



Available online at <http://scik.org>

Commun. Math. Biol. Neurosci. 2026, 2026:89

<https://doi.org/10.28919/cmbn/9851>

ISSN: 2052-2541

MODELING THE INDIRECT TRANSMISSION OF HEPATITIS C VIRUS VIA CONTAMINATED INJECTING EQUIPMENT AND ENVIRONMENTAL RESERVOIRS WITH TREATMENT INTERVENTION

OLUMUYIWA JAMES PETER^{1,2}, S. CONSTANCE ANGELA¹, K. SUDARMOZHI¹, SHARIFAH SAKINAH SYED AHMAD³, DIPO ALDILA^{4,5}

¹Department of Mathematics, Saveetha School of Engineering, SIMATS, Saveetha University, Chennai, Tamil Nadu 602105, India

²Department of Mathematical and Computer Sciences, University of Medical Sciences, Ondo City, Ondo State, Nigeria

³Faculty of Artificial Intelligence and Cyber Security Universiti Teknikal Malaysia Melaka

⁴Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Indonesia, Depok 16424, Indonesia

⁵Innovative Mathematics and Predictive Analytics for Complex System and Technology Laboratory (IMPACT Lab), Universitas Indonesia, Depok 16424, Indonesia

Copyright © 2026 the author(s). This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract.

This study creates and evaluates a deterministic mathematical model for the indirect spread of Hepatitis C virus (HCV) among PWID. The model incorporates behavioural variation by categorising PWID into two separate groups, high-risk and low-risk, with explicit dynamics of contaminated injecting equipment serving as an environmental reservoir. Reinfection, therapy, mobility between behavioural risk categories, and equipment sterilisation are all completely included in the model framework. Analytical results are presented that use the next-generation

*Corresponding author

E-mail address: peterjames4real@gmail.com

Received February 27, 2026

matrix approach to determine the feasible zone, disease-free equilibrium, endemic equilibrium, and the basic reproduction number $\mathcal{R}_0$. The disease-free equilibrium is globally asymptotically stable for $\mathcal{R}_0 < 1$. Bifurcation analyses reveal that the contamination-related transmission rate among high-risk PWID (α_{wh}) and the behavioural transition rate (κ) strongly influence the emergence and stability of endemic states, while the reinfection rate (ψ) influences endemic levels without affecting the threshold quantity $\mathcal{R}_0$. Furthermore, sensitivity analysis utilising Partial Rank Correlation Coefficients (PRCC) revealed that recruitment into high-risk behaviour, contamination rates, and sterilisation of contaminated equipment are important predictors of transmission. The numerical simulations look into the effectiveness of therapy, sterilization, and the availability of clean equipment in reversing infection dynamics. The findings highlight the critical role that environmental contamination and high-risk behaviours play in disease transmission, emphasising the need for integrated harm-reduction strategies that include increased treatment, increased access to sterile equipment, and reduced behavioural risks among PWID. Our findings also show that relapse into high-risk injecting conduct is a critical driver of persistent HCV transmission, emphasizing the relevance of behavioural-risk reduction strategies.

Keywords: hepatitis C model; effective reproduction number; stability analysis; bifurcation analysis.

2020 AMS Subject Classification: 91A40, 34D23.

1. INTRODUCTION

HCV infection continues to be an important public health problem worldwide [1, 2, 3]. The World Health Organization projected that 58 million people had chronic HCV infection, with 1.5 million new infections per year [4, 5]. People who inject drugs have a disproportionately high risk of acquiring and transmitting HCV due to chronic high-risk injecting habits and systemic difficulties in accessing sterile equipment or prompt treatment [6, 7]. HCV transmission among PWID is primarily blood-borne, occurring through the direct sharing of contaminated needles and syringes. However, a growing body of experimental evidence has confirmed the continuous contribution of indirect transmission via contaminated injecting paraphernalia, such as cookers, filters, and water, and environmental reservoirs to sustain transmission [6, 8]. HCV can persist on surfaces or in residual blood for hours or days, depending on temperature, humidity, and injection equipment [9, 10]. This suggests that environmental contamination may still be a latent source of indirect virus transmission, even if direct needle sharing has been eliminated.

Over the past two decades, harm-reduction strategies such as needle and syringe programs and opiate substitution therapy have made significant success in lowering HCV transmission [11, 12]. These programs reduce high-risk injecting behaviors and HCV incidence, although their effectiveness varies depending on coverage, frequency of access, and local injecting norms [13, 14]. Direct-acting antiviral therapies have transformed clinical management of HCV infection, with cure rates of more than 95% achieved utilizing short, well-tolerated regimens. DAA therapy, when appropriately scaled among PWID, improves individual health outcomes while also potentially reducing population-level transmission via a "treatment-as-prevention" impact. Despite these developments, meeting the WHO's HCV elimination targets by 2030 remains a problem. The challenges include low DAA treatment adoption among PWID, reinfection after cure, insufficient NSP coverage, and dirty injecting equipment in the community. These problems underscore the need of mechanistic frameworks that represent both direct and indirect transmission routes and enable the evaluation of integrated intervention efforts.

Mathematical modeling is an effective technique for studying HCV transmission dynamics and assessing the potential impact of public health interventions. Earlier compartmental models focused on the natural history of infection and treatment outcomes, whereas later models included behavioral variability, injection networks, and risk stratification among PWID. Recent modeling frameworks incorporate treatment scale-up, reinfection risk, and harm-reduction treatments to evaluate eradication feasibility in various settings [15, 16]. Few models explicitly consider indirect transmission through contaminated injecting equipment or environmental reservoirs, which may account for a significant fraction of infections within some PWID populations [17].

Few models explicitly consider indirect transmission through contaminated injecting equipment or environmental reservoirs, which may account for a significant fraction of infections within some PWID populations [17, 18]. Despite previous modeling attempts, there still remain many key issues to be addressed regarding the process through which environmental contamination and behavioral dynamics jointly contribute to HCV transmission among PWID. Many previous models have concentrated on direct transmission through needle-sharing behavior but

have not taken explicit account of environmental contamination through equipment use. Further, behavioral relapse to high-risk injecting behavior and reinfection after recovery have been separately addressed but within a unified framework.

The current study proposes a deterministic model in which high-risk and low-risk PWID populations are jointly related to a dynamically changing environmental reservoir of contaminated equipment. This model provides a unified framework in which to analyze behavioral transitions, reinfection, treatment, and equipment sterilization in relation to indirect routes of HCV transmission. This framework enables us to examine the relative contributions of environmental contamination and behavioral dynamics to ongoing HCV transmission and to identify key intervention strategies to control this process.

The rest of this paper is organized as follows. Section 2 describes the model formulation and assumptions. Section 3 describes the mathematical analysis and threshold results. The subsequent sections discuss the epidemiological implications of this model and evaluate the possible effects of a unified intervention strategy to combat HCV transmission among PWID.

2. MODEL FORMULATION

In this section, we present a compartmental model for the indirect transmission of Hepatitis C Virus (HCV) among people who inject drugs. The model accounts for behavioral heterogeneity, reinfection, and dynamic transitions between risk levels. The primary mode of HCV transmission in this population is the sharing of injecting equipment contaminated by infected individuals. To account for the indirect transmission mechanism and heterogeneity in injecting behavior, the PWID population is divided into two subgroups: high-risk and low-risk. High-risk PWID are more likely to share needles and contribute to the contamination of injecting equipment, whereas low-risk PWID engage in safer practices, such as using clean equipment or participating in harm reduction programs.

Each subgroup is further compartmentalized into four epidemiological classes: susceptible, infected, treated, and recovered. Thus, we define S_h, I_h, T_h, R_h for the high-risk group and S_l, I_l, T_l, R_l for the low-risk group. Individuals enter the population at a recruitment rate ϕ_h , with a proportion ρ_h entering the high-risk susceptible class S_h and the remaining proportion $1 - \rho_h$ entering the low-risk susceptible class S_l . All individuals are subject to a natural death rate

μ , and infected individuals face an additional disease-induced death rate τ due to HCV-related complications. The model assumes homogeneous natural mortality across compartments, while the disease-induced mortality only affects the infected classes.

