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BAYESIAN ESTIMATION FOR THE LOMAX DISTRIBUTION: COMPARING LOSS FUNCTIONS UNDER DIFFERENT PRIORS

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Abstract: This study aims to estimate the shape parameter (θ) of the Lomax distribution with a known scale parameter (β) using a Bayesian approach. Three Bayesian Loss Function methods are applied: Square Error Loss Function (SELF), Entropy Loss Function (ELF), and Precautionary Loss Function (PLF). The priors used include the conjugate Gamma prior and the non-informative Jeffrey prior. The estimation was conducted on simulated data with shape parameters (θ = 1.3 and 1.5) and varying sample sizes (n = 30, 150, and 300). The estimation process involves constructing the posterior distribution by combining the prior distribution with the likelihood function of the Lomax distribution. The estimated parameters are evaluated using Akaike Information Criterion (AIC), corrected AIC (AICc), and Bayesian Information Criterion (BIC) to determine the best method under various conditions. Results indicate that the Bayesian SELF method provides the best estimation with the smallest AIC, AICc, and BIC values for small sample sizes (n = 30), regardless of whether the Gamma or Jeffrey prior is used. For larger sample sizes (n = 150 and 300), the Bayesian PLF method performs better. The conjugate Gamma prior consistently produces more stable

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estimates compared to the non-informative Jeffrey prior. This research highlights that the optimal choice of Bayesian Loss Function depends on the sample size and the type of prior. These findings provide valuable insights for improving parameter estimation methods for the Lomax distribution, which has wide applications in heavy-tailed data analysis in fields such as finance, queuing theory, and internet traffic modeling.

Keywords: Lomax distribution; Bayesian estimation; loss function comparison; AIC; Bayesian inference.

2020 AMS Subject Classification: 62F15.

1. INTRODUCTION

The Lomax distribution, a heavy-tailed variant of the Pareto model, has been widely used in reliability engineering, actuarial science, and financial risk 2odelling due to its ability to capture extreme values more effectively than exponential-based distributions data [1], [2], [3]. Accurate estimation of its shape and scale parameters is essential for decision-making in settings such as insurance pricing, equipment failure analysis, and credit risk management. Conventional estimation methods like Maximum Likelihood Estimation (MLE) and the Method of Moments often yield unstable results, especially with small samples or censored data [4], [5]. Asymptotic confidence intervals from these methods may also fail to capture the true uncertainty, particularly in heavy-tailed contexts. These limitations have led to increased interest in Bayesian approaches, which offer more flexible inference by combining prior knowledge with observed data to produce full posterior distributions.

In Bayesian analysis, the choice of loss function plays a critical role in determining the form of the estimator [6], [7]. While the Squared Error Loss Function (SELF) remains the most widely used, alternative loss functions such as the Entropy Loss Function (ELF) and Precautionary Loss Function (PLF) have been proposed for their ability to handle asymmetry and penalize underestimation more heavily, an important consideration in high-risk applications. Yet, these alternatives have rarely been compared directly, particularly in the context of the Lomax distribution.

Recent studies have highlighted the effectiveness of Bayesian approaches under varying loss functions. For instance, Naji and Rasheed [8] demonstrated the utility of the Precautionary Loss Function for estimating Gamma distribution parameters, while Li and Hao [9] used Entropy Loss in the context of Poisson distributions. Al-Bossly [4] developed a compound Linex loss function to estimate the shape parameter of the Lomax distribution. Ijaz [10] presented a Bayesian estimation of the shape parameter of Lomax distribution under Uniform and Jeffery prior. Mehdi et al. [11] considered the Bayesian estimation of transmuted Lomax mixture model (TLMM) for type-I censored samples. Kumari et al. [12] demonstrated the Bayesian analysis for two parameter of Lomax distribution under different loss functions. Ren et al. [13] studied the parameter estimation of the Lomax distribution based on middle censored data. However, no study to date has systematically compared SELF, ELF, and PLF under both Gamma and Jeffrey priors for the Lomax distribution. These limitations highlight a gap in the literature regarding a comprehensive, comparative evaluation of different loss functions and priors within the Bayesian framework for Lomax models. The present study seeks to fill this gap.

This study makes four novel contributions: (i) it is the first to jointly compare SELF, ELF, and PLF within a Bayesian framework for the Lomax shape parameter; (ii) it evaluates both an informative Gamma prior and the objective Jeffreys prior across small, moderate, and large-sample regimes; (iii) it benchmarks estimator performance using AIC, AICc, and BIC on 1 000 replicated datasets for each scenario; and (iv) it provides practitioner guidelines for actuarial, reliability engineering, and fintech applications where heavy-tailed risks predominate.

2. PRELIMINARIES

This section outlines the analytical and empirical procedures employed to evaluate the performance of three Bayesian estimation techniques; SELF, ELF, and PLF, in estimating the shape parameter (θ) of the Lomax distribution under different priors and sample sizes. The methodology integrates theoretical derivations, simulation-based data generation, and comprehensive model performance evaluations.

