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A FRACTIONAL-ORDER SPATIAL-TEMPORAL MODEL FOR THE ADOPTION DYNAMICS OF REMOTE HEALTHCARE SERVICES

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Abstract. This paper presents a fractional-order (FO) spatial-temporal model for the adoption dynamics of Remote Healthcare Services (RHS) in a two-dimensional domain. The model incorporates memory effects and non-local temporal interactions through the Caputo nabla fractional derivative (CNFD), providing a realistic framework for social contagion processes where past states influence current behavior. The spatial domain is discretized using the Method of Lines (MOL) with periodic boundary conditions, transforming the original partial differential equations (PDEs) into a system of fractional ordinary differential equations (ODEs). We derive the equilibrium points (EPs) of the system and establish sufficient conditions for global asymptotic stability (GAS) via a Lyapunov functional (LF) approach. Numerical simulations validate the theoretical stability results, demonstrating convergence to the trivial equilibrium under the chosen parameter set. The proposed model offers a robust framework for analyzing spatial-temporal adoption dynamics with potential applications in public health, technology diffusion, and social behavior modeling.

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1. Introduction

Recent studies have demonstrated the growing importance of advanced analytical and fractional modeling techniques in describing complex dynamical phenomena arising in biological, technological, and social systems. For instance, mathematical frameworks have been successfully applied to epidemiological dynamics and biological processes, providing insight into disease transmission mechanisms and control strategies [1, 2]. In addition, several analytical and semi-analytical methods, including modified differential transform techniques, have been developed to obtain approximate solutions of nonlinear differential and integro-differential systems [3, 4, 5]. More recently, fractional-order and reaction–diffusion models have been widely investigated due to their ability to capture memory effects and spatial interactions in complex systems [6, 7]. Furthermore, theoretical studies on solvability and stability properties of functional and dynamical equations continue to play a crucial role in understanding the behavior of nonlinear systems and their applications in applied mathematics and engineering [8].

The adoption and diffusion of innovative services, particularly in the healthcare sector, are complex socio-dynamical processes that evolve across both space and time [9, 10]. Understanding how RHS spread through populations is of fundamental importance to policymakers and healthcare providers, since adoption levels directly influence accessibility, efficiency, and equity of care [11, 12]. However, human decision-making is not instantaneous: it is shaped by past experiences, delayed information, and persistent social influence [13, 14]. These memory-dependent mechanisms challenge the adequacy of classical diffusion and contagion models based on integer-order derivatives, which assume Markovian and memoryless dynamics [15, 16]. Fractional calculus offers a natural mathematical framework for modeling such memory-driven processes [17, 18]. Fractional derivatives introduce non-local temporal dependence, allowing past states to influence current dynamics through power-law kernels [19, 20]. This makes them particularly suitable for modeling social and behavioral systems, where historical exposure and cumulative experience play critical roles [21, 22]. Among the various formulations, Caputo-type fractional derivatives are especially attractive because they permit physically meaningful initial conditions [23, 24]. In discrete-time settings, the Caputo nabla fractional derivative provides a backward-difference analogue that is well suited to sequential data and digital social processes, where adoption decisions depend on weighted histories of prior interactions [25, 26].

Despite growing interest in fractional models for epidemics and information diffusion, most existing studies focus on temporal memory in well-mixed populations or on spatial models with classical integer-order time derivatives [27, 28]. Spatial–temporal fractional models—particularly those combining two-dimensional diffusion with Caputo nabla memory operators—remain scarce [29, 30]. This gap is especially significant for modeling RHS adoption, where both geographic diffusion and persistent social influence are central [32]. To address this limitation, we develop a spatial–temporal fractional RDs framework for RHS adoption. Specifically, the population is represented by three interacting state variables defined over a two-dimensional spatial domain Ω: $U(x,y,t)$ denotes the density of unaware individuals, $A(x,y,t)$ represents the density of adopters of remote healthcare services, and $H(x,y,t)$ describes the level of healthcare need at spatial location (x,y) and time t.

