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MODELING AND CONTROL OF CRIME USING OPTIMAL CONTROL THEORY

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Abstract. This research introduces a compartmental mathematical model, inspired by epidemiology, to analyze crime dynamics and design reduction strategies. The population is divided into four groups: susceptible individuals, active criminals, prisoners, and rehabilitated individuals. Two control functions are incorporated: one aimed at deterrence through sanction policies, and the other focused on social reintegration via educational, psychological, and professional support programs. Based on optimal control theory, the model identifies relevant intervention strategies to prevent crime and improve its management. The theoretical findings are validated through numerical simulations using MATLAB, comparing the system's evolution with and without intervention. This approach offers a solid analytical framework to support the development of integrated, flexible, and adaptive public policies addressing contemporary crime-related challenges.

Keywords: crime dynamics; compartmental modeling; simulation; optimal control theory; Pontryagin's maximum principle.

2020 AMS Subject Classification: 91D10, 92D25, 65L20, 49K15, 49K21.

1. INTRODUCTION

Crime is a complex social phenomenon influenced by economic, political, and technological factors. Its historical evolution shows a strong ability to adapt, moving from traditional banditry to organized crime, and more recently to cybercrime. To respond to these changes, societies must constantly adjust their strategies to ensure public safety. Worldwide, major threats such as drug trafficking, intentional homicides, and cybercrime require appropriate responses. In 2023, losses related to cyberattacks exceeded 8 trillion dollars, while the global drug trade continues to generate billions in illegal profits[1].

Like many countries, Morocco has seen major changes in its crime landscape, shaped by social and technological shifts. In 2022, a 30.22% drop in criminal cases was recorded, followed by a further 10% reduction in 2023[2], reflecting ongoing efforts to improve security and reduce crime. However, some forms of crime remain persistent, such as urban violence and drug trafficking, highlighting the need for diverse and effective strategies. The use of modern technologies now makes it possible to analyze crime trends and improve policy decisions, making analytical tools essential.

Criminology seeks to understand, prevent, and reduce criminal behavior, with a focus on public safety and social cohesion. However, crime often results from complex interactions between socio-economic, cultural, and environmental factors, making it difficult to predict or control. To better deal with this complexity, mathematical modeling has become a valuable tool. In particular, compartmental models inspired by epidemiological models have been used to represent crime as a dynamic process spreading through a population[3].

These models divide the population into categories based on their involvement in crime: susceptible individuals, active offenders, incarcerated individuals, and reintegrated individuals. This approach makes it possible to track transitions between these states and analyze individual trajectories through the criminal system. Several studies have shown that these models are effective for improving prevention strategies, anticipating crime trends, and guiding public policies[4, 5].

The use of mathematics in criminology dates back several decades. One of the early models was proposed by Cohen and Felson (1979)l[6], based on routine activity theory, which states that crime occurs when a motivated offender meets a vulnerable target in the absence of a capable guardian. Other models, such as Short et al. (2008)[4], use differential equations to describe the spatial spread of criminal activity. Graph-based models have also been developed to study criminal interactions and the structure of delinquent networks. More recently, optimal control theory has been applied to identify the most effective intervention strategies. For example, Grass et al for example, Grass et al. [7] used optimal control theory to analyze nonlinear processes in the context of drugs, corruption, and terror, providing decision-makers with tools to support long-term planning.

In this context, our study introduces a discrete compartmental model with two control variables. The first, noted $u(t)$, represents a strict deterrence and punishment policy targeting active offenders $I(t)$. In many countries, quickly arresting offenders is limited by a lack of resources or slow legal systems. This control aims to discourage criminal acts by creating fear of public sanctions. Even when the actual probability of arrest is low, the perception of risk alone can be enough to reduce illegal behavior, as shown by several studies. This is therefore a psychological control based on legal deterrence rather than direct repression. The second control variable focuses on prisoner reintegration, through professional training, psychological support, and post-prison assistance. These measures aim to reduce the risk of reoffending by helping former inmates return to society and giving them realistic alternatives to crime. The goal of including these two controls in our model is to reduce crime while minimizing the total intervention cost. Optimal control helps identify the most effective strategies by acting in a targeted way on both active offenders and inmates. This approach seeks to apply smart, targeted actions, avoiding harsh or inefficient solutions, while respecting the structure and limits of the modeled system.

The remainder of this article is structured as follows. The first section introduces the compartmental model, inspired by epidemiology, dividing the population into four groups: susceptible individuals, active criminals, prisoners, and rehabilitated individuals. It describes the equations governing transitions between these compartments and the relevant parameters. The second section focuses on the introduction of optimal control, and is divided into three subsections: the modified model incorporating two control functions targeting active criminals and prisoners respectively; the definition of the objective function, which aims to reduce the number of criminals and prisoners while promoting rehabilitation, accounting for the cost of interventions; and the derivation of the optimality conditions based on Pontryagin's Maximum Principle. Finally, the third section presents the results of numerical simulations conducted using MATLAB, comparing system dynamics with and without interventions, before concluding with a summary of the study's contributions and future research perspectives.

