Available online at http://scik.org
Commun. Math. Biol. Neurosci. 2025, 2025:24
https://doi.org/10.28919/cmbn/9041
ISSN: 2052-2541

MODELING AND STABILITY ANALYSIS OF LIQUIDITY RISK CONTAGION IN THE BANKING SYSTEM WITH TIME DELAY

SAID FAHIM*, HAMZA MOURAD, FATIMA AMGHAD, MOHAMED LAHBY

University Hassan II, Higher Normal School Casablanca, Morocco

Abstract. The modeling of contagious spread is a well-established concept in the field of infectious diseases. However, its recent application to the banking system has opened up new avenues for analysis. The theory of delayed differential systems constitutes an essential mathematical tool for addressing this issue. In this study, we specifically focus on liquidity risk within the banking system and investigate the global stability with and without this delayed risk. We formulate a model to examine the potential impact of central bank interventions on the economy, utilizing simulated data from the largest European banks. The methodology emphasizes the importance of research results and specifies key variables or factors considered in the model. Study objectives focus on the modeling and analysis of the stability of liquidity risk contagion in the banking system, with a particular emphasis on the temporal dimension. The connection between modeling contagious spread in infectious diseases and its recent application in the banking system is well-established. To reach these conclusions, the study employed a rigorous methodology, integrating advanced mathematical models and in-depth statistical analysis. This methodological approach led to significant findings that shed new light on the dynamics of liquidity risk contagion in the financial context. The practical implications of these results are crucial for various stakeholders. Risk managers within financial institutions can utilize our findings to identify potential vulnerabilities and implement more effective risk management strategies. Policymakers and regulators can use our results to shape monetary and macroprudential policies aimed at stabilizing the financial system. The study's global stability perspectives also

*Corresponding author
E-mail address: saidfahim66@gmail.com
Received November 27, 2024

provide a basis for improving crisis management practices, ensuring that institutions are better prepared to handle liquidity shocks. Additionally, integrating advanced modeling techniques encourages innovation in financial risk management, equipping institutions with enhanced predictive and responsive capabilities. By specifying key variables such as asset liquidity, interest rates, and other relevant factors, the model provides a solid foundation for a more comprehensive understanding of the underlying mechanisms of liquidity risk contagion in the banking system.

Keywords: mathematical analysis; liquidity risk; time delay; dynamic contagion model; banking system; global stability.

2020 AMS Subject Classification: 92D30.

1. INTRODUCTION

The global banking crisis, which began in the summer of 2007 [1, 2], exposed significant deficiencies in banking risk management, especially concerning general and liquidity risks. These risks were often overlooked in favor of other types such as credit risk or market risk.

Liquidity, defined as an institution's ability to fulfill its obligations as they come due, is a critical component of the banking sector. The interbank market, which facilitates connections between banks through interbank deposits and credit lines, plays a vital role in maintaining the overall liquidity of the banking system [3].

Financial markets have become more interconnected due to globalization and technological advancements, leading to greater market efficiency but also increased risks of financial contagion and systemic risk. This interconnectedness means that shocks in one market can quickly spread to others, as evidenced by recent events such as the collapse of Silicon Valley Bank (SVB). Recent studies on the SVB collapse have highlighted its negative impact on global stock markets and the increase in abnormal volatility, underscoring the fragility of the financial system. The SVB bankruptcy, the second largest bank failure in U.S. history, caused significant disruptions globally, affecting markets in the UK, Australia, China, Japan, South Korea, and Europe.[4, 5, 6, 7]are notable studies on this topic.

Contagion of liquidity risk often spreads through credit lines between banks. When certain banks are financially dependent on these credit lines, their depletion can lead to contagion of liquidity risk throughout the banking system [8].

According to the reference study [9], contagion refers to the probability of a crisis in one country spreading to other countries due to a similar event.

Mathematical modeling of liquidity risk contagion in the banking system has gained increasing interest among researchers and practitioners. The use of delayed differential equations provides a precise approach to study the propagation of liquidity risk with delays. These equations allow for the consideration of temporal dependencies of state variables and a better understanding of the dynamics of the banking system [8].

The mathematical modeling of liquidity risk contagion in the banking system is an important area of research with practical applications in risk management and financial stability. Delayed differential equations offer a solid mathematical framework to study this complex phenomenon and provide valuable insights for decision-makers and practitioners in the banking sector [8, 10, 11, 12, 13, 14].

In this article, we propose a mathematical model with delay to study this problem. The rest of this article is organized as follows: Section 2 presents a review of related work concerning the techniques used for modeling liquidity risk contagion in the banking system. Section 3 addresses the boundedness and positivity of the solutions. Section 4 describes the stability analysis of the solutions. In Section 5, we illustrate the theoretical results with numerical simulations, while Section 6 enumerates the conclusions.

2. Literature Review

In recent years, several studies have explored the modeling of liquidity risk contagion using delayed differential equations. In references [10, 11, 12, 13, 14], the authors have used the SIR (Susceptible-Infectious-Recovered) model for modeling liquidity risk contagion in the banking system.