Their classical (integer-order) spatial–temporal dynamics are governed by the reaction-diffusion system (RDs)

$$(1)\qquad \begin{cases} \dfrac{\partial U(x,y,t)}{\partial t} = D_U \Delta U - \beta UA - \alpha UH + \gamma A, & (x,y,t) \in \Omega \times \mathbb{R}^+, \\ \dfrac{\partial A(x,y,t)}{\partial t} = D_A \Delta A + \beta UA + \alpha UH - \gamma A, & (x,y,t) \in \Omega \times \mathbb{R}^+, \\ \dfrac{\partial H(x,y,t)}{\partial t} = D_H \Delta H - \delta AH + \theta (H_0 - H), & (x,y,t) \in \Omega \times \mathbb{R}^+, \end{cases}$$

where $\Delta = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$ denotes the two-dimensional Laplacian. The diffusion coefficients D_U, D_A, and D_H describe spatial mobility, while the nonlinear reaction terms represent social contagion, need-driven adoption, relapse, consumption of healthcare need, and its regeneration toward a baseline H_0. The principal novelty of this work lies in replacing the classical time derivatives in (1) with Caputo nabla fractional derivatives of order $\wp \in (0,1]$, allowing past adoption and healthcare-need levels to influence present behavior.The spatial domain is discretized via the MOL, yielding a large-scale system of coupled fractional ODEs that combine spatial diffusion with discrete-time memory.

The main contributions of this paper are summarized as follows:

- We introduce the first two-dimensional fractional RDs model for RHS adoption based on the Caputo nabla operator, capturing spatial diffusion and long-range temporal memory.
- A hybrid discretization combining MOL and fractional nabla dynamics is developed, leading to a lattice system of fractional ODEs.
- An LF adapted to the Caputo nabla operator and the discrete Laplacian is constructed, providing explicit algebraic conditions for GAS.
- Non-trivial homogeneous EPs are characterized and validated through numerical simulations.

The remainder of the paper is organized as follows. Section 2 presents the fractional spatial–temporal model and its discretization via the MOL. Section 3 develops the EP analysis and the main GAS theorem. Section 4 provides numerical simulations validating the theoretical results. Section 5 concludes the paper and outlines directions for future research.

2. MODEL DESCRIPTION

To solve the system of PDEs governing the RHS adoption dynamics, we employ the MOL. This technique involves discretizing the spatial domain Ω while leaving the time variable t continuous, thereby transforming the original PDE system into a large system of coupled ODEs. We consider a two-dimensional rectangular domain $\Omega = [0, L_x] \times [0, L_y]$, which is discretized into a uniform grid with N_x points in the x-direction and N_y points in the y-direction. The spatial step sizes are defined as $\Delta x = L_x/(N_x - 1)$ and $\Delta y = L_y/(N_y - 1)$. The grid points are denoted by (x_i, y_j), where

$$x_i = (i-1)\Delta x, \quad y_j = (j-1)\Delta y, \quad i = 1, \dots, N_x,\ j = 1, \dots, N_y. \tag{2}$$

The state variables at each grid point are approximated as time-dependent functions:

$$U_{i,j}(t) \approx U(x_i, y_j, t), \quad A_{i,j}(t) \approx A(x_i, y_j, t), \quad H_{i,j}(t) \approx H(x_i, y_j, t). \tag{3}$$

The spatial diffusion terms are approximated using a second-order central finite-difference scheme for the Laplacian operator Δ. For a generic field variable Φ, the discrete Laplacian $\mathscr{L}_{i,j}[\Phi]$ at an interior node (i,j) is given by

$$\begin{aligned}\Delta\Phi \approx \mathscr{L}_{i,j}[\Phi] &= \frac{\Phi_{i+1,j}-2\Phi_{i,j}+\Phi_{i-1,j}}{(\Delta x)^2}+\frac{\Phi_{i,j+1}-2\Phi_{i,j}+\Phi_{i,j-1}}{(\Delta y)^2}\\ &= \frac{\Delta_x^2\Phi_{i,j}}{(\Delta x)^2}+\frac{\Delta_y^2\Phi_{i,j}}{(\Delta y)^2}, \qquad \Phi\in\{U,A,H\},\end{aligned} \tag{4}$$

where

$$\Delta_x^2\Phi_{i,j} := \Phi_{i+1,j}-2\Phi_{i,j}+\Phi_{i-1,j}, \qquad \Delta_y^2\Phi_{i,j} := \Phi_{i,j+1}-2\Phi_{i,j}+\Phi_{i,j-1}. \tag{5}$$