2. PRESENTAION OF THE MODEL

We consider a discrete compartmental model to analyze the dynamics of crime within a closed population. Inspired by epidemiological models, this approach assumes that criminal behavior can spread through social influence and contextual interactions. The population is divided into four compartments: individuals who are susceptible to committing crimes, denoted by S; active criminals, denoted by I; incarcerated individuals, denoted by P; and rehabilitated individuals, denoted by R.

The following figure illustrates the transitions between these different states, as well as the parameters that describe the movement between compartments.

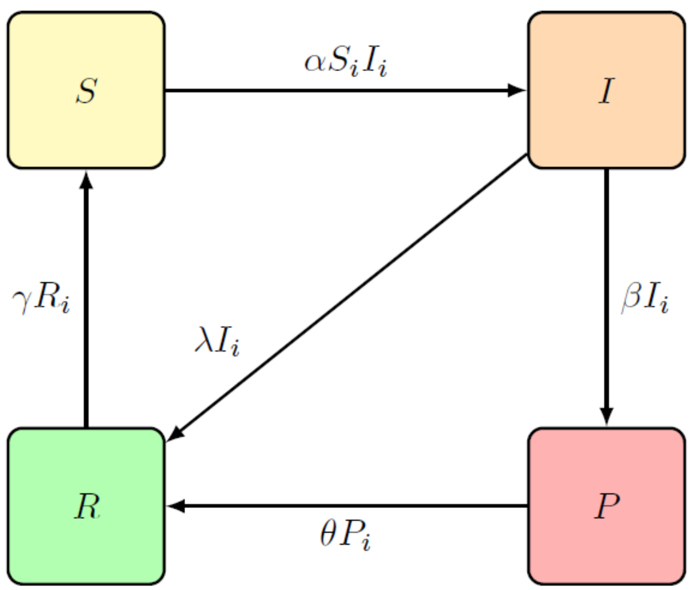

FIGURE 1. Flow chart illustrating transitions between compartments S, I, P, and R

The evolution of the susceptible compartment S is described by the equation:

$$S_{i+1} = S_i - \alpha S_i I_i + \gamma R_i \tag{1}$$

The term $-\alpha S_i I_i$ represents the reduction of susceptible individuals as they are influenced by active criminals. The parameter α represents the rate of criminal influence.The term $+\gamma R_i$ reflects the relapse of rehabilitated individuals who become susceptible again. Here, γ is the recidivism rate.

The compartment I, which includes active criminals, evolves according to:

$$I_{i+1} = I_i + \alpha S_i I_i - \beta I_i - \lambda I_i \tag{2}$$

The number of active criminals increases through the recruitment of susceptible individuals ($+\alpha S_i I_i$) and decreases through arrests ($-\beta I_i$) and direct reintegration into society ($-\lambda I_i$). The parameter β is the arrest rate, and λ is the reintegration rate for individuals who are not incarcerated.

The prisoner compartment P evolves as follows:

$$P_{i+1} = P_i + \beta I_i - \theta P_i \tag{3}$$

New prisoners come from arrested active criminals ($+\beta I_i$), while exits are due to rehabilitation programs ($-\theta P_i$). The parameter θ represents the **rehabilitation rate**, i.e., the fraction of prisoners successfully reintegrated into society.

Finally, the rehabilitated compartment R evolves according to:

$$R_{i+1} = R_i + \theta P_i - \gamma R_i + \lambda I_i \tag{4}$$

This compartment receives individuals from two sources: rehabilitated prisoners ($+\theta P_i$) and directly reintegrated active criminals ($+\lambda I_i$). Losses occur through relapse into susceptibility, captured by the term $-\gamma R_i$. The parameter γ thus represents the **recidivism rate**, indicating the proportion of rehabilitated individuals who return to a state of vulnerability to crime.

Combining all the equations describing the evolution of the four compartments, the complete system can be written as:

$$S_{i+1} = S_i - \alpha S_i I_i + \gamma R_i \tag{5}$$

$$I_{i+1} = I_i + \alpha S_i I_i - \beta I_i - \lambda I_i \tag{6}$$

$$P_{i+1} = P_i + \beta I_i - \theta P_i \tag{7}$$

$$R_{i+1} = R_i + \theta P_i - \gamma R_i + \lambda I_i \tag{8}$$

This discrete system provides a rigorous description of the transitions between different population states and offers a solid framework for analyzing and simulating crime control strategies.

The table below summarizes the parameters used in the model, along with their meanings and roles in the transitions between compartments.