Susceptible individuals in each group may acquire infection through contact with contaminated injecting equipment. The probability of infection depends on the force of infection, which differs between behavioral groups. The force of infection for high-risk PWID is defined as $\lambda_h = \frac{\alpha_{w_h} E_i}{E_u + E_i}$, while that for low-risk PWID is $\lambda_l = \frac{\alpha_{w_l} E_i}{E_u + E_i}$, where α_{w_h} and α_{w_l} represent the effective transmission rates through contaminated equipment for high-risk and low-risk individuals, respectively. The denominator $E_u + E_i$ represents the total injecting equipment in circulation, and the transmission likelihood increases with a higher proportion of contaminated equipment E_i .

Once infected, individuals may begin treatment at a rate δ , entering the treated class T . Following successful treatment, individuals recover at rate γ and move to the recovered class R . However, due to behavioral relapse or continued exposure, recovered individuals may become reinfected via contact with contaminated equipment. This reinfection occurs at a rate $\psi \frac{RE_i}{E_u + E_i}$, where ψ denotes the reinfection rate. It is important to note the distinction between primary infection (acquisition of infection from the susceptible class) and reinfection (re-entry into the infected class from the recovered class), both driven by exposure to environmental reservoirs but representing different stages of disease history.

In addition to disease progression, the model includes behavioral transitions between risk groups. Individuals in the low-risk group may revert to high-risk behavior through external influence or behavioral fatigue. In this formulation, we assume a one-way transition from low-risk to high-risk individuals at a rate κ , representing behavioral relapse.

This shift is applied to the vulnerable and infected classes equally, in reference to the reality that once infected, individuals can return to risky behavior. The model also incorporates the environmental reservoir of injecting equipment, dividing into *clean (uninfected)* equipment $E_u(t)$ and *contaminated (infected)* equipment $E_i(t)$. Both of these compartments cover the indirect pathway of HCV transmission. Clean equipment enters the system at a steady rate θ , which captures interventions such as needle exchange programs or harm reduction interventions that

supply clean needles to PWID. Clean equipment can, however, become infected once it is used by infected individuals. Specifically:

- Equipment becomes contaminated when used by infected high-risk PWID at a rate σ_h , weighted by the proportion of clean equipment in circulation:

$$\sigma_h I_h \cdot \frac{E_u}{E_u + E_i}$$

- Similarly, equipment becomes contaminated when used by infected low-risk PWID at a rate σ_l , also weighted by the proportion of clean equipment:

$$\sigma_l I_l \cdot \frac{E_u}{E_u + E_i}$$

These concepts represent the reality that the risk of contamination is determined not only by the number of affected people, but also by the availability of clean equipment. All injecting equipment (clean or contaminated) is eventually removed from the system at a rate ω , indicating disposal, natural decay, or decontamination by sterilization. Thus, changes in the environment are regulated by the following differential equations:

$$\begin{aligned} \frac{dE_u}{dt} &= \theta - \sigma_h I_h \cdot \frac{E_u}{E_u + E_i} - \sigma_l I_l \cdot \frac{E_u}{E_u + E_i} - \omega E_u, \\ \frac{dE_i}{dt} &= \sigma_h I_h \cdot \frac{E_u}{E_u + E_i} + \sigma_l I_l \cdot \frac{E_u}{E_u + E_i} - \omega E_i. \end{aligned}$$

In this framework:

- The first two terms in $\frac{dE_u}{dt}$ represent losses due to contamination by infected PWID,
- The corresponding terms in $\frac{dE_i}{dt}$ represent gains in the contaminated pool,
- The final terms $-\omega E_u$ and $-\omega E_i$ model the removal of equipment from circulation.

The dynamics of the model are described by the following system of nonlinear differential equations below, the model diagram is presented in Figure 1. The model variable and parametre descriptions are presented in the Table 1 and Table 2 respectively:

$$(1a) \quad \frac{dS_h}{dt} = \rho_h \phi_h - \lambda_h S_h - \mu S_h + \kappa S_l,$$

$$(1b) \quad \frac{dI_h}{dt} = \lambda_h S_h + \psi \frac{R_h E_i}{E_u + E_i} - (\delta + \mu + \tau) I_h + \kappa I_l,$$

$$(1c) \quad \frac{dT_h}{dt} = \delta I_h - (\gamma + \mu) T_h,$$

$$\begin{aligned}
(1d) \quad & \frac{dR_h}{dt} = \gamma T_h - \mu R_h - \psi \frac{R_h E_i}{E_u + E_i}, \\
(1e) \quad & \frac{dS_l}{dt} = (1 - \rho_h) \phi_h - \lambda_l S_l - \mu S_l - \kappa S_l, \\
(1f) \quad & \frac{dI_l}{dt} = \lambda_l S_l + \psi \frac{R_l E_i}{E_u + E_i} - (\delta + \mu + \tau) I_l - \kappa I_l, \\
(1g) \quad & \frac{dT_l}{dt} = \delta I_l - (\gamma + \mu) T_l, \\
(1h) \quad & \frac{dR_l}{dt} = \gamma T_l - \mu R_l - \psi \frac{R_l E_i}{E_u + E_i}, \\
(1i) \quad & \frac{dE_u}{dt} = \theta - \sigma_h I_h \frac{E_u}{E_u + E_i} - \sigma_l I_l \frac{E_u}{E_u + E_i} - \omega E_u, \\
(1j) \quad & \frac{dE_i}{dt} = \sigma_h I_h \frac{E_u}{E_u + E_i} + \sigma_l I_l \frac{E_u}{E_u + E_i} - \omega E_i.
\end{aligned}$$