Substituting the discrete Laplacian into the original model yields the semi-discrete system of integer-order ODEs at each interior grid point (i,j):

$$\begin{cases}\dfrac{dU_{i,j}(t)}{dt} = D_U\mathscr{L}_{i,j}[U](t)-\beta U_{i,j}(t)A_{i,j}(t)-\alpha U_{i,j}(t)H_{i,j}(t)+\gamma A_{i,j}(t),\\ \dfrac{dA_{i,j}(t)}{dt} = D_A\mathscr{L}_{i,j}[A](t)+\beta U_{i,j}(t)A_{i,j}(t)+\alpha U_{i,j}(t)H_{i,j}(t)-\gamma A_{i,j}(t),\\ \dfrac{dH_{i,j}(t)}{dt} = D_H\mathscr{L}_{i,j}[H](t)-\delta A_{i,j}(t)H_{i,j}(t)+\theta\big(H_0-H_{i,j}(t)\big).\end{cases} \tag{6}$$

To capture the non-local temporal dynamics inherent in social contagion—where past influences affect current behavior—we generalize the above system to the fractional-order (FO) setting. The transition from the classical integer-order model to the FO model proceeds as follows:

(1) **Discretization of the time domain:** The continuous time variable is replaced by the discrete time scale $\mathbb{N}_0=\{0,1,2,\ldots\}$, which is suitable for nabla difference calculus.
(2) **Definition of the difference operator:** The instantaneous rate of change is replaced by the backward difference operator ∇, defined in [33] by $\nabla f(t)=f(t)-f(t-1)$.
(3) **Generalization to FO:** We introduce the CNFD ${}^{C}\nabla^{\wp}$ of order $0<\wp\leq 1$, defined in [33] via a convolution sum with a power-law memory kernel:

$${}^{C}\nabla^{\wp}f(t) = \frac{1}{\Gamma(1-\wp)}\sum_{k=1}^{t}(t-\rho(k))^{-\wp}\nabla f(k), \tag{7}$$

where $\rho(k)=k-1$ and $\Gamma(\cdot)$ denotes the Gamma function.

(4) **System replacement:** Replacing the integer-order time derivatives in (6) with the CNFD incorporates memory effects directly into the dynamics.

Applying these steps yields the following system of FO-ODEs:

$$\begin{cases} {}^{C}\nabla^{\wp}U_{i,j}(t) = D_U\mathscr{L}_{i,j}[U](t) - \beta U_{i,j}(t)A_{i,j}(t) - \alpha U_{i,j}(t)H_{i,j}(t) + \gamma A_{i,j}(t), \\ {}^{C}\nabla^{\wp}A_{i,j}(t) = D_A\mathscr{L}_{i,j}[A](t) + \beta U_{i,j}(t)A_{i,j}(t) + \alpha U_{i,j}(t)H_{i,j}(t) - \gamma A_{i,j}(t), \\ {}^{C}\nabla^{\wp}H_{i,j}(t) = D_H\mathscr{L}_{i,j}[H](t) - \delta A_{i,j}(t)H_{i,j}(t) + \theta\big(H_0 - H_{i,j}(t)\big). \end{cases} \tag{8}$$

To close the system and ensure mass conservation within the computational domain, periodic boundary conditions are imposed. These conditions connect opposite boundaries of the domain and are implemented via ghost points:

$$\Phi_{0,j}(t) = \Phi_{N_x,j}(t), \quad \Phi_{N_x+1,j}(t) = \Phi_{1,j}(t), \tag{9}$$

with analogous conditions in the y-direction:

$$\Phi_{i,0}(t) = \Phi_{i,N_y}(t), \quad \Phi_{i,N_y+1}(t) = \Phi_{i,1}(t). \tag{10}$$

Finally, the system is fully specified by the initial conditions at $t = 0$ over the entire domain Ω:

$$U_{i,j}(0) = U_0(x_i, y_j), \quad A_{i,j}(0) = A_0(x_i, y_j), \quad H_{i,j}(0) = H_0(x_i, y_j). \tag{11}$$

3. Stability Analysis

In this section, we analyze the stability of the discrete FO system described by Eq. (8). We begin by determining the EPs of the system by setting the CNFDs to zero. This leads to the algebraic system given by Eq. (14). From this system, we derive the non-trivial equilibrium points as expressed in Eq. (18). To investigate the GS of these equilibria, we employ a LF approach tailored to the CNFD and the discrete Laplacian. We construct a candidate LF and use it to derive sufficient conditions for GS.