Symbol	Description	Interpretation
α	Influence rate	Probability that susceptible individuals (S) become active criminals (I) under criminogenic influence
β	Arrest rate	Probability that active criminals (I) are arrested and moved to the prisoner compartment (P)
θ	Rehabilitation rate	Rate at which prisoners (P) are rehabilitated and transferred to the rehabilitated compartment (R)
γ	Recidivism rate	Probability that rehabilitated individuals (R) become susceptible to committing crimes again (S)
λ	Reintegration rate	Rate at which active criminals (I) are reintegrated into society without incarceration

TABLE 1. Summary of Rates in the SIPR System

3. THE OPTIMAL CONTROL PROBLEM

3.1. Presentation of the controls. In many dynamic systems, using control strategies is essential to limit the spread of harmful behaviors and reduce their negative impact. In criminology, such strategies help lower crime rates, better manage high-risk groups, and support social reintegration efforts [8, 9].

Research has shown that targeted deterrence policies can lead to real improvements. For instance, the "Operation Ceasefire" program in Boston led to a 63% drop in youth gang-related homicides. This result was achieved through a mix of clear threats of punishment, stronger police presence, and direct communication with the individuals involved [10, 11]. These types of strategies rely on the perception of risk and the certainty of consequences, which are key elements in preventing criminal behavior [12].

In this context, we suggest a control strategy based on two decision variables: $u(t)$ and $v(t)$, each aimed at different subgroups to both reduce crime and support rehabilitation.

The first control variable, $u(t)$, refers to a policy of deterrence and strict punishment targeting active offenders $I(t)$. In some countries, quickly arresting offenders is difficult due to limited resources or slow legal systems. This measure focuses on discouraging criminal actions by creating a strong fear of public and harsh consequences, even if the actual chance of getting caught is low. This idea is supported by studies showing that the perceived risk of being punished can be enough to reduce illegal behavior [13].

The second control variable, $v(t)$, focuses on the rehabilitation and social reintegration of former inmates $P(t)$. This involves providing support such as job training, psychological counseling, help with finding employment, and educational programs. The goal is to lower the chances of reoffending by improving the individuals' prospects after release [14, 9].

This type of approach is already being applied in several countries, including Morocco. There, His Majesty King Mohammed VI launched a number of reintegration initiatives through the Mohammed VI Foundation for the Reintegration of Prisoners. These centers help former inmates through training, education, social mediation, and job placement programs, helping them return to society with dignity [15].

This post-prison support policy aligns perfectly with the goal of $v(t)$, which is to promote long-term reintegration, reduce reoffending, and strengthen former inmates' social ties.

Given these controls, the modified system of equations is expressed as follows:

$$S_{i+1} = S_i - \alpha S_i I_i + \gamma R_i \tag{9}$$

$$I_{i+1} = I_i + \alpha S_i I_i - \beta I_i - \lambda I_i - u I_i \tag{10}$$

$$P_{i+1} = P_i + \beta I_i - \theta P_i - v P_i \tag{11}$$

$$R_{i+1} = R_i + \theta P_i - \gamma R_i + \lambda I_i + u I_i + v P_i \tag{12}$$

3.2. Objective Functional. The objective of this optimal control strategy is to reduce the number of active criminals in compartment I and incarcerated individuals in compartment P, while increasing the number of rehabilitated individuals in compartment R. The control variables u and v are applied to achieve these goals with minimal implementation cost. In this context, u represents a deterrence-based control strategy involving intimidation and strict punitive measures, intended to discourage criminal behavior through perceived risk. In contrast, v supports rehabilitation and social reintegration of former offenders. This combined approach balances deterrence and reintegration, while promoting cost-effective intervention [16, 17].

To formalize this, the objective functional $J(u,v)$ is defined as follows:

$$J(u,v) = \alpha_I I_N + \alpha_P P_N - \alpha_R R_N + \sum_{i=0}^{N-1} \left(\alpha_A I_i + \alpha_P P_i - \alpha_R R_i + \frac{A}{2} u_i^2 + \frac{B}{2} v_i^2 \right) \tag{13}$$

Here, $\alpha_I > 0$, $\alpha_P > 0$, and $\alpha_R > 0$ are weight parameters associated with the importance of each compartment. A higher value of α_I emphasizes reducing the active criminal population using deterrent control u, which captures the effect of fear of legal punishment. The parameter α_P emphasizes reducing incarceration levels, while α_R promotes an increase in the number of rehabilitated individuals through control v.

The constants $A > 0$ and $B > 0$ represent the cost of applying controls u and v, respectively. The parameter N defines the final time step of the control period, at which point the final values I_N, P_N, and R_N are evaluated. Separating the final-state terms from the summation allows for consideration of both cumulative and long-term outcomes.