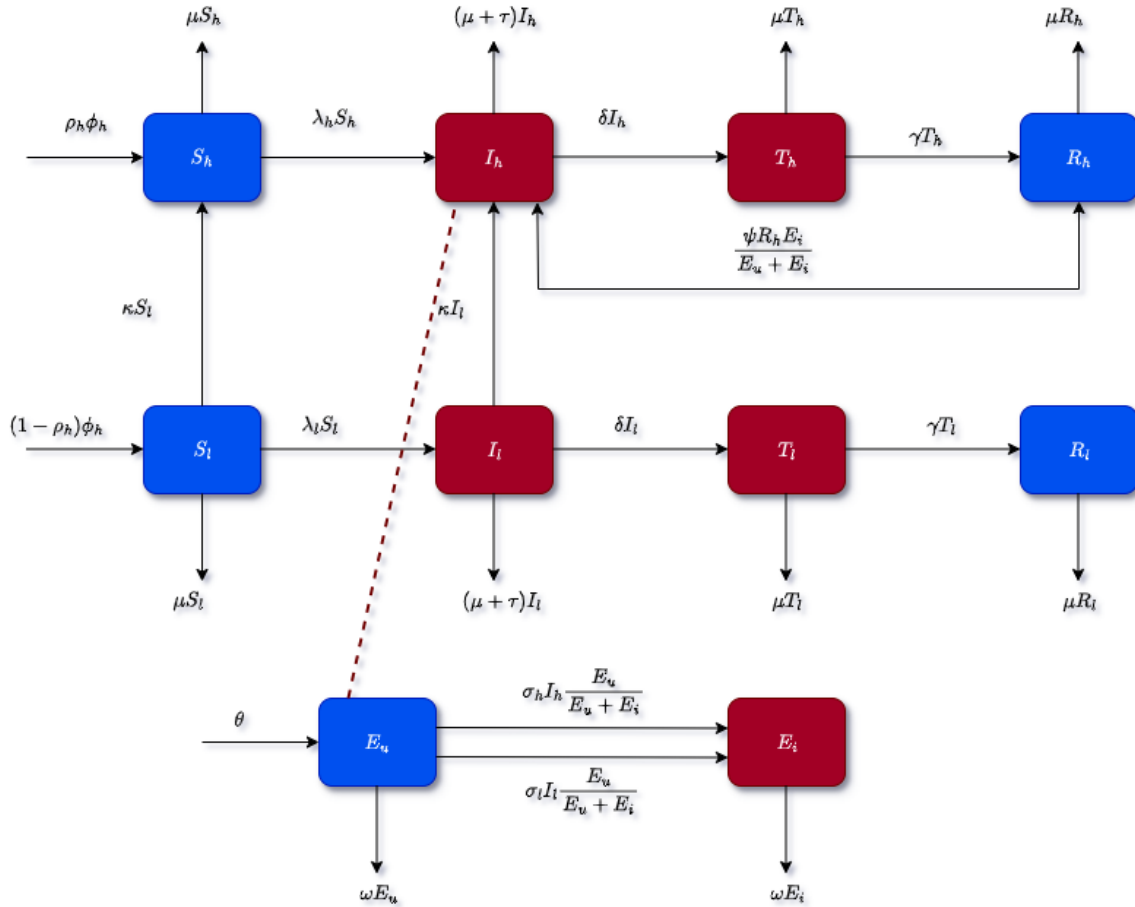


FIGURE 1. Schematic diagram of the model

Model Variables and Parameters.

TABLE 1. Model State Variables

Symbol	Description
$S_h(t)$	Susceptible high-risk PWID at time t
$I_h(t)$	Infected high-risk PWID at time t
$T_h(t)$	Treated high-risk PWID at time t
$R_h(t)$	Recovered high-risk PWID at time t
$S_l(t)$	Susceptible low-risk PWID at time t
$I_l(t)$	Infected low-risk PWID at time t
$T_l(t)$	Treated low-risk PWID at time t
$R_l(t)$	Recovered low-risk PWID at time t
$E_u(t)$	Quantity of clean injecting equipment at time t
$E_i(t)$	Quantity of contaminated injecting equipment at time t

TABLE 2. Model Parameters

Symbol	Description
ϕ_h	Recruitment rate into the PWID population
ρ_h	Proportion of new PWID entering the high-risk group
μ	Natural death rate for all human compartments
τ	Disease-induced death rate for infected individuals
δ	Rate of initiation of antiviral treatment
γ	Recovery rate for treated individuals
α_{w_h}	Transmission rate via contaminated equipment for high-risk PWID
α_{w_l}	Transmission rate via contaminated equipment for low-risk PWID
κ	Behavior change rate from low-risk to high-risk group
ψ	Reinfection rate for recovered individuals
θ	Rate of clean injecting equipment distribution
σ_h	Contamination rate of equipment by infected high-risk PWID
σ_l	Contamination rate of equipment by infected low-risk PWID
ω	Sterilization/removal rate of injecting equipment
$N(t)$	Total PWID population: $N = S_h + I_h + T_h + R_h + S_l + I_l + T_l + R_l$

3. MODEL ANALYSIS

3.1. Feasible Region. To ensure biological and mathematical well-posedness, we identify a positively invariant region in which all solutions remain non-negative and bounded.

Let the total human population be

$$(2) \quad N(t) = S_h(t) + I_h(t) + T_h(t) + R_h(t) + S_l(t) + I_l(t) + T_l(t) + R_l(t).$$

Summing the corresponding human compartments, and ignoring disease transitions, we get:

$$(3) \quad \frac{dN}{dt} = \phi_h - \mu N(t) - \tau(I_h + I_l) \leq \phi_h - \mu N(t).$$

Solving this differential inequality gives:

$$(4) \quad N(t) \leq N(0)e^{-\mu t} + \frac{\phi_h}{\mu} (1 - e^{-\mu t}), \quad \Rightarrow \quad \limsup_{t \rightarrow \infty} N(t) \leq \frac{\phi_h}{\mu}.$$

Similarly, for the environmental compartments:

$$(5) \quad E(t) = E_u(t) + E_i(t),$$

we obtain:

$$(6) \quad \frac{dE}{dt} = \theta - \omega(E_u + E_i) = \theta - \omega E(t),$$

so that:

$$(7) \quad \limsup_{t \rightarrow \infty} E(t) \leq \frac{\theta}{\omega}.$$

Thus, the feasible region is given by:

$$(8) \quad \Omega = \left\{ (S_h, I_h, T_h, R_h, S_l, I_l, T_l, R_l, E_u, E_i) \in \mathbb{R}_+^{10} : N(t) \leq \frac{\phi_h}{\mu}, \quad E_u + E_i \leq \frac{\theta}{\omega} \right\}.$$

3.2. Disease-Free Equilibrium (DFE). At the DFE, all infection-related compartments are zero:

$$(9) \quad I_h^* = I_l^* = T_h^* = T_l^* = R_h^* = R_l^* = E_i^* = 0.$$

Solving the remaining equations, we obtain:

$$(10) \quad S_h^* = \frac{\rho_h \phi_h}{\mu + \kappa},$$

$$(11) \quad S_l^* = \frac{(1 - \rho_h)\phi_h + \kappa S_h^*}{\mu},$$

$$(12) \quad E_u^* = \frac{\theta}{\omega}.$$

Thus, the disease-free equilibrium is given by:

$$(13) \quad \mathcal{E}_0 = (S_h^*, 0, 0, 0, S_l^*, 0, 0, 0, E_u^*, 0).$$

3.3. Endemic Equilibrium (EE). This is the equilibrium state $\mathcal{E}^*$ where infection persist in the population, here, all time derivatives are zero and infection persists. Let:

$$\mathcal{E}^* = (S_h^*, I_h^*, T_h^*, R_h^*, S_l^*, I_l^*, T_l^*, R_l^*, E_u^*, E_i^*).$$

Then the equilibrium values satisfy:

$$(14) \quad S_h^* = \frac{\rho_h \phi_h}{\lambda_h + \mu + \kappa},$$

$$(15) \quad I_h^* = \frac{\lambda_h S_h^*}{\delta + \mu + \tau},$$

$$(16) \quad T_h^* = \frac{\delta I_h^*}{\gamma + \mu}, \quad R_h^* = \frac{\gamma T_h^*}{\mu + \psi \lambda_h},$$

$$(17) \quad S_l^* = \frac{(1 - \rho_h)\phi_h + \kappa S_h^*}{\lambda_l + \mu},$$

$$(18) \quad I_l^* = \frac{\lambda_l S_l^*}{\delta + \mu + \tau},$$

$$(19) \quad T_l^* = \frac{\delta I_l^*}{\gamma + \mu}, \quad R_l^* = \frac{\gamma T_l^*}{\mu + \psi \lambda_l}.$$

For the environment:

$$(20) \quad E_u^* = \frac{\theta(E_u^* + E_i^*)}{\omega(E_u^* + E_i^*) + (\sigma_h I_h^* + \sigma_l I_l^*)},$$

$$(21) \quad E_i^* = \frac{(\sigma_h I_h^* + \sigma_l I_l^*)E_u^*}{\omega(E_u^* + E_i^*)}.$$

The endemic equilibrium can be obtained by solving the above system numerically, ensuring consistency with the infection pressure:

$$\lambda_h = \frac{\alpha_{wh} E_i^*}{E_u^* + E_i^*}, \quad \lambda_l = \frac{\alpha_{wl} E_i^*}{E_u^* + E_i^*}.$$

3.4. Basic Reproduction Number $\mathcal{R}_0$. The basic reproduction number is the expected number of secondary infections arising from the presence of a single infected individual introduced into a completely susceptible population. It serves as the key threshold parameter in the disease model, which determines whether a disease would invade and persist in a population.

To compute $\mathcal{R}_0$, we use the next-generation matrix approach [19, 20, 21]. We consider the infected compartments I_h and I_l , representing high-risk and low-risk infected individuals, respectively.

