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A MODEL OF AN OPTIMAL CONTROL FOR A DISCRETE-TIME OF BRAIN DRAIN

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Abstract. In this paper, we focus on the study of a mathematical model of the phenomenon of professional skills emigration in a region, by proposing a dynamic system model of non-linear differential equations in discrete time, considering four types of variables named: Permanents, Candidates, Emigrants, and Returnees. Our relevant objective is to find an optimal strategy to minimize the number of qualified individuals leaving their territory as well as candidates considering leaving their territory. The characterization of the optimal control analysis is based on Pontryagin's maximum principle, aimed at characterizing an optimal control that minimizes the number of Emigrants and potential Candidates for emigration, in order to maximize the number of Returnees and Permanents. Numerical simulation was performed using MATLAB. Consequently, numerical illustrations of the obtained results are presented, confirming the effectiveness of the optimization strategy followed.

Keywords: brain drain; optimal control; mathematical modeling.

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1. Introduction

Skill migration has become a major global phenomenon, shaping both countries of origin and destination. According to the International Organization for Migration (IOM), approximately 272 million people, representing 3.5 percent of the world's population, lived outside their country of origin in 2019. This figure includes a significant number of skilled workers, contributing to the workforce of developed economies [1].

The phenomenon of migration in Morocco is widespread, with over 3 million Moroccans having emigrated and 42% of the population expressing an intention to emigrate. Men constitute 48% of prospective migrants, while women make up 35%. The regions of Agadir and Marrakech are most affected by emigration, with 52% and 49% of the intentions to leave, respectively. The main destinations for migrants are in Europe, with over 70% of preferences, including 32% for France, 21% for Spain, and 15% for Italy. Migrants are predominantly young, and the intention to emigrate is higher among those with higher levels of education. Approximately 40% of Moroccan migration is female. Despite difficult socio-economic conditions in Morocco, migration encompasses all social categories. However, the financial outcomes of migration do not always meet expectations, with 66% of returning migrants living in good social conditions but 73% facing economic difficulties. Migrants often have precarious jobs abroad, with 44% of men and 46% of women working without contracts. Only 35% of men and 20% of women have acquired rights to retirement or other social benefits during their stay abroad. Additionally, only 33% of returning migrants had their qualifications officially recognized abroad, and 26% held jobs requiring a lower level of education than theirs.

Preparation for migration is often limited, with only 14% of men and 24% of women having undergone specific training before departure. Return is typically organized unofficially, with only 7% of individuals informed about programs for returning migrants. Approximately one-third of migrants plan to re-emigrate, primarily due to difficulties in finding employment. [2]

The mathematical modeling of skill migration has been a topic of growing interest for many researchers. Studies have been conducted to understand migration trends, predict future flows, and assess the impact of these migrations on both origin and destination economies. Highlighting potential benefits and associated challenges with these migrations. These research efforts provide valuable insights to guide public policies and decisions regarding skill migration.Our work is similar to other mathematical models that have addressed social or epidemiological phenomena (see[3][4][5][6]).

Discrete time modeling is based on the collection of statistical data in discrete moments days weeks months or years this type is preferred for many researchers [7][8]. In this study, we aim to construct a discrete mathematical model of Potential Returning Emigrated Skills (PCER) by incorporating saturation incidence coefficients. We will also introduce two essential control factors. The first will focus on administrative and financial facilitation of scientific research for Moroccan emigrants, aiming to encourage their return to the country. The second control factor will involve raising awareness of the crucial importance of local skills to contribute to Morocco's socio-economic development. These two controls are fundamental for influencing skill flows and maximizing benefits for national development. By integrating these elements into the PCER model, we aim to evaluate their impact on migration trends and formulate relevant policy recommendations to enhance local skills and attract returning Moroccan talents. In section 2, we propose the mathematical model. Then in Section 3, we will apply optimal control to our discrete model. section 4 will be dedicated to numerically simulating the results, with the conclusion as the final section.

