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MATHEMATICAL MODELING AND SIMULATION OF STRESS PHENOMENA FOR MEDICAL STUDENTS WITH OPTIMAL CONTROL

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Abstract. Stress is a pervasive issue among medical students, arising from demanding academic workloads, psychological pressures, and limited recovery opportunities. The consequences of unmanaged stress include adverse mental health outcomes, impaired academic performance, and increased burnout rates. This paper presents a novel mathematical model that encapsulates stress dynamics, workload impact, secondary symptoms, and resilience. The model introduces two control variables: $u(t)$, modulating the influence of workload, and $v(t)$, enhancing resilience. A system of nonlinear differential equations describes the interaction among these factors. The framework leverages optimal control strategies to minimize stress levels and improve resilience sustainably. Numerical simulations demonstrate the effectiveness of these strategies, highlighting the potential for precise interventions to mitigate stress. This work provides a foundation for evidence-based decision making in academic and psychological support systems, offering insights into the effective management of stress in demanding educational environment.

Keywords: Stress, Mathematical modeling, Medical analysis, Medical education, Optimal control theory.

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1. Introduction

Stress is a well-known reality for students and educational professionals alike. Popularized as early as 1936 by Hans Selye as "the non-specific response of the body to any demand for change" [2], the term has evolved to encompass psychological and emotional dimensions beyond the simple physiological response. Stress is now recognized as a complex phenomenon with a significant impact on mental and physical health, academic performance and general well-being. Research shows that this type of stress can lead to disorders such as anxiety, depression and even long-term chronic health problems, directly affecting students' quality of life and personal fulfillment [3]. In this context, many factors contribute to increasing student stress, including high workload, pressure to perform, perceived difficulty of subject and perceived social support. This stress, often accompanied by symptoms such as insomnia, fatigue, irritability, and lack of concentration, negatively impacts students ability to prepare effectively and maintain emotional, psychological, and health balance [2, 3, 4].

Mathematical modelling and simulation of stress phenomena for medical students with optimal control. In the context of medical education, the ability to model and simulate stress phenomena can be a valuable tool for students to better understand the physiological and psychological effects of stress. Stress is a pervasive aspect of the medical profession, and effective stress management is critical to the well-being and performance of medical students and practitioners. Mathematical modelling of stress phenomena can involve the use of various techniques such as differential equations, stochastic processes and optimization methods to capture the complex dynamics of the stress response. These models can incorporate factors such as physiological markers (e.g. heart rate, cortisol levels), psychological assessments (e.g. perceived stress, anxiety), and environmental or situational influences (e.g. workload, social support). Simulation of stress phenomena, on the other hand, can provide medical students with the opportunity to explore the effects of different stress-inducing scenarios and the effectiveness of different stress management strategies. Through simulation, students can experiment with different coping mechanisms, evaluate their outcomes and develop personalized approaches to stress management [5, 6].

Optimal control theory can be particularly relevant in this context, as it allows for the identification of the most effective interventions or strategies to manage stress. By formulating the problem as an optimization problem, where the goal is to minimize the negative impacts of stress while maximizing the positive outcomes, optimal control techniques can guide the development of personalized stress management plans for medical students [1]. The integration of mathematical modeling, simulation, and optimal control in the context of stress phenomena for medical students can have several benefits, including: Enhanced understanding of the physiological and psychological mechanisms underlying the stress response; improved ability to identify risk factors and early warning signs of stress-related issues; development of effective stress management strategies tailored to the unique needs and characteristics of individual medical students; opportunities for virtual experimentation and evaluation of stress management interventions, without the risks associated with real-world implementation; promotion of self-awareness and self-regulation skills among medical students, empowering them to proactively manage stress throughout their academic and professional careers. By leveraging these interdisciplinary approaches, medical education can better prepare students to navigate the challenges of the medical field while maintaining their well-being and resilience [7, 8]. The goal of the optimal control problem is to determine the best possible control inputs (such as adjustments to workload, interventions on resilience, or other factors) that minimize the stress experienced by the student over time. The control inputs could include interventions like support systems, study schedules, relaxation practices, or other external factors that can influence the dynamics of the model, see for example [22]. Stress is a common and natural response to challenging situations, but when prolonged or unmanaged, it can lead to significant health problems. Medical students, in particular, are exposed to high stress levels due to the intense academic and emotional demands of their studies [5]. Managing this stress is crucial not only for their health but also for their academic success and professional development. This study uses mathematical modeling and optimal control theory to explore the dynamics of stress and identify optimal interventions to manage stress levels effectively [6]. By simulating stress response systems and applying optimal control methods, we aim to provide actionable strategies for stress management tailored to medical students.

Medical school is known for its intense workload, emotionally taxing demands, and high levels of responsibility. From early in their studies, medical students face a cascade of academic pressures, including examinations, clinical practice, research, and the emotional stress of interacting with patients [9]. As they progress through their academic careers, the cumulative stress can lead to burnout, anxiety, depression, and even physical health problems, such as cardiovascular issues and weakened immune systems [10]. According to studies, stress levels among medical students are significantly higher than in other student populations, with a substantial number reporting that their stress interferes with their well-being and academic performance.

The cognitive load, emotional strain, and time pressures associated with medical education create a unique stress environment. Unlike many other professions, medical students are also required to deal with life-and-death situations early in their training, which adds an extra dimension to the stress they experience [11, 12]. While stress is a natural and sometimes necessary response to challenge, chronic or poorly managed stress can become maladaptive, negatively affecting students learning capabilities, decision-making, and emotional regulation [13, 14, 15].

The physiological response to stress involves multiple systems in the body, primarily the sympathoadrenal system and the HPA axis [23]. These systems trigger the release of stress hormones such as adrenaline and cortisol, which prepare the body for ”fight or flight”. However, if stress persists over time, these systems can have harmful effects on the body, contributing to a range of health problems. Additionally, recovery from stress is an essential process to restore balance in the body, and understanding how to optimize recovery can help mitigate the negative effects of stress. Given the central role that stress plays in medical education, it is essential to understand how stress manifests in the body and how it can be managed effectively. Mathematical modeling offers a powerful tool for analyzing complex systems like stress. By representing the biological processes that underlie stress and recovery, mathematical models can simulate how stress evolves over time and how interventions can influence this evolution. In this context, we turn to an optimal control theory, which provides a framework for determining the best strategies to manage stress. Optimal control theory involves finding control inputs that minimize a predefined cost function, which in this case represents the negative effects of stress on both the individual’s health and academic performance. The cost function includes terms for stress levels, hormone concentrations, and recovery, with penalties applied when these variables exceed desired thresholds [26, 24, 25].

