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NUMERICAL MODELING OF HIV DYNAMICS IN $CD4^+$ T-CELLS USING FUZZY DIFFERENTIAL EQUATIONS

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Abstract. In this study, we use fuzzy differential equations (FDEs) to present a mathematical model for the dynamics of HIV infection in CD4+ T-cells. Uninfected cells, infected cells, and viral particles, all of which are represented as fuzzy dynamical systems, are the three main components of the model. We use ρ-cut techniques to convert the fuzzy system into a corresponding crisp system of differential equations to analyze the spread and control of HIV. Equilibrium points and eigenvalue-based criteria are used in stability analysis. In addition, we derive approximate solutions using the fifth-order Runge-Kutta numerical approach. The suggested method provides a more adaptable and practical framework for understanding the dynamics of HIV infection in an environment of uncertainty.

Keywords: fuzzy dynamical system; stability analysis; equilibrium points; numerical approximation.

2020 AMS Subject Classification: 74H15, 34A07.

1. INTRODUCTION

One of the deadliest epidemic diseases is the Human Immuno Virus by which Acquired Immuno Deficiency Syndrome (HIV-AIDS) is caused. The various types of mathematical models

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such as Ordinary Differential Equations modelling, Fractional Differential equations modelling, Stochastic differential equation modelling etc., are arising to analyze the spread and control of such deadly diseases. Also, there are statistical models, Probabilistic models, etc which are helpful in dealing with real data or clinical data. Mathematicians are still struggling to find the best optimal strategy to overcome the fastness of the spread of HIV infections. Now modern computer scientists use some advanced techniques like Artificial intelligence technique [2], Machine learning technique [1], Neural Network technique [3], etc for their computational contributions. Whatever the techniques are there it is not possible to skip the traditionally famous technique for computation such as the Euler method, Runge-Kutta method etc.,[15]. Also, rather than the above-mentioned techniques, there is one famous technique is called fuzzification by which the fuzziness such as randomness and vagueness in mathematical models can be rectified. After the arrival of fuzzy set around the 1960s contributed by Zadeh [29], there are huge different topics and fields had been found such as fuzzy algebra, fuzzy logic, fuzzy topology, fuzzy differential equations and many more. Here we have considered the new modified viral model for analysing the HIV dynamics by means of fuzzy differential equations and then the numerical solutions by the fifth-order Runge-Kutta method.

The Ho et al. (1995) model,

$$\frac{dE}{dt} = Q - cE$$

where Q denoted a source of infectious enzymes and c indicated the infectious clearance rate, was a basic linear first-order equation that explained viral creation and viral decline. Although several mechanisms, including immune cells, fluidity, and diffusion into other cells, play a part in the disposal of infectious enzymes, c can never make any distinction between them. It was believed that the enzyme inhibitor, the particular piece of treatment used on the sufferers, would be entirely effective upon introduction, or to put it another way, that it would prevent all infectious growth. Hence $Q = 0$, and we are left with the simple equation

$$\frac{dE}{dt} = -cE \Rightarrow E(\kappa) = E_0 e^{ct},$$

where E_0 is measured as the mean infectious concentration in the plasma prior to therapy.

Several different models have been constructed to investigate HIV-I [5] and CD4+ [24]-[28] as a consequence of the results of Ho et al.(1995). One of the basic fuzzy differential equations that might occur in different applications is first-order linear equations. Despite the fact that the form of this equation is quite uncomplicated, it creates numerous issues since the solutions behave differently under various fuzzy differential equation theories (depending on the interpretation used). Researchers have investigated this kind of equation. In recent years Muthukumar et.al., [13, 14] discussed the fifth-order RK method for various types of FDEs.

Many researchers have discussed HIV infection of $CD4^+$ cells. Perelson [16] explained a basic model for HIV (Human immunodeficiency virus) infections Even though the relationship between the human immune system and HIV is incredibly complicated, we really don't understand the pathogenesis. Once more, Perelson et al. [17] expanded on the HIV model and investigated its behaviour. They discussed four aspects: free virus, latently infected $CD4+$ T-cells, infected $CD4+$ T-cells, and uninfected CD4+ T-cells. The variations in the density of the target cells (U_κ^*), infected cells ($I^*{}_\kappa$), and free viruses ($F^*{}_\kappa$) were described in the basic differential equation model used to study viral dynamics. Many of the researchers [6],[8], [11],[24],[25],[28] described the model using the tools of ordinary differential equation (ODE). Especially the factional ordered models are studied in [22, 23]

Nowak and May's [12] basic model of virus dynamics served as the foundation for our investigation of the numerical solution of the fuzzy differential modelling in HIV dynamics using the Runge-Kutta Method of Order Five. Furthermore, the convergence and stability analysis of the proposed fuzzy systems were discussed.

2. Fuzzy Differential Equation Approach to Modeling HIV Infection Dynamics

To comprehend the dynamics of infectious diseases like HIV, mathematical modeling is essential. Ordinary differential equations (ODEs) are used in traditional models to explain how uninfected CD4+ T-cells, infected cells, and viral particles interact. However, these models might not adequately represent the complexity of HIV development because of biological data errors, patient heterogeneity, and environmental influences. Fuzzy Differential Equations (FDEs) have been proposed as an alternate method to deal with this uncertainty. FDEs provide

a more adaptable and realistic depiction of HIV infection dynamics by incorporating imprecise or unknown parameters. In this section we consider the differential equation model of HIV Dynamics [12],

$$
(1)\qquad \begin{cases} \dfrac{d}{dt}U_\kappa{}^* = r - aU_\kappa{}^* - \beta U_\kappa{}^* F_v(\kappa), \\ \dfrac{d}{dt}I_\kappa^* = \beta F_v(\kappa)U_\kappa{}^* - bI_\kappa^*, \\ \dfrac{d}{dt}F_\kappa^* = kI_\kappa{}^* - sF_v(\kappa), \\ U^*(0) = 1000,\ I^*(0) = 5,\ F^*(0) = 7000, \end{cases}
$$