2. Formulation of the Mathematical Model

2.1. Description of the Model. This work addresses the issue of talent drain using a discrete mathematical model, the PCER. The population studied in this subject is classified into four compartments: the qualified population (professionally educated individuals) $\mathscr{P}_k$, candidates (graduates seeking to leave the country) $\mathscr{C}_k$, migrants (skills abroad) $\mathscr{E}_k$, and those returning to their home country $\mathscr{R}_k$. Contact between the populations of two compartments $\mathscr{C}_k$ and $\mathscr{E}_k$ leads to the conversion of a candidate into an emigrant. We denote this contact as $\mathscr{C}_k\mathscr{E}_k$, and the rate of change is α.

- The compartment $\mathscr{P}_k$ consists of individuals with professional and creative skills. It increases due to new graduates at a rate of Λ, as well as candidates changing their minds (having found more favorable conditions abroad...) at a rate of δ. This compartment also decreases due to individuals preparing their immigration files (candidates with a long preparation time) or direct migrants without application (with very short immigration file preparation), at a rate of γ. Additionally, a portion of the population leaves this compartment due to mortality, at a rate of μ.

- The compartment $\mathscr{C}_k$ is formed by qualified professionals and nearly graduated students preparing their immigration files to work abroad. It increases due to a portion of the population $\mathscr{P}_k$ at a rate of $(1-\mu)\gamma$, and decreases due to contact population $\mathscr{C}_k\mathscr{E}_k$ with a rate of α. Thus, candidates changing their minds $\delta\mathscr{C}_k$, and candidates leaving life $\mu\mathscr{C}_k$.

- The compartment $\mathscr{E}_k$ is composed of high-level professionals leaving the country for a permanent job abroad. It increases due to contact individuals $\mathscr{C}_k\mathscr{E}_k$ accounted for at a rate of α, as well as a portion of the population $\mathscr{P}_k$ at a rate of $\mu\gamma$. Decrease in this compartment occurs through the return of migrants at a rate of ρ or due to cessation of life $\mu\mathscr{E}_k$.

- The compartment $\mathscr{R}_k$ contains returning migrants. It increases due to returning migrants at a rate of ρ, but decreases due to loss of life members $\mu\mathscr{R}_k$.

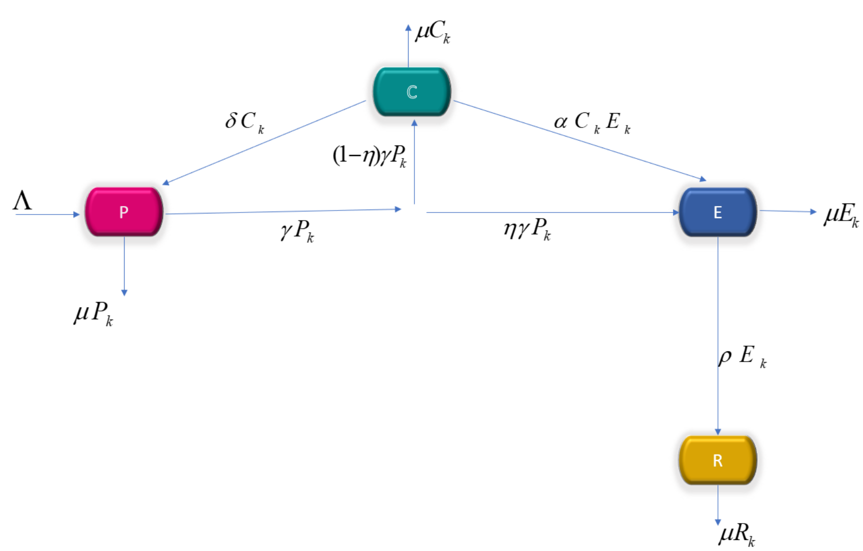

FIGURE 1. diagram

The variables , and are the numbers of the individuals in the four classes at time , respectively. The unit of can correspond to periods, phases, or years. It depends on the frequency of the survey studies as needed. The graphical representation of the proposed model is shown in Figure 1 The total population size at time is denoted by with $\mathcal{N}_k = \mathcal{P}_k + \mathcal{C}_k + \mathcal{E}_k + \mathcal{R}_k$.