The purpose of this study is to model the stress response of medical students throughout their academic journey and determine optimal strategies for stress management using control theory. We aim to provide an understanding of how different factors contribute to stress and recovery, and how interventions such as relaxation techniques, exercise, and medication can be timed and adjusted for maximum benefit. By applying this model, we can guide students and educators in designing evidence-based strategies for mitigating stress, ultimately leading to better health outcomes and academic success[27]. Overview of the study approach is divided into two main parts: The first part of the study involves developing a set of differential equations that describe the dynamics of stress and recovery in medical students. These equations will take into account the key physiological variables such as: Stress level, academic workload, secondary symptoms and resilience at time. The system of equations represents how stress develops, how workload and symptoms are released in response to stress, and how recovery mechanisms counteract stress over time. In the second part, we apply an optimal control theory to the system of equations. The goal is to determine the best control strategies (e.g., when and how much to apply relaxation techniques or pharmacological treatments) to minimize the overall cost function, which incorporates the negative impacts of prolonged stress. The optimal control will be computed using numerical methods such as the Runge-Kutta integration algorithm for the differential equations and optimization algorithms to find the best intervention policies. Key questions addressed by the study: How does stress evolve over the course of a medical student's education? This involves modeling the physiological responses to academic and clinical pressures and their cumulative impact over time. What is the role of recovery mechanisms in mitigating stress? Understanding how different factors, such as sleep, relaxation techniques, and social support, influence the recovery process and help reduce stress. How can optimal control theory help in finding effective stress management strategies? By optimizing interventions based on physiological responses, we aim to minimize the detrimental effects of stress, optimize recovery, and improve overall student health and performance. Relevance to Medical students are often exposed to a unique set of stressors, including intense workload, long hours, emotional encounters, and the pressure of making critical decisions. Managing these stressors is crucial not only for their health but also for their future ability to care for patients effectively. By applying a mathematical approach to stress management, we provide a quantitative and evidence-based framework for students and educators to understand and mitigate the effects of stress. This study is particularly relevant as it offers medical students a structured way to manage their own stress, using strategies derived from mathematical modeling and optimal control, which could be applied not only to their academic careers but also to their future roles as healthcare providers. In the end, we presented a few numerical simulations to validate the proposed strategies.

2. Mathematical Model Description

Here's a detailed model that follows the steps you requested: Introduction, Modeling, Mathematical study, Optimal control, and Numerical simulation. This model is inspired by system dynamics concepts similar to the SITR model, but adapted to student stress. The proposed model divides students into four groups. Where the variables are defined as follows:

- $X(t)$: Represents the level of stress at time t for a medical student. This variable captures the intensity of stress.
- $Y(t)$: Represents the workload at time t, which could include study hours, academic tasks, exams, etc. This variable influences stress directly.
- $Z(t)$: Represents the secondary effects of stress at time t, such as physical or psychological symptoms (e.g., fatigue, anxiety, burnout,depression).
- $K(t)$: Represents resilience at time t, which is the student's capacity to cope with stress. This could include coping mechanisms, social support, or other factors that moderate the impact of stress (see, FIGURE 1).

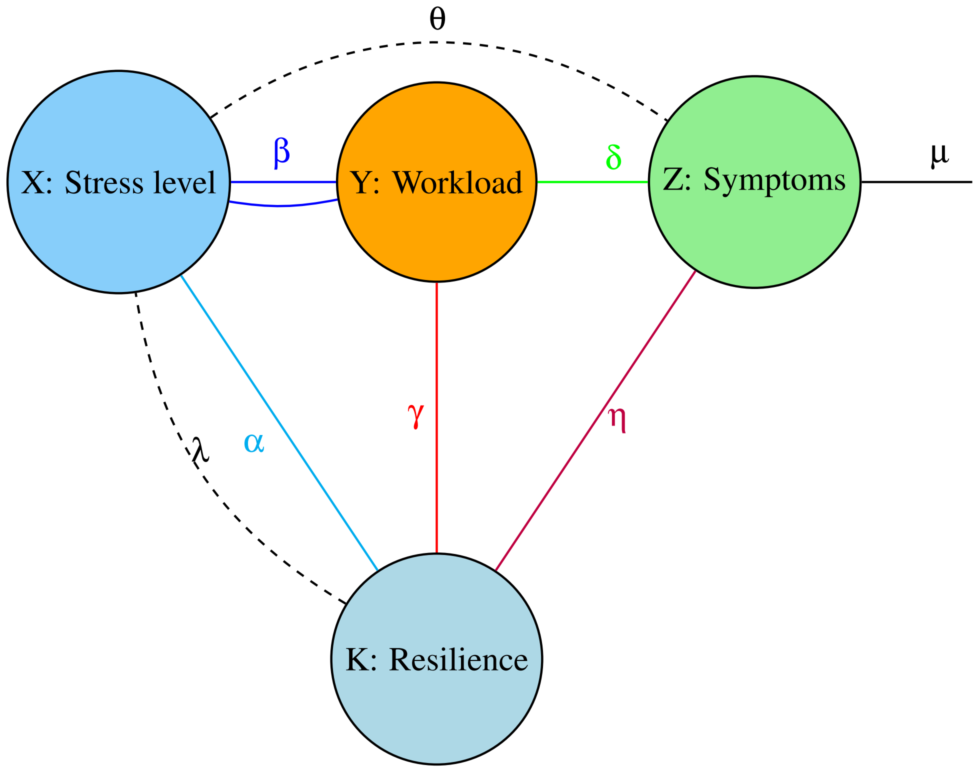

FIGURE 1. Schematic representation of the academic stress model.