The optimal control problem is then formulated as:

$$J(u^*, v^*) = \min \{ J(u,v) : u \in \mathscr{U},\ v \in \mathscr{V} \} \tag{14}$$

Subject to the admissible control sets:

$$\mathscr{U} = \{ u : u_{\min} \le u_i \le u_{\max},\ i = 0, \ldots, N-1 \} \tag{15}$$

$$\mathscr{V} = \{ v : v_{\min} \le v_i \le v_{\max},\ i = 0, \ldots, N-1 \} \tag{16}$$

With bounds defined by:

$$0 < u_{\min} < u_{\max} < 1, \quad 0 < v_{\min} < v_{\max} < 1 \tag{17}$$

In summary, this optimal control framework seeks to minimize the prevalence of criminal activity and incarceration, maximize rehabilitation outcomes, and limit intervention costs. Control

u serves as a deterrent by imposing the risk of strict sanctions, while control v fosters social reintegration. This dual strategy supports both immediate suppression of criminal behavior and long-term social reinsertion.

3.3. Sufficient Conditions. Theorem 1. There exists an optimal control $(u^*, v^*) \in \mathscr{U} \times \mathscr{V}$ such that

$$J(u^*, v^*) = \min_{u \in \mathscr{U}, v \in \mathscr{V}} J(u, v), \tag{18}$$

subject to the control system (5)–(8) and initial conditions.

Proof. Since the parameters of the system are bounded and there are a finite number of time steps, that is, I, S, R, and F are uniformly bounded for all (u, v) in the control set $\mathscr{U} \times \mathscr{V}$, thus $J(u, v)$ is also bounded for all $(u, v) \in \mathscr{U} \times \mathscr{V}$, which implies that $\inf_{(u,v) \in \mathscr{U} \times \mathscr{V}} J(u, v)$ is finite, and there exists a sequence $(u^n, v^n) \in \mathscr{U} \times \mathscr{V}$ such that

$$\lim_{n \to +\infty} J(u^n, v^n) = \inf_{(u,v) \in \mathscr{U} \times \mathscr{V}} J(u, v), \tag{19}$$

and corresponding sequences of states are I^n, S^n, R^n, and F^n.

Since there are a finite number of uniformly bounded sequences, there exists $(u^*, v^*) \in \mathscr{U} \times \mathscr{V}$ and I^*, S^*, R^*, and F^* such that, on a sequence,

$$(u^n, v^n) \to (u^*, v^*), \quad I^n \to I^*, \quad S^n \to S^*, \quad R^n \to R^*, \quad F^n \to F^*. \tag{20}$$

Finally, due to the finite dimensional structure of system (5)–(8), the objective function $J(u, v)$, (u^*, v^*) is an optimal control with corresponding states I^*, S^*, R^*, and F^*, which complete the proof.[18]

3.4. Necessary Conditions. By using a discrete version of Pontryagin's maximum principle [19, 21, 20], we derive the necessary conditions for the optimal controls. For this purpose, we define the Hamiltonian at time step i as:

$$\begin{aligned} H_i = {} & \alpha_I I_i + \alpha_P P_i + \frac{A}{2} u_i^2 + \frac{B}{2} v_i^2 + \lambda_{1,i+1}(S_i - \alpha S_i I_i + \gamma R_i) \\ & + \lambda_{2,i+1}(I_i + \alpha S_i I_i - \beta I_i - \lambda I_i + u_i I_i) + \lambda_{3,i+1}(P_i + \beta I_i - \theta P_i - v_i P_i) \\ & + \lambda_{4,i+1}(R_i + \theta P_i - \gamma R_i + \lambda I_i + u_i I_i + v_i P_i) \end{aligned} \tag{21}$$

where $\lambda_{k,i}$, for $k=1,2,3,4$, are the adjoint variables associated with the state variables S,I,P,R, respectively.

Theorem 2. Given optimal controls u^*,v^* and corresponding state trajectories S^*,I^*,P^*,R^*, there exist adjoint variables $\lambda_{k,i}$, $k=1,2,3,4$, for $i=0,\dots,\mathscr{N}-1$, satisfying the following backward difference equations:

$$\Delta\lambda_{1,i}=\frac{\partial H_i}{\partial S_i}=(\lambda_{2,i+1}-\lambda_{1,i+1})\alpha I_i, \tag{22}$$

$$\Delta\lambda_{2,i}=\frac{\partial H_i}{\partial I_i}=\alpha_I-\lambda_{1,i+1}\alpha S_i+\lambda_{2,i+1}(\alpha S_i-\beta-\lambda+u_i)+\lambda_{3,i+1}\beta+\lambda_{4,i+1}(\lambda+u_i), \tag{23}$$

$$\Delta\lambda_{3,i}=\frac{\partial H_i}{\partial P_i}=\alpha_P-(\lambda_{3,i+1}-\lambda_{4,i+1})(\theta+v_i), \tag{24}$$

$$\Delta\lambda_{4,i}=\frac{\partial H_i}{\partial R_i}=(\lambda_{1,i+1}-\lambda_{4,i+1})\gamma. \tag{25}$$