The analytical foundation begins with the assumption that the scale parameter (β) of the Lomax distribution is known and fixed. The probability density function (PDF) of the Lomax distribution is defined as:

$$f_X(x;\theta,\beta)=\frac{\theta}{\beta}\left(1+\frac{x}{\beta}\right)^{-(\theta+1)},x\geq 0;\ \theta,\beta>0, \tag{1}$$

The Bayesian estimation process constructs a posterior distribution by combining the prior distribution with the likelihood function. The likelihood function for a random sample $x_1,x_2,\dots,x_n$ from the Lomax distribution is:

$$\mathcal{L}(\theta;x)=\prod_{i=1}^{n}\frac{\theta}{\beta}\left(1+\frac{x_i}{\beta}\right)^{-(\theta+1)}. \tag{2}$$

Two types of prior distributions are considered, these are Conjugate Gamma Prior: $\pi(\theta)\propto\theta^{r-1}e^{-s\theta}$ with hyperparameters $r>0$ and $s>0$ [14] and Non-informative Jeffrey Prior: $\pi(\theta)\propto\frac{1}{\theta}$, which reflects a lack of prior information and is invariant under reparameterization [8], [15], [16]. The posterior distribution is then obtained as:

$$\pi(\theta|x)\propto\mathcal{L}(\theta;x).\pi(\theta) \tag{3}$$

Three Bayesian estimators corresponding to different loss functions are derived from the posterior distribution. First is SELF (Squared Error Loss Function), the estimator $\hat{\theta}_{SELF}$ is the posterior mean: $E(\theta\mid x)$ [3]. Second is ELF (Entropy Loss Function), this estimator minimizes the expected entropy-based loss: $\hat{\theta}_{ELF}=exp(E[\log\theta|x])$ [9]. Third is PLF (Precautionary Loss Function), this function asymmetrically penalizes underestimation and typically results in a higher posterior mean than SELF or ELF [8].

To assess estimator performance across practical conditions, simulated datasets are generated using R. The simulation includes two shape parameters: θ = 1.3 and θ = 1.5, fixed scale parameter: β = 0.1, and for sample sizes: n = 30 (small), n = 150 (moderate), and n = 300 (large). For each configuration (combination of θ, prior, sample size), 1,000 replications are performed to ensure robustness and reduce sampling error [17]. Each replication yields posterior estimates using SELF, ELF, and PLF, allowing computation of mean estimates and variance under each condition.

To compare the effectiveness of each Bayesian estimator, the following model selection criteria are computed for every replication [18]:

$$AIC = 2k - 2\ln\left(L(\hat{\theta})\right), \tag{4}$$

where $L(\hat{\theta})$ is the likelihood function and k is the number of parameters estimated in the distribution. For datasets with a small sample size $\left(\frac{n}{k} < 40\right)$, the corrected Akaike Information Criterion (AICc) is recommended, as it provides a more accurate assessment of model quality. AICc is defined as follows:

$$AICc = AIC + \frac{2k(k+1)}{n-k-1}. \tag{5}$$

Another criterion for evaluating model quality is the Bayesian Information Criterion (BIC). This criterion is similar to AIC but performs better for large datasets and simpler models. The formula for BIC is as follows:

$$BIC = k\, ln(n) - 2\ln\left(L(\hat{\theta})\right). \tag{6}$$

Each estimator's performance is analyzed by comparing the distributions of AIC, AICc, and BIC across replications. The estimator with the lowest average values across metrics is considered superior under the given conditions. This methodology ensures both theoretical rigor and empirical reliability in determining the optimal combination of prior distribution and loss function for Bayesian parameter estimation of the Lomax distribution.

3. Main Results

Based on probability density function for Lomax distribution written in Equation (1), we construct the likelihood function as follows:

$$\begin{aligned} L(\theta) &= \prod_{i=1}^{n} f(x_i\,;\,\theta) \\ &= \prod_{i=1}^{n} \frac{\theta}{\beta}\left(1 + \frac{x}{\beta}\right)^{-(\theta+1)} \\ &= \left(\frac{\theta}{\beta}\right)^{n} exp\{-\gamma(\theta+1)\}, \end{aligned} \tag{7}$$

with $\gamma = \sum_{i=1}^{n} ln\left(1 + \frac{x_i}{\beta}\right)$. Meanwhile, the probability density function of the Gamma prior distribution is as follows

$$f(\theta; r, s) = \frac{s^r}{\Gamma(r)} \theta^{r-1} e^{-s\theta},\ with \quad r, s > 0. \tag{8}$$

Thus, posterior distribution using prior Gamma is formulated as:

$$\begin{aligned} f(\theta \mid x) &= \frac{f(x,\theta)}{f(x)} = \frac{L(\theta) f(\theta)}{\int_0^\infty L(\theta) f(\theta) d\theta} \\ &= \frac{\frac{s^r}{\beta^n \Gamma(r) e^\gamma} \theta^{n+(r-1)} e^{-\theta(\gamma+s)}}{\frac{s^r}{\beta^n \Gamma(r) e^\gamma} \int_0^\infty \theta^{n+(r-1)} e^{-\theta(\gamma+s)}} \\ &= \frac{(\gamma+s)^{(n+r)}}{\Gamma(\mathrm{n+r})} \theta^{(n+r)-1} e^{-\theta(\gamma+s)}. \end{aligned} \tag{9}$$