Then, this model is modeled by the following differential equation:

$$
\left\{
\begin{aligned}
\frac{dX(t)}{dt} &= -\beta X(t)Y(t) - (\alpha+\theta)X(t) + \lambda K(t)\\
\frac{dY(t)}{dt} &= \beta X(t)Y(t) - \gamma Y(t) - \delta Y(t)\\
\frac{dZ(t)}{dt} &= \theta X(t) + \delta Y(t) - (\eta+\mu)Z(t)\\
\frac{dK(t)}{dt} &= \alpha X(t) + \gamma Y(t) + (\eta+\mu)Z(t) - \lambda K(t)
\end{aligned}
\right. \tag{1}
$$

with $X(t_0) = X_0 \geq 0$, $Y(t_0) = Y_0 \geq 0$, $Z(t_0) = Z_0 \geq 0$ and $K(t_0) = K_0 \geq 0$ are the initial conditions, and $t \in \left[t_0, T_f\right]$, and the coefficients are interpreted by:

- β: A coefficient representing the strength of the influence of workload $Y(t)$ on stress $X(t)$. It quantifies how much stress increases as workload increases.
- α: A coefficient that represents the effect of resilience $K(t)$ on stress $X(t)$. A larger value of α means that greater resilience helps reduce the stress.

- η: A coefficient that represents the effect of resilience $K(t)$ on the secondary symptoms of stress $Z(t)$. This describes how resilience influences the impact of stress on secondary symptoms.
- γ: A coefficient representing the effect of the secondary stress symptoms $Z(t)$ on workload $Y(t)$. This captures how stress symptoms (e.g., fatigue, anxiety) reduce productivity or academic performance.
- δ: A coefficient representing the effect of workload $Y(t)$ on the secondary stress symptoms $Z(t)$. It describes how increasing workload leads to more stress-related symptoms.
- θ: A coefficient that represents the effect of the secondary symptoms $Z(t)$ on resilience $K(t)$. This captures how stress symptoms may reduce the student's ability to recover and cope.
- λ: A coefficient representing the effect of resilience $K(t)$ on stress $X(t)$. It describes how a student's coping mechanisms or social support may reduce stress.
- μ: A coefficient representing the rate at which the secondary stress symptoms $Z(t)$ decay over time. This coefficient captures how a student's coping strategies, recovery, or external interventions reduce the impact of stress symptoms such as fatigue, anxiety, or burnout. A larger value of μ would mean faster recovery from stress-related symptoms.

Each equation represents the rate of change of a variable over time, reflecting the dynamics of stress, workload, symptoms, and resilience. Then, the term $\frac{dX(t)}{dt}$ describes how stress $X(t)$ changes over time. Stress is influenced by workload $Y(t)$, secondary symptoms $Z(t)$, and resilience $K(t)$. The term $\frac{dY(t)}{dt}$ describes how workload $Y(t)$ changes over time. It is influenced by stress $X(t)$, the effect of symptoms $Z(t)$, and the decrease in workload due to fatigue or anxiety. The term $\frac{dZ(t)}{dt}$ describes how the secondary stress symptoms $Z(t)$ evolve over time, influenced by stress $X(t)$, workload $Y(t)$, and resilience $K(t)$, and the term $\frac{dK(t)}{dt}$ describes how resilience $K(t)$ evolves, influenced by stress $X(t)$, workload $Y(t)$, and secondary symptoms $Z(t)$.

3. Mathematical analysis of the model

3.1. Positivity of solution.

Theorem 3.1. *If $X(t_0) \geq 0$, $Y(t_0) \geq 0$,$Z(t_0) \geq 0$ and $K(t_0) \geq 0$, the solutions $X(t)$, $Y(t)$, $Z(t)$ and $K(t)$ of System* (1) *are positive for all $t \geq t_0$.*

Proof. It follows from the first equation of system (1) that

$$\dot{X}(t) = \frac{dX(t)}{dt} = -\beta X(t)Y(t) - (\alpha+\eta)X(t) + \lambda K(t) \geq -\beta X(t)Y(t) - (\alpha+\eta)X(t),$$

$$\Longrightarrow \frac{dX(t)}{dt} + (\beta Y(t) + (\alpha+\eta))X(t) \geq 0,$$

by multiplying the both side of the last equality by $\exp\left(\int_{t_0}^{t} (\beta Y(s) + (\alpha+\eta))ds\right)$, we obtain

$$\frac{dX}{dt}\exp\left(\int_{t_0}^{t} (\beta Y(s) + (\alpha+\eta))\,ds\right) + \beta X(t)Y(t)\left(\exp\left(\int_{t_0}^{t} (\beta Y(s) + (\alpha+\eta))\,ds\right)\right) \geq 0.$$

Then,

$$\frac{d}{dt}\left(X(t)\exp\left(\int_{t_0}^{t} (\beta Y(s) + (\alpha+\eta))\,ds\right)\right) \geq 0,$$

integrating this inequality from t_0 to t gives

$$X(t)\exp\left(\int_{t_0}^{t} (\beta Y(s) + (\alpha+\eta))\,ds\right) - X(t_0) \geq 0.$$

Then, $X(t)\exp\left(\int_{t_0}^{t} (\beta Y(s) + (\alpha+\eta))\,ds\right) \geq X(t_0)$. Hence,

$$X(t) \geq X(t_0)\exp\left(-\int_{t_0}^{t} (\beta Y(s) + (\alpha+\eta))\,ds\right).$$

i.e., $X(t) \geq 0$. Similarly, we show the positiveness of $Y(t)$, $Z(t)$ and $K(t)$. □

3.2. Boundedness of solutions.

Theorem 3.2. *The set $\Omega = \left\{(X,Y,Z,K) \in \mathbb{R}^4 / 0 \leq X+Y+Z+K \leq C\right\}$ positively invariant under system* (1) *with initial conditions $X(t_0) \geq 0$, $Y(t_0) \geq 0$, $Z(t_0) \geq 0$ and $K(t_0) \geq 0$.*

Proof. We presume that, $N(t) = X(t) + Y(t) + Z(t) + K(t)$.
So, $\frac{dN}{dt} = 0$ then $N(t) = C = N(t_0)$. Hence, we will have $0 \leq N(t) \leq C$ it implies the region Ω is positively invariant set for the system. □

3.3. Solution existence.

Theorem 3.3. *The system* (1) *satisfies a given initial condition, then it has a unique solution.*

Proof. Let, $\phi(t) = \begin{pmatrix} X(t) \\ Y(t) \\ Z(t) \\ K(t) \end{pmatrix}$ and $\phi_t(t) = \begin{pmatrix} \frac{dX}{dt} \\ \frac{dY}{dt} \\ \frac{dZ}{dt} \\ \frac{dK}{dt} \end{pmatrix}$.