Now, consider the initial value problem (IVP)(1)

$$
(2)\qquad \begin{cases} \frac{d}{dt}U^* = f_1(\kappa, U_\kappa^*, I_\kappa^*, F_\kappa^*) \\ \frac{d}{dt}I^* = f_2(\kappa, U_\kappa^*, I_\kappa^*, F_\kappa^*) \\ \frac{d}{dt}F^* = f_3(\kappa, U_\kappa^*, I_\kappa^*, F_\kappa^*) \\ U_c(\kappa_0) = U_{c_0},\ I_c(\kappa_0) = I_{c_0},\ F_v(\kappa_0) = F_{v_0}. \end{cases}
$$

where $f : [0,\infty) \times R \to R$ is continuous. We would like to interpret (2) using the Seikkala derivative and $U_0^*, I_0^*, F^*{}_0 \in F$. Let $[U_0^*]^\rho = \left[[\underline{U^*}0]^\rho, [\overline{U^*}0]^\rho\right]$, $[I_0^*]^\rho = \left[[\underline{I^*}0]^\rho, [\overline{I^*}0]^\rho\right]$, $[F_0^*]^\rho = \left[[\underline{F^*}0]^\rho, [\overline{F^*}0]^\rho\right]$ and $[U_\kappa^*]^\rho = \left[[\underline{U^*}\kappa]^\rho, [\overline{U^*}\kappa]^\rho\right]$, $[I_\kappa^*]^\rho = \left[[\underline{I^*}\kappa]^\rho, [\overline{I^*}\kappa]^\rho\right]$, $[F_\kappa^*]^\rho = \left[[\underline{F^*}\kappa]^\rho, [\overline{F^*}\kappa]^\rho\right]$ By the Zadeh's extension principle we get $f : [0,\infty) \times F^1 \times F^1 \times F^1 \to F^1$ where

$$
\begin{cases} f_1(\kappa, [U_\kappa^*]^\rho, [I_\kappa^*]^\rho, [F_\kappa^*]^\rho) = [\{\underline{f}_1(\kappa,u,v,w) : u \in [\underline{[U_\kappa^*]^\rho}), \overline{[U_\kappa^*]^\rho})], \\ v \in [\underline{[I_\kappa^*]^\rho}, \overline{[I_\kappa^*]^\rho}],\ w \in [\underline{[F_\kappa^*]^\rho}), \overline{[F_\kappa^*]^\rho})]\}, \{\overline{f}_1(\kappa,u,v,w) : u \in [\underline{[U_\kappa^*]^\rho}), \overline{[U_\kappa^*]^\rho})], \\ v \in [\underline{[I_\kappa^*]^\rho}, \overline{[I_\kappa^*]^\rho}], w \in [\underline{[F_\kappa^*]^\rho}), \overline{[F_\kappa^*]^\rho)}]\}], \\ f_2(\kappa, [U_\kappa^*]^\rho, [I_\kappa^*]^\rho, [F_\kappa^*]^\rho) = [\{\underline{f}_2(\kappa,u,v,w) : u \in [\underline{[U_\kappa^*]^\rho}), \overline{[U_\kappa^*]^\rho})], \\ v \in [\underline{[I_\kappa^*]^\rho}, \overline{[I_\kappa^*]^\rho}],\ w \in [\underline{[F_\kappa^*]^\rho}), \overline{[F_\kappa^*]^\rho})]\}, \{\overline{f}_2(\kappa,u,v,w) : u \in [\underline{[U_\kappa^*]^\rho}), \overline{[U_\kappa^*]^\rho})], \\ v \in [\underline{[I_\kappa^*]^\rho}, \overline{[I_\kappa^*]^\rho}], w \in [\underline{[F_\kappa^*]^\rho}), \overline{[F_\kappa^*]^\rho})]\}], \\ f_3(\kappa, [U_\kappa^*]^\rho, [I_\kappa^*]^\rho, [F_\kappa^*]^\rho) = [\{\underline{f}_3(\kappa,u,v,w) : u \in [\underline{[U_\kappa^*]^\rho}), \overline{[U_\kappa^*]^\rho})], \\ v \in [\underline{[I_\kappa^*]^\rho}, \overline{[I_\kappa^*]^\rho}],\ w \in [\underline{[F_\kappa^*]^\rho}), \overline{[F_\kappa^*]^\rho})]\}, \{\overline{f}_3(\kappa,u,v,w) : u \in [\underline{[U_\kappa^*]^\rho}), \overline{[U_\kappa^*]^\rho})], \\ v \in [\underline{[I_\kappa^*]^\rho}, \overline{[I_\kappa^*]^\rho}], w \in [\underline{[F_\kappa^*]^\rho}), \overline{[F_\kappa^*]^\rho})]\}]. \end{cases}
$$

Then $U^*:[0,\infty)\to F^1, I^*:[0,\infty)\to F^1,$ and $F^*:[0,\infty)\to F^1$ are the solution of (2) using the Seikkala derivative and $U_0^*, I_0^*, F_0^* \in F^1$ if

$$\begin{aligned}
\tfrac{d}{dt}\underline{[U_\kappa^*]^\rho} &= [\{\underline{f}_1(\kappa,u,v,w): u\in[\underline{[U_\kappa^*]^\rho}, \overline{[U_\kappa^*]^\rho}], v\in[\underline{[I_\kappa^*]^\rho}, \overline{[I_\kappa^*]^\rho}],\\
&\quad w\in[\underline{[F_\kappa^*]^\rho}, \overline{[F_\kappa^*]^\rho}]\}, \quad \underline{[U^*{}_\kappa]^\rho} = \underline{[U^*{}_\kappa]^\rho},\\
\tfrac{d}{dt}\overline{[U_\kappa^*]^\rho} &= [\{\overline{f}_1(\kappa,u,v,w): u\in[\underline{[U_\kappa^*]^\rho}, \overline{[U_\kappa^*]^\rho})], v\in[\underline{[I_\kappa^*]^\rho}, \overline{[I_\kappa^*]^\rho}],\\
&\quad w\in[\underline{[F_\kappa^*]^\rho}, \overline{[F_\kappa^*]^\rho})]\}, \quad \overline{[U^*{}_\kappa]^\rho} = \overline{[U^*{}_\kappa]^\rho},\\
\tfrac{d}{dt}\underline{[I_\kappa^*]^\rho} &= [\{\underline{f}_2(\kappa,u,v,w): u\in[\underline{[U_\kappa^*]^\rho}, \overline{[U_\kappa^*]^\rho}], v\in[\underline{[I_\kappa^*]^\rho}, \overline{[I_\kappa^*]^\rho}]\\
&\quad w\in[\underline{[F_\kappa^*]^\rho}, \overline{[F_\kappa^*]^\rho)}]]\}, \quad \underline{[I_\kappa^*]^\rho} = \underline{[I^*{}_\kappa]^\rho},\\
\tfrac{d}{dt}\overline{[I_\kappa^*]^\rho} &= \{\overline{f}_2(\kappa,u,v,w): u\in[\underline{[U_\kappa^*]^\rho}, \overline{[U_\kappa^*]^\rho}], v\in[\underline{[I_\kappa^*]^\rho}, \overline{[I_\kappa^*]^\rho}]\\
&\quad w\in[\underline{[F_\kappa^*]^\rho}), \overline{[F_\kappa^*]^\rho}]\}, \quad \overline{[I_\kappa^*]^\rho} = \overline{[I_\kappa^*]^\rho},\\
\tfrac{d}{dt}[\underline{[F_\kappa^*]^\rho}) &= \{\underline{f}_3(\kappa,u,v,w): u\in[\underline{[U_\kappa^*]^\rho}, \overline{[U_\kappa^*]^\rho})], v\in[\underline{[I_\kappa^*]^\rho}, \overline{[I_\kappa^*]^\rho}]\\
&\quad w\in[\underline{[F_\kappa^*]^\rho}, \overline{[F_\kappa^*]^\rho}]\}, \quad \underline{[F_\kappa^*]^\rho} = \underline{[F_\kappa^*]^\rho},\\
\tfrac{d}{dt}\overline{[F_\kappa^*]^\rho} &= [\{\overline{f}_3(\kappa,u,v,w): u\in[\underline{[U_\kappa^*]^\rho}, \overline{[U_\kappa^*]^\rho}], v\in[\underline{[I_\kappa^*]^\rho}, \overline{[I_\kappa^*]^\rho}]\\
&\quad w\in[\underline{[F_\kappa^*]^\rho}, \overline{[F_\kappa^*]^\rho}]\}, \quad \overline{[F^*{}_\kappa]^\rho} = \overline{[F^*{}_\kappa]^\rho},
\end{aligned}$$

