases such as pandemics and conflicts or wars between countries are extreme events or conditions that can occur at any time, so mortality rate modeling is needed that can accommodate these extreme events. Extreme Value Theory (EVT) is an approach often used to model extreme values. EVT was initially introduced to deal with data found at the tail of a distribution [10]. Furthermore, EVT is applied to quantify the risk of an extreme event [11]. In modeling and forecasting mortality, EVT is combined with the Gompertz law of mortality to predict mortality levels at certain thresholds [12]. With the popularity of EVT for modeling extreme events, Gungah and Narsoo [9] proposed a modified Lee-Carter model with EVT to accommodate extreme mortality levels called the modified Lee-Carter EVT model. The EVT modified Lee-Carter model uses the Peak Over Threshold (POT) approach with Generalized Pareto Distribution (GPD) on changes in the Lee-Carter model mortality index. POT and GPD are used to overcome spikes in the change value of the Lee-Carter model mortality index, which causes the value to no longer be normally distributed [9]. With POT and GPD, the cumulative distribution function of changes in the mortality index is determined. Parameters in the GPD are estimated using the maximum likelihood method. After that, the value of changes in the mortality index for the next period is generated by simulation using the cumulative distribution function. The change value of the mortality index that has been generated is used to forecast the mortality level for the next period. There are reasons for the use of POT in the modified Lee-Carter EVT Model.

The POT approach is one of the approaches in EVT. In the POT approach, a threshold value is determined from the entire data first; then values that are above the threshold value are considered extreme values. An approach other than POT in EVT is the Block Maxima (BM) approach. The difference between POT and BM lies in determining extreme values. BM does not use a threshold value but uses blocks to find the local maximum value for each block formed. The local maximum value obtained from each block is considered the extreme value. This causes the extreme values obtained through BM not to represent the extreme values of the entire existing data. The POT approach, which uses a threshold, can provide a more general picture of extreme values compared to BM. Based on this, in the modified Lee-Carter EVT model, the POT approach with GPD was chosen [9]. The choice to use GPD is also based on several reasons. The Generalized Pareto Distribution (GPD) is a family of continuous probability distributions often used to model the tails of a distribution. This distribution is characterized by a thick-tailed distribution. The thick-tailed distribution has large observational values with high frequencies, far from the average value, corresponding to extreme values. Therefore, GPD is used for modeling extreme values. In addition, based on the Pickands-Balkema-de Haan (1974) theorem, the distribution of values above a threshold value is assumed to converge towards the Generalized Pareto Distribution (GPD). Based on these principles, GPD is used in the modified Lee-Carter EVT model [9]. The modified Lee-Carter EVT model and its implementation on Indonesian mortality rate data during the Covid-19 pandemic period from 2019 to 2022. By using the modified Lee-Carter EVT model, Indonesia's mortality rate can be predicted for the following year. Model accuracy is assessed based on the mean absolute error (MAE) and mean absolute percentage error (MAPE).

2. METHODOLOGY

Mortality Rate

The mortality rate is defined as a measure of the number of deaths in a particular population scaled by the size of that population per unit time [1]. The mortality rate of a population can be calculated using the following equation:

$$m_{x,t} = \frac{D_{x,t}}{N_{x,t}}$$

where

$m_{x,t}$ = Age group mortality rate x in the 3rd t-period

$D_{x,t}$ = Number of deaths of people by age group x in the 3rd t-period

$N_{x,t}$ = The total population of the age group x in the 3rd t-period

Lee-Carter Model

Ronald D. Lee and Lawrence R. Carter [2] proposed a model that was used to accommodate and describe the phenomenon regarding the mortality rate of the United States population in 1992. The Lee-Carter model is stated as:

$$\ln(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t} \ , \ x = 1,2,\ldots,p \ ; \ t = 1,2,\ldots,q$$

$$\text{with constraints } \sum_{t=1}^{q} k_t = 0 \text{ and } \sum_{x=1}^{p} b_x = 1$$

where

$m_{x,t}$ = mortality rate in age group x and period t

a_x = parameter of the average mortality rate in the age group x

b_x = parameters of changes in mortality rates in age group x

k_t = Lee-Carter mortality index or trend of mortality rates by year t

$\varepsilon_{x,t}$ = error model

The two constraints above are used in the process of estimating model parameters. This constraint appears to limit the value of the parameter so that the parameter can be estimated.