So, the system (1) can be rewritten in the following form $\phi_t(t) = A\phi + N(\phi)$, with,

$$A = \begin{pmatrix} -(\alpha+\theta) & 0 & 0 & \lambda \\ 0 & -(\gamma+\delta) & 0 & 0 \\ \theta & \delta & -(\eta+\mu) & 0 \\ \alpha & \gamma & \eta+\mu & -\lambda \end{pmatrix}, \quad N(\phi) = \begin{pmatrix} -\beta XY \\ \beta XY \\ 0 \\ 0 \end{pmatrix}$$

and ϕ_t denotes derivative of ϕ with respect to time. The second term on the right-hand side satisfies

$$|N(\phi_1) - N(\phi_2)| = |N_1(\phi_1) - N_1(\phi_2)| + |N_2(\phi_1) - N_2(\phi_2)|,$$

then by applying the triangular inequality, we have

$$\begin{aligned} |N(\phi_1) - N(\phi_2)| &\leq 2\sigma M(|X_1 - X_2| + |Y_1 - Y_2|) \\ &\leq 2\sigma M|X_1 - X_2| + 2\sigma M|Y_1 - Y_2| \\ &\leq C|\phi_1 - \phi_2| \end{aligned}$$

with $C = 2\sigma M > 0$. Thus, it follows that the function N is uniformly Lipschitz continuous, therefore the solution of the system exists. □

4. Optimal Control of the Model

In this section, we define the controlled model linked to the proposed model, the existence of the optimal control and the characterization of he optimal control. Then, to define the optimal control of the model, we aim to find a control strategy that minimizes or maximizes a certain objective, subject to the constraints imposed by the system of differential equations. In the context of your stress model for medical students, the objective could be minimizing the overall stress or workload while optimizing resilience and secondary stress symptoms.

4.1. Optimal control formulation. In an optimal control framework, we need to introduce control inputs that can modify the system's dynamics. The control variables $u(t)$ and $v(t)$ represent the time-dependent decisions that influence the stress level $X(t)$ and the resilience $K(t)$. The coefficients are as follows:

- $u(t)$: Control to modify the impact of the workload on the stress.
- $v(t)$: Control to modify the impact of resilience on the stress and secondary symptoms.

Then, the controlled optimal of the proposed model (1) is given by the following system:

$$
\begin{cases}
\dfrac{dX(t)}{dt} = -\beta(1-u(t))X(t)Y(t) - (\alpha+\theta+v(t))X(t) + \lambda K(t) \\
\dfrac{dY(t)}{dt} = \beta(1-u(t))X(t)Y(t) - \gamma Y(t) - \delta Y(t) \\
\dfrac{dZ(t)}{dt} = \theta X(t) + \delta Y(t) - (\eta+\mu)Z(t) \\
\frac{dK(t)}{dt} = (\alpha+v(t))X(t) + \gamma Y(t) + (\eta+\mu)Z(t) - \lambda K(t)
\end{cases} \tag{2}
$$

with $X(t_0) = X_0 \geq 0$, $Y(t_0) = Y_0 \geq 0$, $Z(t_0) = Z_0 \geq 0$ and $K(t_0) = K_0 \geq 0$ are the initial conditions, and $t \in \left[t_0, T_f\right]$.

The objective of the optimal control problem is to minimize the total stress over time while considering the costs of applying control inputs. The objective function is given by:

$$
J(u,v) = \int_{t_0}^{T_f} \left[\alpha_X X(t)^2 + \alpha_Y Y(t)^2 + \alpha_Z Z(t)^2 + \alpha_K K(t)^2 + \beta_u u(t)^2 + \beta_v v(t)^2\right] dt,
$$

where $\alpha_X, \alpha_Y, \alpha_Z, \alpha_K$ are the weights on the state variables, and β_u and β_v are the weights on the control variables. In other words, we are searching for the optimal values u^* and v^* of the commands u and v such that

$$
J(u^*, v^*) = \min\left\{J(u,v) \,|\, (u,v) \in S\right\},
$$

with S is the admissible controls set defined by

$$
S = \left\{(u,v) \in \left(L^\infty\left(\left[t_0, T_f\right]\right)\right)^2, \forall t \in \left[t_0, T_f\right], 0 \leq u(t), v(t) \leq 1\right\}.
$$

In this section, we will prove the existence of an optimal control by using a result of [18, 19, 17]

Theorem 4.1. *There exists control functions u^*, v^* such that*

$$J(u^*, v^*) = \min_{(u,v)\in S} J(u(t), v(t)).$$

Proof. It is simple to confirm that an optimal control exists in order to demonstrate that:

C_1 : The set of controls, and corresponding state variables, is not empty.

C_2 : The admissible set S is convex and closed.

C_3 : The right-hand side of the state system is bounded by a linear function in the state and control variables.

C_4 : The integrand L of the objective functional defined in (3) is convex on S.

C_5 : There exist constants $\delta_1 \geq 0$, $\delta_2 \geq 0$ and $\eta \geq 1$ such the integrand $L(X,Y,Z,K,u,v)$ of the objective functional satisfies.

$$L(X,Y,Z,K,u,v) \geq \delta_2 + \delta_1(|u^2| + |v^2|)^{\eta/2}$$

with

$$L(X,Y,Z,K,u,v) = \alpha_X X(t)^2 + \alpha_Y Y(t)^2 + \alpha_Z Z(t)^2 + \alpha_K K(t)^2 + \beta_u u(t)^2 + \beta_v v(t)^2. \quad (3)$$

The first condition C_1 is verified using Luke's results [18]. The set S is convex and closed by definition, thus the condition C_2. Our state system is bounded hence the condition C_3. Note that the integrand of our objective function is convex from where conditions C_4. To prove the condition C_5, we have $u + v \geq |u|^2 + |v|^2$ since $0 \leq u, v \leq 1$, from which the last condition is derived. The result follows directly from (Fleming and Rishel [19]). □

4.2. Optimal solution and characterization.

4.2.1. *Hamiltonian formulation.* To apply the Pontryagin's Maximum Principle, we define the Hamiltonian H as the Lagrangian of the system (for more details, see for example, [8, 20, 16, 21, 8]). The Hamiltonian is given by:

$$\begin{aligned} H =& \lambda_X \left(-\beta(1-u(t))X(t)Y(t) - (\alpha+\theta+v(t))X(t) + \lambda K(t)\right) \\ &+ \lambda_Y \left(\beta(1-u(t))X(t)Y(t) - \gamma Y(t) - \delta Y(t)\right) + \lambda_Z \left(\theta X(t) + \delta Y(t) - (\eta+\mu)Z(t)\right) \\ &+ \lambda_K \left((\alpha+v(t))X(t) + \gamma Y(t) + (\eta+\mu)Z(t) - \lambda K(t)\right) + \beta_u u(t)^2 + \beta_v v(t)^2, \end{aligned}$$

where $\lambda_X, \lambda_Y, \lambda_Z, \lambda_K$ are the costate variables corresponding to $X(t), Y(t), Z(t), K(t)$, respectively. The Hamiltonian is maximized with respect to the control variables $u(t)$ and $v(t)$.