Ware the transversality conditions. $\lambda_{1,\mathscr{N}}=\alpha_I,\quad \lambda_{2,\mathscr{N}}=0,\quad \lambda_{3,\mathscr{N}}=\alpha_P,\quad \lambda_{4,\mathscr{N}}=-\alpha_R$. In addition,

$$u_i^*=\min\left(\max\left(u_{\min},-\frac{(\lambda_{2,i+1}+\lambda_{4,i+1})I_i}{A}\right),u_{\max}\right), \tag{26}$$

$$v_i^*=\min\left(\max\left(v_{\min},\frac{(\lambda_{3,i+1}-\lambda_{4,i+1})P_i}{B}\right),v_{\max}\right). \tag{27}$$

Proof. Using the discrete version of Pontryagin's maximum principle [19, 21], we obtain the following adjoint equations:

$$\begin{aligned}\Delta\lambda_{1,i}=\frac{\partial H_i}{\partial S_i}&=-\lambda_{1,i+1}\alpha I_i+\lambda_{2,i+1}\alpha I_i\\ &=(\lambda_{2,i+1}-\lambda_{1,i+1})\alpha I_i,\end{aligned} \tag{28}$$

$$\begin{aligned}\Delta\lambda_{2,i}=\frac{\partial H_i}{\partial I_i}&=\alpha_I-\lambda_{1,i+1}\alpha S_i+\lambda_{2,i+1}(\alpha S_i-\beta-\lambda+u_i)\\ &\quad+\lambda_{3,i+1}\beta+\lambda_{4,i+1}(\lambda+u_i),\end{aligned} \tag{29}$$

$$\begin{aligned}\Delta\lambda_{3,i}=\frac{\partial H_i}{\partial P_i}&=\alpha_P-\lambda_{3,i+1}(\theta+v_i)+\lambda_{4,i+1}(\theta+v_i)\\ &=\alpha_P-(\lambda_{3,i+1}-\lambda_{4,i+1})(\theta+v_i),\end{aligned} \tag{30}$$

$$\Delta\lambda_{4,i}=\frac{\partial H_i}{\partial R_i}=\lambda_{1,i+1}\gamma-\lambda_{4,i+1}\gamma$$

$$= (\lambda_{1,i+1} - \lambda_{4,i+1})\gamma. \tag{31}$$

Assuming no terminal cost, the transversality conditions are:

$$\lambda_{k,\mathcal{N}} = 0, \quad \text{for } k = 1,2,3,4. \tag{32}$$

To determine the optimal controls, we differentiate the Hamiltonian with respect to u_i and v_i, and equate the results to zero:

$$\frac{\partial H_i}{\partial u_i} = Au_i + (\lambda_{2,i+1} + \lambda_{4,i+1})I_i = 0, \tag{33}$$

$$\frac{\partial H_i}{\partial v_i} = Bv_i + (\lambda_{4,i+1} - \lambda_{3,i+1})P_i = 0. \tag{34}$$

Solving these equations yields the unconstrained optimal controls:

$$u_i^* = -\frac{(\lambda_{2,i+1} + \lambda_{4,i+1})I_i}{A}, \tag{35}$$

$$v_i^* = \frac{(\lambda_{3,i+1} - \lambda_{4,i+1})P_i}{B}. \tag{36}$$

Considering the admissible bounds on the controls $u_i \in [u_{\min}, u_{\max}]$, $v_i \in [v_{\min}, v_{\max}]$, the optimal controls are projected as:

$$u_i^* = \min\left(\max\left(u_{\min}, -\frac{(\lambda_{2,i+1} + \lambda_{4,i+1})I_i}{A}\right), u_{\max}\right), \tag{37}$$

$$v_i^* = \min\left(\max\left(v_{\min}, \frac{(\lambda_{3,i+1} - \lambda_{4,i+1})P_i}{B}\right), v_{\max}\right). \tag{38}$$

These conditions define the full set of necessary conditions for the discrete-time optimal control problem.

4. Numerical Simulation and Discussion

This section presents a comprehensive numerical analysis of the proposed model, aiming to evaluate the overall impact of a combined control strategy on the dynamics of the different subpopulations. Two scenarios are compared: one without any intervention, in which behaviors evolve freely, and another integrating two control functions simultaneously, targeting both active criminals and incarcerated individuals. The total simulated population includes 1000 individuals, initially distributed as follows: $S(0) = 700$, $I(0) = 200$, $P(0) = 50$, and $R(0) = 50$.

These initial conditions are kept constant across both scenarios to ensure a fair comparison. Simulations are carried out using MATLAB software, with the following parameters: $\alpha = 2.1 \times 10^{-2}$, $\beta = 4.1 \times 10^{-3}$, $\gamma = 10^{-4}$, $\theta = 4 \times 10^{-2}$, and $\lambda = 10^{-4}$.

The control functions $u(t)$ and $v(t)$ are progressively intensified during the simulation, starting respectively at 0.1 and 0.15. This choice reflects a gradual strengthening of prevention and reintegration policies, simulating an adaptive adjustment to the dynamics observed in the system. The goal is to examine the impact of this combined approach on the evolution of criminal behavior, by observing separately the effects on the susceptible, criminal, incarcerated, and rehabilitated populations. The following results illustrate how these intervention levers influence the system as a whole, providing concrete insights into the effectiveness of such an integrated strategy.