for all $t\in[0,\infty)$ and $\rho\in[0,1]$. Now, the system of equations (1) can be written as

$$\text{(3)}\qquad \begin{cases} \dfrac{d}{dt}[U_\kappa^*]^\rho = r(\rho) - a[U_\kappa^*]^\rho - \beta[U_\kappa^*]^\rho[F_\kappa^*]^\rho,\\ \dfrac{d}{dt}[I_\kappa^*]^\rho = \beta[F_\kappa^*]^\rho[U_\kappa^*]^\rho - b[I_\kappa^*]^\rho,\\ \dfrac{d}{dt}[F_\kappa^*]^\rho = k[I_\kappa^*]^\rho - s[F_\kappa^*]^\rho, \end{cases}$$

where the initial conditions of the fuzzy model are

$$\begin{aligned}
[U^*\kappa_0]^\rho &= F_1(\rho) = [F_{1\ min}(\rho), F_{1\ max}(\rho)] = [1000\rho + 850(1-\rho), 1000\rho + 1150(1-\rho)],\\
[I^*\kappa_0]^\rho &= F_2(\rho) = [F_{2\ min}(\rho), F_{2\ max}(\rho)] = [5\rho + 4(1-\rho),\ 5\rho + 6(1-\rho)],\\
[F^*\kappa_0]^\rho &= F_3(\rho) = [F_{3\ min}(\rho), F_{3\ max}(\rho)]\\
&= [7000\rho + 6750(1-\rho), 7000\rho + 7250(1-\rho)],\ \rho\in[0,1].
\end{aligned}$$

where $[U_\kappa^*]^\rho$, $[I_\kappa^*]^\rho$ and $[F_\kappa^*]^\rho$ indicates the quantity of normal, affected, and unaffected virus components at period t. Non-infected cells produce at a constant rate of r, die at a constant rate of $a[U_\kappa^*]^\rho$, and unaffected cells with free viruses grow at a constant rate of $\beta[U_\kappa^*]^\rho[F_\kappa^*]^\rho$ the affected cells .Unaffected cells perish at a speed of $b[I_\kappa^*]^\rho$ in the second equation, and normal viruses are generated from affected cells at a rate of $k[I_\kappa^*]^\rho$ and die at rate $s[F_\kappa^*]^\rho$ in the third equation. The respective values of all the parameters are given in table 1

TABLE 1. Values for viral spread parameters

Terms	Description of variables and constants	Values
	Dependent variables	
U_c	The number of uninfected $CD4^+T$ cells	1000mm^{-3}
I_c	The concentration of $CD4^+T$ cells was affected	0
F_c	Initial density of HIV RNA	10^{-3}
	Parameters and Constants	
a	rate of CD4+ T-cells dying naturally	0.007day^{-1}
β	Rate infected cells become active	$42163\times 10^{-11}\text{mm}^3\text{day}^{-1}$
r	Source term for uninfected $CD4^+$ T-cells	7day^{-1}
b	Blanket death rate of infected $CD4^+$ T-cells	0.0999day^{-1}
k	Lytic death rate for infected cells	90.67day^{-1}
s	$CD4^+$ T-cells from uninfected sources	$0.2(\text{day})^{-1}(\text{mm}^{-3})$
	derived quantities	
U_{c_0}	An HIV-negative population of CD4+ T cells	1000mm^{-3}

By the following steps shows how the non-linear system of fuzzy differential equations(1) converts into the fuzzy linear system.

(i) To find equilibrium points

(ii) To find Jacobian matrix at equilibrium points

(i) To find the equilibrium points. The system (1) has two steady states: (i) The unaffected steady state $E_0 = \left([U_{\kappa 0}^*]^\rho, 0, 0\right)$,where $[U_{\kappa 0}^*]^\rho$ is given by $[U_{\kappa 0}^*]^\rho = \left[\frac{r}{a}\right][F_1(\rho)]$.

(ii) The positively affected steady state $\overline{E} = \left([\overline{U_\kappa^*}]^\rho, [\overline{I_\kappa}^*]^\rho, [\overline{F_\kappa^*}]^\rho \right)$ where $[\overline{U_\kappa^*}]^\rho, [\overline{I_\kappa}^*]^\rho, [\overline{F_\kappa^*}]^\rho$ are given by $[\overline{U_\kappa^*}]^\rho = \left[\frac{bs}{k\beta}\right][F_1(\rho)]$, $[\overline{I_\kappa}^*]^\rho = \left[\frac{sF_\kappa^*}{k}\right][F_2(\rho)]$, $[\overline{F_\kappa^*}]^\rho = \left[\frac{a}{\beta} - \frac{rk}{bs}\right][F_3(\rho)]$.

We find the three fixed (or) equilibrium points of the system (1),

(i) Initially: $E_0 = \left([U_{\kappa 0}^*]^\rho, 0, 0 \right) = (1000, 0, 0)$

(ii) $\overline{E} = \left([\overline{U_\kappa^*}]^\rho, [\overline{I_\kappa}^*]^\rho, [\overline{F_\kappa^*}]^\rho \right) = (522.64[F_1(\rho)],\ 350.35[F_2(\rho)],\ 31766.27[F_3(\rho)])$.