4.2.2. *First-Order Optimality Conditions.* To find the optimal controls $u(t)$ and $v(t)$, we differentiate the Hamiltonian with respect to $u(t)$ and $v(t)$, and set these derivatives equal to zero.

Control $u(t)$:

$$\frac{\partial H}{\partial u(t)} = 0 \Longrightarrow \frac{\partial}{\partial u(t)}\left[\lambda_X\left(-\beta(1-u(t))X(t)Y(t)\right) + \lambda_Y\left(\beta(1-u(t))X(t)Y(t)\right) + \beta_u u(t)^2\right] = 0.$$

$$\Longrightarrow \lambda_X \beta X(t)Y(t) - \lambda_Y \beta X(t)Y(t) + 2\beta_u u(t) = 0.$$

$$\Longrightarrow (\lambda_X - \lambda_Y)\beta X(t)Y(t) + 2\beta_u u(t) = 0.$$

$$\Longrightarrow 2\beta_u u(t) = (\lambda_Y - \lambda_X)\beta X(t)Y(t).$$

$$\Longrightarrow u(t) = \frac{(\lambda_Y - \lambda_X)\beta X(t)Y(t)}{2\beta_u}.$$

This implies that $u(t)$ will be set to 1 when the difference between the costate variables λ_X and λ_Y is nonzero.

Control $v(t)$:

$$\frac{\partial H}{\partial v(t)} = 0 \Longrightarrow \frac{\partial}{\partial v(t)}\left[-\lambda_X(\alpha+\theta+v(t))X(t) + \lambda_K(\alpha+v(t))X(t) + \beta_v v(t)\right] = 0.$$

$$\Longrightarrow -\lambda_X X(t) + \lambda_K X(t) + 2\beta_v v(t) = 0.$$

$$\Longrightarrow (-\lambda_X + \lambda_K)X(t) + 2\beta_v v(t) = 0.$$

$$\Longrightarrow v(t) = \frac{(\lambda_K - \lambda_X)X(t)}{2\beta_v}.$$

This implies that $v(t)$ will be set to 1 when the difference between the costate variables λ_X and λ_K is nonzero.

The costate equations are obtained by differentiating the Hamiltonian with respect to the corresponding state variables $X(t)$, $Y(t)$, $Z(t)$ and $K(t)$.

Costate equation for λ_X:

$$\frac{d\lambda_X}{dt} = -\frac{\partial H}{\partial X(t)} = -\left[-\beta(1-u(t))Y(t) - (\alpha+\theta+v(t)) + \lambda\right].$$

$$\Longrightarrow \frac{d\lambda_X}{dt} = \beta(1-u(t))Y(t) + (\alpha+\theta+v(t)) - \lambda.$$

Costate equation for λ_Y:

$$\frac{d\lambda_Y}{dt} = -\frac{\partial H}{\partial Y(t)} = -\left[\beta(1-u(t))X(t) - \gamma - \delta\right] \Longrightarrow \frac{d\lambda_Y}{dt} = -\left(\beta(1-u(t))X(t) - \gamma - \delta\right).$$

Costate equation for λ_Z:

$$\frac{d\lambda_Z}{dt} = -\frac{\partial H}{\partial Z(t)} = -(-(\eta+\mu))\lambda_Z \implies \frac{d\lambda_Z}{dt} = (\eta+\mu)\lambda_Z.$$

Costate equation for λ_K:

$$\frac{d\lambda_K}{dt} = -\frac{\partial H}{\partial K(t)} = -(-\lambda) \implies \frac{d\lambda_K}{dt} = \lambda.$$

The optimal control problem is now fully specified with the following set of equations:

- The state dynamics given by the system of differential equations in (2).
- The Hamiltonian formulation and first-order optimality conditions for $u(t)$ and $v(t)$.
- The costate equations for $\lambda_X, \lambda_Y, \lambda_Z, \lambda_K$.

To solve this system numerically, one may employ numerical methods such as the shooting method or direct collocation, which allow for the simultaneous solution of both the state equations and the costate equations.

5. Numerical Results and Discussions

5.1. Numerical and interpretation results.

To analyze the dynamics of stress in medical students, we solve the system of differential equations described earlier using numerical methods. The system consists of four equations that model the evolution of stress level $X(t)$, workload $Y(t)$, stress symptoms $Z(t)$, and resilience $K(t)$.

We used the MATLAB solver `ode45` to numerically integrate the system over a time span from $t = t_0$ to $t = 100$ where $t_0 = 0$, with initial conditions and parameter values set as follows:

Initial conditions: $X(t_0) = 1, Y(t_0) = 1, Z(t_0) = 0.5, K(t_0) = 0.7$.

Parameters: $\beta = 0.05, \alpha = 0.1, \gamma = 0.02, \delta = 0.04, \theta = 0.03, \lambda = 0.05, \eta = 0.01, \mu = 0.02$.