The dynamics of this model are governed by the following nonlinear system of diference equations:

$$
\begin{cases}
\mathcal{P}_{k+1} = \Lambda + (1-\gamma-\mu)\mathcal{P}_k + \delta.\mathcal{C}_k \\
\mathcal{C}_{k+1} = (1-\delta-\mu).\mathcal{C}_k - \alpha.\mathcal{C}_k\mathcal{E}_k + (1-\eta).\gamma.\mathcal{P}_k \\
\mathcal{E}_{k+1} = (1-\rho-\mu)\mathcal{E}_k + \alpha.\mathcal{C}_k\mathcal{E}_k + \eta.\gamma.\mathcal{P}_k \\
\mathcal{R}_{k+1} = (1-\mu).\mathcal{R}_k + \rho\mathcal{E}_k
\end{cases} \tag{1}
$$

3. Treatment the Problem With an Optimal Control

We have addressed two strategic controls: raising awareness of the high importance of local skills to contribute to the socio-economic development of the country and the second control is based on administrative and financial facilitation for scientific research intended for Moroccan emigrants, during the time steps $k = 0$ to T and also minimizing the cost spent in applying the two strategies. In this model, we include the two controls u_{1k} and u_{2k}, that represent consecutively the awareness program through media and education, treatment, and psychological support with follow-up as measures at time k. So, the controlled mathematical system is given by the following system of difference equations:

$$
\begin{cases}
\mathcal{P}_{k+1} = \Lambda + (1-\gamma-\mu)\mathcal{P}_k + \delta.\mathcal{C}_k + r_1u_1\mathcal{C}_k \\
\mathcal{C}_{k+1} = (1-\delta-\mu).\mathcal{C}_k - \alpha.\mathcal{C}_k\mathcal{E}_k + (1-\eta).\gamma.\mathcal{P}_k - r_1u_1\mathcal{C}_k \\
\mathcal{E}_{k+1} = (1-\rho-\mu)\mathcal{E}_k + \alpha.\mathcal{C}_k\mathcal{E}_k + \eta.\gamma.\mathcal{P}_k - r_2u_2\mathcal{E}_k \\
\mathcal{R}_{k+1} = (1-\mu).\mathcal{R}_k + \rho\mathcal{E}_k + r_2u_2\mathcal{E}_k
\end{cases} \tag{2}
$$

where $:r_i = \begin{cases} 1 \\ \\ 0 \end{cases} \quad for\ i = 1,2.$

r_1	r_2	Interpretations
1	1	Discrete Emigration model with controls u_1 and u_2
1	0	Discrete Emigration model with control u_1
0	1	Discrete Emigration model with control u_2
0	0	Discrete Emigration model without controls

Table1: Interpretations according to the values of r_i.

There are two controls $u_1 = (u_{1,0}, u_{1,1}, \dots, u_{1,T})$ and $u_2 = (u_{2,0}, u_{2,1}, \dots, u_{2_T})$. The first control represents the proportion for administrative and financial facilitation of scientific research aimed at emigrants, with the goal of encouraging their return to the country. Thus, we note that $u_{1,k}\mathscr{C}_k$ is the proportion of candidate individuals transitioning to the category of individuals permanently not involved in immigration at time step k. The second control can be interpreted as the proportion to raise awareness of the crucial importance of local skills to contribute to socio-economic development. Therefore, we note that $u_{2,k}\mathscr{E}_k$ is the proportion of individuals transitioning from the category of emigrants to that of individuals definitively returning to their country at time step k. Indeed, the system above (S') presents four different models, as explained in Table 1.

The problem that we face here is how to minimize the objective functional:

$$J(u_1,u_2) = \sum_{k=0}^{T-1}\left(A_k\mathscr{C}_k + B_k\mathscr{E}_k + \frac{\Phi_k}{2}r_1u_{1,k}^2 + \frac{\psi_k}{2}r_2u_{2,k}^2\right) + A_T\mathscr{C}_T + B_T\mathscr{E}_T \tag{3}$$

Where the parameters $A_k > 0$, $B_k > 0$, $\phi_k > 0$, and $\psi_k > 0$ are the cost coefficients; they are selected to weigh the relative importance of $\mathscr{C}_k$, $\mathscr{E}_k$, $u_{1,k}$, and $u_{2,k}$ at time k. T is the final time.