(ii) To find the Jacobian matrix at the equilibrium points

The nonlinear system (1) can be follow as,

$$(4) \qquad \begin{cases} \frac{d}{dt}[U_\kappa^*]^\rho &= [r]^\rho - a[U_\kappa^*]^\rho + \beta[U_\kappa^*]^\rho[F_\kappa^*]^\rho, = f_1([U_\kappa^*]^\rho, [I_\kappa^*]^\rho, [F_\kappa^*]^\rho), \\ \frac{d}{dt}[I_\kappa^*]^\rho &= \beta[F_\kappa^*]^\rho[U_\kappa^*]^\rho - b[I_\kappa^*]^\rho, = f_2([U_\kappa^*]^\rho, [I_\kappa^*]^\rho, [F_\kappa^*]^\rho), \\ \frac{d}{dt}[F_\kappa^*]^\rho &= k[I_\kappa^*]^\rho - s[F_\kappa^*]^\rho, = f_3([U_\kappa^*]^\rho, [I_\kappa^*]^\rho,\ [F_\kappa^*]^\rho). \end{cases}$$

The nonlinear fuzzy system (4) can be approximated into a fuzzy linear system as follows:

$$\begin{cases} \frac{d}{dt}[U_\kappa^*]^\rho &= f_1(([U_\kappa^*]^\rho, [I_\kappa^*]^\rho, [F_\kappa^*]^\rho)) \approx ([U_\kappa^*]^\rho, [I_\kappa^*]^\rho, [F_\kappa^*]^\rho) \\ &\quad + \frac{\partial f_1}{\partial U_c}([U_\kappa^*]^\rho, [I_\kappa^*]^\rho, [F_\kappa^*]^\rho)([U_\kappa^*]^\rho - [\overline{U_\kappa^*}]^\rho) \\ &\quad + \frac{\partial f_1}{\partial I}([U_\kappa^*]^\rho, [I_\kappa^*]^\rho, [F_\kappa^*]^\rho)([I_\kappa^*]^\rho - [\overline{I_\kappa}^*]^\rho) \\ &\quad + \frac{\partial f_1}{\partial F_c}([U_\kappa^*]^\rho, [I_\kappa^*]^\rho, [F_\kappa^*]^\rho)([F_\kappa^*]^\rho - [\overline{F_\kappa^*}]^\rho), \\ \frac{d}{dt}[I_\kappa^*]^\rho &= f_2(([U_\kappa^*]^\rho, [I_\kappa^*]^\rho, [F_\kappa^*]^\rho)) \approx ([U_\kappa^*]^\rho, [I_\kappa^*]^\rho, [F_\kappa^*]^\rho) \\ &\quad + \frac{\partial f_2}{\partial U_c}([U_\kappa^*]^\rho, [I_\kappa^*]^\rho, [F_\kappa^*]^\rho)([U_\kappa^*]^\rho - [\overline{U_\kappa^*}]^\rho) \\ &\quad + \frac{\partial f_2}{\partial I}([U_\kappa^*]^\rho, [I_\kappa^*]^\rho, [F_\kappa^*]^\rho)([I_\kappa^*]^\rho - [\overline{I_\kappa}^*]^\rho) \\ &\quad + \frac{\partial f_2}{\partial V}([U_\kappa^*]^\rho, [I_\kappa^*]^\rho, [F_\kappa^*]^\rho)([F_\kappa^*]^\rho - [\overline{F_\kappa^*}]^\rho), \\ \frac{d}{dt}[F_\kappa^*]^\rho &= f_3(([U_\kappa^*]^\rho, [I_\kappa^*]^\rho, [F_\kappa^*]^\rho)) \approx ([U_\kappa^*]^\rho, [I_\kappa^*]^\rho, [F_\kappa^*]^\rho) \\ &\quad + \frac{\partial f_3}{\partial U_c}([U_\kappa^*]^\rho, [I_\kappa^*]^\rho, [F_\kappa^*]^\rho)([U_\kappa^*]^\rho - [\overline{U_\kappa^*}]^\rho) \\ &\quad + \frac{\partial f_3}{\partial I}([U_\kappa^*]^\rho, [I_\kappa^*]^\rho, [F_\kappa^*]^\rho)([I_\kappa^*]^\rho - [\overline{I_\kappa}^*]^\rho) \\ &\quad + \frac{\partial f_3}{\partial V}([U_\kappa^*]^\rho, [I_\kappa^*]^\rho, [F_\kappa^*]^\rho)([F_\kappa^*]^\rho - [\overline{F_\kappa^*}]^\rho). \end{cases}$$

Then the equilibrium points,

$$f_1(([T(\kappa)]^\rho,[I_\kappa^*]^\rho,[F_\kappa^*]^\rho)) = f_2(([U_\kappa^*]^\rho,[I_\kappa^*]^\rho,[F_\kappa^*]^\rho)) = f_3(([U_\kappa^*]^\rho,[I_\kappa^*]^\rho,[F_\kappa^*]^\rho)) = 0$$

$$\begin{cases} \frac{d}{dt}[U_\kappa^*]^\rho &\approx \frac{\partial f_1}{\partial U_c}([U_\kappa^*]^\rho,[I_\kappa^*]^\rho,[F_\kappa^*]^\rho)([U_\kappa^*]^\rho - [\overline{U_\kappa^*}]^\rho) \\ &\quad + \frac{\partial f_1}{\partial I_c}([U_\kappa^*]^\rho,[I_\kappa^*]^\rho,[F_\kappa^*]^\rho)([I_\kappa^*]^\rho - [\overline{I_\kappa}^*]^\rho) \\ &\quad + \frac{\partial f_1}{\partial V}([U_\kappa^*]^\rho,[I_\kappa^*]^\rho,[F_\kappa^*]^\rho)([F_\kappa^*]^\rho - [\overline{F_\kappa^*}]^\rho), \\ \frac{d}{dt}[I_\kappa^*]^\rho &\approx \frac{\partial f_2}{\partial U_c}([U_\kappa^*]^\rho,[I_\kappa^*]^\rho,[F_\kappa^*]^\rho)([U_\kappa^*]^\rho - [\overline{U_\kappa^*}]^\rho) \\ &\quad + \frac{\partial f_2}{\partial I_c}([U_\kappa^*]^\rho,[I_\kappa^*]^\rho,[F_\kappa^*]^\rho)([I_\kappa^*]^\rho - [\overline{I_\kappa}^*]^\rho) \\ &\quad + \frac{\partial f_2}{\partial V}([U_\kappa^*]^\rho,[I_\kappa^*]^\rho,[F_\kappa^*]^\rho)([F_\kappa^*]^\rho - [\overline{F_\kappa^*}]^\rho), \\ \frac{d}{dt}[F_\kappa^*]^\rho &\approx f_3\frac{\partial f_3}{\partial U_c}([U_\kappa^*]^\rho,[I_\kappa^*]^\rho,[F_\kappa^*]^\rho)([U_\kappa^*]^\rho - [\overline{U_\kappa}^*]^\rho) \\ &\quad + \frac{\partial f_3}{\partial I_c}([U_\kappa^*]^\rho,[I_\kappa^*]^\rho,[F_\kappa^*]^\rho)([I_\kappa^*]^\rho - [\overline{I_\kappa}^*]^\rho) \\ &\quad + \frac{\partial f_3}{\partial V}([U_\kappa^*]^\rho,[I_\kappa^*]^\rho,[F_\kappa^*]^\rho)([F_\kappa^*]^\rho - [\overline{F_\kappa^*}]^\rho). \end{cases}$$

is linearized system.