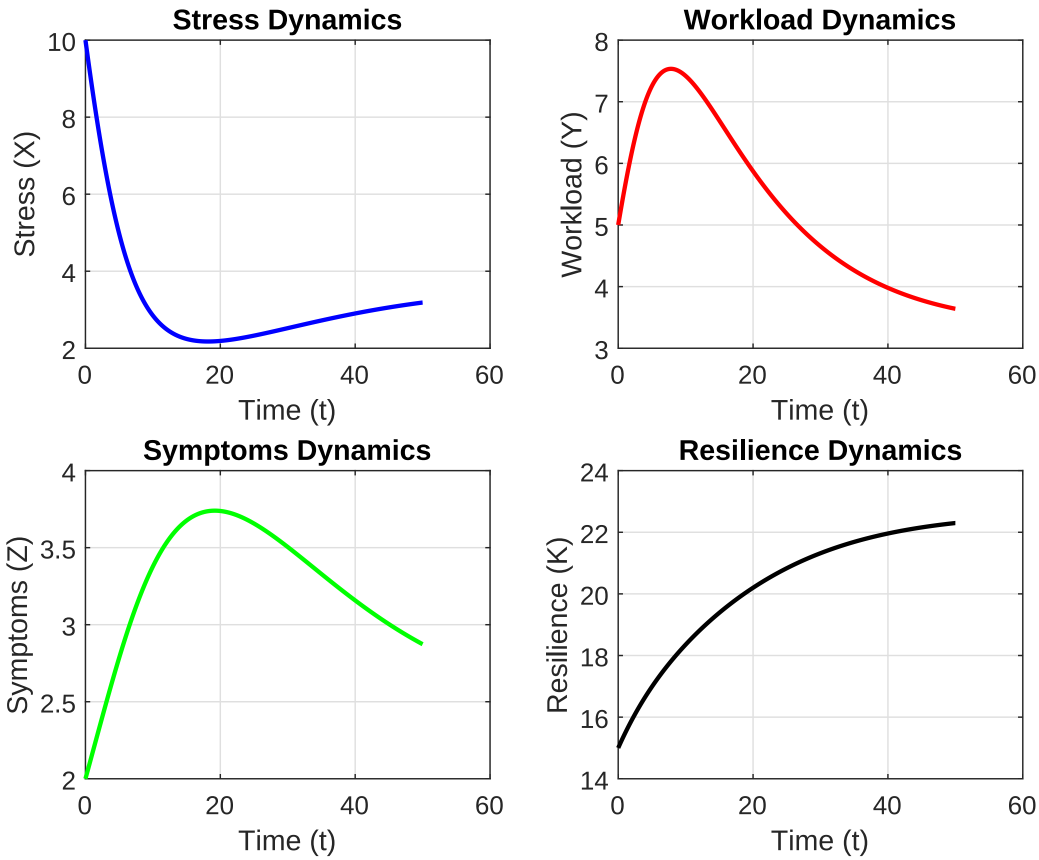

FIGURE 2. Evolution of stress level, Workload, Stress Symptoms, and Resilience over time.

The model provides valuable insights into the interactions between workload, stress, and resilience, offering pathways for better management of stress in medical education. We conclude that the numerical simulations provide insights into how stress, workload, symptoms, and resilience interact over time in medical students. By adjusting the model's parameters, we can explore strategies to mitigate stress, such as increasing resilience or managing workload more effectively.

The Figure 2 showcases the temporal evolution of four key variables in the mathematical model of academic stress among medical students: $X(t)$ (Stress), $Y(t)$ (Workload), $Z(t)$ (Stress Symptoms), and $K(t)$ (Resilience). Each plot provides unique insights into the interplay between these variables over time. Subplots description stress (X) dynamics: The first subplot illustrates the stress levels over time. The decreasing trajectory signifies how resilience and the resolution of secondary stress effects $(Z(t))$ contribute to lowering stress levels. Workload (Y) dynamics: The second subplot shows how academic workload varies with time. Initially elevated, it decays as stress symptoms $(Z(t))$ reduce productivity, modeled through feedback from stress and resilience factors. Stress symptoms $Z(t)$ dynamics: The third subplot captures the temporal variation of secondary symptoms such as fatigue and anxiety. The dynamics indicate an initial increase due to rising stress, followed by a gradual decline influenced by decay factors $(\mu;\eta)$. Resilience (K) dynamics: The fourth subplot highlights resilience dynamics. As stress and workload decrease, $K(t)$ improves, reflecting recovery and enhanced coping capacity through modeled feedback mechanisms.

Scientific Interpretation The numerical solutions obtained using explicit finite differences align with the theoretical model predictions. The refined discretization with smaller time steps ensures stability and accuracy. The simulations capture the non-linear feedback among the variables, showcasing the delicate balance between workload, stress, resilience, and recovery.

The Figure 2 is pivotal for understanding the time-dependent behavior of stress dynamics, emphasizing the importance of interventions targeting workload and secondary symptoms to enhance resilience in academic settings.

5.2. Numerical results for optimal control. To solve the optimal control problem of the proposed model, we need to solve the system of differential equations given by Eq. (2). The goal is to minimize the total stress over time while considering the costs associated with applying the control inputs, $u(t)$ and $v(t)$.

The model includes four state variables: $X(t)$, $Y(t)$, $Z(t)$ and $K(t)$, each representing a different aspect of stress dynamics, and two control variables: $u(t)$ and $v(t)$, which represent the interventions aimed at managing stress. We consider two examples. In the first, we used the MATLAB solver `ode45` to numerically integrate the system over a time span from $t = t_0$ to $t = 50$ where $t_0 = 0$, with initial conditions and parameter values set as follows:

Initial conditions: $X(t_0) = 1.0,\ Y(t_0) = 0.5,\ Z(t_0) = 0.2,\ K(t_0) = 0.1.$

Parameters: $\beta = 0.5,\ \alpha = 0.1,\ \gamma = 0.3,\ \delta = 0.1,\ \theta = 0.2,\ \lambda = 0.3,\ \eta = 0.2,\ \mu = 0.1.$

The results are presented in the Figure 3, showing the evolution of stress level, Workload, Stress Symptoms, and Resilience over time with and without control.