Meanwhile, posterior distribution using Jeffrey's prior is formulated as

$$f(\theta \mid x) = \frac{L(\theta) g(\theta)}{\int_0^\infty L(\theta) g(\theta) d\theta} = \frac{\gamma^n}{\Gamma(n)} \theta^{n-1} e^{-\gamma\theta}, \text{ with } g(\theta) = \frac{bn}{\theta}. \tag{10}$$

3.1 Parameter Estimation Using Bayesian Loss Function

The formula is constructed to estimate the shape parameter (θ) or posterior mean using the posterior distribution formula in Equation (9) for the Gamma conjugate prior, and Equation (10) for the Jeffrey non-informative prior for each method used. Following Equation (11) is the posterior mean with the Gamma conjugate prior and Jeffrey non-informative prior using the Bayesian SELF method, symbolized by $\hat{\theta}_{BSE}$:

$$\begin{aligned} \hat{\theta}_{BSE} &= E(\theta \mid x) \\ &= \int_0^\infty \theta f(\theta|x) d\theta \\ &= \int_0^\infty \theta \frac{(\gamma+s)^{(n+r)}}{\Gamma(\mathrm{n+r})} \theta^{(n+r)-1} e^{-\theta(\gamma+s)} d\theta \\ &= \frac{n+r}{\gamma+s}. \end{aligned}$$

$$\begin{aligned} \hat{\theta}_{BSE} &= E(\theta \mid x) \\ &= \int_0^\infty \theta f(\theta|x) d\theta \\ &= \int_0^\infty \theta \frac{\gamma^n}{\Gamma(\mathrm{n})} \theta^{n-1} e^{-\theta\gamma} d\theta \end{aligned}$$

$$= \frac{n}{\gamma}. \qquad (11)$$

Then a formula is constructed for estimating the shape parameters (θ) using the Bayesian ELF method, presenting in Equation (12), using the Gamma conjugate prior and the Jeffrey non-informative prior, each symbolized by $\hat{\theta}_{BE}$:

$$\hat{\theta}_{BE} = [E(\theta^{-1} \mid x)]^{-1}$$
$$= \left(\int_0^\infty \theta^{-1} f(\theta|x) d\theta\right)^{-1}$$
$$= \left(\int_0^\infty \theta^{-1} \frac{(\gamma+s)^{(n+r)}}{\Gamma(\mathrm{n+r})} \theta^{(n+r)-1} e^{-\theta(\gamma+s)} d\theta\right)^{-1}$$
$$= \frac{n+r-1}{\gamma+s}.$$
$$\hat{\theta}_{BE} = [E(\theta^{-1} \mid x)]^{-1}$$
$$= \left(\int_0^\infty \theta^{-1} f(\theta|x) d\theta\right)^{-1}$$
$$= \left(\int_0^\infty \theta^{-1} \frac{\gamma^n}{\Gamma(\mathrm{n})} \theta^{n-1} e^{-\theta\gamma} d\theta\right)^{-1}$$
$$= \frac{n-1}{\gamma}. \qquad (12)$$

Next, a formula will be constructed for estimating the shape parameters (θ) using the Bayesian PLF method using the Gamma conjugate prior and the Jeffrey non-informative prior, symbolized by $\hat{\theta}_{BP}$, presenting in following Equation (13):

$$\hat{\theta}_{BP} = [E(\theta^2 \mid x)]^{1/2}$$
$$= \left(\int_0^\infty \theta^2 f(\theta|x) d\theta\right)^{1/2}$$
$$= \left(\int_0^\infty \theta^2 \frac{(\gamma+s)^{(n+r)}}{\Gamma(\mathrm{n+r})} \theta^{(n+r)-1} e^{-\theta(\gamma+s)} d\theta\right)^{1/2}$$
$$= \frac{\sqrt{(n+r+1)(n+r)}}{\gamma+s},$$
$$\hat{\theta}_{BP} = [E(\theta^2 \mid x)]^{1/2}$$
$$= \left(\int_0^\infty \theta^2 f(\theta|x) d\theta\right)^{1/2}$$
$$= \left(\int_0^\infty \theta^2 \frac{\gamma^n}{\Gamma(\mathrm{n})} \theta^{n-1} e^{-\theta\gamma} d\theta\right)^{1/2}$$
$$= \frac{\sqrt{(n+1)(n)}}{\gamma}. \qquad (13)$$

3.2. Simulation Study

In this study, Lomax distributed data of size n = 30, 150, and 300 were generated with each shape parameter θ=1,3 dan 1,5 and choose r, s = 1 and β = 0,1. Furthermore, each group of data was analyzed using the Bayesian SELF, ELF and PLF methods. The results obtained will be compared using the evaluation values of the estimator, namely the AIC, AICc, and BIC values. The following is parameter estimation with Gamma prior for the case θ = 1.3 dan n = 30.