Setting the time derivatives to zero yields the algebraic system:

$$0 = -\beta U^*A^* - \alpha U^*H^* + \gamma A^*, \tag{12}$$

$$0 = \beta U^* A^* + \alpha U^* H^* - \gamma A^*, \tag{13}$$

$$0 = -\delta A^* H^* + \theta (H_0 - H^*). \tag{14}$$

From (13) we obtain, assuming $A^* \neq 0$,

$$U^* = \frac{\gamma}{\beta + \alpha \cdot \frac{H^*}{A^*}}. \tag{15}$$

From (14),

$$A^* = \frac{\theta(H_0 - H^*)}{\delta H^*}. \tag{16}$$

Substituting $\frac{H^*}{A^*} = \frac{\delta (H^*)^2}{\theta(H_0 - H^*)}$ gives

$$U^* = \frac{\gamma \theta (H_0 - H^*)}{\beta \theta (H_0 - H^*) + \alpha \delta (H^*)^2}. \tag{17}$$

Thus the family of non-trivial equilibria is

$$(U^*, A^*, H^*) = \left(\frac{\gamma \theta (H_0 - H^*)}{\beta \theta (H_0 - H^*) + \alpha \delta (H^*)^2},\ \frac{\theta(H_0 - H^*)}{\delta H^*},\ H^* \right), \quad 0 < H^* < H_0. \tag{18}$$

In the special case $H_0 = 0$, the equilibrium set reduces to

$$(U^*, A^*, H^*) = \left\{ (0,0,0),\ \left(\frac{\alpha}{\beta}, \frac{\theta}{\delta}, 0 \right) \right\}. \tag{19}$$

Theorem 3.1 ([34])**.** *Suppose there exists a positive definite and decrescent scalar function $V(t)$ such that ${}^{C}\nabla^{\wp} V(t) \leq 0$. Then, the EP of the system is GAS.*

Lemma 3.2 ([35])**.** *The following inequality holds:*

$${}^{C}\nabla^{\wp} U^2(t) \leq U(t) {}^{C}\nabla^{\wp} U(t), \quad t \in \mathbb{N}_1. \tag{20}$$

Theorem 3.3. *The EP (U^*, A^*, H^*) of the discrete FO system* (8) *is GAS if there exist positive constants K_1, K_2, K_3 such that*

$$\min\left\{ D_U \lambda_1 + K_1,\ D_A \lambda_1 + K_2,\ D_H \lambda_1 + K_3 \right\} > 0, \tag{21}$$

where $\lambda_1 > 0$ is the first non-zero eigenvalue of the discrete Laplacian operator under periodic boundary conditions.

Proof. Consider the LF candidate

$$V(t)=\sum_{i,j}V_{i,j}(t)=\frac{1}{2}\sum_{i,j}\big(u_{i,j}^2(t)+a_{i,j}^2(t)+h_{i,j}^2(t)\big). \tag{22}$$

Applying the CNFD to $V(t)$ and using the inequality for the fractional derivative of a square (Lemma 3.2), we obtain

$${}^{C}\nabla^{\wp}V(t)\le\sum_{i,j}\big(u_{i,j}\,{}^{C}\nabla^{\wp}u_{i,j}+a_{i,j}\,{}^{C}\nabla^{\wp}a_{i,j}+h_{i,j}\,{}^{C}\nabla^{\wp}h_{i,j}\big). \tag{23}$$

Substituting the system (8) for the fractional derivatives of the deviations gives

$$\begin{aligned}{}^{C}\nabla^{\wp}V(t)\le\sum_{i,j}\Big[&u_{i,j}\big(D_U\mathscr{L}_{i,j}[u]+F_U(u_{i,j},a_{i,j},h_{i,j})\big)\\&+a_{i,j}\big(D_A\mathscr{L}_{i,j}[a]+F_A(u_{i,j},a_{i,j},h_{i,j})\big)\\&+h_{i,j}\big(D_H\mathscr{L}_{i,j}[h]+F_H(u_{i,j},a_{i,j},h_{i,j})\big)\Big],\end{aligned} \tag{24}$$

where the nonlinearities F_U,F_A,F_H are

$$\begin{aligned}F_U(u,a,h)&=-\beta(U^*a+uA^*+ua)-\alpha(U^*h+uH^*+uh)+\gamma a,\\F_A(u,a,h)&=\beta(U^*a+uA^*+ua)+\alpha(U^*h+uH^*+uh)-\gamma a,\\F_H(u,a,h)&=-\delta(A^*h+aH^*+ah)-\theta h.\end{aligned}$$