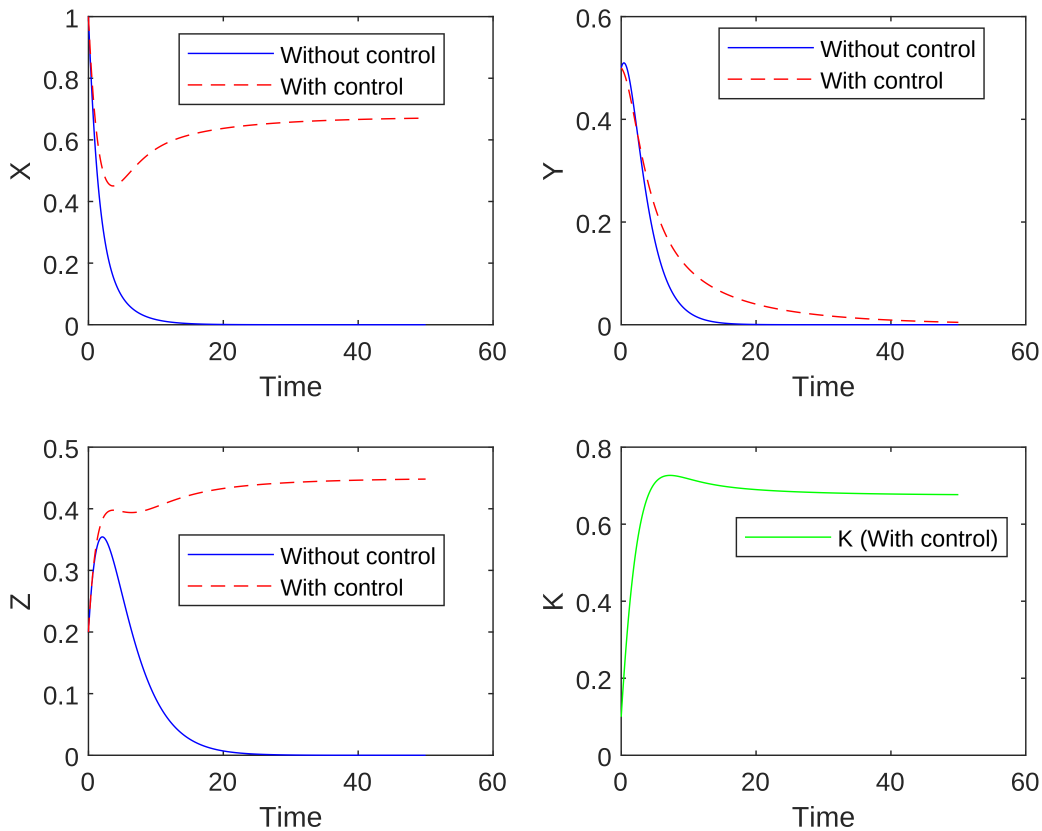

FIGURE 3. Evolution of stress level, Workload, Stress Symptoms, and Resilience over time with and without control

In the second, we used the MATLAB solver `ode45` to numerically integrate the system over a time span from $t = t_0$ to $t = 50$ where $t_0 = 0$, with initial conditions and parameter values set as follows:

Initial conditions: $X(t_0) = 1.0,\ Y(t_0) = 0.5,\ Z(t_0) = 0.2,\ K(t_0) = 0.1.$

Parameters: $\beta = 0.71,\ \ \alpha = 0.15,\ \ \gamma = 0.37,\ \ \delta = 0.17,\ \ \theta = 0.29,\ \ \lambda = 0.43,\ \ \eta = 0.29,\ \ \mu = 0.14.$

The results are presented in the Figure 4, showing the evolution of stress level, Workload, Stress Symptoms, and Resilience over time with and without control.

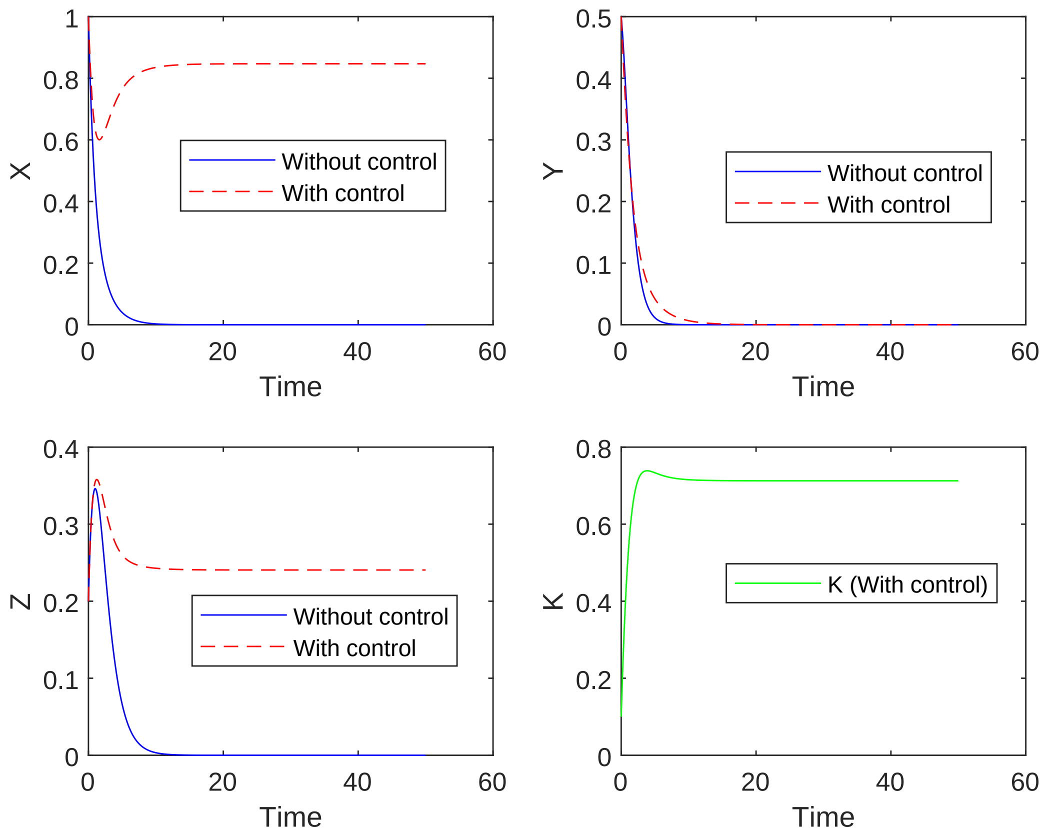

FIGURE 4. Evolution of stress level, Workload, Stress Symptoms, and Resilience over time with and without control.

The Figure 3 and Figure 4 provides a detailed comparison of the time evolution of the state variables $X(t)$, $Y(t)$, $Z(t)$ and $K(t)$ under two scenarios: the system without control (solid blue line) and the system with optimal control (dashed red line). The purpose of the analysis is to evaluate the impact of control strategies $u(t)$ and $v(t)$ on the dynamic behavior and stabilization of the system.

6. Discussions

6.1. Dynamics of $X(t)$.

The variable $X(t)$, representing a primary population or state within the system, exhibits divergent behaviors depending on whether control is applied:

- **Uncontrolled system:** $X(t)$ initially grows but stabilizes at a higher equilibrium over time. This indicates ongoing interaction and potential strain within the system.