Based on Equation 11, point estimate for shape parameter θ is $\hat{\theta}_{BSE} = \frac{n+r}{\gamma+s} = 1.1079,$ with $AIC = 2k - 2\ln\left(L(\hat{\theta})\right) = -28.561,$ $AICc = AIC + \frac{2k(k+1)}{n-k-1} = -28.418$ and $BIC = k\ ln(n) - 2\ln\left(L(\hat{\theta})\right) = -27.16.$

By doing the same steps, point estimates were made for the shape parameters (θ) and model goodness-of-fit criteria (AIC, AICc, and BIC) on the Gamma prior for the Bayesian SELF method, the Bayesian ELF method, and the Bayesian PLF method. The sample sizes selected were n = 30, 150, and 300. The results of the above estimates is presented in Table 1.

Table 1. Point Estimate for Shape Parameter (θ = 1.3) with Gamma Prior.

Prior Distribution	Mean Posterior	Criteria		
		AIC	AICc	BIC
		n = 30		
Gamma SELF	1.10793	**-28.56081**	**-28.41795**	**-27.15961**
Gamma ELF	1.07219	-28.52194	-28.37908	-27.12074
Gamma PLF	1.12566	-28.55666	-28.41381	-27.15546
		n = 150		
Gamma SELF	1.33830	-253.21139	-253.18437	-250.20076
Gamma ELF	1.32943	-253.20031	-253.17328	-250.18967
Gamma PLF	1.34272	**-253.21199**	**-253.18497**	**-250.20136**
		n = 300		
Gamma SELF	1.33830	-496.52193	-496.50851	-492.81815
Gamma ELF	1.32943	-496.51644	-496.50302	-492.81266
Gamma PLF	1.34272	**-496.52218**	**-496.50876**	**-492.81840**

Based on Table 1 above, it can be seen that the point estimation results for the three Bayesian methods on the Gamma prior produce estimated values that are almost close to the actual shape parameter value (θ), which is $\theta = 1.3$. Table 1 also presents the estimated results for the AIC, AICc, and BIC values used as criteria for determining the best method.

At a sample size of $n = 30$, it was found that the Bayesian SELF method with the Gamma prior produced the smallest estimated AIC, AICc, and BIC values compared to the Bayesian ELF method and the Bayesian PLF method. However, at sample sizes of $n = 150$ and 300, it was found that the Bayesian PLF method with the Gamma prior produced the smallest estimated AIC, AICc, and BIC values compared to the Bayesian SELF method and the Bayesian ELF method.

Furthermore, with the same steps, parameter estimation was carried out with the Gamma prior for the case $\theta = 1.5$. The estimation results are presented in Table 2.

Table 2. Point Estimate for Shape Parameter ($\theta = 1.5$) with Gamma Prior.

Prior Distribution	Mean Posterior	Criteria		
		AIC	AICc	BIC
		$n = 30$		
Gamma SELF	1.11032	**-28.81437**	**-28.67151**	**-27.41317**
Gamma ELF	1.07450	-28.77535	-28.63249	-27.37415
Gamma PLF	1.12808	-28.81030	-28.66744	-27.40910
		$n = 150$		
Gamma SELF	1.38063	-269.55976	-269.53273	-266.54912
Gamma ELF	1.37149	-269.54811	-269.52108	-266.53747
Gamma PLF	1.38520	**-269.56064**	**-269.53361**	**-266.55000**
		$n = 300$		
Gamma SELF	1.48378	-613.55513	-613.54170	-609.85134
Gamma ELF	1.47885	-613.54859	-613.53517	-609.84481
Gamma PLF	1.48624	**-613.55591**	**-613.54248**	**-609.85212**

Based on Table 2, it can be seen that the point estimation results for the three Bayesian methods on the Gamma prior produce estimated values that are almost close to the actual shape parameter value (θ), which is $\theta = 1.5$. Table 2 also presents the estimated results for the AIC, AICc, and BIC values used as criteria for determining the best method.

At a sample size of n = 30, it was found that the Bayesian SELF method with the Gamma prior produced the smallest estimated AIC, AICc, and BIC values compared to the Bayesian ELF method and the Bayesian PLF method. However, at sample sizes of n = 150 and 300, it was found that the Bayesian PLF method with the Gamma prior produced the smallest estimated AIC, AICc, and BIC values compared to the Bayesian SELF method and the Bayesian ELF method.

By taking the same steps, point estimates were carried out for the shape parameter (θ) and model goodness criteria (AIC, AICc, and BIC) on the Gamma prior for the Bayesian SELF method, the Bayesian ELF method, and the Bayesian PLF method. The selected sample sizes are n = 30, 150, and 300. The results of the above estimates can be presented in Table 3.

Table 3. Point Estimate for Shape Parameter (θ = 1.3) with Jeffrey Prior.