In reference [14], the authors have proposed an effective model for the propagation of liquidity shocks in the interbank market, using a contagion mechanism similar to epidemic models. The primary goal of this model is to assess systemic liquidity risk, particularly funding liquidity risk, by incorporating the specific properties of banks and the structure of the European interbank network. The model measures potential systemic risk using simplified dynamics and aggregated balance sheet data, offering a less complex alternative to detailed models. The study also emphasizes the importance of understanding and monitoring systemic liquidity risk for macroprudential policy and financial stability. However, the model uses a simplified contagion process to represent liquidity shocks. This simplification may not capture all the complexities and nuances of real liquidity dynamics, especially during a financial crisis. The model relies on aggregated balance sheet data, which may not provide the necessary granularity to accurately assess systemic risk. More detailed data, such as exposures broken down by seniority or contract type, could offer better insights. The model does not account for how banks might reallocate assets in response to shocks, nor for interventions by central banks or governments. These factors can significantly influence real outcomes during a liquidity crisis. The model depends on several assumptions and parameters, which may not be entirely accurate or reflect real-world conditions. These assumptions could affect the reliability of the results. The study focuses on the European interbank market, which may limit the generalization of the results to other regions or global financial systems. While the model is simpler and requires less detailed data, it may not be as accurate or comprehensive as more sophisticated approaches. The trade-off between simplicity and accuracy could be a limitation.

On the other hand, references [15, 16] both use the SIR model but differ in the details of the studies and the types of epidemiological models discussed. They discuss the theory of optimal control, with more details on specific interventions in the first article. This one focuses on the various applications of epidemiological models and only mentions the crisis at the beginning of the 21st century. Meanwhile, the second draws an analogy with infectious diseases and emphasizes globalization and the 2008 financial crisis, favoring a robust theoretical framework.

This methodological approach is often limited to a local perspective of stability, placing less emphasis on complex temporal dependencies and delays in risk transmission, as well as less importance on the impact of central bank policies and overall stability. A model that may lack details on precise temporal dynamics and delayed effects. We can summarize the weaknesses of the approach used in these studies in the following three points:

(1) Although epidemiological models like SIR are useful, their direct application to financial systems can overly simplify the complexity of economic interactions. Financial

dynamics can be influenced by many factors not accounted for by these simple epidemiological models,

(2) Monte Carlo simulations and empirical studies may not fully capture the reality of financial crises, especially if they rely on simplified assumptions or limited datasets. The simulation results heavily depend on the initial assumptions and chosen parameters,

(3) Theoretical optimal control measures may not be easily applicable in the real world due to political, economic, and social constraints,

To deal with limitations, we propose a mathematical model based on delayed differential equations to model the propagation of liquidity risk. This model allow us to identify conditions of global stability within the banking system, evaluate the impact of central bank interventions on economic stability, and highlight the significance of temporal variables in risk propagation.

3. Mathematics Model

We will consider the differential system is an extension of the research work [8] and based on the research work [17]

$$\begin{cases} \frac{df_1(t)}{dt} = -\alpha_1 f_1(t) f_2(t-\tau) - \alpha_2 f_1(t), \\ \frac{df_2(t)}{dt} = \alpha_1 f_1(t) f_2(t-\tau) - \alpha_3 f_2(t), \\ \frac{df_3(t)}{dt} = \alpha_2 f_1(t) + \alpha_3 f_2(t). \end{cases} \tag{3.1}$$

with

$f_1(t)$: The set of banks at time t whose general ratio is greater than 1.

$f_2(t)$: The set of banks at time t whose general ratio is less than 1.

$f_3(t)$: The set of banks that have gone bankrupt at time t.

where with the initial conditions,

$$t = 0, f_1(0) = a \neq 0, f_2(0) = 0, f_3(0) = 0. \tag{3.2}$$

$$f_1(t) + f_2(t) + f_3(t) = N$$

N denotes the number of banking system at time t, and

α_1: Spread rate. Parameter α_1 indicates the speed at which contagion spreads. It is a measure of the contagiousness of an infected bank, where a susceptible bank in contact with a single infected bank will become infected with probability α_1.

α_2: Solution rate. This second parameter, represents the speed of recovery. It is a measure of the resistance to contagion. Accordingly, an infected bank will recover with probability α_2.

α_3: Bankruptcy rate. This third parameter represents the rate at which an infected bank will go bankrupt. Therefore a distressed bank will go bankrupt with a probability α_3.

and

τ: delay, which represents the time lag between the infection of a bank and the transmission of liquidity risk to other banks.

The first equation, $\frac{df_1(t)}{dt} = -\alpha_1 f_1 f_2(t-\tau) - \alpha_2 f_1(t)$, represents the rate of change of susceptible banks over time. The first term, $-\alpha_1 f_1 f_2(t-\tau)$, indicates the loss of susceptible banks due to infection from infectious banks with a delay of τ. The parameter α_1 represents the transmission rate, which is the probability that a susceptible bank gets infected by an infectious bank. The second term, $-\alpha_2 f_1(t)$, represents the rate at which susceptible banks recover, with α_2 being the recovery rate.