For the diffusion terms we invoke the discrete summation-by-parts identity together with periodic boundary conditions:

$$\sum_{i,j}\Phi_{i,j}\mathscr{L}_{i,j}[\Phi]=-\sum_{i,j}|\nabla_d\Phi_{i,j}|^2\le-\lambda_1\sum_{i,j}\Phi_{i,j}^2,\qquad\Phi\in\{u,a,h\}. \tag{25}$$

Hence

$$\sum_{i,j}\big(D_Uu_{i,j}\mathscr{L}_{i,j}[u]+D_Aa_{i,j}\mathscr{L}_{i,j}[a]+D_Hh_{i,j}\mathscr{L}_{i,j}[h]\big)\le-\sum_{i,j}\big(D_U\lambda_1u_{i,j}^2+D_A\lambda_1a_{i,j}^2+D_H\lambda_1h_{i,j}^2\big). \tag{26}$$

For the reaction terms we follow the expansion and bounds derived in the continuous case. After linearising around the EP and applying Young's inequality to the cubic and cross terms, we obtain, for each grid point (i, j),

$$
\begin{aligned}
S = & -(\beta A^* + \alpha H^*)\, u^2 + (\beta U^* - \gamma)\, a^2 - (\delta A^* + \theta)\, h^2 \\
& + (-\beta U^* + \gamma + \beta A^* + \alpha H^*) ua - \alpha U^* uh + (\alpha U^* - \delta H^*) ah \\
& + (-\beta u^2 a - \alpha u^2 h + \beta u a^2 + \alpha uah - \delta a h^2).
\end{aligned} \tag{27}
$$

For any $r > 0$, if $|u|, |a|, |h| \leq r$ in a neighborhood of the EP, we have:

$$
\begin{aligned}
|-\beta u^2 a| &\leq \frac{\beta}{2} u^4 + \frac{\beta}{2} a^2 \leq \frac{\beta}{2} r^2 u^2 + \frac{\beta}{2} a^2, \\
|-\alpha u^2 h| &\leq \frac{\alpha}{2} u^4 + \frac{\alpha}{2} h^2 \leq \frac{\alpha}{2} r^2 u^2 + \frac{\alpha}{2} h^2, \\
|\beta u a^2| &\leq \frac{\beta}{2} a^4 + \frac{\beta}{2} u^2 \leq \frac{\beta}{2} r^2 a^2 + \frac{\beta}{2} u^2, \\
|\alpha uah| &\leq \frac{\alpha}{2} u^2 a^2 + \frac{\alpha}{2} h^2 \leq \frac{\alpha}{2} r^2 (u^2 + a^2) + \frac{\alpha}{2} h^2, \\
|-\delta a h^2| &\leq \frac{\delta}{2} h^4 + \frac{\delta}{2} a^2 \leq \frac{\delta}{2} r^2 h^2 + \frac{\delta}{2} a^2.
\end{aligned} \tag{28}
$$

Define constants:

$$
M_1 = \left(\frac{\beta}{2} + \frac{\alpha}{2}\right) r^2, \quad M_2 = \frac{\beta}{2} r^2, \quad M_3 = \frac{\delta}{2} r^2, \quad M_4 = \frac{\alpha}{4} r^2.
$$

Then the cubic terms satisfy:

$$
\begin{aligned}
-\beta u^2 a - \alpha u^2 h + \beta u a^2 + \alpha uah - \delta a h^2 \leq & \left(\frac{\beta}{2} + M_1 + M_4\right) u^2 \\
& + \left(\frac{\beta}{2} + \frac{\delta}{2} + M_2 + M_4\right) a^2 \\
& + (\alpha + M_3) h^2.
\end{aligned} \tag{29}
$$