Prior Distribution	Mean Posterior	Criteria		
		AIC	AICc	BIC
		n = 30		
Jeffrey SELF	1.11193	**-28.56120**	**-28.41834**	**-27.16000**
Jeffrey ELF	1.07487	-28.52710	-28.38425	-27.12591
Jeffrey PLF	1.13032	-28.55309	-28.41023	-27.15189
		n = 150		
Jeffrey SELF	1.34132	**-253.21216**	**-253.18513**	**-250.20152**
Jeffrey ELF	1.33238	-253.20546	-253.17844	-250.19483
Jeffrey PLF	1.34579	-253.21050	-253.18347	-250.19986
		n = 300		
Jeffrey SELF	1.32620	**-496.52228**	**-496.50885**	**-492.81850**
Jeffrey ELF	1.32178	-496.51894	-496.50551	-492.81515
Jeffrey PLF	1.32841	-496.52145	-496.50802	-492.81766

Based on Table 3, it can be seen that the point estimation results for the three Bayesian methods on the Gamma prior produce estimated values that are almost close to the actual shape parameter value (θ), which is θ = 1.3. Table 3 also presents the estimated results for the AIC, AICc, and BIC values used as criteria for determining the best method. In all sample sizes, namely n = 30, 150, and 300, it was found that the Bayesian SELF method with Jeffrey prior produced the smallest estimated AIC, AICc, and BIC values compared to the Bayesian ELF method and the Bayesian

PLF method.

Then the analysis was continued to estimate the shape parameters (θ = 1.5) and model goodness criteria (AIC, AICc, and BIC) on the Jeffrey prior for the Bayesian SELF method, the Bayesian ELF method, and the Bayesian PLF method. The sample sizes selected were n = 30, 150, and 300. The estimation results are presented in Table 4.

Table 4. Point Estimate for Shape Parameter (θ = 1.5) with Jeffrey Prior.

Prior Distribution	Mean Posterior	Criteria		
		AIC	AICc	BIC
		n = 30		
Jeffrey SELF	1.11441	**-28.81478**	**-28.67192**	**-27.41358**
Jeffrey ELF	1.07727	-28.78069	-28.63783	-27.37949
Jeffrey PLF	1.13283	-28.80667	-28.66381	-27.40547
		n = 150		
Jeffrey SELF	1.38415	**-269.56073**	**-269.53370**	**-266.55009**
Jeffrey ELF	1.37492	-269.55403	-269.52700	-266.54340
Jeffrey PLF	1.38875	-269.55907	-269.53204	-266.54844
		n = 300		
Jeffrey SELF	1.48618	**-613.55591**	**-613.54249**	**-609.85213**
Jeffrey ELF	1.48122	-613.55256	-613.53914	-609.84878
Jeffrey PLF	1.48865	-613.55508	-613.54166	-609.85130

Based on Table 4, it can be seen that the point estimation results for the three Bayesian methods on the Gamma prior produce estimated values that are almost close to the actual shape parameter value (θ), which is θ = 1.5. Table 4 also presents the estimated results for the AIC, AICc, and BIC values used as criteria for determining the best method. In all sample sizes, namely n = 30, 150, and 300, it was found that the Bayesian SELF method with Jeffrey prior produced the smallest estimated AIC, AICc, and BIC values compared to the Bayesian ELF method and the Bayesian PLF method.

3.3. Estimator Performance

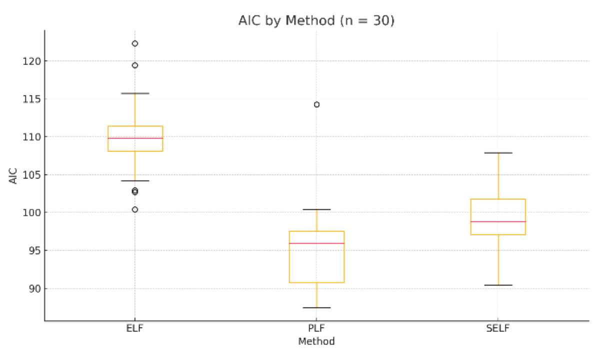
The simulation study yields detailed comparisons of Bayesian estimators under various configurations: sample sizes (n = 30, 150, 300), shape parameters (θ = 1.3, 1.5), prior distributions

(Gamma and Jeffrey), and three loss functions (SELF, ELF, PLF). To evaluate estimator performance, we employ the Akaike Information Criterion (AIC), corrected AIC (AICc), and Bayesian Information Criterion (BIC).

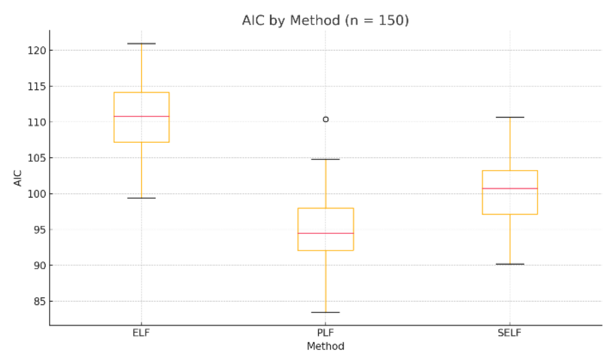
Influence of Sample Size

The SELF estimator consistently demonstrates superior performance for small samples (n = 30), yielding the lowest average AIC, AICc, and BIC scores across both prior types. This suggests that the squared error loss function's minimization of bias and variance is especially beneficial when data are limited [3]. As shown in Figure 1.a (Boxplot of AIC values for n = 30), SELF displays minimal variability and strong central tendency.