The Jacobian matrix at the equilibrium point $\overline{E}$ is given by

$|J|=$

$$\begin{pmatrix} \frac{\partial f_1}{\partial U_c}([\overline{U_\kappa^*}]^\rho,[\overline{I_\kappa}^*]^\rho,[\overline{F_\kappa^*}]^\rho) & \frac{\partial f_1}{\partial I}([\overline{U_\kappa^*}]^\rho,[\overline{I_\kappa}^*]^\rho,[\overline{F_\kappa^*}]^\rho) & \frac{\partial f_1}{\partial V}([\overline{U_\kappa^*}]^\rho,[\overline{I_\kappa}^*]^\rho,[\overline{F_\kappa^*}]^\rho) \\ \frac{\partial f_2}{\partial U_c}([\overline{U_\kappa^*}]^\rho,[\overline{I_\kappa}^*]^\rho,[\overline{F_\kappa^*}]^\rho) & \frac{\partial f_2}{\partial I}([\overline{U_\kappa^*}]^\rho,[\overline{I_\kappa}^*]^\rho,[\overline{F_\kappa^*}]^\rho) & \frac{\partial f_2}{\partial F_c}([\overline{U_\kappa^*}]^\rho,[\overline{I_\kappa}^*]^\rho,[\overline{F_\kappa^*}]^\rho) \\ \frac{\partial f_3}{\partial U_c}([\overline{U_\kappa}^*]^\rho,[\overline{I_\kappa}^*]^\rho,[\overline{F_\kappa^*}]^\rho) & \frac{\partial f_3}{\partial I_c}([\overline{U_\kappa^*}]^\rho,[\overline{I_\kappa}^*]^\rho,[\overline{F_\kappa^*}]^\rho) & \frac{\partial f_3}{\partial F_c}([\overline{U_\kappa}^*]^\rho,[\overline{I_\kappa}^*]^\rho,[\overline{F_\kappa^*}]^\rho) \end{pmatrix}$$

$$(5) \qquad \begin{bmatrix} [U_\kappa'^*]^\rho \\ [I_\kappa'^*]^\rho \\ [F_\kappa'^*]^\rho \end{bmatrix} = \begin{bmatrix} -a+\beta\overline{F_\kappa^*} & 0 & -\beta\overline{U_\kappa^*} \\ \beta\overline{F_\kappa^*} & -b & \beta\overline{U_\kappa^*} \\ 0 & k & -s \end{bmatrix} \begin{bmatrix} [U_\kappa^* - \overline{U_\kappa^*}] \\ [I_\kappa^* - \overline{I_\kappa}^*] \\ [F_\kappa^* - \overline{F_\kappa^*}] \end{bmatrix} \begin{bmatrix} F_1(\rho) \\ F_2(\rho) \\ F_3(\rho) \end{bmatrix},$$

where the jacobian matrix is given by

$$[J]^{\rho} = \begin{pmatrix} -a+\beta\overline{F_{\kappa}^{*}} & 0 & -\beta\overline{U_{\kappa}^{*}} \\ \beta\overline{F_{\kappa}^{*}} & -b & \beta\overline{U_{\kappa}^{*}} \\ 0 & k & -s \end{pmatrix} \tag{6}$$

Thus the system (5) can be written as,

$$\begin{cases} [U_{\kappa}^{'*}]^{\rho} &= -\left(a+\beta[\overline{F_{\kappa}^{*}}]^{\rho}\right)\left([U_{\kappa}^{*}]^{\rho}-[\overline{U_{\kappa}^{*}}]^{\rho}+\beta[F_{\kappa}^{*}]^{\rho}-[\overline{F_{\kappa}^{*}}]^{\rho}\right), \\ [I_{\kappa}^{'*}]^{\rho} &= \beta[\overline{F_{\kappa}^{*}}]^{\rho}[(U_{\kappa}^{*}]^{\rho}-[\overline{U_{\kappa}^{*}}]^{\rho}-b\left([I_{\kappa}^{*}]^{\rho}-[\overline{I_{\kappa}}^{*}]^{\rho}\right) \\ & \quad +\beta[\overline{U_{\kappa}^{*}}]^{\rho}\left([F_{\kappa}^{*}]^{\rho}-[\overline{F_{\kappa}^{*}}]^{\rho}\right), \\ [F_{\kappa}^{'*}]^{\rho} &= k\left([I_{\kappa}^{*}]^{\rho}-[\overline{I_{\kappa}}^{*}]^{\rho}\right)-s\left([F_{\kappa}^{*}]^{\rho}-[\overline{F_{\kappa}^{*}}]^{\rho}\right), \end{cases} \tag{7}$$

System (7) is a linear fuzzy system. Around the fuzzy fixed points $(1000,0,0)$ and $(522.64[F_1(\rho)]\ \ 350.35[F_2(\rho)],\ \ 31766.27[F_3(\rho)])$ the fuzzy linear system (5) can be written as by using the parameter values

$$\begin{bmatrix} [U_{\kappa}^{'*}]^{\rho} \\ [I_{\kappa}^{'*}]^{\rho} \\ [F_{\kappa}^{'*}]^{\rho} \end{bmatrix} = \begin{bmatrix} -0.02039361 & 0 & -0.0002037 \\ 0.01339 & -0.0999 & 0.0002037 \\ 0 & 90.67 & -0.2 \end{bmatrix} \begin{bmatrix} [U_{\kappa}^{*}-1000] \\ [I_{\kappa}^{*}] \\ [F_{\kappa}^{*}] \end{bmatrix} \begin{bmatrix} F_1(\rho) \\ F_2(\rho) \\ F_3(\rho) \end{bmatrix}, \tag{8}$$

and

$$\begin{bmatrix} [U_{\kappa}^{'*}]^{\rho} \\ [I_{\kappa}^{'*}]^{\rho} \\ [F_{\kappa}^{'*}]^{\rho} \end{bmatrix} = \begin{bmatrix} -0.02039361 & 0 & -0.0002037 \\ 0.01339 & -0.0999 & 0.0002037 \\ 0 & 90.67 & -0.2 \end{bmatrix} \begin{bmatrix} [U_{\kappa}^{*}-522.64] \\ [I_{\kappa}^{*}-350.35] \\ [F_{\kappa}^{*}-31766.27] \end{bmatrix} \begin{bmatrix} F_1(\rho) \\ F_2(\rho) \\ F_3(\rho) \end{bmatrix}, \tag{9}$$

Equation (9) can be written in the form

$$\begin{bmatrix} [U_{\kappa}^{'*}]^{\rho} \\ [I_{\kappa}^{'*}]^{\rho} \\ [F_{\kappa}^{'*}]^{\rho} \end{bmatrix} = \begin{bmatrix} -0.02039361 & 0 & -0.0002037 \\ 0.01339 & -0.0999 & 0.0002037 \\ 0 & 90.67 & -0.2 \end{bmatrix} \begin{bmatrix} [U_{\kappa}^{*}] \\ [I_{\kappa}^{*}] \\ [F_{\kappa}^{*}] \end{bmatrix} \begin{bmatrix} F_1(\rho) \\ F_2(\rho) \\ F_3(\rho) \end{bmatrix}$$

$$+\begin{bmatrix} -17.6586 \\ -21.0002 \\ 41034.52 \end{bmatrix}\begin{bmatrix} F_1(\rho) \\ F_2(\rho) \\ F_3(\rho) \end{bmatrix}, \tag{10}$$