In other words, we seek the optimal controls $u_{1,k}$ and $u_{2,k}$ such that

$$J(u_1^*,u_2^*) = \min_{(u_1,u_2)\in\mathscr{U}_{\text{ad}}} J(u_1,u_2) \tag{4}$$

$$\mathscr{U}_{\text{ad}} = \{(u_{1,k},u_{2,k}) : a \le u_{1,k} \le b, c \le u_{2,k} \le d; k = 0,1,2,\dots,T-1\} \tag{5}$$

The sufficient condition for the existence of optimal controls (u_1,u_2) for problem (S') and (2) comes from the following theorem.

Theorem 3.1. *There exists an optimal control* $(u_1^*,\, u_2^*)$ *such that*

$$J(u_1^*,u_2^*) = \min_{(u_1,u_2)\in\mathscr{U}_{ad}} J(u_1,u_2) \tag{6}$$

Subject to the control system (S') with initial conditions.

Proof. Since the coefficients of the state equations are bounded and there are a finite number of time steps, $\mathscr{P} = (\mathscr{P}_0, \mathscr{P}_1, \ldots, \mathscr{P}_T)$, $\mathscr{C} = (\mathscr{C}_0, \mathscr{C}_1, \ldots, \mathscr{C}_T)$, $\mathscr{E} = (\mathscr{E}_0, \mathscr{E}_1, \ldots, \mathscr{E}_T)$, $\mathscr{R} = (\mathscr{R}_0, \mathscr{R}_1, \ldots, \mathscr{R}_T)$ are uniformly bounded for all (u_1,u_2) in the control set $\mathscr{U}_{\text{ad}}$,and thus (u_1,u_2) is bounded for all $(u_1,u_2)\in\mathscr{U}_{\text{ad}}$. Since $J(u_1,u_2)$ is bounded, $\inf_{(u_1,u_2)\in\mathscr{U}_{\text{ad}}} J(u_1,u_2)$ is finite, and there exists a sequence $(u_{1,j},u_{2,j},)\in\mathscr{U}_{\text{ad}}$ such that $\lim_{j\to+\infty} J(u_{1,j},u_{2,j}) = inf_{(u_1,u_2)\in\mathscr{U}_{\text{ad}}}$ $J(u_1,u_1)$ And corresponding sequences of states $\mathscr{P}_j$, $\mathscr{C}_j$, $\mathscr{E}_j$,and $\mathscr{R}_j$. Since there is a finite number of uniformly bounded sequences, there exist $(u_1^*,u_2^*)\in\mathscr{U}_{\text{ad}}$ and $\mathscr{P}_j^*$, $\mathscr{C}_j^*$, $\mathscr{E}_j^*$, $\mathscr{R}_j^*$ $\in\mathbb{R}^{T+1}$ such that, on a subsequence, $(u_{1,j},u_{2,j})\to(u_1^*,u_2^*)$, $\mathscr{P}_j\to\mathscr{P}^*$, $\mathscr{C}_j\to\mathscr{C}^*$, $\mathscr{E}_j\to\mathscr{E}^*$, and $\mathscr{R}_j\to\mathscr{R}^*$. Finally, due to the finite-dimensional structure of system (2) and the objective function $J(u_1,u_1)$, (u_1^*,u_2^*) is an optimal control with corresponding states $\mathscr{P}^*$,$\mathscr{C}^*$,$\mathscr{C}^*$,$\mathscr{R}^*$. Therefore, $\inf_{(u_1,u_2)\in\mathscr{U}_{\text{ad}}} J(u_1,u_2)$ is achieved. □

In order to derive the necessary condition for optimal control, the Pontryagin's maximum principle in discrete time given in [9][10][11][12][13] was used. This principle converts into a problem of minimizing a Hamiltonian H_k at time step k defined by

$$H_k = A_k\mathscr{C}_k + B_k\mathscr{E}_k + \frac{\phi_k}{2} r_1 u_{1,k}^2 + \frac{\psi_k}{2} r_2 u_{2,k}^2 + \sum_{j=1}^{4} \lambda_{j,k+1} f_{j,k+1}, \tag{7}$$

where $f_{j,k+1}$ is the right side of the system of difference equations (S') for the j-th state variable at time step $k+1$.