The second equation, $\frac{df_2(t)}{dt} = \alpha_1 f_1 f_2(t-\tau) - \alpha_3 f_2(t)$, describes the rate of change of infectious banks $f_2(t)$ over time. The first term, $\alpha_1 f_1 f_2(t-\tau)$, represents the gain of infectious banks from the transmission of infectious banks with a delay of τ. The second term, $-\alpha_3 f_2(t)$, indicates the rate at which infectious banks recover directly without going through the susceptible phase. The parameter α_3 is the transition rate from infectious to bankrupt.

The third equation, $\alpha_2 f_1(t) + \alpha_3 f_2(t)$, describes the rate of change of banks transitioning to bankruptcy $f_3(t)$ over time. The first term, $\alpha_2 f_1(t)$, represents the rate at which susceptible banks recover and transition to the recovered phase. The second term, $\alpha_3 f_2(t)$, indicates the rate at which infectious banks transition to bankruptcy.

By solving these simultaneous differential equations with appropriate initial conditions for $f_1(t)$, $f_2(t)$, and $f_3(t)$, you can study the propagation of liquidity risks in the banking system with a delay. The values of the parameters α_1, α_2, α_3, and τ need to be determined based on the specific context of the study and the characteristics of the considered banking system.

Our Model:

We propose a bank dynamics model inspired by the research work [8]. This model is based on a system of differential equations that describes the evolution of three sets of banks according to their financial health:

$f_1(t)$: The number of banks at time t with a general ratio greater than 1 (healthy).

$f_2(t)$: The number of banks at time

t with a general ratio less than 1 (in distress).

$f_3(t)$: The number of banks at time

t that have gone bankrupt.

N representing the total number of banks.

The parameters:

α_1: Contagion rate.

α_2: Recovery rate.

α_3: Bankruptcy rate.

Common Aspect with the Model of [8]: Our model shares fundamental characteristics with the model presented in [8], which also examines the dynamics of banks based on their financial health. Both models utilize differential equations to model the transition between the states of banks. For instance, the structure of the equations and the classification of banks into three groups (healthy, at risk, bankrupt) are common elements.

New Aspect:

The main novelty of our model lies in the introduction of a delay (τ) in the first equation. This delay represents the time elapsed between when a bank is infected by liquidity risk and when this contagion spreads to other banks.

Temporal Reality: Unlike the model in [8], which assumes instant transmission of risks, our model captures the temporal lag that may exist in the propagation of banking crises. This delay makes our modeling more realistic by considering situations where the effects of a crisis do not manifest immediately.

Modified Dynamics: The introduction of this delay can alter the overall dynamics of the system, allowing for a better understanding of how a financial crisis can develop and intensify over time, rather than spreading instantaneously.

In summary, our model differs from that of [8] by incorporating a delay in the propagation of risks, thus adding an essential temporal dimension for more accurately modeling contagion in the banking system.

4. Bornitude and Positivity of Solutions

In this section, we will establish the positivity and the Bornitude of the solutions of the model (3.1).

4.1. Positivity of solutions.

Proposition 4.1.
For any positive initial condition $f_1(0)$, $f_2(0)$, $f_3(0)$ model variables (3.1) $f_1(t)$, $f_2(t)$,and $f_3(t)$ will remain positive for everything $t > 0$.

Proof. We will prove that for positive initial conditions, i.e., $f_1(0) = a > 0$, $f_2(0) = 0$, and $f_3(0) = 0$, the solutions $f_1(t)$, $f_2(t)$, and $f_3(t)$ remain positive for all $t > 0$.

To simplify the notations, we will denote f_1, f_2, and f_3 instead of $f_1(t)$, $f_2(t)$, and $f_3(t)$, respectively.

(1) Positivity of f_1:

The derivative of f_1 with respect to time is given by:

$$\frac{df_1}{dt} = -\alpha_1 f_1 f_2(t-\tau) - \alpha_2 f_1.$$

Since $f_1(0) = a > 0$ and $f_2(t) \geq 0$ for all t, the terms $-\alpha_1 f_1 f_2(t-\tau)$ and $-\alpha_2 f_1$ are both negative or zero. Therefore, $\frac{df_1}{dt} \leq 0$. Let's assume that there exists a $T > 0$ such that $f_1(T) = 0$. This would mean that f_1 reaches its minimum at T. However, assuming the continuity of f_1, this would imply that $\frac{df_1}{dt}$ must be strictly positive at T to move away from the minimum value of f_1. This contradicts $\frac{df_1}{dt} \leq 0$. Therefore, $f_1(t) > 0$ for all $t > 0$.

(2) Positivity of f_2:

The derivative of f_2 with respect to time is given by:

$$\frac{df_2}{dt} = \alpha_1 f_1 f_2(t-\tau) - \alpha_3 f_2.$$

Since $f_1 > 0$ for all $t > 0$ and $f_2(t) \geq 0$ for all t, the terms $\alpha_1 f_1 f_2(t-\tau)$ and $-\alpha_3 f_2$ are both positive or zero. Therefore, $\frac{df_2}{dt} \geq 0$. Let's assume that there exists a $T > 0$ such that $f_2(T) = 0$. This would mean that f_2 reaches its minimum at T. However, assuming the continuity of f_2, this would imply that $\frac{df_2}{dt}$ must be strictly negative at T to move away from the minimum value of f_2. This contradicts $\frac{df_2}{dt} \geq 0$. Therefore, $f_2(t) > 0$ for all $t > 0$.