Substituting the bounds into S yields:

$$
\begin{aligned}
S \leq & \left(-\beta A^* - \alpha H^* + \frac{\beta}{2} + M_1 + M_4\right) u^2 \\
& + \left(\beta U^* - \gamma + \frac{\beta}{2} + \frac{\delta}{2} + M_2 + M_4\right) a^2 \\
& + (-\delta A^* - \theta + \alpha + M_3) h^2 \\
& + (-\beta U^* + \gamma + \beta A^* + \alpha H^*) ua - \alpha U^* uh + (\alpha U^* - \delta H^*) ah.
\end{aligned} \tag{30}
$$

Using the inequalities:

$$
\begin{aligned}
|(-\beta U^* + \gamma + \beta A^* + \alpha H^*)ua| &\leq \frac{|-\beta U^* + \gamma + \beta A^* + \alpha H^*|}{2}(u^2 + a^2), \\
|-\alpha U^* uh| &\leq \frac{\alpha U^*}{2}(u^2 + h^2), \\
|(\alpha U^* - \delta H^*)ah| &\leq \frac{|\alpha U^* - \delta H^*|}{2}(a^2 + h^2),
\end{aligned} \tag{31}
$$

and collecting all terms, we obtain:

(32)

$$
\begin{aligned}
S \leq &- \left[\beta A^* + \alpha H^* - \frac{\beta}{2} - M_1 - M_4 - \frac{|-\beta U^* + \gamma + \beta A^* + \alpha H^*|}{2} - \frac{\alpha U^*}{2}\right] u^2 \\
&- \left[-\beta U^* + \gamma - \frac{\beta}{2} - \frac{\delta}{2} - M_2 - M_4 - \frac{|-\beta U^* + \gamma + \beta A^* + \alpha H^*|}{2} - \frac{|\alpha U^* - \delta H^*|}{2}\right] a^2 \\
&- \left[\delta A^* + \theta - \alpha - M_3 - \frac{\alpha U^*}{2} - \frac{|\alpha U^* - \delta H^*|}{2}\right] h^2.
\end{aligned}
$$

Define the constants:

$$
\begin{aligned}
K_1 &= \beta A^* + \alpha H^* - \frac{\beta}{2} - M_1 - M_4 - \frac{|-\beta U^* + \gamma + \beta A^* + \alpha H^*|}{2} - \frac{\alpha U^*}{2}, \\
K_2 &= -\beta U^* + \gamma - \frac{\beta}{2} - \frac{\delta}{2} - M_2 - M_4 - \frac{|-\beta U^* + \gamma + \beta A^* + \alpha H^*|}{2} - \frac{|\alpha U^* - \delta H^*|}{2}, \\
K_3 &= \delta A^* + \theta - \alpha - M_3 - \frac{\alpha U^*}{2} - \frac{|\alpha U^* - \delta H^*|}{2}.
\end{aligned}
$$

Then we have:

$$
uF_U + aF_A + hF_H \leq -K_1 u^2 - K_2 a^2 - K_3 h^2. \tag{33}
$$

Summation over all grid points yields

$$
\sum_{i,j}(u_{i,j}F_U + a_{i,j}F_A + h_{i,j}F_H) \leq -\sum_{i,j}(K_1 u_{i,j}^2 + K_2 a_{i,j}^2 + K_3 h_{i,j}^2). \tag{34}
$$

Combining the diffusion and reaction estimates,

$$
{}^{C}\nabla^{\wp} V(t) \leq -\sum_{i,j}\Big[(D_U\lambda_1 + K_1)u_{i,j}^2 + (D_A\lambda_1 + K_2)a_{i,j}^2 + (D_H\lambda_1 + K_3)h_{i,j}^2\Big]. \tag{35}
$$

Since $V(t) = \frac{1}{2}\sum_{i,j}(u_{i,j}^2 + a_{i,j}^2 + h_{i,j}^2)$, we have

$$
{}^{C}\nabla^{\wp} V(t) \leq -2\min\big\{D_U\lambda_1 + K_1,\ D_A\lambda_1 + K_2,\ D_H\lambda_1 + K_3\big\} V(t). \tag{36}
$$

If condition (21) holds, the coefficient is positive, and by the fractional Lyapunov stability theorem 3.1 the EP (U^*, A^*, H^*) is GAS. □

4. Numerical Simulation

To validate the theoretical GS analysis established in Theorem 3.3, we conducted comprehensive numerical simulations of the FO system. The computational implementation employed the same MOL spatial discretization together with the CNFD time-stepping scheme. The spatial domain $\Omega = [0,20] \times [0,20]$ was discretized using $N_x = N_y = 100$ grid points. The temporal evolution was computed with a time step $\Delta t = 0.3$ up to the final time $T = 30$. The parameter values, carefully calibrated to satisfy the GS conditions of Theorem 3.3, are summarized in Table 1.