Theorem 3.2. *Given an optimal control* $(u_1^*,u_2^*)\in\mathscr{U}_{ad}$ *and the solutions* ${\mathscr{P}^*}_k, {\mathscr{C}^*}_k, {\mathscr{E}^*}_k$ *and* ${\mathscr{R}^*}_k$ *of the corresponding state system (S'), there exist an adjoint functions* $\lambda_{1,k}, \lambda_{2,k}, \lambda_{3,k}$ *and* $\lambda_{4,k}$ *satisfying:*

$$\lambda_{1,k} = \lambda_{1,k+1}(1-\gamma-\mu) + \lambda_{2,k+1}(1-\eta)\gamma + \lambda_{3,k+1}\eta\gamma \tag{8}$$

$$\lambda_{2,k} = A_k + \lambda_{1,k+1}(\delta + r_1 u_{1,k}) + \lambda_{2,k+1}((1-\delta-\mu) - \alpha\mathscr{E}_k - r_1 u_{1,k}) + \lambda_{3,k+1}\alpha\mathscr{E}_k \tag{9}$$

$$\lambda_{3,k} = B_k - \lambda_{2,k+1}\alpha\mathscr{C}_k + \lambda_{3,k+1}((1-\rho-\mu)+\alpha\mathscr{C}_k - r_2u_{2,k}) + \lambda_{4,k+1}(\rho + r_2u_{2,k}) \tag{10}$$

$$\lambda_{4,k} = \lambda_{4,k+1}(1-\mu) \tag{11}$$

with the transversality conditions at time T, $\lambda_{1,T} = \lambda_{4,T} = 0$*,* $\lambda_{2,T} = A_T$ *and* $\lambda_{3,T} = B_T$*.*

Furthermore, for $k = 0,1,2,\ldots,(T-1)$ *and* $r_1 = r_2 = 1$*, the optimal controls* u_1^* *and* u_2^* *are given by:*

$$\begin{aligned} u_{1,k+1} &= \min\left[b, \max\left(a, \frac{1}{\phi_k}(\lambda_{1,T-k+1} - \lambda_{2,T-k+1})\mathscr{C}_k\right)\right] \\ u_{2,k+1} &= \min\left[d, \max\left(c, \frac{1}{\psi_k}(\lambda_{3,T-k+1} - \lambda_{4,T-k+1})\mathscr{E}_k\right)\right] \end{aligned} \tag{12}$$

Proof: The Hamiltonian at time step t is given by:

(13)

$$\begin{aligned} \mathscr{H}_k &= A_k\mathscr{C}_k + B_k\mathscr{E}_k + \frac{\phi_k}{2}r_1u_{1,k}^2 + \frac{\psi_k}{2}r_2u_{2,k}^2 + \lambda_{1,k+1}.f_{1,k+1} + \lambda_{2,k+1}.f_{2,k+1} + \lambda_{3,k+1}.f_{3,k+1} \\ &\quad + \lambda_{4,k+1}.f_{4,k+1} \\ &= A_k\mathscr{C}_k + B_k\mathscr{E}_k + \frac{\phi_k}{2}r_1u_{1,k}^2 + \frac{\psi_k}{2}r_2u_{2,k}^2 + \lambda_{1,k+1}[\Lambda + (1-\gamma-\mu)\mathscr{P}_k + \delta.\mathscr{C}_k + r_1u_1\mathscr{C}_k] \\ &\quad + \lambda_{2,k+1}[(1-\delta-\mu).\mathscr{C}_k - \alpha.\mathscr{C}_k\mathscr{E}_k + (1-\eta).\gamma.\mathscr{P}_k - r_1u_1\mathscr{C}_k] + \lambda_{3,k+1}[(1-\lambda-\mu)\mathscr{E}_k \\ &\quad + \alpha.\mathscr{C}_k\mathscr{E}_k + \eta.\gamma.\mathscr{P}_k - r_2u_2\mathscr{E}_k] + \lambda_{4,k+1}[(1-\mu).\mathscr{R}_k + \rho\mathscr{E}_k + r_2u_2\mathscr{E}_k] \end{aligned}$$