Equations (10) can be written

$$A=\begin{bmatrix} -0.02039361 & 0 & -0.0002037 \\ 0.01339 & -0.0999 & 0.0002037 \\ 0 & 90.67 & -0.2 \end{bmatrix} \text{ and } B=\begin{bmatrix} -17.6586 \\ -21.0002 \\ 41034.52 \end{bmatrix}, \tag{11}$$

The analytical solution is of the form $[X(\kappa)]^{\rho}=\Phi^{*}_{\kappa}C+\Phi^{*}_{\kappa}\int_{t_0}^{t}\Phi^{-1}(s)B(s)ds.$, the fundamental matrix of the fuzzy system is given by,

$[\Phi(\kappa)]^{\rho}$

$$=\begin{bmatrix} e^{\rho t}\left(a_1cos(\beta\kappa)-b_1sin(\beta\kappa)\right) & e^{\rho t}\left(a_1sin(\beta\kappa)+b_1cos(\beta\kappa)\right) & \gamma_1e^{\gamma\kappa} \\ e^{\rho t}\left(a_2cos(\beta\kappa)-b_2sin(\beta\kappa)\right) & e^{\rho t}\left(a_2sin(\beta\kappa)+b_2cos(\beta\kappa)\right) & \gamma_2e^{\gamma\kappa} \\ e^{\rho t}\left(a_3cos(\beta\kappa)-b_3sin(\beta\kappa)\right) & e^{\rho t}\left(a_3sin(\beta\kappa)+b_3cos(\beta\kappa)\right) & \gamma_3e^{\gamma\kappa} \end{bmatrix}\begin{bmatrix} F_1(\rho) \\ F_2(\rho) \\ F_3(\rho) \end{bmatrix}, \tag{12}$$

where $a_1=-0.00286233,\ a_2=0.00211605,\ a_3=0.997895,\ b_1=0.00663557,$
$b_2=0.000313352,\ b_3=0\ \gamma_1=0.000788392\ \gamma_2=-0.00114721,\ \gamma_3=0.997895$

Let $B=\begin{bmatrix} -17.65856 \\ -21.0002 \\ 41034.52 \end{bmatrix}$ and $[\Phi_{\kappa}^{-1}]^{\rho}B=\frac{1}{M}\begin{bmatrix} [e^{-\rho\kappa}(m_1sin(\beta\kappa)+m_2cos(\beta\kappa))]] \\ [-e^{-\rho t}(m_1cos(\beta\kappa)-m_2sin(\beta\kappa))] \\ [m_3e^{-\gamma\kappa}] \end{bmatrix}\begin{bmatrix} [F_1(\rho)] \\ [F_2(\rho)] \\ [F_3(\rho)] \end{bmatrix},$

where, $M=[a_1h_2-b_1h_1+\gamma_1h_3]^{\rho},\ m_1=n_1h_1-n_2h_5+n_3h_8,$

$m_2=n_1h_2+n_2h_4-n_3h_7,\ m_3=n_1h_3-n_2h_6+n_3h_9,$

$n_1=-17.65856,\ n_2=-21.0002,\ n_3=41034.52$

The analytical solution of the linear system (7) is given by,

$$[U^*_\kappa]^\rho = (a_1 cos\beta\kappa - b_1 sin\beta\kappa)\left[c_1 e^{\rho t} + \frac{1}{M(\rho^2+\beta^2)}\left[m_1(\xi_1(\kappa)) + m_2(\xi_2(\kappa))\right]\right]$$

$$+ (a_1 sin\beta\kappa + b_1 cos\beta\kappa)\left[c_2 e^{\rho t} - \frac{1}{M(\rho^2+\beta^2)}\left[m_1(\xi_2(\kappa)) - m_2(\xi_1(\kappa))\right]\right]$$

$$+ \gamma_1 e^{\gamma\kappa}\left[c_3 + \tfrac{m_3}{M\gamma}(1-e^{-\gamma\kappa})\right][F_1(\rho)],$$

$$[I^*_\kappa]^\rho = (a_2 cos\beta\kappa - b_2 sin\beta\kappa)\left[c_1 e^{\rho t} + \frac{1}{M(\rho^2+\beta^2)}\left[m_1(\xi_1(\kappa)) + m_2(\xi_2(\kappa))\right]\right]$$

$$+ (a_2 sin\beta\kappa + b_2 cos\beta\kappa)\left[c_2 e^{\rho t} - \frac{1}{M(\rho^2+\beta^2)}\left[m_1(\xi_2(\kappa)) - m_2(\xi_1(\kappa))\right]\right]$$

$$+ \gamma_2 e^{\gamma\kappa}\left[c_3 + \tfrac{m_3}{M\gamma}(1-e^{-\gamma\kappa})\right][F_2(\rho)],$$

$$[F^*_\kappa]^\rho = (a_3 cos\beta\kappa - b_3 sin\beta\kappa)\left[c_1 e^{\rho t} + \frac{1}{M(\rho^2+\beta^2)}\left[m_1(\xi_1(\kappa)) + m_2(\xi_2(\kappa))\right]\right]$$

$$+ (a_3 sin\beta\kappa + b_3 cos\beta\kappa)\left[c_2 e^{\rho t} - \frac{1}{M(\rho^2+\beta^2)}\left[m_1(\xi_2(\kappa)) - m_2(\xi_1(\kappa))\right]\right]$$

$$+ \gamma_3 e^{\gamma\kappa}\left[c_3 + \tfrac{m_3}{M\gamma}(1-e^{-\gamma\kappa})\right][F_3(\rho)], \quad \rho \in [0,1].$$

where,

$$\begin{cases} h_1 = \gamma_3 a_2 - \gamma_2 a_3, & h_2 = \gamma_3 b_2 - \gamma_2 b_3, & h_3 = a_2 b_3 - b_2 a_3, \\ h_4 = \gamma_1 b_3 - \gamma_3 b_1, & h_5 = \gamma_3 a_1 - \gamma_1 a_3, & h_6 = a_1 b_3 - a_3 b_1, \\ h_7 = \gamma_1 b_2 - \gamma_2 b_1, & h_8 = \gamma_2 a_1 - \gamma_1 a_2, & h_9 = a_1 b_2 - b_1 a_2. \end{cases}$$

$$\xi_1(\kappa) = -\rho sin\beta\kappa - \beta cos\beta\kappa + \beta e^{\rho\kappa}, \ \xi_2(\kappa) = -\rho cos\beta\kappa + \beta sin\beta\kappa + \rho e^{\rho\kappa}.$$

3. Stability Analysis of the FDEs Model of HIV Dynamics

The stability of the fuzzy differential equations model for HIV dynamics in CD4+ T-cells is examined in this section. This stability analysis provides useful insights into the behavior of the HIV infection model, allowing us to better understand how the system responds to perturbations and changes in parameter settings. By studying the system's eigenvalues, we may evaluate

if the infection dynamics are stable or unstable, which has crucial consequences for disease progression and control efforts.