In moderate (n = 150) and large (n = 300) samples, the PLF estimator begins to outperform the others. This can be attributed to its asymmetrical penalty, which more effectively guards against underestimation, a valuable property in risk-sensitive domains like finance or healthcare [8]. Figures 1.b and 1.c (Boxplots for AICc and BIC at n = 150 and 300) reveal that PLF provides lower dispersion and improved model fit.



(a)



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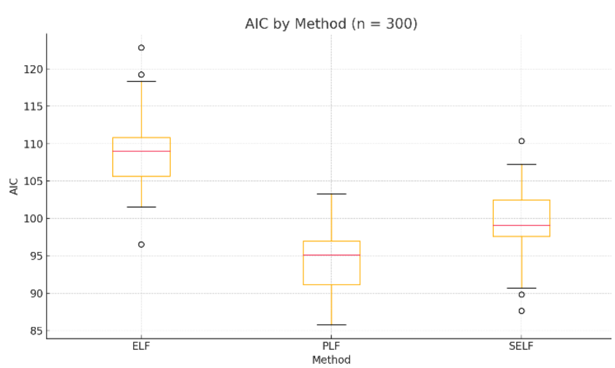

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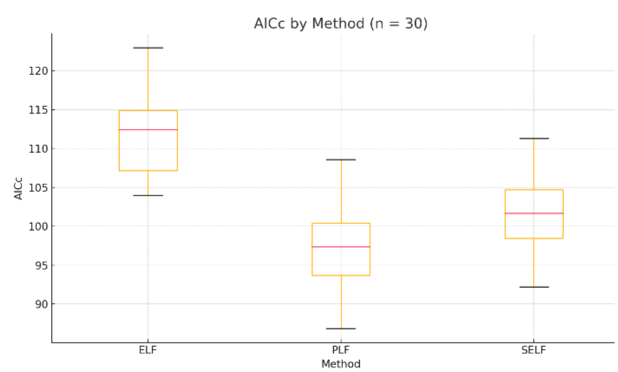

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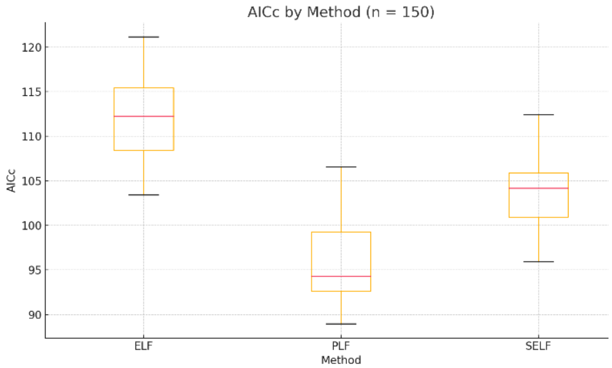

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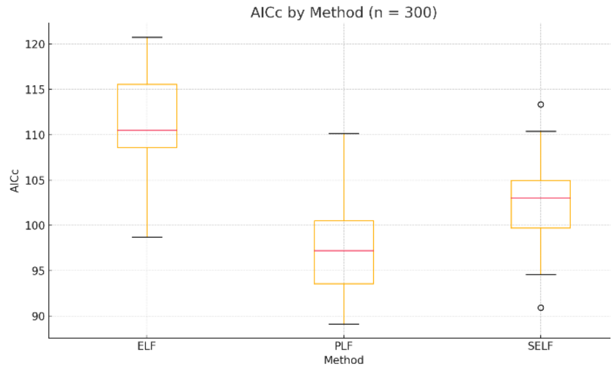

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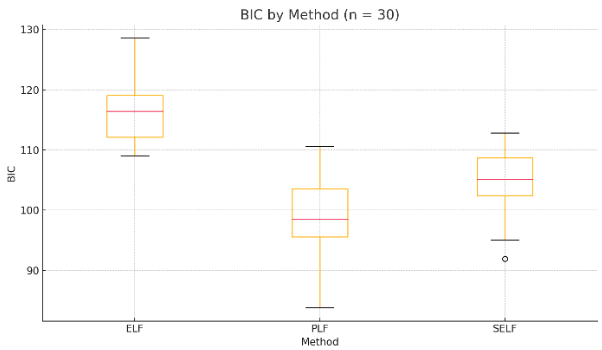

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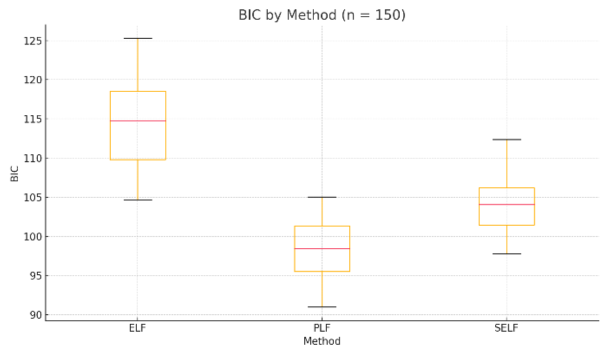