(3) Positivity of f_3:

The derivative of f_3 with respect to time is given by:

$$\frac{df_3}{dt} = \alpha_2 f_1 + \alpha_3 f_2.$$

Since $f_1 > 0$ and $f_2 \geq 0$ for all $t > 0$, the terms $\alpha_2 f_1$ and $\alpha_3 f_2$ are both positive or zero. Therefore, $\frac{df_3}{dt} \geq 0$. Let's assume that there exists a $T > 0$ such that $f_3(T) = 0$. This would mean that f_3 reaches its minimum at T. However, assuming the continuity of f_3, this would imply that $\frac{df_3}{dt}$ must be strictly negative at T to move away from the minimum value of f_3. This contradicts $\frac{df_3}{dt} \geq 0$. Therefore, $f_3(t) > 0$ for all $t > 0$.

In summary, we have shown that the solutions $f_1(t)$, $f_2(t)$, and $f_3(t)$ of the system remain positive for all $t > 0$ with the initial conditions $f_1(0) = a > 0$, $f_2(0) = 0$, and $f_3(0) = 0$. □

4.2. Bornitude of solutions.

Proposition 4.2.

the solutions f_1, f_2 and f_3, of the system (3.1) *are bounded.*

Proof. To demonstrate the boundedness of the system, we need to show that the variables of the system remain bounded for all values of time t.

Let's assume that we have a solution $(f_1(t), f_2(t), f_3(t))$ of the given differential system. We will demonstrate that each component of this solution remains bounded as t varies within a given interval.

Let's consider the first equation of the system:

$$\frac{df_1(t)}{dt} = -\alpha_1 f_1(t) f_2(t-\tau) - \alpha_2 f_1(t)$$

The terms $-\alpha_1 f_1(t) f_2(t-\tau)$ and $-\alpha_2 f_1(t)$ lead to a decrease in $f_1(t)$ over time. Since α_1 and α_2 are positive, these terms act as decay factors. Consequently, $f_1(t)$ remains bounded.

Moving to the second equation:

$$\frac{df_2(t)}{dt} = \alpha_1 f_1(t) f_2(t-\tau) - \alpha_3 f_2(t)$$

The term $\alpha_1 f_1(t) f_2(t-\tau)$ results in an increase in $f_2(t)$, while the term $-\alpha_3 f_2(t)$ leads to a decrease. As α_1 and α_3 are positive, the growth term outweighs the decay term, implying that $f_2(t)$ remains bounded.

Finally, the third equation is:

$$\frac{df_3(t)}{dt} = \alpha_2 f_1(t) + \alpha_3 f_2(t)$$

Both terms on the right contribute to an increase in $f_3(t)$, as α_2 and α_3 are positive. Thus, $f_3(t)$ remains bounded.

In conclusion, by examining each equation individually, we observe that all variables $f_1(t)$, $f_2(t)$, and $f_3(t)$ remain bounded. Consequently, the solutions of the system remain bounded for all values of time t. This demonstrates the boundedness of the system. □

5. Stability Analysis

We will analyze the global stability [18, 19, 20, 21, 22, 23, 24, 25] of the differential system given by equations (3.1). The system models the evolution of three interconnected banks in the presence of contagion and bankruptcy. Our objective is to examine the equilibrium points of the system and analyze their global stability.

5.1. Equilibrium Points.

To find the equilibrium points, we need to solve the system of differential equations [26, 27] by setting the rates of change to zero:

$$\frac{df_1}{dt} = -\alpha_1 f_1 f_2(t-\tau) - \alpha_2 f_1 = 0, \tag{5.1}$$

$$\frac{df_2}{dt} = \alpha_1 f_1 f_2(t-\tau) - \alpha_3 f_2 = 0, \tag{5.2}$$

$$\frac{df_3}{dt} = \alpha_2 f_1 + \alpha_3 f_2 = 0. \tag{5.3}$$

From the third equation, we can express f_2 in terms of f_1:

$$f_2 = -\frac{\alpha_2}{\alpha_3} f_1. \tag{5.4}$$

Substituting this expression into the second equation, we get:

$$\alpha_1 f_1 \left(-\frac{\alpha_2}{\alpha_3}\right) f_1(t-\tau) - \alpha_3 f_2 = 0. \tag{5.5}$$

Simplifying and solving for f_1, we find:

$$f_1(t-\tau) = \frac{\alpha_3}{\alpha_1}. \tag{5.6}$$

This means that the equilibrium points are characterized by a constant f_1 with a value of $\frac{\alpha_3}{\alpha_1}$, f_2 proportional to f_1 with a proportionality constant of $-\frac{\alpha_2}{\alpha_3}$, and $f_3 = 0$.