The new infection terms are:

$$(22) \quad \mathcal{F} = \begin{bmatrix} \frac{\alpha_{wh}E_i}{E_u+E_i}S_h \\ \frac{\alpha_{wl}E_i}{E_u+E_i}S_l \end{bmatrix},$$

while the transition terms (outflow due to treatment, natural death, and disease-induced death) are:

$$(23) \quad \mathcal{V} = \begin{bmatrix} (\delta + \mu + \tau)I_h \\ (\delta + \mu + \tau)I_l \end{bmatrix}.$$

At the disease-free equilibrium (DFE), we have:

$$I_h = I_l = T_h = T_l = R_h = R_l = E_i = 0,$$

and the DFE values for the remaining compartments are:

$$(24) \quad S_h^* = \frac{\rho_h \phi_h}{\mu + \kappa},$$

$$(25) \quad S_l^* = \frac{(1 - \rho_h)\phi_h + \kappa S_h^*}{\mu} = \frac{\phi_h [(1 - \rho_h)(\mu + \kappa) + \kappa \rho_h]}{\mu(\mu + \kappa)},$$

$$(26) \quad E_u^* = \frac{\theta}{\omega}, \quad E_i^* = 0.$$

Using the next generation matrix method, the basic reproduction number is given by the spectral radius of FN^{-1} , where:

$$(27) \quad \mathcal{R}_0 = \mathcal{R}_h + \mathcal{R}_l,$$

with:

$$(28) \quad \mathcal{R}_h = \frac{S_h^* \cdot \alpha_{wh} \cdot \sigma_h}{(\delta + \mu + \tau)\omega} = \frac{\rho_h \phi_h}{\mu + \kappa} \cdot \frac{\alpha_{wh} \sigma_h}{(\delta + \mu + \tau)\omega},$$

$$(29) \quad \mathcal{R}_l = \frac{S_l^* \cdot \alpha_{wl} \cdot \sigma_l}{(\delta + \mu + \tau)\omega} = \left[\frac{\phi_h [(1 - \rho_h)(\mu + \kappa) + \kappa \rho_h]}{\mu(\mu + \kappa)} \right] \cdot \frac{\alpha_{wl} \sigma_l}{(\delta + \mu + \tau)\omega}.$$

Thus, the final expression for the basic reproduction number is:

$$(30) \quad \mathcal{R}_0 = \underbrace{\frac{\rho_h \phi_h}{\mu + \kappa} \cdot \frac{\alpha_{wh} \sigma_h}{(\delta + \mu + \tau)\omega}}_{\text{High-risk contribution}} + \underbrace{\left[\frac{\phi_h [(1 - \rho_h)(\mu + \kappa) + \kappa \rho_h]}{\mu(\mu + \kappa)} \right] \cdot \frac{\alpha_{wl} \sigma_l}{(\delta + \mu + \tau)\omega}}_{\text{Low-risk contribution}}.$$

Interpretation. The value of $\mathcal{R}_0$ captures the contributions from both high-risk and low-risk groups. If $\mathcal{R}_0 < 1$, the disease will die out in the long term. If $\mathcal{R}_0 > 1$, the infection can spread and become endemic. Control measures such as increased treatment rate δ , behavioral transition κ , reduced contamination rates σ_h, σ_l , or enhanced equipment sterilization ω can effectively reduce $\mathcal{R}_0$ below unity.

4. GLOBAL STABILITY OF THE DISEASE-FREE EQUILIBRIUM

In this section, we analyze the global stability of the Disease-Free Equilibrium (DFE) of the model using the approach proposed by Castillo-Chavez et al. [22]. To apply this method, we rewrite the model system by separating the compartments into uninfected and infected groups.

Let the vector of uninfected compartments be

$$\mathbf{X} = [S_h, S_l, R_h, R_l, E_u],$$

and the vector of infected compartments be

$$\mathbf{Z} = [I_h, I_l, T_h, T_l, E_i].$$

The model system can then be expressed in the following general form:

$$(31) \quad \frac{d\mathbf{X}}{dt} = f(\mathbf{X}, \mathbf{Z}),$$

$$(32) \quad \frac{d\mathbf{Z}}{dt} = g(\mathbf{X}, \mathbf{Z}), \quad \text{with } g(\mathbf{X}, 0) = 0.$$

According to the geometric approach by Castillo-Chavez et al. [22], the Disease-Free Equilibrium is globally asymptotically stable in the feasible region Ω if the following two conditions are satisfied:

(H1) The subsystem (31) given by $\frac{d\mathbf{X}}{dt} = f(\mathbf{X}, 0)$ has a globally asymptotically stable equilibrium $\mathbf{X}^0$.

(H2) The subsystem (32) given by $\frac{d\mathbf{Z}}{dt} = g(\mathbf{X}, \mathbf{Z})$ satisfies:

- (a) $g(\mathbf{X}, 0) = 0$ for all $\mathbf{X} \in \Omega$,
- (b) The Jacobian matrix $D_{\mathbf{Z}}g(\mathbf{X}, \mathbf{Z})$ is a Metzler matrix (i.e., all its off-diagonal elements are non-negative),
- (c) The basic reproduction number $\mathcal{R}_0 < 1$.

Verification of the Conditions. Condition (H1): When $\mathbf{Z} = 0$, the uninfected subsystem becomes:

$$(33) \quad \frac{dS_h}{dt} = \rho_h \phi_h - (\mu + \kappa) S_h,$$

$$(34) \quad \frac{dS_l}{dt} = (1 - \rho_h) \phi_h + \kappa S_h - \mu S_l,$$

$$(35) \quad \frac{dR_h}{dt} = -\mu R_h,$$

$$(36) \quad \frac{dR_l}{dt} = -\mu R_l,$$

$$(37) \quad \frac{dE_u}{dt} = \theta - \omega E_u.$$

This system is linear with constant coefficients and has a unique equilibrium given by:

$$S_h^0 = \frac{\rho_h \phi_h}{\mu + \kappa}, \quad S_l^0 = \frac{(1 - \rho_h) \phi_h + \kappa S_h^0}{\mu}, \quad R_h^0 = R_l^0 = 0, \quad E_u^0 = \frac{\theta}{\omega}.$$

Since the system is linear and all eigenvalues of the Jacobian matrix have negative real parts, the DFE of the uninfected subsystem is globally asymptotically stable.

Condition (H2):

- The infection subsystem $g(\mathbf{X}, \mathbf{Z})$ vanishes when $\mathbf{Z} = 0$, i.e., $g(\mathbf{X}, 0) = 0$ for all $\mathbf{X} \in \Omega$.
- The infection terms involve linear or bilinear functions of infected states (e.g., transmission, treatment), so the Jacobian matrix $D_{\mathbf{Z}}g(\mathbf{X}, \mathbf{Z})$ has non-negative off-diagonal entries, hence it is a Metzler matrix.
- From the next-generation matrix approach, the basic reproduction number $\mathcal{R}_0$ has been computed. When $\mathcal{R}_0 < 1$, the spectral radius of the next-generation matrix is less than one.

Thus, all the required conditions of the theorem are satisfied.

Conclusion.

Theorem 1. *The Disease-Free Equilibrium (DFE) of the Hepatitis C virus transmission model is globally asymptotically stable in the feasible region Ω whenever the basic reproduction number satisfies $\mathcal{R}_0 < 1$.*

This result implies that, regardless of the initial levels of infection, the disease will eventually be eliminated from the population if the reproduction number is brought below unity.

5. BIFURCATION DIAGRAM

In this section, we conduct numerical experiments to examine how α_{wh} and κ affect the size of the endemic equilibrium using bifurcation diagram analysis. The bifurcation diagrams are generated with *Matcont*, a MATLAB-based toolbox for continuation. Other continuation tools commonly used in the literature include COCO, AUTO (via MATLAB interfaces), and DDE-Biftool for delay systems.