The first figure (Figure 2) illustrates the evolution of the susceptible population. In the absence of intervention, this population decreases from 700 to 550 individuals, corresponding to a loss of 150 people, or 21.4% of the initially non-criminal population. This decline reflects the lack of psychological or social barriers to entering criminality and highlights the vulnerability of a society where delinquent influences are neither mitigated nor counterbalanced. In contrast, when control measures are implemented, the population decreases only to 620 individuals, a reduction of 80 individuals (11.4%). This result indicates that 70 additional individuals were kept out of the criminal circuit thanks to the preventive policies in place. This preservation is not merely numerical: it reflects the success of awareness campaigns, educational efforts, and increased surveillance in high-risk areas. The gain confirms the protective effect of deterrent strategies which, by reducing transitions to delinquency, contribute to the stabilization of social structure and the preservation of community cohesion.

The evolution of the active criminal population is illustrated in Figure 3. In the scenario without control, this population gradually increases from 200 to 240 individuals, representing a 20% rise, which reflects a continuous spread of delinquent behaviors due to the absence of deterrent mechanisms. This situation reveals a concerning social dynamic: in the absence of intervention, criminal acts tend to proliferate through mimicry, a sense of impunity, or a lack of viable

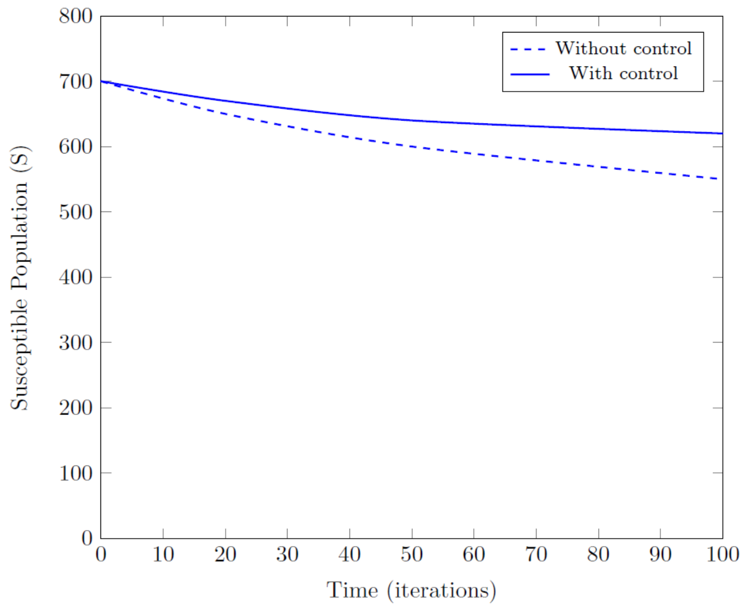

FIGURE 2. Susceptible population (S) before and after control

alternatives. In contrast, when both control measures are applied simultaneously, this population sharply drops to only 60 individuals, marking a 70% reduction. This significant decrease demonstrates the effectiveness of the dual approach combining deterrence and rehabilitation: on one hand, deterrence amplifies the fear of legal consequences, thereby discouraging criminal behavior; on the other hand, rehabilitation provides pathways for reintegration, offering former offenders real opportunities for change. This result highlights the value of an integrated strategy to sustainably manage criminal activity, by simultaneously reducing both the inflow into crime and the risk of recidivism.

The prison population, presented in Figure 4, shows a sharp increase in the scenario without intervention, rising from 50 to 110 individuals, which represents a 120% increase. This trend reflects a penal system characterized by numerous incarcerations and limited releases, indicating an accumulation of sentences due to the absence of effective reintegration mechanisms or parole programs. In contrast, under the control scenario, the prison population initially rises slightly to 60, then gradually declines to 40 individuals. This reversed trend reveals two combined effects of the implemented controls: on the one hand, a more selective incarceration process that targets individuals who truly require judicial intervention; on the other hand, an efficient reintegration policy that facilitates social reinsertion and reduces the average duration of