5.2. Jacobian Matrix. The Jacobian matrix J of the system is given by [17, 28, 29]

$$J = \begin{bmatrix} \alpha_1 f_2(t-\tau) - \alpha_2 & -\alpha_1 f_1(t-\tau) \\ \alpha_1 f_2(t-\tau) & \alpha_1 f_1(t-\tau) - \alpha_3 \\ \alpha_2 & \alpha_3 \end{bmatrix}. \tag{5.7}$$

Evaluated at the equilibrium point $(f_1, f_2, f_3) = \left(\frac{\alpha_3}{\alpha_1}, -\frac{\alpha_2}{\alpha_3} f_1, 0\right)$, the Jacobian matrix becomes:

$$J_{\text{eq}} = \begin{bmatrix} \alpha_2 & -\alpha_1\left(\frac{\alpha_3}{\alpha_1}\right) \\ \alpha_1\left(-\frac{\alpha_2}{\alpha_3}\right) & \alpha_1\left(\frac{\alpha_3}{\alpha_1}\right) - \alpha_3 \\ \alpha_2 & \alpha_3 \end{bmatrix}. \tag{5.8}$$

Let's now calculate the eigenvalues of J_{eq}.

5.3. Calculation of Eigenvalues. The characteristic equation of J_{eq} is:

$$\det(J_{\text{eq}} - \lambda I) = 0, \tag{5.9}$$

where I is the identity matrix. Let's solve this equation for λ:

$$\begin{vmatrix} \alpha_2 - \lambda & -\alpha_1\left(\frac{\alpha_3}{\alpha_1}\right) & \alpha_1\left(-\frac{\alpha_2}{\alpha_3}\right) & \alpha_1\left(\frac{\alpha_3}{\alpha_1}\right) - \alpha_3 - \lambda \end{vmatrix} = 0. \tag{5.10}$$

After calculations, we obtain the eigenvalues:

$$\lambda_1 = -\alpha_2, \tag{5.11}$$

$$\lambda_2 = \frac{\alpha_1 \alpha_3}{\alpha_3} - \alpha_3 = -\alpha_2, \tag{5.12}$$

$$\lambda_3 = \frac{\alpha_1 \alpha_3}{\alpha_3} - \alpha_3 - \alpha_2. \tag{5.13}$$

5.4. Interpretation of Eigenvalues. The eigenvalues provide insights into the stability of the equilibrium point. In our case, all three eigenvalues are equal to $-\alpha_2$, except for the third one which is $-\alpha_2 - \alpha_3$.

5.4.1. *Eigenvalue* λ_1. The first eigenvalue $\lambda_1 = -\alpha_2$ is repeated twice. This indicates that the system has a direction in which the dynamics are slower, but all trajectories remain close to the equilibrium point. This slowness is due to the variable f_1.

5.4.2. *Eigenvalue* λ_2. The second eigenvalue $\lambda_2 = -\alpha_2$ also indicates marginal stability in the direction related to f_1. Trajectories stay close to the equilibrium point, but with slower convergence.

5.4.3. *Eigenvalue* λ_3. The third eigenvalue $\lambda_3 = -\alpha_2 - \alpha_3$ is different from the first two. If α_3 is small compared to α_2, then λ_3 is close to $-\alpha_2$, meaning that the dynamics in the f_1 direction are also stable.

This concludes our analysis of the global stability of the differential system. The eigenvalues indicate that trajectories stay close to the equilibrium point despite minor variations in initial conditions. The stability of the system depends on the parameters α_1, α_2, and α_3. A more detailed interpretation of the results would require specific numerical values for these parameters.

6. Numerical Simulation

In this section, we present numerical results to demonstrate the performance of the proposed approach and compare it with the traditional method, which uses fixed weighting coefficients in the cost function. The model defined by the system of equations is analyzed.

For the numerical simulation in a real-world context, the propagation and recovery rates, denoted as α_1 and α_2, are taken from the work of Philipps, Koutelidakis, and Leontitsis [31].

The data, based on 169 major European banks in 2012, were computed in [31]. Thus, in our simulation, we assume $N = 169$, representing the total number of banks, with 10 banks initially affected by liquidity risk at $t = 0$.

Three initial liquidity crisis scenarios have been considered for each case: one in Spain, one in France, and one in Germany. The data for these scenarios are provided in the following tables: (1), (2), and (3).

Parameters	Fig.2a	Fig.3a	Source
α_1	0.005	0.005	[32]
α_2	0.013	0.013	[32]
α_3	0.1	0.0001	Assumed
$f_1(0)$	158.0	158.0	[32]
$f_2(0)$	10.0	10.0	[32]
$f_3(0)$	0.0	0.0	[32]
τ_1	5.0	5.0	Assumed
τ_2	50.0	50.0	Assumed

TABLE 1. The initial parameter conditions for the model (3.1) in the context of Spain.

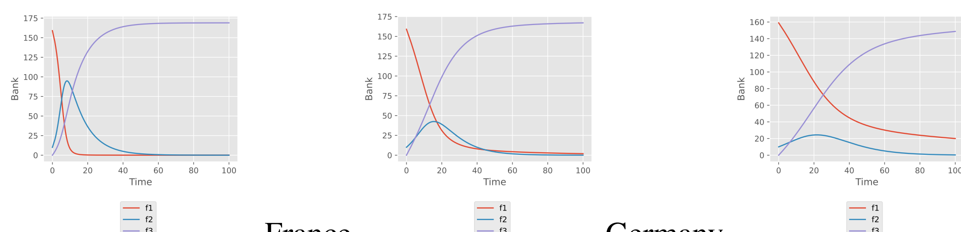
Parameters	Fig.2b	Fig.3b	Source
α_1	0.002	0.002	[32]
α_2	0.02	0.02	[32]
α_3	0.1	0.0001	Assumed
$f_1(0)$	158.0	158.0	[32]
$f_2(0)$	10.0	10.0	[32]
$f_3(0)$	0.0	0.0	[32]
τ_1	5.0	5.0	Assumed
τ_2	50.0	50.0	Assumed

TABLE 2. The initial parameter conditions for the model (3.1) in the context of France.

