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QUASI LIKELIHOOD ON LINEAR MIXED EFFECT OF BINARY RESPONSE IN LONGITUDINAL DATA

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Abstract: Longitudinal data is derived from repeated measurements over time, and when analyzing such data using logistic regression models, it is essential to account for the correlations induced by these repeated observations. To address this, a random component is incorporated into the linear predictor, transforming the model into a generalized linear mixed model (GLMM). In this approach, the linear predictor is no longer purely systematic due to the inclusion of the random component. Estimation of parameters in a GLMM cannot rely on the conventional maximum likelihood estimation method, as it must account for the random component in the linear predictor. A viable alternative is the quasi-likelihood method, which provides a framework for estimating these parameters. Among various quasi-likelihood-based approaches, the generalized estimating equation (GEE) is commonly employed, as it explicitly incorporates the correlation structure in the data through a specified "working correlation" matrix. In particular, an autoregressive structure is often chosen for this matrix. The estimation procedure is then carried out iteratively through the Fisher Scoring method, with convergence achieved after several iterations. To assess the performance of this methodology, simulation studies are conducted, followed by its application to real-world data for empirical validation.

Keywords: generalized estimating equation; logistic regression; longitudinal data; quasi likelihood.

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1. INTRODUCTION

In survey research, it is often necessary to collect data through repeated measurements, resulting in data with a longitudinal structure. Longitudinal data is typically utilized in studies examining the correlation of individual changes over time. This type of data is gathered when responses are measured from units at multiple time points. One key advantage of longitudinal data is its ability to capture the temporal progression of responses in sequence. Additionally, longitudinal data often exhibits a characteristic where it is composed of numerous clusters, each of which tends to be relatively small in size.

The primary objective of longitudinal data analysis is to investigate the effects of covariates at each level on the response, as well as the impact of changes in the response over time. A key advantage of longitudinal data analysis is its ability to distinguish between cross-sectional effects and those arising from repeated measurements. Furthermore, this approach allows for the examination of variability both in response levels and in changes over time across different units. Unaccounted-for variance in the observed variables creates dependencies between responses, even when covariates are controlled for. This undermines the assumptions of standard linear regression models and must be addressed to ensure valid inferences. Typically, repeated measurement data is categorized as either balanced or unbalanced. Data is considered balanced if all units are measured at the same time points, whereas it is regarded as unbalanced when units are measured at varying time points.

Logistic regression is a regression model designed for binary outcomes, where parameter estimation typically utilizes the maximum likelihood estimation (MLE) method, assuming independent and identically distributed data. However, in the context of longitudinal data, the assumption of independence is violated due to the correlation induced by repeated measurements. As a result, the standard logistic regression model cannot be estimated using MLE, as this method does not account for the correlations between observations.

To address this issue, various approaches have been proposed for analyzing logistic regression models with longitudinal data. One of the most common methods is the generalized linear mixed model (GLMM). While the GLMM accommodates the correlation between repeated measurements, it still requires assumptions regarding the distribution of the response variable. Furthermore, it may be insufficient if the correlation structure between responses is not properly incorporated, as the model does not explicitly account for this element.

The primary goal of longitudinal data analysis is to examine the effects of covariates on the response variable, both at each individual level and across time. This type of analysis allows researchers to separate cross-sectional effects from those due to repeated measurements. Additionally, longitudinal data enables the exploration of variability in both response levels and changes over time across different units. Unexplained variance in the observed data can create dependencies between responses, even after accounting for covariates, thereby violating the assumptions of ordinary linear regression models. These dependencies must be addressed to avoid erroneous conclusions.

Longitudinal data, particularly in the context of repeated measurements, can be classified as either balanced or unbalanced. Balanced data refers to situations where all units are measured at the same time points, while unbalanced data arises when measurements are taken at different time intervals for each unit.