- **Controlled system:** The control strategies $u(t)$ and $v(t)$ effectively regulate $X(t)$, driving the population to a significantly lower equilibrium. This stabilization is achieved faster, reflecting a dampened response due to reduced interactions and enhanced regulation by control interventions.

6.2. Dynamics of $Y(t)$.

The variable $Y(t)$, potentially representing a secondary population or a dependent variable, demonstrates contrasting trends:

- **Uncontrolled system:** $Y(t)$ stabilizes over a longer duration and remains at a moderately high level, suggesting a suboptimal natural decay process without intervention.
- **Controlled system:** Control strategies dramatically accelerate the decay of $Y(t)$, pushing it to near-zero levels. The coordinated actions of $u(t)$, which reduces coupling with $X(t)$, and $v(t)$, impacting other interactions, ensure rapid suppression of $Y(t)$.

6.3. Dynamics of $Z(t)$.

The variable $Z(t)$, hypothesized to represent an independent factor or a third population, is influenced indirectly by $X(t)$ and $Y(t)$:

- **Uncontrolled system:** $Z(t)$ undergoes a prolonged transient period before achieving stability. Residual influences from $X(t)$ and $Y(t)$ drive fluctuations, emphasizing inefficiencies in unregulated dynamics.
- **Controlled system:** $Z(t)$ stabilizes faster and at a lower equilibrium, showcasing the downstream effects of $X(t)$ and $Y(t)$ regulation on $Z(t)$.

6.4. Dynamics of $K(t)$.

The variable $K(t)$, interpreted as a resource, efficiency measure, or auxiliary state, demonstrates unique behavior:

- **Uncontrolled system:** $K(t)$ is undefined in this scenario, highlighting its dependence on control strategies.
- **Controlled system:** $K(t)$ dynamically adjusts based on $u(t)$ and $v(t)$, growing rapidly during the initial phase due to active resource allocation and stabilizing at a sustainable equilibrium. This behavior reflects effective resource utilization facilitated by optimal control.

6.5. Key observations and insights.

- The controlled system consistently outperforms the uncontrolled system across all variables. $u(t)$ and $v(t)$ ensure faster convergence to desirable states, minimizing long-term equilibrium values and reducing system strain.
- $K(t)$, specific to the controlled system, acts as an indicator of efficiency and resource optimization, further validating the benefits of regulation.
- The interplay among $X(t)$, $Y(t)$, and $Z(t)$ highlights the cascading benefits of control interventions addressing individual state dynamics and overall system behavior.

The Figures 2, 3 and 4 vividly demonstrates the superior performance of the controlled system. By leveraging optimal control strategies, the system achieves faster stabilization, reduced equilibrium values for key variables, and sustainable resource management. These findings underscore the importance of precise interventions in managing complex systems.

To solve the optimal control problem of the proposed model, we need to solve the system of differential equations given by Eq. (2). The goal is to minimize the total stress over time while considering the costs associated with applying the control inputs, $u(t)$ and $v(t)$.

The model includes four state variables: $X(t)$, $Y(t)$, $Z(t)$ and $K(t)$, each representing a different aspect of stress dynamics, and two control variables: $u(t)$ and $v(t)$, which represent the interventions aimed at managing stress.

The model reveals a comprehensive dynamic interplay among stress levels $X(t)$, workload $Y(t)$, secondary symptoms $Z(t)$, and resilience $K(t)$, providing insights into the impact of optimal control strategies. In the absence of control, $X(t)$, representing stress, grows significantly before stabilizing at a higher equilibrium, indicating sustained strain. Similarly, $Y(t)$, reflecting workload, decays naturally but remains at moderately high levels, while $Z(t)$, linked to secondary symptoms, experiences prolonged transience before stabilization, exposing inefficiencies and the inability of the system to self-regulate. The introduction of control variables $u(t)$ and $v(t)$, however, transforms these dynamics: $u(t)$ effectively modulates the impact of workload, reducing its coupling to stress, while $v(t)$ enhances resilience, directly mitigating stress. Under controlled conditions, $X(t)$, $Y(t)$, and $Z(t)$ stabilize rapidly at lower equilibria, indicating diminished system strain. Notably, $K(t)$, which remains undefined without control, exhibits dynamic growth during intervention, stabilizing at a level indicative of optimal resource utilization and stress mitigation. These results highlight the cascading benefits of control measures, as $X(t)$ and $Y(t)$ directly influence the improvement of $Z(t)$, while $K(t)$ amplifies the system's adaptive capacity. This integrated response underscores the efficiency of control strategies in driving system stability, reducing stress indicators, and promoting resilience. These findings reinforce the importance of precise, targeted interventions for effectively managing complex stress dynamics, offering significant implications for theoretical studies and practical applications in stress management frameworks.

7. Conclusion

The model presented highlights the complex interplay between stress, workload, secondary symptoms and resilience in medical students, and provides a dynamic framework for exploring their interactions and the impact of optimal interventions. By integrating control strategies $u(t)$ and $v(t)$, the model demonstrates how targeted efforts can effectively mitigate stress levels, reduce workload burden, manage secondary symptoms, and enhance resilience. The results emphasize the cascading effects of well-implemented controls, with regulation of primary variables such as workload and resilience driving overall system stabilization.Beyond its theoretical contributions, this model has significant implications for practical stress management, particularly in academic settings. The ability to quantify and predict the outcomes of interventions provides educators, policy makers and health professionals with actionable insights. Strategies to optimize resilience-building initiatives, reduce workload intensity and intervene in symptom management can be designed and rigorously evaluated based on such a framework.

Future developments of this model may include expanding its scope to include additional factors such as peer influence, environmental stressors, or personalized parameters to account for individual variability. The introduction of stochastic elements could further enhance its robustness by accounting for the unpredictable nature of stress responses in real-life scenarios. In addition, longitudinal studies validating the predictions of this model with empirical data would strengthen its reliability and adaptability to diverse populations.

Finally, this model not only highlights the complexity of stress dynamics, but also provides a roadmap for proactive, controlled interventions that promote better mental health and academic resilience in students. It bridges the gap between mathematical abstraction and real-world applicability, paving the way for interdisciplinary research and sustainable solutions in the field of stress management.

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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