To perform this simulation, we use the following parameter values, unless stated otherwise:

$$\begin{aligned} \phi_h = 0.129, \rho_h = 0.23, \mu = 0.09, \tau = 0.035, \delta = 0.04, \gamma = 0.02, \alpha_{wh} = 0.85, \alpha_{wl} = 0.05, \\ \kappa = 0.00274, \psi = 0.000047, \theta = 0.27, \sigma_h = 0.59, \sigma_l = 0.1, \omega = 0.143. \end{aligned}$$

To produce the bifurcation diagram, we first calculate the existing equilibrium points using the above parameter values. By direct calculation, we obtain $\mathcal{R}_0 = 2.05$, which yields a stable endemic equilibrium given by:

$$\begin{aligned} S_h = 0.107, I_h = 0.13, T_h = 0.05, R_h = 0.01, S_l = 0.945, I_l = 0.069, T_l = 0.025, R_l = 0.005, \\ E_u = 1.42, E_i = 0.465. \end{aligned}$$

In the same time, the disease-free equilibrium

$$S_h = 0.362, I_h = 0, T_h = 0, R_h = 0, S_l = 1.07, I_l = 0, T_l = 0, R_l = 0, E_u = 1.88, E_i = 0,$$

is unstable. This results are in line with our previous analytical results, that the endemic equilibrium is always stable if $\mathcal{R}_0 > 1$. At the same time, the disease-free equilibrium is unstable.

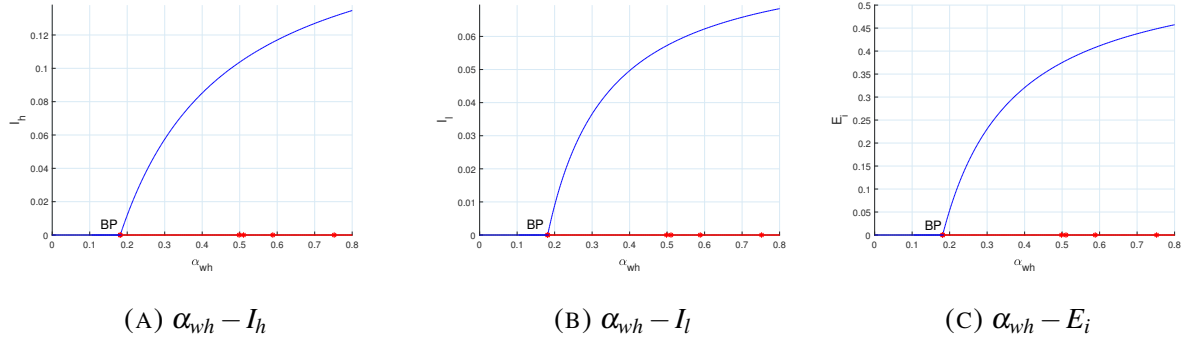


FIGURE 2. Bifurcation diagram of our model with α_{wh} as the bifurcation parameter with respect to the equilibrium level of I_h, I_l , and E_i . The red and blue curve represent the unstable and stable equilibrium point.

We use α_{wh} and κ as our bifurcation parameters, and the continuation results are depicted in Figure 2 and 3. In Figure 2, we can see the impact of α_{wh} on the size of the endemic equilibrium of the system, which is represented by the variables I_h, I_l , and E_i . It can be seen clearly that for all $\alpha_{wh} < \text{BP}$, there is no endemic equilibrium point, and the disease-free equilibrium remains stable. Here, BP is the value of α_{wh} such that $\mathcal{R}_0 = 1$. This stability is maintained until the system reaches the branching point BP. After passing this point, the disease-free equilibrium loses its stability, and at the same time the endemic equilibrium begins to emerge. This stability behaviour for both equilibrium points is preserved for all larger values of α_{wh} . Furthermore, it can be seen that all infected compartments I_h, I_l , and E_i increase in the endemic equilibrium as α_{wh} increases, indicating that a more intense infection rate leads to a higher number of infected individuals at equilibrium.

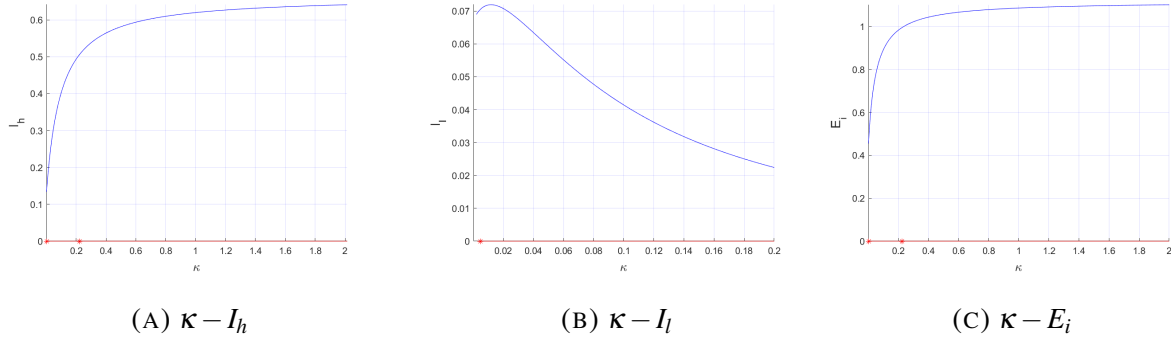
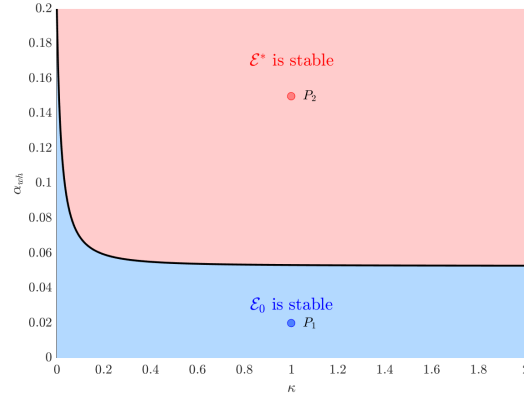


FIGURE 3. Bifurcation diagram of our model with κ as the bifurcation parameter with respect to the equilibrium level of I_h , I_l , and E_i . The red and blue curve represent the unstable and stable equilibrium point.

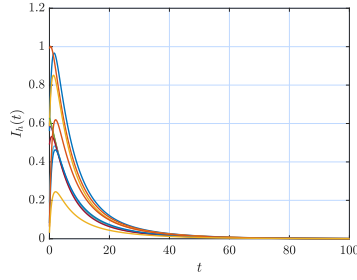
The next bifurcation diagram is given in Figure 3, where we use κ as the bifurcation parameter. It can be seen that, with the same parameter values, there is no value of κ such that $\mathcal{R}_0 = 1$. Therefore, we have no branching point BP in the figure. This implies that for all $\kappa > 0$, we have $\mathcal{R}_0$ always larger than one. Consequently, a stable endemic equilibrium always exists, and the disease-free equilibrium remains unstable. Among the infected compartments, only I_l exhibits a non-monotonic response to κ : it increases over a small initial range of κ and then declines as κ becomes larger. In contrast, both I_h and E_i increase monotonically with κ in the endemic equilibrium. This behaviour is reasonable since κ represents the transition rate from the low-risk group to the high-risk group.