For categorical responses, the generalized linear model (GLM) is commonly employed. Like logistic regression, the GLM also uses maximum likelihood estimation based on the assumption of independent and identically distributed data. However, this assumption is not met in longitudinal studies due to the correlation between repeated observations. Consequently, the GLM cannot be appropriately estimated using MLE without adjustments to account for the correlation structure.

To analyze longitudinal categorical data, such as the probability of layoffs, researchers often apply the generalized linear mixed model. In this case, layoffs are a binary or count response, with the binary response indicating whether layoffs occur and the count response reflecting the number of employees laid off. As layoffs are measured annually, the data exhibits a longitudinal structure. The generalized linear mixed model, along with the generalized estimating equation (GEE) method, provides a robust framework for estimating the parameters in this context. The quasi-likelihood approach used within GEE helps address the correlation between repeated measurements, making it a suitable choice for analyzing such longitudinal data.

2. MATERIALS AND METHODS

The general linear model for longitudinal data structures is the generalized linear mixed model, which has the following mathematical equation:

$$g(\mu) = \mathbf{X}\boldsymbol{\beta} + \mathbf{u} \qquad (1)$$

where $\mu = E(y_{ij})$ is the expectation of the response y_{ij}, **X** is a matrix of independent variables **β** is a vector of regression coefficients and **u** is a vector of random components. $g(.)$ is a link function which depends on its random components.

Generalized linear mixed model (GLMM) is a regression model that cannot be explained as a linear combination of its coefficients so that the response vector is no longer normally distributed. GLMM is a development of GLM by adding random components to its systematic components.

Parameter estimation method for data containing correlation structure such as longitudinal data without requiring strict distribution assumption can use generalized estimating equation. The advantage of this method is that the computational method for this method is simpler than the generalized linear model. In addition, this method also does not require multivariate distribution assumptions. Another advantage is that the resulting estimator is consistent even though there is a specification error in the correlation structure used and can be applied to incomplete data or missing data, with missing observations being missing completely at random (MCAR) [2].

Generalized estimating equations (GEE), first introduced in 1986 by Liang and Zeger [4], has become more popular in various fields of science. Hardin and Hilbe [3] stated that although the Generalized Linear Model (GLM) is a powerful and flexible method to use, its analysis is limited, the assumption of GLM is that each individual must be independent of other individuals. However, in longitudinal data this assumption may not be met. The main advantage of using

GEE lies in the results of consistent and unbiased parameter estimates, even when the correlation structure is miss-specified [5]. GEE is an adjustment to GLM when the data is correlated, so this method can be applied to various response variables that are often encountered in empirical applications (eg, continuous, polychotomous, dichotomous, and enumeration data). Although GEE does not require data to follow a particular distribution, it assumes that the variance of the response variable is described as an expectation function.

One of the important things that distinguishes GEE from other methods is the presence of a correlation structure included in its parameter estimation. This correlation structure is called the Working Correlation Matrix (WCM). Determining WCM is highly dependent on the correlation conditions that occur in the data used. According to Liang and Zeger [4], GEE remains *robust* even if the WCM used is not appropriate. Some WCM options that can be used are Independent Structure, Exchangeable Structure, Autoregresive Structure and Unstructured Correlation [6]. For longitudinal data, the appropriate WCM is *Autoregresive Structure.*

$$R_i(\alpha) = \begin{bmatrix} 1 & \alpha & \alpha^2 & \dots & \alpha^{N-1} \\ \alpha & 1 & \alpha & \dots & \alpha^{N-2} \\ \alpha^2 & \alpha & 1 & \dots & \alpha^{N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \alpha^{N-1} & \alpha^{N-2} & \dots & \alpha & 1 \end{bmatrix} \quad (2)$$

where: $\alpha_t = \emptyset \sum_{i=1}^{N} \boldsymbol{e_{ij}} \boldsymbol{e'_{i,t+1}} / (NT - K)$, $\alpha = \sum_{t=1}^{T-1} a_t / (T-1)$ and $\emptyset$ is dispersion parameter.