Extreme Value Theory

EVT is a theory that accommodates extreme values located at the tail of the distribution. EVT is usually used to predict extreme values by looking for a distribution that fits these extreme values. The POT approach uses a threshold. Values of data that are above a predetermined threshold are categorized as extreme values. These extreme values are modeled by a distribution. According to the Pickands-Balkema-de Haan theorem (1974), the distribution of values above the threshold is assumed to converge towards the Generalized Pareto Distribution (GPD). The POT approach is widely used, especially in financial risk modeling. In this thesis, the approach used for EVT is the POT approach using GPD.

Peak Over Threshold (POT)

Approach categorizes a data point into an extreme value based on a predetermined threshold value. If the data point is more than or above the threshold value, then the data point is categorized as an extreme value, and vice versa. Suppose $X_1, X_2, \ldots, X_n$ is a collection of observations of data points that follow a distribution called F. A value is determined u which represents the threshold value of a set of data points. The value of the $X - u$ conditional probability $X > u$ can be defined as follows $F(x) = P(X - u \leq x | X > u)$ where $x \geq 0$. Absolute values that X are above or greater than u are said to follow the Generalized Pareto Distribution (GPD) with two parameters.

Generalized Pareto Distribution (GPD)

Generalized Pareto Distribution (GPD) is a family of continuous probability distributions. This distribution is often used to model the tails of a distribution. In general, GPD has three parameters, namely location parameters μ, shape parameters γ, and scale parameters σ. GPD can also be determined by just two parameters, namely scale and shape parameters and can be determined by just one parameter, namely shape parameters. In the POT approach, the type of GPD used is GPD with two parameters, namely shape and scale parameters. The cumulative distribution function of GPD distribution is:

$$F(x) = \begin{cases} 1-\left(1+\frac{\gamma x}{\sigma}\right) & ,x \in (0,\infty), \gamma > 0 \\ 1-e^{-\frac{x}{\sigma}} & ,x \in (0,\infty), \gamma = 0 \\ 1-\left(1+\frac{\gamma x}{\sigma}\right)^{-\frac{1}{\gamma}} & ,x \in \left(0,-\frac{\sigma}{\gamma}\right), \gamma < 0 \end{cases}$$

and the probability density function of GPD is:

$$f(x) = \begin{cases} \frac{1}{\sigma}\left(1+\frac{\gamma x}{\sigma}\right)^{-\frac{1}{\sigma}-1} & ,\gamma \neq 0 \\ \frac{1}{\sigma}\exp\left(-\frac{x}{\sigma}\right) & ,\gamma = 0 \end{cases}$$

Extreme Value Theory Modified Lee-Carter Model

The modified Lee-Carter model is a model for forecasting extreme event mortality rates which refers to the journal Gungah and Narsoo in 2022 [9]. The model parameters are estimated using the least squares and Singular Value Decomposition (SVD) methods and evaluated using the Mean Absolute Error (MAE) assessment metric. and Mean Absolute Percentage Error (MAPE). After obtaining the estimated parameter values for the Lee-Carter model, further modeling was carried out to estimate the parameters k_t. This is because in forecasting the next period a value is needed k_{t+1} first. In the modified Lee-Carter EVT model, the value is obtained by modeling the cumulative distribution function for Δk_t the EVT approach using GPD and generating values Δk_{t+1} from the cumulative distribution function with the help of random numbers from the uniform distribution $(0,1)$. Forecasting the mortality rate for the next period is carried out using the Lee-Carter model equation. The values $\widehat{\Delta k_{q+1}}$ is used in the Lee-Carter model equation are values derived from $\widehat{k_q} + \Delta k_q$.

3. Result and Discussion

The data used is Indonesian mortality rate data (https://esa.un.org/undp/wpp) which consists of 21 age groups starting from the age group zero years, $1-4$ years, $5-10$ years to $96-100$ years. For each age group, the mortality rate values were observed for 25 years, starting from 1998 to 2022. The following is table 1 which displays a snapshot of Indonesia's mortality rate data for the last 5 years, starting from 2018 to 2022 in the zero to 2022 age group $36-40$.