TABLE 1. Parameter values for numerical simulation of FO–RHS adoption dynamics

Parameter	Symbol	Value
Diffusion coefficient (unaware)	D_U	0.00015
Diffusion coefficient (adopters)	D_A	0.00015
Diffusion coefficient (healthcare need)	D_H	0.00015
Social contagion rate	β	0.25
External influence coefficient	α	0.5
Relapse rate	γ	0.5
Need consumption rate	δ	0.5
Need regeneration rate	θ	0.5
Baseline healthcare need	H_0	0
Fractional order	$\wp$	0.998

Initial conditions were defined as spatially varying distributions:

$$\begin{aligned} U(x,y,0) &= 5 \times 10^{12} \exp\left(-\frac{2\pi(x+y)}{L_x}\right), \\ A(x,y,0) &= 8 \times 10^{14} \exp\left(-\frac{\pi(x+y)}{L_x}\right), \\ H(x,y,0) &= 12 \times 10^{18} \exp\left(-\frac{3\pi(x+y)}{L_x}\right). \end{aligned} \tag{37}$$

The GS conditions from Theorem 3.3 were numerically verified using the computed EP $(U^*, A^*, H^*) = (0,0,0)$ obtained from the simulations. With the first non-zero eigenvalue of the discrete Laplacian $\lambda_1 \approx 1000$ under periodic boundary conditions and neighborhood radius $r = 0.01$, the stability constants were computed as

$$\begin{aligned} &K_1 = 0.1249375, \quad K_2 = -0.125025, \quad K_3 = -1.25 \times 10^{-5}, \\ &\min\{D_U\lambda_1 + K_1,\, D_A\lambda_1 + K_2,\, D_H\lambda_1 + K_3\} = 0.024975 > 0, \end{aligned} \tag{38}$$

where $M_1 = 3.75 \times 10^{-5}$, $M_2 = 2.5 \times 10^{-5}$, $M_3 = 2.5 \times 10^{-5}$, and $M_4 = 2.5 \times 10^{-5}$. Since the minimum value is strictly positive, the conditions of Theorem 3.3 are satisfied, confirming the GS of the EP (U^*, A^*, H^*). The spatial and temporal evolution of the system variables is illustrated in Figures 1–3.

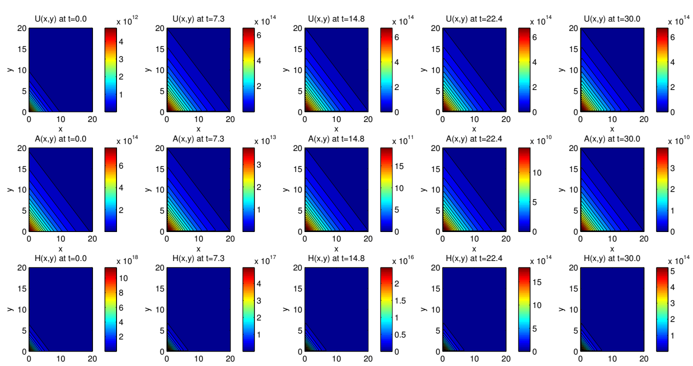

FIGURE 1. Spatial distribution of the unaware population $U(x,y,t)$ at $t = 0$ (left) and $t = 30$ (right).

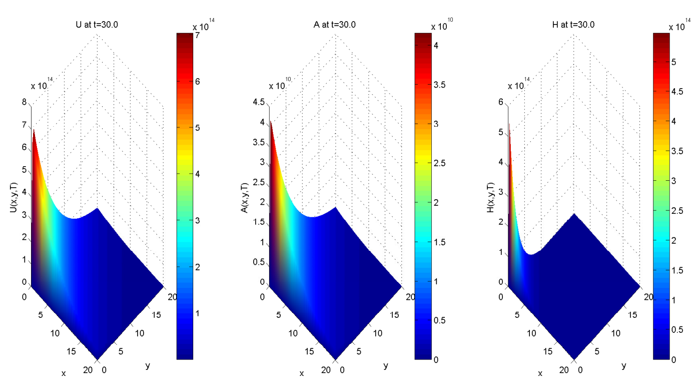

FIGURE 2. Spatial distribution of the adopter population $A(x,y,t)$ at $t = 0$ (left) and $t = 30$ (right).