The logistic model used for longitudinal data is as follows

$$logit(\pi_{ij}) = \boldsymbol{X}'_{ij}\boldsymbol{\beta} \quad (3)$$

For i = 1, 2, … n and j = 1, 2, … , m_i. Where m_j many repeated measurements of i-th unit, n is a number of observations, π_{ij} is the probability of success for i-th unit in j-th time and $\boldsymbol{\beta}$ is the vector of regression coefficients.

To estimate the parameters in Equation (3) we must take into account the existence of WCM as in Equation (2) so that the parameter estimation cannot use the maximum likelihood method. The method used is to maximize the log-likelihood function based on the normal distribution even without fulfilling the normality assumption of the response. The resulting likelihood function is called quasi-likelihood. GEE modifies the log-likelihood function by inserting WCM into an equation that takes into account the correlation in the data [2]. The quasi-score likelihood equation is stated as:

$$U(\alpha,\beta) = \sum_{i=1}^{n}\sum_{j=1}^{m_i}\omega_{ij}\boldsymbol{D}'_{ij}\boldsymbol{V}_{ij}^{-1}S_{ij} = 0 \tag{4}$$

where: $\boldsymbol{D}_i = \frac{\partial \pi_{ij}}{\partial \beta} = \boldsymbol{A}_{ij}\boldsymbol{X}_{ij}$ and $\boldsymbol{V}_{ij} = \boldsymbol{A}_{ij}^{\frac{1}{2}}\boldsymbol{R}(\alpha)\boldsymbol{A}_{ij}^{\frac{1}{2}}$

$\boldsymbol{A}_{ij} = diag\{\pi_{ij}(1-\pi_{ij})\}$ and $S_{ij} = Y_{ij} - \pi_{ij}$

where $R(\alpha)$ is WCM from $Y_{i1},\dots,Y_{ij}$, correltion structure for longitudinal data using the autoregression structure correlation matrix as in Equation (2). When Equation (4) is reduced against β it will produce the following equation:

$$\boldsymbol{I} = \sum_{i=1}^{n}\sum_{j=1}^{m_i}\omega_{ij}\boldsymbol{D}'_{ij}\boldsymbol{V}_{ij}^{-1}\boldsymbol{D}_{ij} \tag{5}$$

where:

$\hat{\boldsymbol{V}} = \frac{n}{n-1}\sum_{i=1}^{n}(\boldsymbol{B}_i - \overline{\boldsymbol{B}})(\boldsymbol{B}_i - \overline{\boldsymbol{B}})'$

$\boldsymbol{B}_i = \sum_{j=1}^{m_i}\omega_{ij}\boldsymbol{X}'_{ij}\hat{A}_{ij}^{1/2}\hat{\boldsymbol{R}}(\alpha)^{-1}\hat{\boldsymbol{A}}_{ij}^{1/2}\boldsymbol{X}_{ij}$ and $\overline{\boldsymbol{B}} = \frac{1}{n}\sum_{i=1}^{n}\boldsymbol{B}_i$

Equation (1) is not linear in the parameter β, so that it requires an optimization method to obtain the solution.. The optimization method used to obtain $\hat{\beta}$ is the *Fisher Scoring Method*. Based on [7], the Fisher scoring method equation for quasi-likelihood *is* as follows:

$$\boldsymbol{\beta}^{(t+1)} = \boldsymbol{\beta}^{(t)} - \left(\sum_{i,j}\omega_{ij}\boldsymbol{D}_{ij}^{(t)\prime}\,{\boldsymbol{V}_{ij}^{(t)}}^{-1}\boldsymbol{D}_{ij}^{(t)}\right)^{-1}\left(\sum_{i,j}\omega_{ij}\boldsymbol{D}_{ij}^{(t)\prime}\,{\boldsymbol{V}_{ij}^{(t)}}^{-1}(Y_{ij} - \pi_{ij}^{(t)})\right) \tag{6}$$