Consider the characteristic equation of the linearized systems (5)

$$|A^{\rho} - \lambda I| = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3 = 0$$

where

$$\begin{aligned} c_1 &= \rho + \beta\overline{V} + b + s \\ c_2 &= \rho b + \rho s + (\beta b + \beta s)\overline{V} + bs \\ c_3 &= \rho bs + \beta bs\overline{V} + k\beta\overline{U_c}(1 - \beta\overline{V}) \end{aligned}$$

The coefficient value from the given initial coefficient value

$$c_1 = 0.32029, \quad c_2 = 0.02068, \quad c_3 = 0.02012$$

The eigenvalues of the matrix A are $\rho \pm \beta$ and γ,

$$\rho = -0.00813793, \ \beta = 0.0284116, \ \gamma = -0.304018$$

A system is considered stable if and only if all eigenvalues have negative real parts. Conversely, if at least one eigenvalue has a positive real part, the system is unstable. Our model contains three linearly independent eigenvalues: one real eigenvalue and two complex conjugate eigenvalues. Since all eigenvalues possess negative real portions, the system is confirmed to be stable, meaning that tiny disturbances in the system would diminish over time, leading to a steady-state equilibrium.

4. Numerical Simulation of SIR Model

This section uses the fifth-order Runge-Kutta method to obtain approximate solutions. By iteratively computing the system's evolution over time, this high-precision numerical method sheds light on how the infection develops in hazy environments. By choosing the parameter values listed in Table 1.1 and the designated initial circumstances, the numerical simulation is performed. We found values of U_K^*, I_K^*, and F_K^* at h=0.1, for $\rho \in [0,1]$ the best approximation.

Using above mention method to plot U_κ^*, I_κ^*, and F_κ^* for the fuzzy valued model (1) are given in figure 1, 2, 3.

$$[U_c(r+1)]^\rho = [U_\kappa^*]^\rho + \left(\frac{1}{90}(7[K_1]^\rho + 32[K_3]^\rho + 12[K_4]^\rho + 32[K_5]^\rho + 7[K_6]^\rho)\right),$$
$$[I_c(r+1)]^\rho = [I_\kappa^*]^\rho + \left(\frac{1}{90}(7[L_1]^\rho + 32[L_3]^\rho + 12[L_4]^\rho + 32[L_5]^\rho + 7[L_6]^\rho)\right),$$
$$[F_c(r+1)]^\rho = [F_\kappa^*]^\rho + \left(\frac{1}{90}(7[M_1]^\rho + 32[M_3]^\rho + 12[M_4]^\rho + 32[M_5]^\rho + 7[M_6]^\rho)\right).$$

where,

$$[K_1]^\rho = h(r - \rho[U_\kappa^*]^\rho + \beta[U_\kappa^*]^\rho[F_\kappa^*]^\rho),$$

$$[L_1]^\rho = h(\beta[F_\kappa^*]^\rho[U_\kappa^*]^\rho - b[I_\kappa^*]^\rho),$$

$$[M_1]^\rho = h(k[I_\kappa^*]^\rho - s[[I_\kappa^*]^\rho(\kappa)]^\rho).$$

$$[K_2]^\rho = h(r - \rho\left([U_\kappa^*]^\rho + 1/2[K_1]^\rho\right) + \beta\left([U_\kappa^*]^\rho + 1/2[K_1]^\rho\right)\left([F_\kappa^*]^\rho + 1/2[M_1]^\rho\right)),$$

$$[L_2]^\rho = h(\beta\left([F_\kappa^*]^\rho + 1/2[M_1]^\rho\right)\left([U_\kappa^*]^\rho + 1/2[K_1]^\rho\right) - b\left([I_\kappa^*]^\rho + 1/2[L_1]^\rho\right)),$$

$$[M_2]^\rho = h(k\left([I_\kappa^*]^\rho + 1/2[L_1]^\rho\right) - s\left([I_\kappa^*]^\rho(\kappa) + 1/2[L_1]^\rho\right)).$$

$$[K_3]^\rho = h(r - \rho\left([U_\kappa^*]^\rho + 3/16[K_1]^\rho + 1/16[K_2]^\rho\right) + \beta\left([U_\kappa^*]^\rho + 3/16[K_1]^\rho + 1/16[K_2]^\rho\right)\left([F_\kappa^*]^\rho + 3/16[M_1]^\rho + 1/16[M_2]^\rho\right)),$$

$$[L_3]^\rho = h(\beta\left([F_\kappa^*]^\rho + 3/16[M_1]^\rho + 1/16[M_2]^\rho\right)\left([U_\kappa^*]^\rho + 3/16[K_1]^\rho + 1/16[K_2]^\rho\right) - b\left([I_\kappa^*]^\rho + 3/16[L_1]^\rho + 1/16[L_2]^\rho\right)),$$

$$[M_3]^\rho = h(k\left([I_\kappa^*]^\rho + 3/16[L_1]^\rho + 1/16[L_2]^\rho\right) - s\left([I_\kappa^*]^\rho(\kappa) + 3/16[L_1]^\rho + 1/16[L_2]^\rho\right)).$$