For k = 0, 1, ...,(T-1) the optimal controls u_1^* and u_2^* can be solved from the optimaly condition,

$$\frac{\partial H_k}{\partial u_{1,k}} = 0$$
$$\frac{\partial H_k}{\partial u_{2,k}} = 0$$

That are

$$\frac{\partial H_k}{\partial u_{1,k}} = \phi_k r_1 u_{1,k} + (\lambda_{1,k+1} - \lambda_{2,k+1})r_1\mathscr{C}_k = 0$$
$$\frac{\partial H_k}{\partial u_{2,k}} = \psi_k r_2 u_1 1,k + (\lambda_{4,k+1} - \lambda_{3,k+1})r_1\mathscr{E}_k = 0$$

Thus, for $r_1 = r_2 = 1$ we have:

$$u_{1,k} = \frac{(\lambda_{2,k+1}-\lambda_{1,k+1})\mathscr{C}_k}{\phi_k}$$
$$u_{2,k} = \frac{(\lambda_{2,k+1}-\lambda_{1,k+1})\mathscr{E}_k}{\phi_k}$$

However, if $r_i = 0$ for $i = 1,2$, the control attached to this case will be eliminated and removed.

By the bounds in $\mathscr{U}_{ad}$ of the controls, it is easy to obtain u_1^* and u_2^* in the form (8).

4. Numerical Simulation

4.1. Algorithm. In this section, we present the results obtained by solving numerically the optimality system.

This system consists of the state system, adjoint system, initial and final time conditions, and the controls characterization.

$\mathscr{P}_0$	2.10^6
$\mathscr{C}_0$	9.10^5
$\mathscr{E}_0$	18.10^4
$\mathscr{R}_0$	10^4
ρ	$0,003$
γ	$0,05$
η	$0,003$
δ	$0,02$
μ	$0,001$
α	0.15

Table 2 : The description of parameters used for the definition of discrete time system (S).

We just used an academic data.

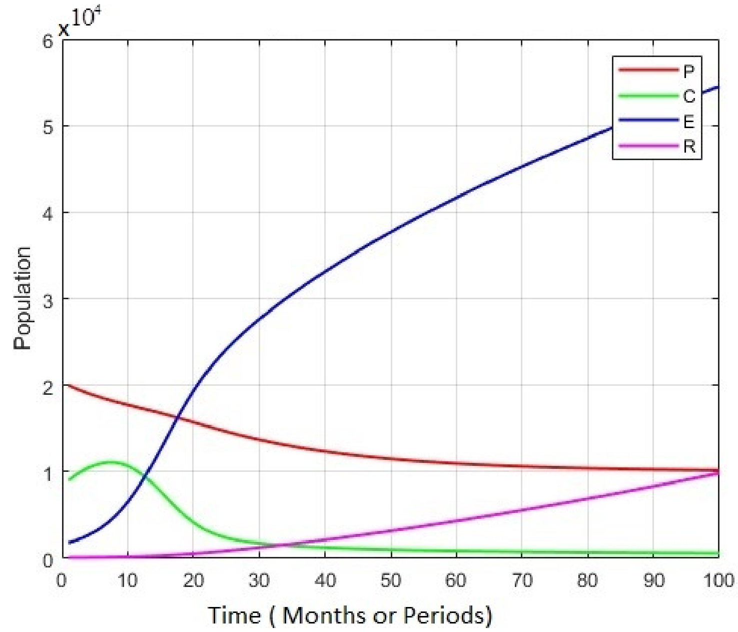

FIGURE 2. Simulation of $\mathscr{P}, \mathscr{C}, \mathscr{E}$ and $\mathscr{R}$ over time without controls