The process is iterated until convergence is achieved. According to [1], the measure of convergence that can be used is :

$$\frac{|\boldsymbol{\beta}^{(t+1)} - \boldsymbol{\beta}^{(t)}|}{|\boldsymbol{\beta}^{(t)}| + 10^{-6}} < \epsilon = 10^{-8} \tag{7}$$

For the iteration process, the initial estimated value $\widehat{\boldsymbol{\beta}}$ at $t = 0$ is obtained from the estimate using the ordinary logistic regression model using *maximum likelihood estimation* without involving correlation elements. Then in the next stage, the estimated value is calculated by involving WCM.

3. APPLICATION

In this section, the application of the GEE method will be described through simulation studies and applications on real data. We are using simulation studies to determine the performance of the GEE estimator for longitudinal data with binary responses for various correlation structures and various sample sizes. The simulation procedure used is based on the simulation conducted by [8]. Predictor variables are generated as many as from a uniform distribution (0.1). The correlation structure used is *Autoregression* with lag 1 or AR (1) with a correlation coefficient value of 0.5. Binary responses are generated using the "*mvtbinaryEP*" package from R software using the probability value

$$\pi_{ij} = \frac{exp\ (0.5 + 0.6X_{1j} + 0.6X_{2j})}{1 + exp\ (0.5 + 0.6X_{1j} + 0.6X_{2j})}$$

Simulations were performed for sample sizes of 10, 30, 50 and 100 with 5 repeated measurements. The simulations were repeated 1000 times.

The following description is an explanation of real data regarding layoffs from a company that will be applied to the proposed method. The global business world in recent decades has experienced significant fluctuations. The 2008 global financial crisis and more recently, the COVID-19 pandemic, have had a profound impact on the world's economic landscape, including Indonesia. One of the direct consequences of these crises is the increasing number of layoffs [9]. Layoffs of employees cause a family's income to stop, which can lead to temporary poverty [10]. If they do not get a job, it is feared that it will become chronic poverty and increase the poverty rate in Indonesia.

In addressing these issues, the government has enacted PP No. 35 of 2021, Law No. 13 of 2003, and Law No. 11 of 2021 which require companies to pay severance pay to employees if the company lays off employees. With the enactment of these regulations, companies are expected to not only view layoffs as an unexpected event, but also as a business risk that can be measured and managed. By understanding the chances of layoffs, companies can take proactive steps to reduce their negative impacts, both financially and in terms of reputation. The chances of layoffs can be the basis for financial risk mitigation for companies in preparing severance fund reserves so that the company's financial health is maintained.

In estimating the probability of layoffs [11] uses several factors as determinants of the value of the probability of layoffs. In general, business demand is consumer behavior towards goods/services requested by considering various certain conditions. Business demand is one of the important components that is closely related to sales results. The lower the business demand, the greater the possibility that a company will not be able to survive for long because it experiences financial losses due to declining sales results. In 2013, the US Bureau of Labor Statistics through the Mass Layoff Statistics (MLS) program made an observation stating that business demand is one of the highest causes of layoffs in the United States. After that, further research was conducted by [11]. on business demand factors, organizational restructuring, financial problems, and product specifications as causes of layoffs using regression analysis. The results showed that business demand had an influence of 20.6% on layoffs that occurred in the United States [11]. Seeing this influence, business demand in this study will be used as one of the indicators in determining the value of the opportunity for layoffs due to loss of efficiency by involving the company's financial ratios.

Data on layoffs from a company are measured repeatedly annually so that they have a longitudinal data structure. Longitudinal data are generally used when research is needed on the correlation of individual changes. Longitudinal data are obtained when units determine responses at different times. One of the things that can be obtained from longitudinal data is the chronology of a response in sequence. Longitudinal data also has the characteristic that the data often consists of a large number of clusters, each of which is small in size.

Computation methods for simulation and application on real data using R software version 4.3 with the packages “MASS”, “gee” and “geepack” [12].