To understand the impact of the combination between α_{wh} and κ on the size of $\mathcal{R}_0$ and the stability of each equilibrium point, we produce a codimension-2 bifurcation diagram as shown in Figure 4. The black curve represents a combination between α_{wh} and κ which gives $\mathcal{R}_0 = 1$. The blue region represents the region where we always have $\mathcal{R}_0 < 1$ for any combination of α_{wh} and κ . We choose a combination of α_{wh} and κ as shown in P_1 , and use it to solve our system. We have demonstrated that the trajectories of system (1) tend to the disease-free equilibrium for various initial conditions (see Figure 4b, 4c, and 4d). On the other hand, the red region represents combinations of κ and α_{wh} that give $\mathcal{R}_0 > 1$. Therefore, a stable endemic equilibrium always exists for any parameter values chosen within this region. For example,

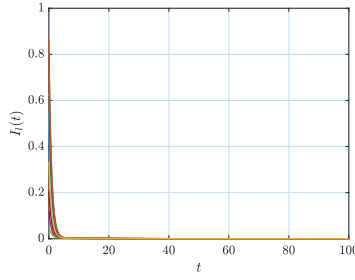
when we choose a combination of κ and α_{wh} as in point P_2 , the dynamics of system (1) tend to the endemic equilibrium for all different initial conditions (see Figure 4e, 4f, and 4g).



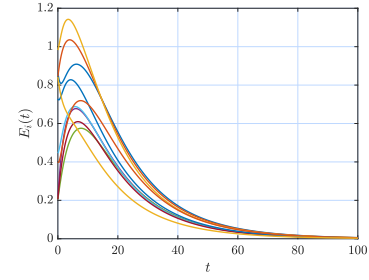
(A) Codimension-2 diagram of system (1) for κ and α_{wh} as the bifurcation parameter.



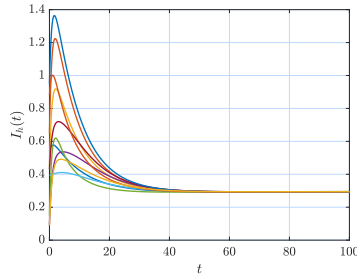
(B) Dynamic of $I_h(t)$ using point P_1



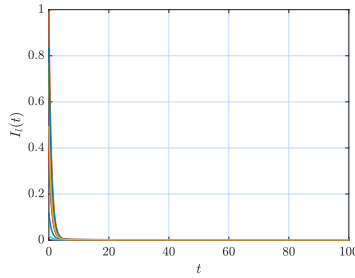
(C) Dynamic of $I_l(t)$ using point P_1



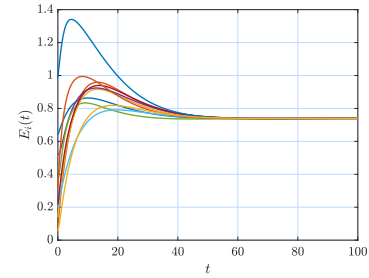
(D) Dynamic of $E_i(t)$ using point P_1



(E) Dynamic of $I_h(t)$ using point P_2



(F) Dynamic of $I_l(t)$ using point P_2



(G) Dynamic of $E_i(t)$ using point P_2

FIGURE 4. Codimension-2 diagram of system (1) with dynamic of solutions using sample point in P_1 which gives $\mathcal{R}_0 = 0.639$ and P_2 which gives $\mathcal{R}_0 = 1.649$.

For the last bifurcation diagram, we choose ψ as the bifurcation parameter. This choice is important since ψ does not appear in $\mathcal{R}_0$. Therefore, understanding the behaviour of system (1) solely through the expression of $\mathcal{R}_0$ is not possible. As mentioned earlier, with the same parameter values, we have $\mathcal{R}_0 = 2.05$, which always gives a stable endemic equilibrium point. Consequently, in Figure 5 we always observe a stable endemic equilibrium. Furthermore, we can see that the sizes of I_h , I_l , and E_i in the endemic equilibrium increase as ψ increases. Therefore, we may conclude that as reinfection of HCV becomes more frequent, although it does not change the value of $\mathcal{R}_0$, it does alter the size of the endemic equilibrium of the system.

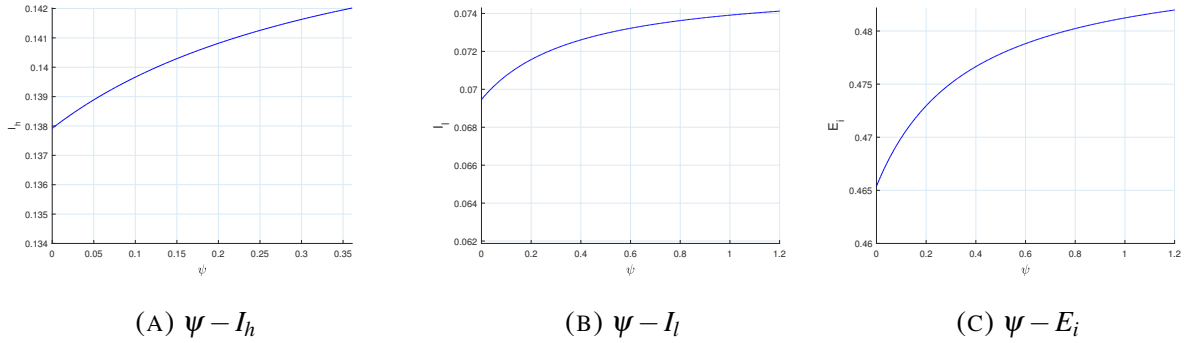


FIGURE 5. Bifurcation diagram of our model with ψ as the bifurcation parameter with respect to the equilibrium level of I_h , I_l , and E_i .

In conclusion, from our continuation results using bifurcation diagrams, our analysis demonstrates that the parameters α_{wh} , κ , and ψ play distinct and important roles in shaping the long-term behaviour of system (1). We observe that α_{wh} and κ directly influence the size of $\mathcal{R}_0$, and therefore determine the stability type of each equilibrium as these parameters change. As the values of these parameters increase, $\mathcal{R}_0$ also increases, leading to a higher chance of obtaining a stable endemic equilibrium point. On the other hand, although ψ does not change the value of $\mathcal{R}_0$, it plays a strong role in determining the sizes of I_h , I_l , and E_i at the endemic equilibrium. Increasing ψ leads to higher levels of persistent infection in the population. These results collectively highlight that while $\mathcal{R}_0$ provides the global dynamics of the system, additional parameters not expressed in $\mathcal{R}_0$ can substantially modify the disease burden at equilibrium.

6. PARTIAL RANK CORRELATION COEFFICIENT (PRCC) BASED SENSITIVITY ANALYSIS

To determine the relative influence of model parameters on the transmission potential of HCV among PWID, we performed a global sensitivity analysis using the PRCC. The PRCC is a measure that quantifies the strength and direction of monotonic relationships between each parameter and the basic reproduction number, $\mathcal{R}_0$, conditional upon the remaining parameters. Unlike local measures of sensitivity, which are concerned with small perturbations around a fixed point, PRCC is a robust global measure that detects nonlinear and non-additive effects across a broad range of parameters.

Latin Hypercube Sampling was used to generate a large ensemble of parameter sets, each independently drawn from biologically plausible intervals around their baseline values. The associated reproduction number $\mathcal{R}_0$ was then calculated for each sampled set using the analytical expression obtained in Section 3.4. The resulting PRCC values are displayed in Figure 6.

Among the parameters analyzed, the recruitment rate of new PWID and the recruitment rate into the high-risk group had very strong positive associations with $\mathcal{R}_0$ with the values +0.8424 and +0.8309, respectively, which points to increased inflow into the injecting population as being of primary importance in substantially raising the transmission potential. The effective transmission rate through contaminated equipment among high-risk PWID, α_{wh} , and the corresponding contamination rate σ , were also strongly positively associated with $\mathcal{R}_0$ with PRCC of 0.8397 and 0.8576, respectively. These suggest that the processes underlying equipment contamination and injecting behavior in the high-risk group provide the dominant contribution to sustained transmission.