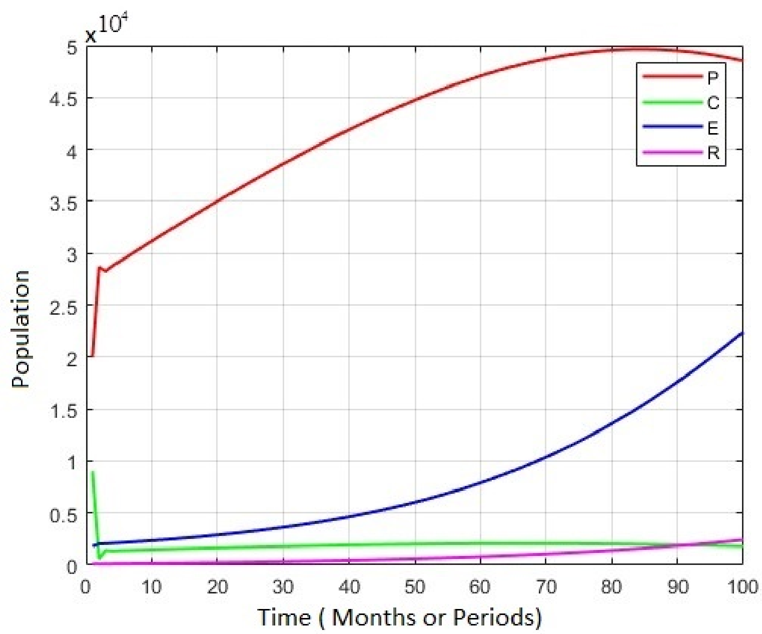

FIGURE 3. Simulation of $\mathscr{P}, \mathscr{C}, \mathscr{E}$ and $\mathscr{R}$ over time with only u_1 control

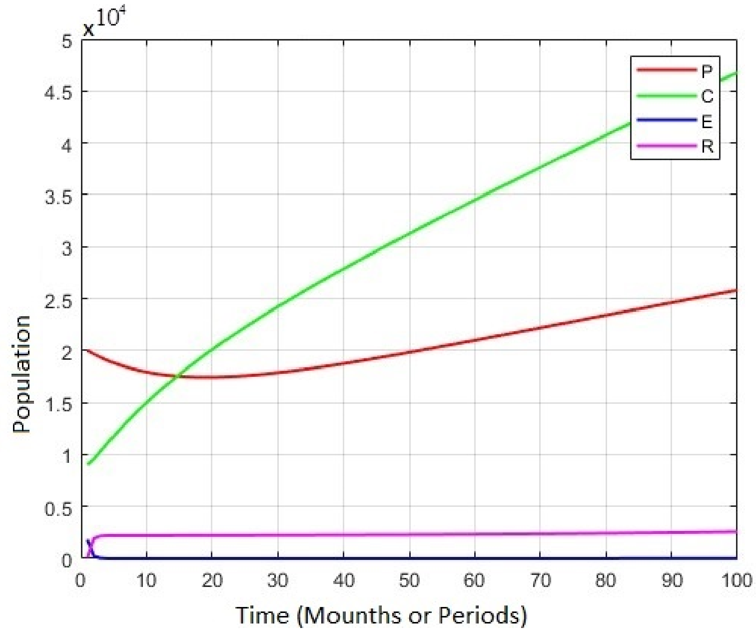

FIGURE 4. Simulation of $\mathscr{P}, \mathscr{C}, \mathscr{E}$ and $\mathscr{R}$ over time with only u_2 control