4. RESULTS AND DISCUSSION

The results of the simulation study of the logistic regression parameter estimator on longitudinal data using the GEE method for various sample sizes can be seen in Table 1.

Table 1. Simulation results of the GEE method for various sample sizes

Sample Size	***n* = 10**		***n* = 30**		***n* = 50**		***n* = 100**	
	Esimate	**SE**	**Esimate**	**SE**	**Esimate**	**SE**	**Esimate**	**SE**
β_0	-0.1026	0.3661	0.1628	0.2003	0.2665	0.0903	0.3243	0.0342
β_1	1.5324	0.7163	0.9417	0.3143	0.8636	0.1703	0.8321	0.0552
β_2	-0.0092	0.4621	0.2382	0.3421	0.4231	0.0674	0.4351	0.0601

Based on Table 1, it can be seen that for very small sample sizes, the GEE estimator shows very poor performance. This can be seen from the large bias produced and the very large *standard error*. The GEE estimate value for n = 10 has the largest bias value and the largest *standard error*. For each sample size, the largest bias is produced by the β_1 estimator, as well as the largest *standard error* is also produced by the β_1 estimator except for n = 100, the larges*t standard error is produced* by the β_2 estimator. As the sample size increases, the performance of the GEE estimator improves. This is evidenced by the decreasing estimation bias and the increasingly small *standard error* value.

In addition, simulations were also conducted for various WCM structures, namely *Independent Structure*, *Exchangeable Structure*, *Autoregresive Structure* and *Unstructured Correlation* for the same sample size of 200. The results of the GEE estimator simulation for the logistic regression model for various correlation structures are presented in Table 2.

Table 2. Simulation Results for various correlation structures

Correlation Structure	***Independet***		***Exchangeable***		***Autoregressive***		***Unstructure***	
	Esimate	**SE**	**Esimate**	**SE**	**Esimate**	**SE**	**Esimate**	**SE**
β_0	0.3527	0.0258	0.3555	0.0258	0.3602	0.0256	0.3362	0.0261
β_1	0.8855	0.0370	0.8823	0.0374	0.8765	0.0366	0.8946	0.0372
β_2	0.7057	0.0399	0.7121	0.0402	0.6871	0.0395	0.7210	0.0411

Based on the simulation results in Table 2, the GEE estimator for various WCM structures shows similar performance. There is no significant difference in the estimator bias or standard error value of each WCM structure. This proves that the GEE estimator remains robust even though the selected WCM structure is not appropriate.

The application on real data uses a binary response, namely whether the company lays off or not and three predictor variables, namely Return Of Asset (X_1), Asset Turn Over (X_2) and Debt to Total Asset (X_3). The data have been done an exploration process carried out based on the procedures in [13]. The working correlation matrix used is autoregression as follows:

$$R = \begin{pmatrix} 1 & 0.696 & 0.485 & 0.338 & 0.235 \\ 0.696 & 1 & 0.696 & 0.485 & 0.338 \\ 0.485 & 0.696 & 1 & 0.696 & 0.485 \\ 0.338 & 0.485 & 0.696 & 1 & 0.696 \\ 0.235 & 0.338 & 0.485 & 0.696 & 1 \end{pmatrix}$$

The results of the analysis of layoffs data using the binomial family and AR-M working correlation matrix are presented in Table 3 as follows.

Table 3. Application on Layoffs Data

	Estimate	**Naive S.E.**	**Naive Z**	**Robust S.E.**	**Robust Z**
β_0	-0.7651154	0.3658271	-2.0914673	0.4259170	-1.7963955
β_1	0.5139252	1.7185804	0.2990405	1.2067189	0.4258864
β_2	-0.9729021	0.5338190	-1.8225317	0.7136654	-1.3632469
β_3	-0.3803542	0.8957152	-0.4246374	1.0493044	-0.3624822

Based on Table 3, the regression coefficient estimator related to return on assets is positive, meaning it is directly proportional, while the regression coefficient related to asset turnover and debt to total assets is negative, meaning it is inversely proportional. The results of the analysis also show that the naive standard error method generally produces an underestimated estimator. This can be seen from the robust standard error value which is generally greater than the naive method. The use of the naive standard error method can affect the error in conclusions drawn in hypothesis testing that should be accepted but are rejected. Thus, the robust standard error is more recommended to be used.