By contrast, the natural mortality rate, μ , had the strongest negative correlation with $\mathcal{R}_0$ (PRCC = -0.8987), which occurred because it decreases the size of the infectious pool. The sterilization or removal rate of injecting equipment, ω , also had a strong negative effect (PRCC = -0.7834), as one would expect from a mechanism that eliminates infectious equipment from circulation. Disease-induced mortality τ and treatment initiation rate δ both have smaller, but still negative, correlations (PRCC = -0.1717 and -0.1170 , respectively) that are still consistent with a belief that they may reduce transmission, although to a lesser extent than

the other parameters. In general, the PRCC results provide evidence that the most influential determinants of HCV transmission in the model are those involving the recruitment and behavior of high-risk PWID, the contamination of injecting equipment, and effectiveness of equipment sterilization. These findings reinforce the importance of interventions such as reducing high-risk initiation, limiting equipment sharing, and expanding access to clean or sterilized equipment as key strategies for reducing $\mathcal{R}_0$ and controlling HCV transmission among PWID.

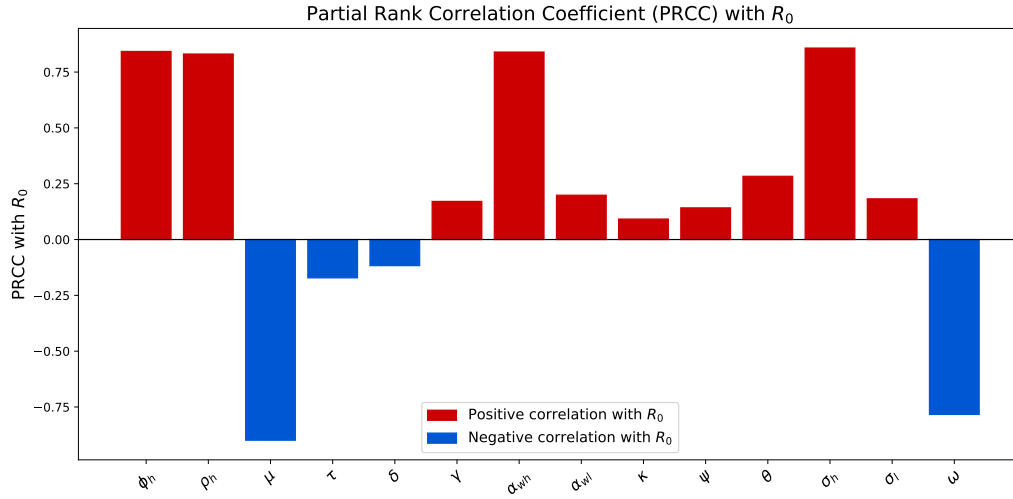


FIGURE 6. Sensitivity analysis of $\mathcal{R}_e$

TABLE 3. Model Parameters, Values, and Sources

Parameter	Value (per week)	Source
ϕ_h	0.129	[17]
ρ_h	0.23	Estimated
μ	0.09	[23]
τ	0.035	[23]
δ	0.04	[23]
γ	0.02	[23]
α_{wh}	0.00085	Estimated
α_{wl}	0.00005	Estimated
κ	0.00274	Estimated
ψ	0.000047	Estimated
θ	0.27	Estimated
σ_h	0.59	Estimated
σ_l	0.10	Estimated
ω	0.143	[17]

PARAMETER ESTIMATION

The transmission probability via contaminated injecting equipment for low-risk PWID, α_{w_l} , is estimated at 0.0057 per event and scaled by average injecting frequency (e.g., 2 times per week), yielding a daily rate of $\alpha_{w_l} = 0.00005$. For high-risk PWID, assuming twice the exposure, α_{w_h} is set to 0.00085 per week [24]. The contamination rates of injecting equipment by infected users are assumed to be 0.1 per week for low-risk PWID (σ_l), and based on behavioral studies showing 5.9-fold higher sharing risk, σ_h is estimated as 0.59 per week [25]. The reinfection rate ψ is derived from follow-up studies reporting 1.7 reinfections per 100 person-years among treated PWID, which converts to approximately 0.000047 per week [26]. The rate of behavioral transition from low-risk to high-risk PWID, κ , is assumed as 1.0 per year (typical relapse time), equivalent to 0.00274 per week [27]. The distribution rate of clean injecting equipment, θ , is taken as 0.27 syringes per person per week based on WHO guidelines for needle and syringe programs [28]. Lastly, the proportion ρ_h of new PWID entering the high-risk group is estimated at 0.23 from modeling studies [29].

7. NUMERICAL SIMULATION

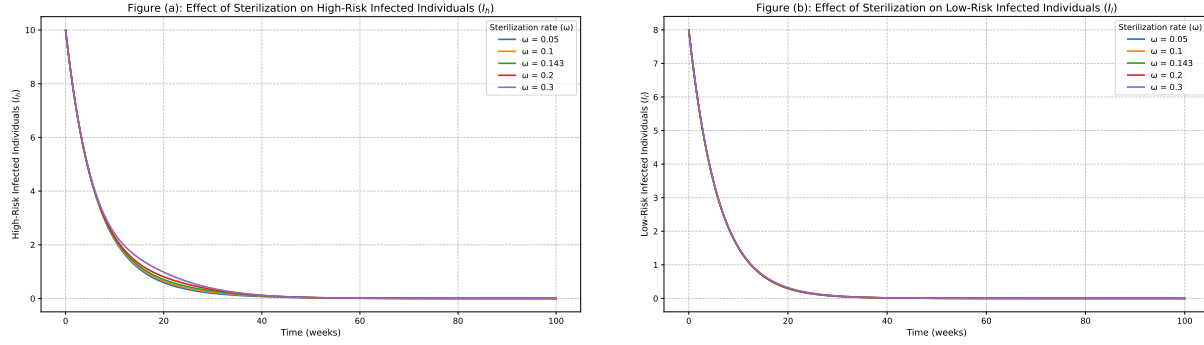
In this section, we present numerical simulations to investigate the dynamics of the infected compartments ($I_h(t)$, $I_l(t)$, and $E_i(t)$) under various control strategies, namely sterilization (ω), equipment cleanliness (θ), treatment (δ), and relapse prevention (κ). The simulations are performed in MATLAB using the *ODE45* solver for ordinary differential equations. Unless stated otherwise, all parameter values are adopted from Table 3. The system is initialized with $[700, 10, 0, 0, 900, 8, 0, 0, 200, 2]$, and the simulation is run over a period of 500 days.

7.1. Effect of Sterilization Control. Figure 7a illustrates how varying sterilization rates (ω) affect the dynamics of *high-risk infected individuals* $I_h(t)$. As sterilization rates rise, the number of infected high-risk persons fluctuates due to shifts in the relative balance of infection and sterilizing processes. Over time, however, all pathways converge on the same endemic equilibrium point. The higher the sterilizing rate, the smaller the early infection peaks due to less exposure

to contaminated injection equipment. Sterilization removes both infected and uninfected equipment, therefore the overall exposure ratio $\frac{E_i}{E_u+E_i}$ remains roughly the same, explaining why all infection curves eventually converge.

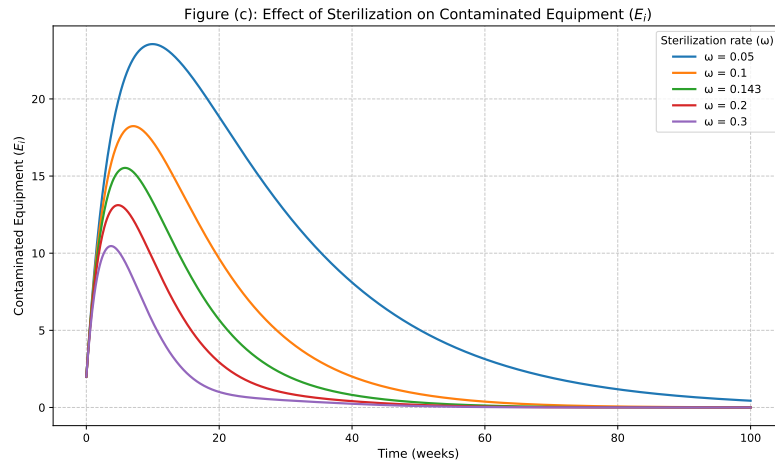
Figure 7b illustrates the dynamics of infected low-risk people ($I_l(t)$). Figure 7a shows a qualitatively similar trend, albeit with a lower overall infection size. This is due to the group's reduced risk behaviors and lower transmission rate (α_{wl}). Smaller, smoother infection peaks at higher sterilizing rates result in speedier convergence to equilibrium. These findings suggest that sterilization has a slight but significant protective effect on the infected low-risk PWID group; sterilization reduces infection burden while maintaining the long-term endemic level.