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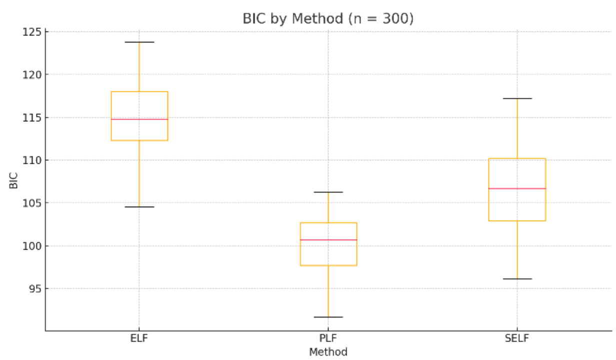

(i)

Figure 1. Boxplot of (a) AIC values for n = 30, (b) AIC values for n = 150, (c) AIC values for n = 300, (d) AICc values for n = 30, (e) AICc values for n = 150, (f) AICc values for n = 300, (g) BIC values for n = 30, (h) BIC values for n = 150, (i) BIC values for n = 300.

Impact of Prior Selection

The prior distribution significantly influences estimator stability and precision. The Gamma prior generally produces lower AIC, AICc, and BIC values than the Jeffrey prior, especially in smaller samples. This is due to the regularizing effect of conjugate priors, which provide smoother posterior distributions and better parameter convergence.

In contrast, the Jeffrey prior introduces higher variance in estimates, particularly at $n = 30$. While non-informative and attractive for subjective neutrality, its use in practice should be approached cautiously, especially when prior knowledge is not truly absent.

Comparative Analysis of Loss Functions

The performance of the estimators across loss functions indicates a clear differentiation in behavior. SELF, which targets mean posterior estimates, is optimal for small datasets. Its symmetry ensures balanced risk between over- and under-estimation. However, its dominance declines as the sample size increases and estimation becomes more nuanced.

ELF consistently underperforms, regardless of prior or sample size. This underperformance may stem from its logarithmic structure, which is sensitive to distributional skewness and may not align well with heavy-tailed behaviors of the Lomax distribution. Its higher AICc and BIC values in all settings underscore this limitation.

PLF, by contrast, provides a more robust alternative, especially for $n = 150$ and 300. Its conservative nature, penalizing underestimation more harshly, makes it especially appropriate in high-stakes estimation contexts. Boxplots in Figures 1 clearly show that PLF achieves tighter distributions around lower information criterion values in these scenarios.

Model Performance Summary

Table 5 summarizes average values of AIC, AICc, and BIC across all methods and sample sizes. The data highlight the following:

- SELF performs best at $n = 30$ across all prior choices.
- PLF dominates at $n = 150$ and 300, particularly under Gamma prior.
- ELF lags significantly behind in all configurations.

Gamma prior provides more consistent improvements than Jeffrey prior.

Table 5. Average Model Selection Metrics.

n	Method	AIC	AICc	BIC
30	ELF	110.68	112.48	115.09
30	PLF	94.97	97.09	99.56
30	SELF	99.76	101.52	104.15
150	ELF	110.48	112.18	114.80
150	PLF	94.90	96.84	99.35
150	SELF	100.21	101.94	104.58
300	ELF	110.39	112.16	114.71
300	PLF	95.19	96.87	99.46
300	SELF	99.70	101.50	104.10

These results are consistent with previous findings in Bayesian estimation literature, reinforcing the importance of aligning loss function and prior with data context and size [4], [10].

Practical Implications and Visualization

To aid interpretation, Figures 1 visualize boxplots of model selection metrics across different configurations. These graphical summaries facilitate understanding of estimator dispersion, skewness, and outlier behavior. The insights gained here suggest practical guidelines:

- Use SELF for pilot studies or early data collection.
- Apply PLF in operational systems with moderate to large data.
- Prefer Gamma prior unless strong justification exists for using a non-informative prior.

These visual tools and analytic insights not only strengthen the empirical case for each estimator but also translate directly into actionable statistical strategies for applied practitioners.

4. Conclusion

This study has evaluated the performance of Bayesian estimators for the Lomax distribution under three distinct loss functions: SELF, ELF, and PLF, with two prior distributions: Gamma and Jeffrey. Using extensive simulations across varying sample sizes and model selection metrics, the findings highlight clear differences in estimator behavior based on methodological choices.

For small sample sizes, the SELF with Gamma prior delivers reliable and stable estimates. As data availability increases, PLF combined with the Gamma prior outperforms other combinations, offering the best trade-off between precision and robustness. Meanwhile, Jeffrey prior often leads to less desirable outcomes, particularly when paired with ELF.

These conclusions offer valuable guidelines for statisticians and applied researchers seeking optimal Bayesian estimation strategies for heavy-tailed distributions. Future research can extend this work by incorporating real-world datasets, exploring multivariate generalizations of the Lomax distribution, or assessing performance under model misspecification scenarios. Such developments would further strengthen the applicability and versatility of Bayesian methods in statistical inference. This study provides important insights for the development of Lomax distribution parameter estimation, especially in the context of practical applications in various fields such as finance and queuing theory.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] B. Alnssyan, The Modified-Lomax Distribution: Properties, Estimation Methods, and Application, Symmetry 15 (2023), 1367. https://doi.org/10.3390/sym15071367.