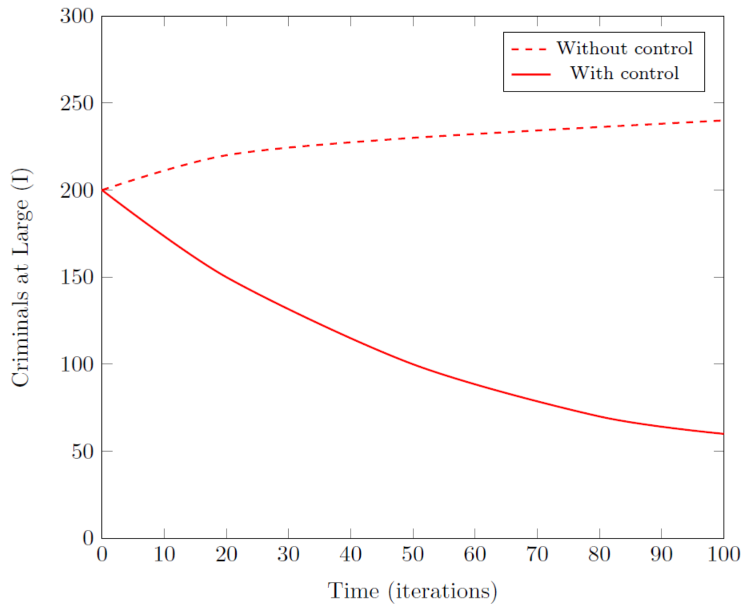

FIGURE 3. Active criminals (I) before and after control

detention. This outcome highlights the effectiveness of rehabilitation measures in streamlining the prison system, helping to ease prison overcrowding while providing former inmates with a second chance. Such behavior of the correctional system contributes to limiting the costs associated with overpopulation and preventing the detrimental effects of prolonged incarceration, such as recidivism and social alienation.

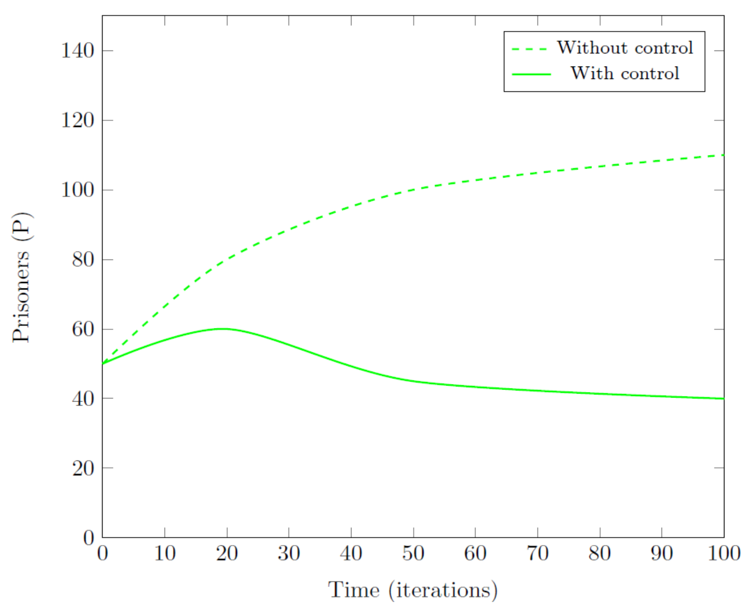

FIGURE 4. Prisoner population (P) before and after control

As shown in Figure 5, the evolution of the rehabilitated population differs markedly between scenarios. Without intervention, this population increases from 50 to 100 individuals, representing a modest doubling (+100%), which reflects the limited effectiveness of spontaneous reintegration mechanisms. This outcome indicates a lack of structured institutional support and unfavorable conditions for reintegration into active life after incarceration. In contrast, under the scenario integrating both control functions, the rehabilitated population reaches 270 individuals a remarkable increase of 440%. This growth highlights the substantial impact of rehabilitation policies that promote the empowerment of former inmates through targeted support programs (such as vocational training, social assistance, psychological counseling, etc.). Strengthening social reintegration not only helps break the cycle of recidivism but also contributes to restoring community cohesion by valuing individual recovery paths. Such a dynamic demonstrates that investment in rehabilitation generates beneficial multiplier effects, both for individuals and for society as a whole.