Parameters	Fig.2c	Fig.3c	Source
α_1	0.0012	0.0012	[32]
α_2	0.008	0.008	[32]
α_3	0.1	0.0001	Assumed
$f_1(0)$	158.0	158.0	[32]
$f_2(0)$	10.0	10.0	[32]
$f_3(0)$	0.0	0.0	[32]
τ_1	5.0	5.0	Assumed
τ_2	50.0	50.0	Assumed

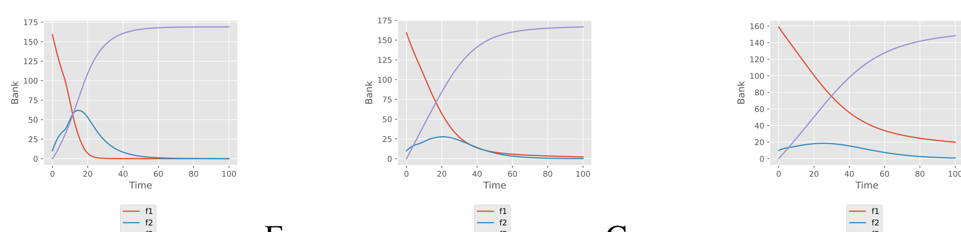
TABLE 3. The initial parameter conditions for the model (3.1) in the context of Germany.

Data on the behavior of the system liquidity risk model (3.1) during the same period is illustrated in Figures 1, 2, 3, and 4.



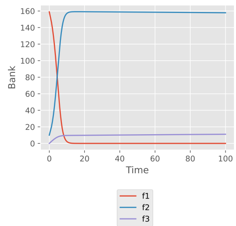
Spain France Germany

FIGURE 1. Liquidity risk with time delay in Spain, France and Germany behavior over time with $\alpha_3 = 0.1$, $\tau_1 = 5$ and $t = 100$.

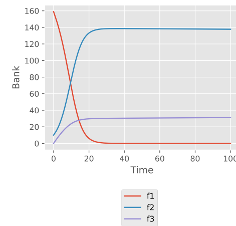


Spain France Germany

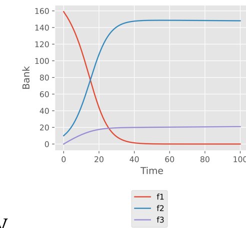
FIGURE 2. Liquidity risk with time delay in Spain, France and Germany behavior over time with $\alpha_3 = 0.1$, $\tau_2 = 50$ and $t = 100$.



Spain

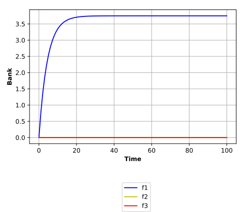


France



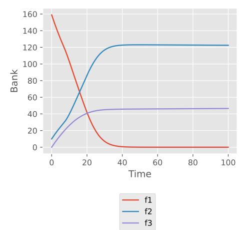
Germany

FIGURE 3. Liquidity risk with time delay in Spain, France and Germany behavior over time with $\alpha_3 = 0.0001$, $\tau_1 = 5$ and $t = 100$.

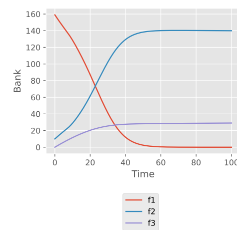


(A)

Spain



France



Germany

FIGURE 4. Liquidity risk with time delay in Spain, France and Germany behavior over time with $\alpha_3 = 0.0001$, $\tau_2 = 50$ and $t = 100$.

- First Figure (1) with a delay of $\tau_1 = 5$:

  A short delay ($\tau_1 = 5$) means that the period during which the central bank reacted to liquidity risk was limited. In Spain, with an insufficient delay, liquidity risk affected 100 banks, indicating a high vulnerability. In France and Germany, the risk reached nearly 25 banks, suggesting relatively better but still significant management.
- Second Figure (2) with a delay of $\tau_2 = 50$:

  A longer delay ($\tau_2 = 50$) indicates that central banks had enough time to implement effective solutions. Liquidity risk decreased, reaching approximately 60 banks in Spain and around 18 banks in France and Germany. This suggests that longer delays allow for more effective management.
- Third Figure (3) with $\alpha_3 = 0.0001$ and $\tau_1 = 5$:

Introducing $\alpha_3 = 0.0001$ (a parameter) and a short delay ($\tau_1 = 5$), liquidity risk remains stable in the three countries due to a negligible failure rate. However, the $\tau_1 = 5$ delay remains insufficient for effective liquidity risk management.

- Fourth Figure (4) with a delay of $\tau_2 = 50$:

  With a longer delay ($\tau_2 = 50$), the fourth figure shows improved management through macroprudential policies, resulting in a reduction of liquidity risk in all three countries.