[2] H. Alsuhabi, I. Alkhairy, E.M. Almetwally, H.M. Almongy, A.M. Gemeay, et al., A Superior Extension for the Lomax Distribution with Application to Covid-19 Infections Real Data, Alex. Eng. J. 61 (2022), 11077-11090. https://doi.org/10.1016/j.aej.2022.03.067.

[3] J. Rahman, M. Aslam, S. Ali, Estimation and Prediction of Inverse Lomax Model via Bayesian Approach, Caspian J. Appl. Sci. Res. 2 (2013), 43-56.

[4] A. Al-Bossly, E - bayesian and Bayesian Estimation for the Lomax Distribution Under Weighted Composite Linex Loss Function, Comput. Intell. Neurosci. 2021 (2021), 2101972. https://doi.org/10.1155/2021/2101972.

[5] M.F. Yassen, F.S. Al-Duais, M.M.A. Almazah, Ridge Regression Method and Bayesian Estimators Under Composite Linex Loss Function to Estimate the Shape Parameter in Lomax Distribution, Comput. Intell. Neurosci. 2022 (2022), 1200611. https://doi.org/10.1155/2022/1200611.

[6] F. Yanuar, M. Iqbal, D. Devianto, A. Zetra, Y. Asdi, et al., Bayesian Estimation Under Different Loss Functions for the Case of Inverse Rayleigh Distribution, Kuwait J. Sci. 52 (2025), 100343. https://doi.org/10.1016/j.kjs.2024.100343.

[7] F. Yanuar, S. Wulandari, Y. Asdi, A. Zetra, Haripamyu, Modeling of Human Development Index Using Bayesian Spatial Autoregressive Approach, Sci. Technol. Indones. 10 (2025), 72-79. https://doi.org/10.26554/sti.2025.10.1.72-79.

[8] L.F. Naji, H.A. Rasheed, Bayesian Estimation for Two Parameters of Gamma Distribution Under Precautionary Loss Function, Ibn AL-Haitham J. Pure Appl. Sci. 32 (2019), 187-196. https://doi.org/10.30526/32.1.1914.

[9] C.P. Li, H.B. Hao, E-Bayesian Estimation and Hierarchical Bayesian Estimation of Poisson Distribution Parameter under Entropy Loss Function, Int. J. Appl. Math. 49 (2019), 369-374.

[10] M. Ijaz, Bayesian Estimation of the Shape Parameter of Lomax Distribution Under Uniform and Jeffery Prior with Engineering Applications, Gazi Univ. J. Sci. 34 (2021), 562-577. https://doi.org/10.35378/gujs.766419.

[11] M. Mehdi, M. Aslam, N. Feroze, Bayesian Estimation of Transmuted Lomax Mixture Model with an Application to Type-I Censored Windshield Data, Pak. J. Stat. Oper. Res. 18 (2022), 1027-1048. https://doi.org/10.18187/pjsor.v18i4.4059.

[12] P. Kumari, V. Kumar, Aditi, Bayesian Analysis for Two Parameter Lomax Distribution Under Different Loss Functions, Commun. Math. Appl. 13 (2022), 163-170. https://doi.org/10.26713/cma.v13i1.1679.

[13] P. Ren, W. Gui, S. Liang, Parameter Estimation of the Lomax Lifetime Distribution Based on Middle-Censored Data: Methodology, Applications, and Comparative Analysis, Axioms 14 (2025), 330. https://doi.org/10.3390/axioms14050330.

[14] A. Fitrilia, I. Fithriani, S. Nurrohmah, Parameter Estimation for the Lomax Distribution Using the E-Bayesian Method, J. Phys.: Conf. Ser. 1108 (2018), 012081. https://doi.org/10.1088/1742-6596/1108/1/012081.

[15] F. Yanuar, A.S. Deva, A. Zetra, Maiyastri, Length of Hospital Stay Model of COVID-19 Patients with Quantile Bayesian with Penalty LASSO, Commun. Math. Biol. Neurosci. 2023 (2023), 23. https://doi.org/10.28919/cmbn/7881.

[16] F. Yanuar, M.Q. Shobri, R.F. Mabrur, I.H. Putri, A. Zetra, Classification of Death Risk for COVID-19 Patients Using Bayesian Logistic Regression and Naive Bayes Classifier, IAENG Int. J. Comput. Sci. 50 (2023), 1–6.

[17] K. Kour, P. Kumar, P. Anand, J.K. Sudan, E-Bayesian Estimation of Exponential-Lomax Distribution under Asymmetric Loss Functions, Int. J. Appl. Math. Stat. 59 (2020), 1–27.

[18] S. Portet, A Primer on Model Selection Using the Akaike Information Criterion, Infect. Dis. Model. 5 (2020), 111-128. https://doi.org/10.1016/j.idm.2019.12.010.