$$
\begin{aligned}
[K_4]^{\rho} &= h(r-\rho\left([U_{\kappa}^{*}]^{\rho}+1/2[K_3]^{\rho}\right)+\beta\left([U_{\kappa}^{*}]^{\rho}+1/2[K_3]^{\rho}\right)\left([F_{\kappa}^{*}]^{\rho}+1/2[M_3]^{\rho}\right)),\\
[L_4]^{\rho} &= h(\beta\left([F_{\kappa}^{*}]^{\rho}+1/2[M_3]^{\rho}\right)\left([U_{\kappa}^{*}]^{\rho}+1/3[K_3]^{\rho}\right)-b\left([I_{\kappa}^{*}]^{\rho}+1/2[L_3]^{\rho}\right)),\\
[M_4]^{\rho} &= h(k\left([I_{\kappa}^{*}]^{\rho}+1/2[L_3]^{\rho}\right)-s\left([I_{\kappa}^{*}]^{\rho}(\kappa)+1/2[L_3]^{\rho}\right)).\\
[K_5]^{\rho} &= h(r-\rho\left([U_{\kappa}^{*}]^{\rho}-3/16[K_2]^{\rho}+6/16[K_3]^{\rho}+9/16[K_4]^{\rho}\right)+\beta\left([U_{\kappa}^{*}]^{\rho}-3/16[K_2]^{\rho}\right.\\
&\quad \left.+6/16[K_3]^{\rho}+9/16[K_4]^{\rho}\right)\left([F_{\kappa}^{*}]^{\rho}-3/16[M_2]^{\rho}+6/16[M_3]^{\rho}+9/16[M_4]^{\rho}\right)),\\
[L_5]^{\rho} &= h(\beta\left([F_{\kappa}^{*}]^{\rho}-3/16[M_2]^{\rho}+6/16[M_3]^{\rho}+9/16[M_4]^{\rho}\right)\left([U_{\kappa}^{*}]^{\rho}-3/16[K_2]^{\rho}\right.\\
&\quad \left.+6/16[K_3]^{\rho}+9/16[K_4]^{\rho}\right)-b\left([I_{\kappa}^{*}]^{\rho}-3/16[L_2]^{\rho}+6/16[L_3]^{\rho}+9/16[L_4]^{\rho}\right)),\\
[M_5]^{\rho} &= h(k\left([I_{\kappa}^{*}]^{\rho}-3/16[L_2]^{\rho}+6/16[L_3]^{\rho}+9/16[L_4]^{\rho}\right)\\
&\quad -s\left([I_{\kappa}^{*}]^{\rho}(\kappa)-3/16[L_2]^{\rho}+6/16[L_3]^{\rho}+9/16[L_4]^{\rho}\right)).\\
[K_6]^{\rho} &= h(r-\rho\left([U_{\kappa}^{*}]^{\rho}+1/7[K_1]^{\rho}+4/7[K_2]^{\rho}+6/7[K_3]^{\rho}-12/7[K_4]^{\rho}+8/7[K_5]^{\rho}\right)\\
&\quad +\beta\left([U_{\kappa}^{*}]^{\rho}+1/7[K_1]^{\rho}+4/7[K_2]^{\rho}+6/7[K_3]^{\rho}-12/7[K_4]^{\rho}+8/7[K_5]^{\rho}\right)\\
&\quad \left([F_{\kappa}^{*}]^{\rho}+1/7[M_1]^{\rho}+4/7[M_2]^{\rho}+6/7[M_3]^{\rho}-12/7[M_4]^{\rho}+8/7[M_5]^{\rho}\right)),\\
[L_6]^{\rho} &= h(\beta\left([F_{\kappa}^{*}]^{\rho}+1/7[L_1]^{\rho}+4/7[L_2]^{\rho}+6/7[L_3]^{\rho}-12/7[L_4]^{\rho}+8/7[L_5]^{\rho}\right)\\
&\quad \left([T(\kappa)]^{\rho}+1/7[K_1]^{\rho}+4/7[K_2]^{\rho}+6/7[K_3]^{\rho}-12/7[K_4]^{\rho}+8/7[K_5]^{\rho}\right)\\
&\quad -b\left([I_{\kappa}^{*}]^{\rho}+1/7[L_1]^{\rho}+4/7[L_2]^{\rho}+6/7[L_3]^{\rho}-12/7[L_4]^{\rho}+8/7[L_5]^{\rho}\right)),
\end{aligned}
$$

$$[M_6]^\rho = h(k\left([I^*_\kappa]^\rho + 1/7[L_1]^\rho + 4/7[L_2]^\rho + 6/7[L_3]^\rho - 12/7[L_4]^\rho + 8/7[L_5]^\rho\right)$$

$$-s\left([I^*_\kappa]^\rho(\kappa) + 1/7[L_1]^\rho + 4/7[L_2]^\rho + 6/7[L_3]^\rho - 12/7[L_4]^\rho + 8/7[L_5]^\rho\right)).$$

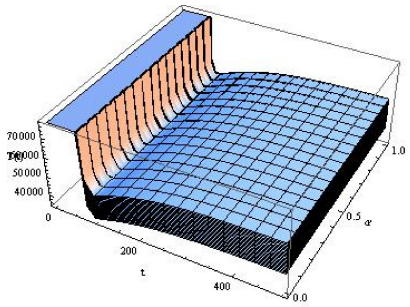

FIGURE 1. Uninfected Cells

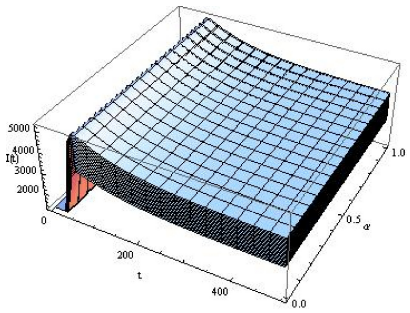

FIGURE 2. Infected Cells

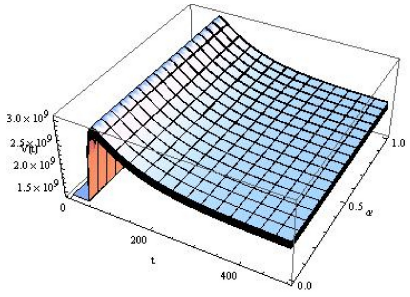

FIGURE 3. Virus Infectious

5. DISCUSSION

In this work, we used the Seikkala derivative and Hukuhara differentiability in fuzzy processes to reconstruct the differential equation model of HIV infection put forward by Nowak and May [12] into a fuzzy system of three equations. Our investigation revealed a requirement that must be met for the infection to continue: the quantity of virus particles released per infected cell. The system displays a positive equilibrium in this situation, which is the infected steady state.

We determined adequate requirements on the model parameters to guarantee the stability of the infected steady state through stability analysis. The numerical simulations supported our theoretical conclusions, and our results verified that these stability requirements are satisfied. The existence and durability of the infected steady state are unaffected by the difference between our calculated value and that reported by Perelson et al.[17] regarding the amount of virus particles released per infected cell. n nonlinear differential equations, analytical solutions are often challenging to obtain. Since most nonlinear differential equations do not have explicit solutions, numerical methods become essential for solving them. The fifth-order Runge-Kutta method is particularly well-suited for approximating solutions to fuzzy differential equations, providing high accuracy and efficiency in modeling the dynamics of HIV infection.

6. CONCLUSION

In this study, we developed a fuzzy differential equation model to analyze the dynamics of HIV infection in CD4+ T-cells. By incorporating uncertainty through fuzzy logic, the model provides a more flexible and realistic representation of the infection process. Using the Seikkala derivative and Hukuhara differentiability, we transformed the classical HIV model into a fuzzy system and analyzed its stability under various conditions. Our stability analysis demonstrated that the system reaches a positive equilibrium when specific parameter constraints are satisfied. The numerical results, obtained using the fifth-order Runge-Kutta method, confirmed the theoretical findings and provided further insights into the behavior of the infection over time. The study highlights the importance of fuzzy differential equations in modeling biological systems where uncertainty plays a crucial role. The proposed approach can be extended to explore other

infectious diseases and optimize treatment strategies by considering real-world uncertainties in parameter estimation.

Future work may focus on incorporating treatment effects, immune response variations, and optimal control strategies into the fuzzy modeling framework to enhance the predictive power of the model.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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