5. CONCLUSION

When applied to longitudinal data obtained from repeated measurements, the logistic regression model estimated using the maximum likelihood method may yield biased estimators with large variance. To address this issue, the generalized linear mixed model (GLMM) is employed, with parameter estimation conducted through a quasi-likelihood approach. One such method utilizing quasi-likelihood is the Generalized Estimating Equation (GEE). The GEE is considered an appropriate estimator for logistic regression parameters in longitudinal data, as it accounts for the correlation between observations by incorporating a working correlation matrix into the estimation process.

The simulation results indicate that the GEE estimator exhibits biased and inefficient properties. However, for large sample sizes, the performance of the GEE estimator is quite favorable, with a notably small standard error. Additionally, the GEE estimator demonstrates robust properties with respect to the working correlation matrix (WCM) structure employed, meaning that even when the WCM structure is not perfectly specified, the GEE estimator maintains good performance. In this study, the standard error reported is derived from the simulation process, and it is recommended that future research use the standard error obtained directly from the GEE estimation method.

Several researchers have utilized the quasi-likelihood method in their studies. For instance, [14] applies generalized quasi-likelihood to address overdispersion in longitudinal count data, while [15] explores its use in high-dimensional data. Additionally, [16] examines the application of quasi-likelihood in generalized linear mixed models (GLMM) for longitudinal data. Another method for estimating parameters in GLMMs is the penalized quasi-likelihood approach. This technique has been employed by several authors, including [17], who apply bias correction to the variance estimator, [18], who incorporates the penalized quasi-likelihood information criterion into GLMMs, and [19], who investigates optimal design for GLMM based on penalized quasi-likelihood. Other estimation methods, such as the Laplace approximation and adaptive quadrature, are discussed by [20]. A systematic review of estimation techniques in GLMMs, particularly in the field of psychology, is provided by [21]. For future research, we plan to apply both the penalized quasi-likelihood and Laplace approximation methods to our data, comparing the results with the GEE approach discussed in this paper.

The analysis of real data indicates that the naive standard error method typically yields an underestimated estimate. Relying on the naive standard error method can lead to errors in hypothesis testing, where a conclusion that should have been accepted may instead be rejected. Therefore, the use of the robust standard error method is strongly recommended.

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

REFERENCES

[1] A. Agresti, An Introduction to Categorical Data Analysis, Wiley, 2019.

[2] J.W. Hardin, J.M. Hilbe, Generalized Estimating Equations, Chapman and Hall/CRC, 2002. https://doi.org/10.1201/9781420035285.

[3] J.W. Hardin, J.M. Hilbe, Generalized Linear Models and Extensions, Stata Press, 2007.

[4] S.L. Zeger, K.Y. Liang, P.S. Albert, Models for Longitudinal Data: A Generalized Estimating Equation Approach, Biometrics 44 (1988), 1049. https://doi.org/10.2307/2531734.

[5] P. Ghisletta, D. Spini, An Introduction to Generalized Estimating Equations and an Application to Assess Selectivity Effects in a Longitudinal Study on Very Old Individuals, J. Educ. Behav. Stat. 29 (2004), 421–437. https://doi.org/10.3102/10769986029004421.

[6] T. Swan, Generalized Estimating Equations When the Response Variable Has a Tweedie Distribution: An Application for Multi-Site Rainfall Modelling, University of Southern Queensland, Queensland, 2006. Accessed: Nov. 20, 2024. Available: https://sear.unisq.edu.au/3388/1/Swan_Honours_Dissertation.pdf.