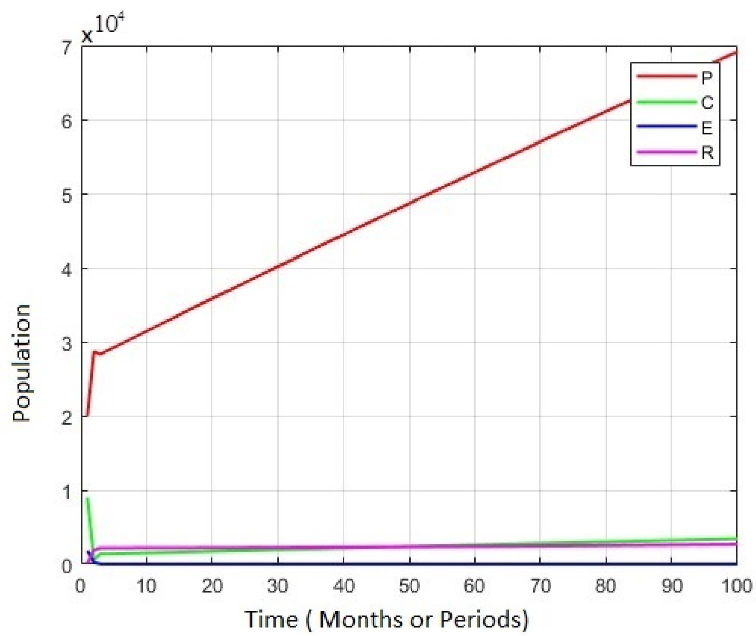

FIGURE 5. Simulation of $\mathscr{P}, \mathscr{C}, \mathscr{E}$ and $\mathscr{R}$ over time with controls u_1 and u_2

In this formulation, there were initial conditions for the state variables and terminal conditions for the adjoints.

That is, the optimality system is a two-point boundary value problem with separated boundary conditions at time steps $k = 0$ and $k = T$. We solve the optimality system by an iterative method with forward solving of the state system followed by backward solving of the adjoint system. We start with an initial guess for the controls at the first iteration and then before the next iteration we update the controls by using the characterization.

We continue until convergence of successive iterates is achieved.

4.2. Discussion. In this section, we study and analyse numerically the effects of optimal control strategies such as awareness of the crucial importance of local skills to contribute to socio-economic development and administrative and financial facilitation of scientific research (Table 2).

4.2.1. Strategy A: Control with awareness of the crucial importance of local skills to contribute to socio-economic development administrative , we propose an optimal strategy for this purpose. Hence, we activate the optimal control variable u_1 which represents awareness program for emigrants candidates. Figure 2 compares the evolution of candidates with and without control u_1 in which the effect of the proposed awareness program through media and education is proven to be positive in decreasing the number of candidates.

4.2.3. Strategy B: Control with administrative and financial facilitation of scientific research aimed at emigrants, with the goal of encouraging their return to the country . In this strategy, we propose an optimal strategy by using the optimal control V in the beginning. We notice that the numbers returned emigrants are creased markedly which leads to satisfactory results.

5. Conclusion

In this current article, we have defined a discrete model of the phenomenon of emigration of Moroccan high-level cadres, with the aim of minimizing the number of emigrants and candidates thinking of emigrating, in order to maximize the number of permanent and returning cadres.

We also imposed two controls which, respectively, represent raising awareness of the high importance of local skills to contribute to the socio-economic development of their countries, and administrative and financial facilitation for scientific research aimed at emigrants to encourage them to return and invest their experience and skills where they belong.

We succeeded in obtaining the expected characterizations of optimal controls after rigorous application of the results of control theory. The effectiveness of the proposed strategies was well demonstrated by numerical simulation of the theoretical results obtained.

And we'd like to emphasize here that all other theoretical controls will no longer do their main job of retaining senior executives in various potential and critical professions - Professors, Doctors, Engineers, Computer Programmers, Finance Specialists, Technicians, etc. - without a real political will to tackle this brain drain crisis, whose consequences are remarkable in both the short and long term, and which is having a destructive impact on the country's social and economic development, which the latter will pay dearly from its time, resources and effort.

And We are preparing a work on the fractional optimal control of a similar system.

Conflict of Interests

The authors declare that there is no conflict of interests.

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