TABLE 1. Indonesian Mortality Rate Data

Age Group	2018	2019	2020	2021	2022
0	0.02091	0.02017	0.01950	0.01887	0.01801
1 – 4	0.00111	0.00104	0.00100	0.00096	0.00093
5 – 10	0.00055	0.00052	0.00051	0.00050	0.00048
11 – 15	0.00050	0.00048	0.00047	0.00047	0.00046
16 – 20	0.00107	0.00105	0.00104	0.00116	0.00109
21 – 25	0.00141	0.00139	0.00138	0.00153	0.00145
26 – 30	0.00148	0.00147	0.00146	0.00164	0.00155
31 – 35	0.00176	0.00174	0.00179	0.00201	0.00190
36 – 40	0.00236	0.00234	0.00252	0.00283	0.00267

From statistics descriptive analysis based on Indonesian mortality rate data, the average value of the mortality rate decreased in 2019, again increased in 2020 and 2021 and decreased again in 2022. This indicates that there has been fluctuation in the value of the mortality rate over the last 5 years. Therefore, it may be necessary to model mortality rates on this data. The Lee-Carter model parameters denoted by a_x, b_x, and k_t were estimated using this data from $1998-2022$, yielding the following results:

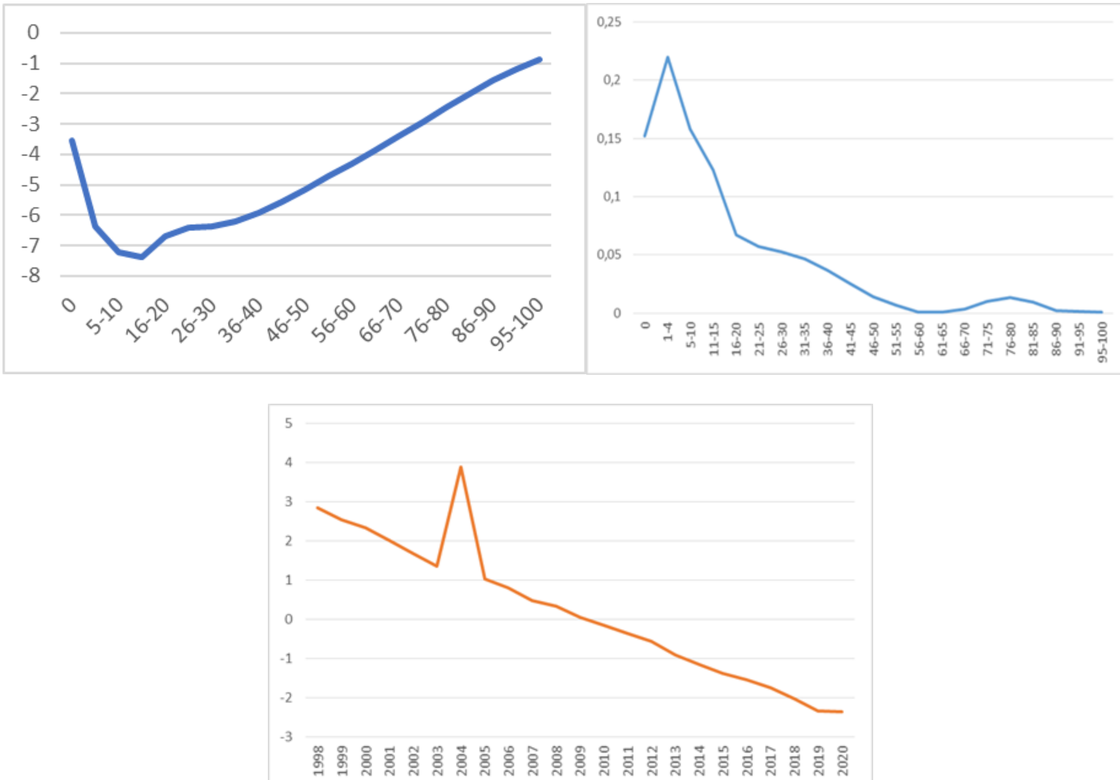

FIGURE 1. Lee-Carter Model Parameter Estimation Results

After obtaining the parameter estimation results for the Lee-Carter model, an evaluation of the Lee-Carter model was carried out by comparing the forecasting results from 1998 – 2022 with the actual values for all age groups. For example, the following is a graphical comparison of forecasting results for the 5 – 10 year age group:

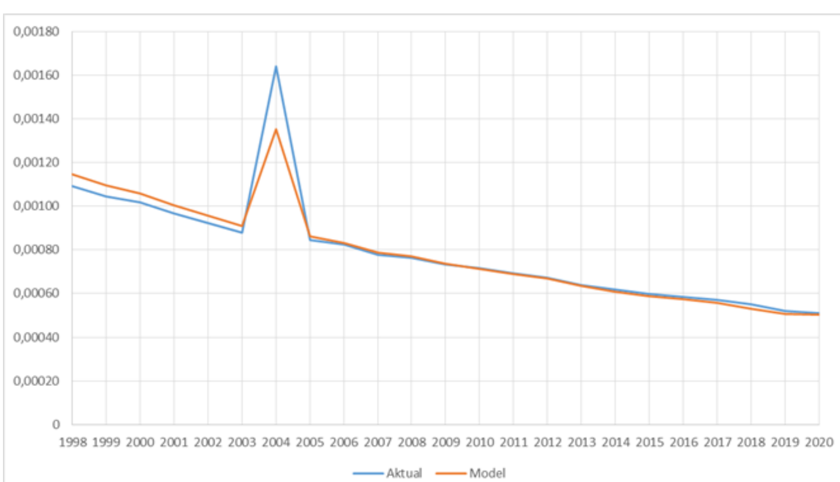

FIGURE 2. Comparison results for the 5 – 10 year age group

From Figure 2 above, the model can accommodate fluctuations in the actual value. When the actual value increased at the 2004 point, the model's estimated value also increased. Apart from that, the actual value which tends to continue to decrease over time is also reflected by the model estimated value. The resulting estimated value is also quite close to the actual value. This is in line with the MAE and MAPE values produced in the age group $5-10$ year is 0.000030 and 2.831%. After it was seen that the parameters had succeeded in forecasting well, further modifications were made to the parameter estimates k_t by building a cumulative Δk_t distribution function. The cumulative distribution function Δk_t is constructed using the POT approach. GPD is used for values above the threshold with a threshold value of 0.02. For values below the threshold, a normal distribution is used for the EVT-modified Lee-Carter model and an empirical distribution for the EVT-modified Lee-Carter model. The cumulative distribution function constructed for a normal distribution as follow:

$$F_x(x) = \begin{cases} H_1(x-0.02)\,(1-F_N(0.02)) + F_N(0.02) & ,x \geq 0.02 \\ F_N = \int_{-\infty}^{x} \frac{1}{0.07717262\sqrt{2\pi}} e^{-\frac{(t+0.24376321)^2}{(2\times 0.07717262^2)}}\, dt & ,-2.846839 < x < 0.02 \\ H_1(-x-0.02)\,(1-F_N(0.02)) + F_N(0.02) & ,x \leq -2.846839 \end{cases}$$

where

$$H_1(x-0.02) = 1 - \left(1 + \frac{-13.52233(x-0.02)}{38.22645}\right)^{-\frac{1}{-13.52233}}$$

For the empirical distribution:

$$F_X(x) = \begin{cases} H_1(x-0.02)\left(1-\hat{F}(0.02)\right) + \hat{F}(0.02) & ,x \geq 0.02 \\ \hat{F}(x) & ,-2.846839 < x < 0.02 \\ H_1(-x-0.02)\left(1-\hat{F}(0.02)\right) + \hat{F}(0.02) & ,x \leq -2.846839 \end{cases}$$

where

$$H_1(x-0.02) = 1 - \left(1 + \frac{-13.52233(x-0.02)}{38.22645}\right)^{-\frac{1}{-13.52233}}$$

of each cumulative distribution function can be generated with the help of random numbers from the uniform distribution Δk_t $(0,1)$ and the inverse function of the cumulative distribution function. After the values Δk_t are generated, these values can be used to obtain estimated parameter values k_t and forecast in 2021 and 2022. The following is a comparison graph of forecasting results for 2021 and 2022 in the $5-10$ year age group:

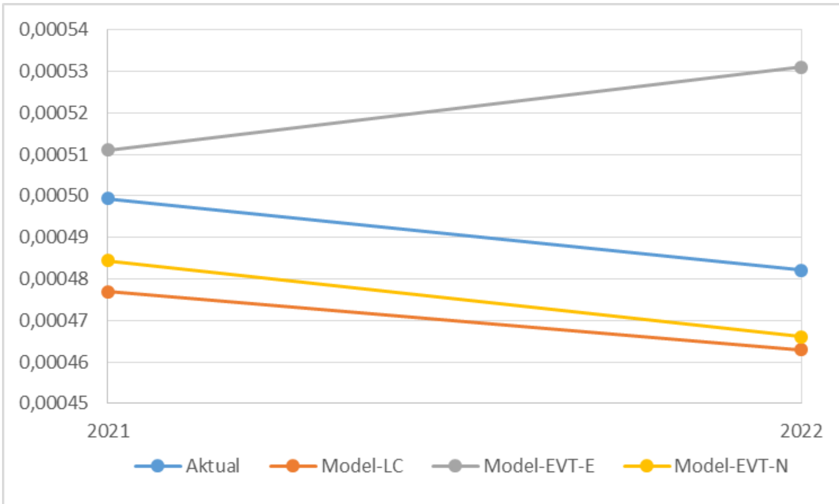

FIGURE 3. Comparison of Forecasting Results in the $5-10$ Year Age Group

In Figure 3, there has been a decrease in the actual mortality rate in 2021 and 2022. The results of forecasting using the Extreme Value Theory Modified Lee-Carter empirical distribution model produce forecasts with the largest values followed by the forecasting results of the Extreme Value Theory Modified Lee-Carter normal distribution model and the regular Lee-Carter Model. Forecasting using the Extreme Value Theory Modified Lee-Carter empirical distribution model is the only forecast those results in an increase in the mortality rate in 2022. This makes the Extreme Value Theory Modified Lee-Carter empirical distribution model have a forecasting value that is very different from the mortality rate value. actual. The forecast results closest to the actual values are produced by the Extreme Value Theory Modified Lee-Carter normal distribution model and followed by the ordinary Lee-Carter Model. Both models produce forecast values that have decreased like actual mortality rates in 2021 and 2022. The MAE and MAPE values for each model are:

TABLE 2. MAE and MAPE of EVT Model

Age Group	MAE EVT-Exponential	MAE EVT-Normal	MAPE EVT-Exponential	MAPE EVT-Normal
0	0.00216	0.00042	11.831%	2.276%
1 – 4	0.00010	0.00002	10.834%	2.530%
5 – 10	0.00003	0.00002	6.247%	3.157%
11 – 15	0.00002	0.00002	3.465%	3.948%
16 – 20	0.00004	0.00008	3.671%	6.780%
21 – 25	0.00005	0.00009	3.265%	6.304%
26 – 30	0.00007	0.00012	4.311%	7.231%
31 – 35	0.00013	0.00018	6.557%	9.071%
36 – 40	0.00026	0.00031	9.348%	11.281%

Extreme Value Theory Modified Lee-Carter empirical distribution model provides the best performance and accuracy in forecasting extreme event mortality rates. The worst performance and accuracy in forecasting extreme event mortality rates is shown by the model Lee-Carter regular. What can be seen clearly are the MAE and MAPE values in the age group 16 – 20 year. The MAPE for the Extreme Value Theory Modified Lee-Carter empirical distribution model is 3.6707% while the MAPE for the regular Lee-Carter model is 7.22%. There is a difference in MAPE values of 3.5491%. Thus, based on a comparison of the MAE and MAPE values in the age group that experiences extreme conditions, namely the age group 11 – 15 year to 36 – 40 year, it can be concluded that the Extreme Value Theory Modified Lee-Carter empirical distribution model has the best performance and accuracy, followed by the Extreme Value Theory Modified Lee-Carter model with normal distribution and the worst performance and accuracy is the model Lee-Carter regular.

4. CONCLUSION

The Modified Lee-Carter Extreme Value Theory model is implemented on Indonesian mortality rate data from 1998 to 2022. Data from 1998 to 2020 are used to estimate the three model parameters, while data from 2021 and 2022 are used to evaluate how the model performs in forecasting. The evaluation of model parameters produces an average MAE value of 0.001863 and an average MAPE of 2.898%. The threshold obtained by Δk_t going through the 90th percentile value, and the qqnorm plot is 0.02. For values above the threshold, they are modeled with GPD. For values below the threshold, two different distributions are modeled, namely the normal and empirical distribution. The results obtained from the MAE and MAPE values in forecasting for 2021 and 2022 show that the EVT Modified Lee-Carter empirical distribution model provides the smallest MAPE value of 12.156% in forecasting mortality rates in age groups with extreme events. Additionally, the Extreme Value Theory Modified Lee-Carter normal distribution model provides the smallest MAPE of 2.844% in predicting mortality rates in age groups with a decrease in mortality rates that is not too large.

Conflict of Interests

The authors declare that there is no conflict of interests.

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