The overall conclusion emphasizes that the delay is a crucial element in liquidity risk management. Central banks need time to stabilize the banking system effectively. The analysis of the different figures highlights how the duration of the delay influences the spread and management of liquidity risk in the three countries, underscoring the importance of an adequate period to react effectively.

7. Conclusion

We investigated the dynamic contagion behavior in the banking sector with respect to liquidity risk and global stability, using a delayed SIR epidemic model. The parameters for transmission and recovery rates were found to vary based on the country being studied, which also influences the onset of liquidity risk.

The analysis revealed significant negative impacts from three contagion scenarios involving liquidity risk within the banking sector. Banks with substantial financial stakes in Europe are more susceptible to banking crises. To mitigate this, a delay mechanism was proposed to reduce the number of affected banks, curb widespread contagion, and avoid severe financial and economic repercussions.

The recommendations are based on the following results:

- Implications of the results for financial and macroeconomic sectors:

  Our study's results suggest that effective liquidity risk management depends on the factor of time, especially regarding central banks' interventions. This could influence monetary and financial policies, emphasizing the need to consider the optimal timing to stabilize the banking system.

- Study limitations:

  We acknowledge that our model relies on simplifying assumptions and uses simulated data from the largest European banks. These limitations provide opportunities for future research to refine the model and make it more representative of reality.

- Potential avenues for future research:

  It would be interesting to further explore the impact of bank-specific factors on the spread of liquidity risk. The integration of real-time data and an in-depth study of central banks' specific interventions could enhance our understanding of the phenomenon.

- Comparison of model predictions with observed results in European banks:

  The comparative analysis of our model's predictions with the observed results in European banks is crucial. This highlights the strengths and limitations of our model, evaluating its practical relevance. We note that our model may well capture certain aspects of liquidity risk contagion, while other cited studies provide complementary or divergent perspectives.

- Comparison with other cited studies:

  In comparison with the cited studies [32],[33], [34], our study stands out for these studies enrich the research landscape by providing additional nuances or addressing specific aspects that complement our approach.

In summary, the comprehensive discussion section provides a more complete and critical perspective on our research, highlighting its contributions, limitations, and context in relation to existing work.

Conflict of Interests

The authors declare that there is no conflict of interests.

References

[1] J.A. Chan-Lau, M. Espinosa, K. Giesecke, et al. Assessing the Systemic Implications of Financial Linkages, IMF Global Financial Stability Report, Vol. 2, 2009. https://ssrn.com/abstract=1417920.

[2] P. Glasserman, H.P. Young, Contagion in Financial Networks, J. Econ. Lit. 54 (2016), 779–831. https://doi.org/10.1257/jel.20151228.

[3] A.C. Huser, Too Interconnected to Fail: A Survey of the Interbank Networks Literature, SAFE Working Paper No. 91. SSRN, (2015). https://doi.org/10.2139/ssrn.2577241.

[4] D.Y. Aharon, S. Ali, M. Naved, Too Big to Fail: The Aftermath of Silicon Valley Bank (SVB) Collapse and Its Impact on Financial Markets, Res. Int. Bus. Finance 66 (2023), 102036. https://doi.org/10.1016/j.ribaf.2023.102036.

[5] S. Ali, M. Naveed, M. Gubareva, X. Vinh Vo, Reputational Contagion from the Silicon Valley Bank Debacle, Res. Int. Bus. Finance 69 (2024), 102275. https://doi.org/10.1016/j.ribaf.2024.102275.

[6] H. El-Chaarani, Determinants of Bank Liquidity in the Middle East Region, Int. Rev. Manag. Market. 9 (2019), 64-75. https://doi.org/10.32479/irmm.7742.

[7] G. Brandi, R. Di Clemente, G. Cimini, Epidemics of Liquidity Shortages in Interbank Markets, Physica A 507 (2018), 255–267. https://doi.org/10.1016/j.physa.2018.05.104.

[8] H. Mourad, S. Fahim, A. Burlea-Schiopoiu, et al. Modeling and Mathematical Analysis of Liquidity Risk Contagion in the Banking System, J. Appl. Math. 2022 (2022), 5382153. https://doi.org/10.1155/2022/5382153.

[9] F. Allen, D. Gale, Financial Contagion, J. Polit. Econ. 108 (2000), 1–33. https://doi.org/10.1086/262109.

[10] G. Brandi, R. Di Clemente, G. Cimini, Epidemics of Liquidity Shortages in Interbank Markets, Physica A 507 (2018), 255–267. https://doi.org/10.1016/j.physa.2018.05.104.

[11] W.O. Kermack, A.G. McKendrick, A Contribution to the Mathematical Theory of Epidemics, Proc. R. Soc. Lond. A 115 (1927), 700–721. https://doi.org/10.1098/rspa.1927.0118.

[12] J. Guo, X. Wu, Y. Yang, Study on Mechanism of Enterprise Group' Crisis Management Based on the Epidemic Contagion Dynamic Model, in: 2009 Third International Conference on Genetic and Evolutionary Computing, IEEE, Guilin, China, 2009: pp. 717–720. https://doi.org/10.1109/WGEC.2009.187.