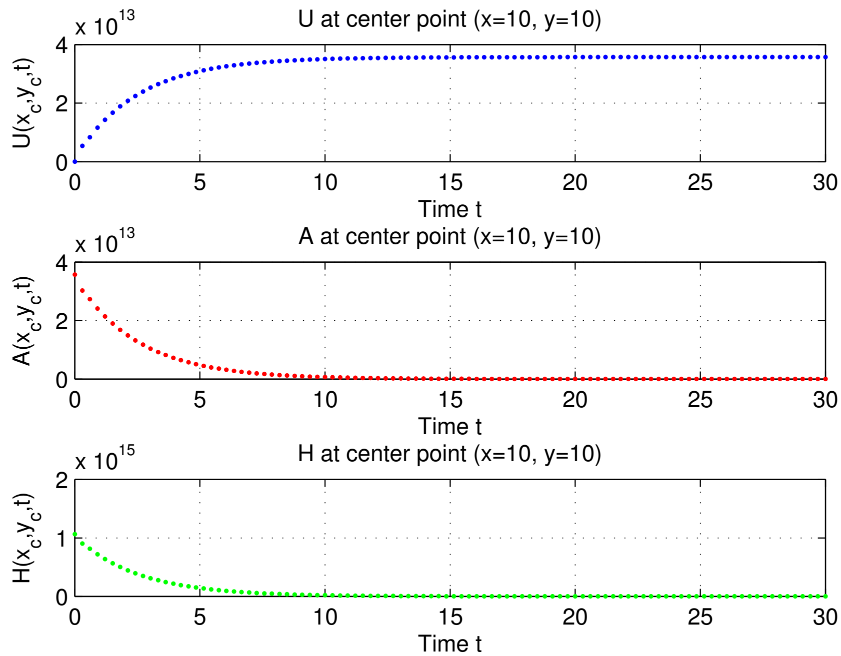

FIGURE 3. Spatial distribution of the healthcare need $H(x,y,t)$ at $t = 0$ (left) and $t = 30$ (right).

The simulations confirm the theoretical prediction that the system converges globally and asymptotically to the trivial EP $(0,0,0)$ under the chosen parameters, thereby validating the stability criteria derived in Section 3.

5. CONCLUSION

In this paper, we have developed and analyzed a fractional-order spatial–temporal model for the adoption dynamics of RHS in a two-dimensional domain. The model incorporates memory effects and non-local interactions via the CNFD, providing a more realistic representation of social contagion processes in which past states influence present behavior. By discretizing the spatial domain using the MOL and applying periodic boundary conditions, we transformed the original PDEs into a system of fractional ordinary differential equations. We derived the EPs of the system and conducted a thorough stability analysis. Using an LF approach, we established sufficient conditions for the GAS of the EP. These conditions depend explicitly on the model parameters and the spectral properties of the discrete Laplacian operator. Numerical simulations were performed to validate the theoretical stability results. The simulations confirmed that, under the chosen parameter set, the system converges globally and asymptotically to the trivial EP $(0,0,0)$. The stability constants computed from the numerical experiments satisfied the conditions of Theorem 3.3, thereby corroborating the theoretical analysis. The spatial and temporal evolution of the system variables illustrated the convergence dynamics and highlighted the effects of diffusion and fractional-order memory. Future research may explore the inclusion of time-varying parameters, heterogeneous diffusion coefficients, or more complex boundary conditions. Extending the model to incorporate stochastic perturbations or network-structured interactions could also provide deeper insights into real-world adoption processes. Additionally, empirical validation with real-world data would be valuable for calibrating the model parameters and assessing its predictive power. In summary, the proposed FO model offers a robust framework for studying the spatial–temporal dynamics of social contagion and adoption processes, with potential applications in public health, technology diffusion, and social behavior modeling.

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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