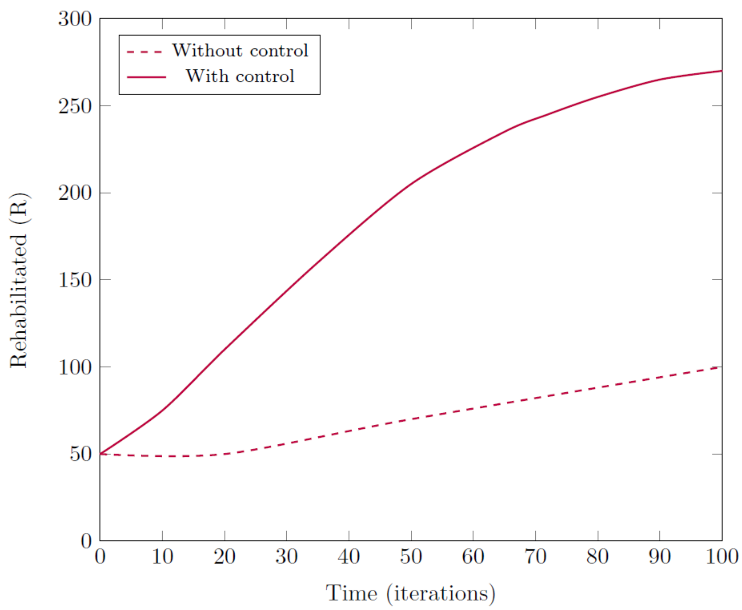

FIGURE 5. Rehabilitated population (R) before and after control

The evolution of the control functions $u(t)$ and $v(t)$, illustrated in Figure 6, highlights a progressive and differentiated intervention strategy. The function $u(t)$, associated with deterrence and punitive policies, increases moderately from 0.1 to 0.35, reflecting a gradual reinforcement of pressure on individuals engaged in criminal activity. This measured deployment allows the targeting of high-risk behaviors while avoiding excessive repression. Meanwhile, the function $v(t)$, representing reintegration policies and post-incarceration support, shows a steeper rise, going from 0.15 to 0.4. This more pronounced increase signals a strong commitment to long-term social investment aimed at sustainably transforming individual trajectories. This overall progression illustrates an adaptive approach to criminal management, in which resources are flexibly allocated according to the system's needs. Such adaptability is essential to maintaining a balance between firmness and inclusion while maximizing the impact of public policy on social stability and recidivism prevention.

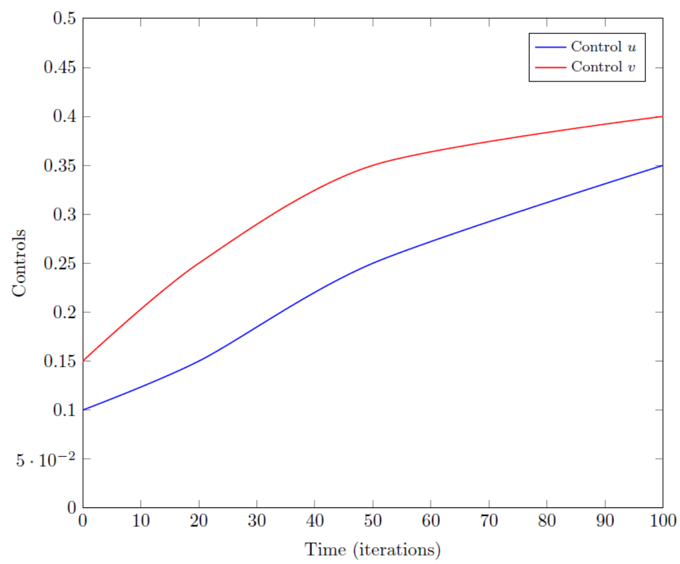

FIGURE 6. Evolution of the control functions $u(t)$ and $v(t)$

In conclusion, the results highlight the effectiveness of an integrated strategy combining deterrence and rehabilitation. The joint application of these two levers leads to a significant reduction in crime: the population of active offenders drops by more than 70% compared to the no-intervention scenario. This improvement is based on a dual action, both preventive and corrective: on the one hand, deterrent measures slow down criminal acts by increasing the perceived risk of punishment; on the other hand, reintegration mechanisms facilitate the return of released individuals into society, thereby reducing the likelihood of recidivism. The system thus becomes more efficient by addressing both the causes and consequences of criminal behavior.

At the same time, the prison dynamics evolve in a more controlled manner. While in the absence of intervention the incarcerated population grows significantly, the control measures not only limit this increase but also initiate a process of prison decongestion through optimized detention management. The number of rehabilitated individuals is multiplied by more than four, underscoring the structural impact of reintegration policies on social stabilization. These findings show that a balanced approach based on a careful mix of targeted repression and human-centered support can not only reduce insecurity but also strengthen social cohesion. This model serves as a relevant reference for the design of public policies in other contexts facing similar challenges.

CONCLUSION

In this article, we addressed the issue of crime by proposing a mathematical framework to analyze its dynamics and optimize intervention strategies. Crime remains a major societal challenge with significant implications for public safety, social stability, and the economy. Given its complexity, it is essential to adopt a multidimensional approach that integrates prevention, enforcement, and rehabilitation to ensure effective and sustainable management.

The model developed in this study provides a structured perspective on crime dynamics by considering the interactions between different groups of individuals and the effects of implemented measures. Using advanced analytical tools, we evaluated the impact of various strategies on crime reduction and social reintegration. The findings suggest that while a strictly repressive policy may lead to an immediate decrease in criminal activity, it can also increase the number of individuals in incarceration, highlighting the need for a complementary approach. Additionally, initiatives focused on rehabilitation and post-incarceration support have proven effective in reducing recidivism and facilitating successful reintegration.

Our findings highlight the importance of adopting a flexible and balanced strategy, capable of adjusting interventions in response to evolving crime patterns and societal changes. Combining preventive and corrective measures enhances long-term public safety while reducing overdependence on punitive policies. This work contributes to the development of decision-support tools for optimizing resources and improving public policy design in crime management. Future research may benefit from integrating socioeconomic and demographic factors to refine analyses, as well as validating the model with real-world data and exploring AI-based approaches to improve predictions and intervention efficiency.

Conflict of Interests

The authors declare that there is no conflict of interests.

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