[13] O. Kostylenko, H.S. Rodrigues, D.F.M. Torres, Banking Risk as an Epidemiological Model: An Optimal Control Approach, in: A.I.F. Vaz, J.P. Almeida, J.F. Oliveira, A.A. Pinto (Eds.), Operational Research, Springer, Cham, 2018: pp. 165–176. https://doi.org/10.1007/978-3-319-71583-4_12.

[14] V. Macchiati, G. Brandi, T. Di Matteo, D. Paolotti, G. Caldarelli, G. Cimini, Systemic Liquidity Contagion in the European Interbank Market, J. Econ. Interact. Coord. 17 (2022), 443–474. https://doi.org/10.1007/s11403-021-00338-1.

[15] O. Kostylenko, H.S. Rodrigues, D.F.M. Torres, Banking Risk as an Epidemiological Model: An Optimal Control Approach, in: A.I.F. Vaz, J.P. Almeida, J.F. Oliveira, A.A. Pinto (Eds.), Operational Research, Springer, Cham, 2018: pp. 165–176. https://doi.org/10.1007/978-3-319-71583-4_12.

[16] O. Kostylenko, H.S. Rodrigues, D.F.M. Torres, The Risk of Contagion Spreading and Its Optimal Control in the Economy, arXiv:1812.06975 [physics.soc-ph] (2018). https://doi.org/10.48550/arXiv.1812.06975.

[17] W. Ma, M. Song, Y. Takeuchi, Global Stability of an SIR Epidemicmodel with Time Delay, Appl. Math. Lett. 17 (2004), 1141–1145. https://doi.org/10.1016/j.aml.2003.11.005.

[18] J.K. Hale, Theory of Functional Differential Equations, Springer, New York, 1977. https://doi.org/10.1007/978-1-4612-9892-2.

[19] Y. Kuang, Delay Differential Equations with Applications in Population Dynamics, Academic Press, San Diego, 1993.

[20] T. Yoshizawa, Stability Theory by Liapunov's Second Method, The Mathematical Society of Japan, Tokyo, 1966.

[21] E. Beretta, Y. Takeuchi, Convergence Results in SIR Epidemic Models with Varying Population Sizes, Nonlinear Anal.: Theory Methods Appl. 28 (1997), 1909–1921. https://doi.org/10.1016/S0362-546X(96)00035-1.

[22] Y. Takeuchi, W. Ma, E. Beretta, Global Asymptotic Properties of a Delay SIR Epidemic Model with Finite Incubation Times, Nonlinear Anal. 42 (2000), 931-947.

[23] E. Beretta, Y. Takeuchi, Global Stability of an SIR Epidemic Model with Time Delays, J. Math. Biol. 33 (1995), 250–260. https://doi.org/10.1007/BF00169563.

[24] E. Beretta, T. Hara, W. Ma, Y. Takeuchi, Global Asymptotically Stability of an SIR Epidemic Model with Distributed Time Delay, Nonlinear Anal. 47 (2001), 4107-4115.

[25] E. Beretta, T. Hara, W. Ma, Y. Takeuchi, Permanence of an SIR Epidemic Model with Distributed Time Delays, Tohoku Math. J. 54 (2002), 581-591. https://doi.org/10.2748/tmj/1113247650.

[26] W. Wang, Global Behavior of an SEIRS Epidemic Model with Time Delays, Appl. Math. Lett. 15 (2002), 423–428. https://doi.org/10.1016/S0893-9659(01)00153-7.

[27] H.I. Freedman, S.G. Ruan, Uniform Persistence in Functional Differential Equations, J. Differ. Equ. 115 (1995), 173–192. https://doi.org/10.1006/jdeq.1995.1011.

[28] G. Butler, H.I. Freedman, P. Waltman, Uniformly Persistent Systems, Proc. Amer. Math. Soc. 96 (1986), 425–430. https://doi.org/10.1090/S0002-9939-1986-0822433-4.

[29] J.K. Hale, P. Waltman, Persistence in Infinite-Dimensional Systems, SIAM J. Math. Anal. 20 (1989), 388–395. https://doi.org/10.1137/0520025.

[31] D. Philippas, Y. Koutelidakis, A. Leontitsis, Insights into European Interbank Network Contagion, Manag. Finance 41 (2015), 754–772. https://doi.org/10.1108/MF-03-2014-0095.

[32] S. Ali, M. Naveed, H. Hanif, M. Gubareva, The Resilience of Shariah-Compliant Investments: Probing the Static and Dynamic Connectedness between Gold-Backed Cryptocurrencies and GCC Equity Markets, Int. Rev. Financ. Anal. 91 (2024), 103045. https://doi.org/10.1016/j.irfa.2023.103045.

[33] M. Naveed, S. Ali, M. Gubareva, A. Omri, When Giants Fall: Tracing the Ripple Effects of Silicon Valley Bank (SVB) Collapse on Global Financial Markets, Res. Int. Bus. Finance 67 (2024), 102160. https://doi.org/10.1016/j.ribaf.2023.102160.

[34] D.Y. Aharon, S. Ali, M. Naved, Too Big to Fail: The Aftermath of Silicon Valley Bank (SVB) Collapse and Its Impact on Financial Markets, Res. Int. Bus. Finance 66 (2023), 102036. https://doi.org/10.1016/j.ribaf.2023.102036.