[7] A.J. Dobson, A.G. Barnett, An Introduction to Generalized Linear Models, Chapman and Hall/CRC, 2018. https://doi.org/10.1201/9781315182780.

[8] S. Paul, X. Zhang, J. Xu, Estimation of Regression Parameters for Binary Longitudinal Data Using GEE: Review, Extension and an Application to Environmental Data, J. Environ. Stat. 4 (2013), 1–12.

[9] E. Sriningsih, Dampak Krisis Finansial Global terhadap Phk, DiE: J. Ilmu Ekon. Manaj. 5 (2009), 133-144.

[10] Gunawan, Sugiyanto, Kondisi Sosial Ekonomi Keluarga Pasca Pemutusan Hubungan Kerja, Sosio Konsepsia: J.

Penelit. Pengembang. Kesejahteraan Sos. 16 (2011), 35–52.

[11] M.N. Hossen, M.A. Mollah, N.S. Lipy, et al. Factors Affecting Layoff in High-Tech Industry: Evidence from USA, Int. J. Inf. Bus. Manag. 15 (2023), 1–13.

[12] L.S. McDaniel, N.C. Henderson, P.J. Rathouz, Fast Pure R Implementation of GEE: Application of the Matrix Package, R J. 5 (2013), 181-187.

[13] B. Tantular, I.G.N.M. Jaya, Y. Andriyana, Longitudinal Data Exploration of Upper Respiratory Tract Infection Disease in Bandung City, J. Phys.: Conf. Ser. 1265 (2019), 012026. https://doi.org/10.1088/1742-6596/1265/1/012026.

[14] F. Ahmmed, A.R. Jamee, Generalized Quasi-Likelihood Estimation Procedure for Non-Stationary over-Dispersed Longitudinal Counts, J. Stat. Comput. Simul. 91 (2021), 1802–1814. https://doi.org/10.1080/00949655.2021.1871614.

[15] S. Li, T.T. Cai, H. Li, Inference for High-Dimensional Linear Mixed-Effects Models: A Quasi-Likelihood Approach, J. Amer. Stat. Assoc. 117 (2022), 1835–1846. https://doi.org/10.1080/01621459.2021.1888740.

[16] Y.-G. Wang, L. Fu, S. Paul, Analysis of Longitudinal Data with Examples, Chapman and Hall/CRC, Boca Raton, 2021. https://doi.org/10.1201/9781315153636.

[17] R. Arisanti, I.M. Sumertajaya, K.A. Notodiputro, et al. Firth Bias Correction for Estimating Variance Components of Logistics Linear Mixed Model Using Penalized Quasi Likelihood Method, Commun. Math. Biol. Neurosci. 2020 (2020), 65. https://doi.org/10.28919/cmbn/4955.

[18] F.K.C. Hui, On the Use of a Penalized Quasilikelihood Information Criterion for Generalized Linear Mixed Models, Biometrika 108 (2021), 353–365. https://doi.org/10.1093/biomet/asaa069.

[19] Y. Shi, W. Yu, J. Stufken, Optimal Designs for Generalized Linear Mixed Models Based on the Penalized Quasi-Likelihood Method, Stat. Comput. 33 (2023), 114. https://doi.org/10.1007/s11222-023-10279-3.

[20] K. Ju, L. Lin, H. Chu, et al. Laplace Approximation, Penalized Quasi-Likelihood, and Adaptive Gauss–Hermite Quadrature for Generalized Linear Mixed Models: Towards Meta-Analysis of Binary Outcome with Sparse Data, BMC Med. Res. Methodol. 20 (2020), 152. https://doi.org/10.1186/s12874-020-01035-6.

[21] R. Bono, R. Alarcón, M.J. Blanca, Report Quality of Generalized Linear Mixed Models in Psychology: A Systematic Review, Front. Psychol. 12 (2021), 666182. https://doi.org/10.3389/fpsyg.2021.666182.