Figure 7c illustrates how sterilization impacts the quantity of *contaminated equipment* ($E_i(t)$). In this scenario, there is a clear inverse relationship with the rate of sterilization. As ω increases, $E_i(t)$ decays faster and reaches extremely low levels with increasing sterilization rates. Environmental sterilizing appears to significantly reduce the availability of previously contaminated reused equipment. As sterilization rates increase, the equilibrium value of E_i decreases significantly, indicating a strong positive association where sterilization efforts directly diminish environmental sources of infection.



(a) Effect of sterilization on high-risk infected individuals (I_h)

(b) Effect of sterilization on low-risk infected individuals (I_l)



(c) Effect of sterilization on contaminated equipment (E_i)

FIGURE 7. Effect of sterilization control (ω) on HCV transmission dynamics among PWID. Increasing sterilization rate reduces the quantity of contaminated equipment and slightly alters infection trajectories, leading all variables to converge toward the same endemic equilibrium.

7.2. Effect of Clean Equipment. The dynamics of Hepatitis C virus among PWID are strongly influenced by the production rate of clean injecting equipment, θ . Figures 8a–c illustrate these effects for three values of θ (0.01, 0.27, 0.54). A smaller θ means fewer clean needles

are available, which leads to more sharing and reuse of contaminated equipment. Consequently, the infection prevalence rises among both high-risk (I_h) and low-risk (I_l) PWID. Moreover, the absolute number of contaminated items (E_i) also increases due to the greater reuse of existing equipment. As θ increases, the supply of clean equipment is sufficient to reduce exposure to contaminated items, leading to lower infection levels in both human subpopulations and a decrease in environmental contamination. In Figure 8a, high-risk infected individuals, $I_h(t)$, experience higher infection peaks when θ is low, while increasing θ reduces both the peak and equilibrium levels. Similarly, Figure 8b shows that low-risk infected individuals, $I_l(t)$, display smaller infection magnitudes, but low θ still causes significant growth. Finally, Figure 8c shows that contaminated equipment, $E_i(t)$, rises most for the smallest θ , reflecting the scarcity of clean equipment. Higher values of θ reduce E_i and allow faster environmental renewal. Overall, these results demonstrate that enhancing clean-equipment production is an effective intervention to reduce human infections and environmental contamination.

7.3. Effect of Treatment. Treatment of infected individuals, represented by the rate δ , is an integral part of controlling Hepatitis C dynamics among PWID. Figures 9a–c depict the effects of different treatment rates ($\delta = 0.01, 0.04, 0.08$ per day) on high-risk and low-risk infected populations and contaminated equipment.

With the increase in treatment rate δ , infected individuals recover faster and are taken out of the infectious class. Hence, $I_h(t)$ decreases more rapidly for higher δ , as can be viewed in Figure 9a. Similarly, $I_l(t)$ also decreases more rapidly with increased treatment rates as depicted in Figure 9b. Thus, both subpopulations' infection prevalence is diminished by improved treatment coverage.

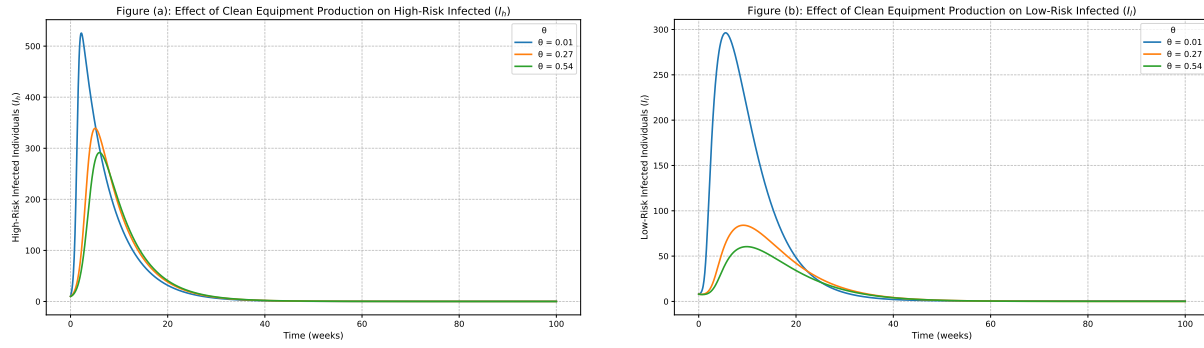
This effect of treatment also spills over into the environmental reservoir. Contaminated equipment, $E_i(t)$, declines more sharply for a high value of δ (Figure 9c), as fewer infected individuals are present to contaminate equipment. If δ is low, however, infections can persist longer, sustaining environmental contamination. Taken together, these results suggest that increased treatment rates can thus reduce not only human infections but also contaminated equipment, a major goal of treatment programs in disease control strategies.

7.4. Effect of relapse. The relapse or behavioural transition rate, κ , is crucially significant in partitioning individuals into low-risk and high-risk injecting groups. Figures 10a–c depict three relapse rates ($\kappa = 0.0007, 0.0014, 0.0027$ per week) that affect the dynamics of infected compartments and contaminated equipment.

As κ grows, more people move from low-risk to high-risk categories. As shown in Figure 10a, higher values of κ result in an increase in the number of high-risk infected individuals, $I_h(t)$. Individuals who move into the high-risk category are more likely to be exposed to contaminated equipment and engage in unsafe injection behaviour. In contrast, the low-risk infected, $I_l(t)$, falls slightly for larger values of κ , as seen in Figure 10b, since fewer individuals remain in the low-risk class to sustain infections there.

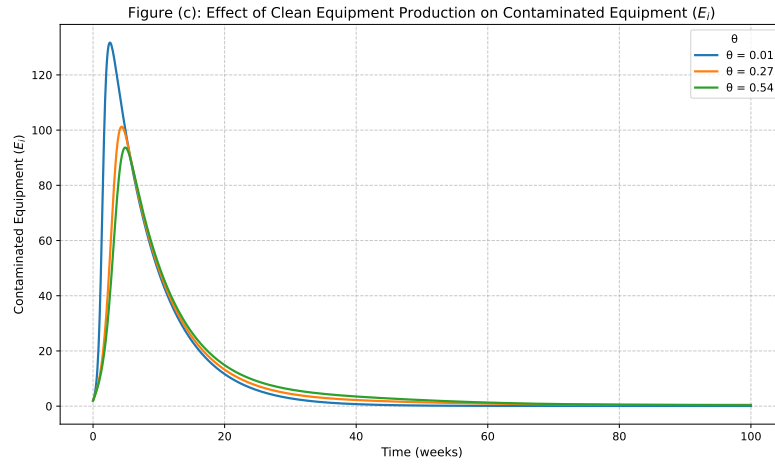
The contamination level of injecting equipment, $E_i(t)$, was very marginally responsive to fluctuations in κ (Figure 10c). While a high relapse rate increases $I_h(t)$ and hence the potential for contamination, this effect is somewhat offset by a drop in $I_l(t)$, yielding only a little net effect on environmental contamination. This study shows that changes in human behavioural transitions influence infection persistence primarily through their impact on the composition of the PWID community, rather than major modifications in environmental transmission dynamics.

Taken together, these findings suggest that preventing relapse into high-risk behaviour may be one effective approach to lowering HCV transmission since it removes the individual from the high-risk setting where infection is most likely. Intervention programs based on behavioural risk reduction may thus complement other harm reduction techniques in order to reduce the illness burden overall.



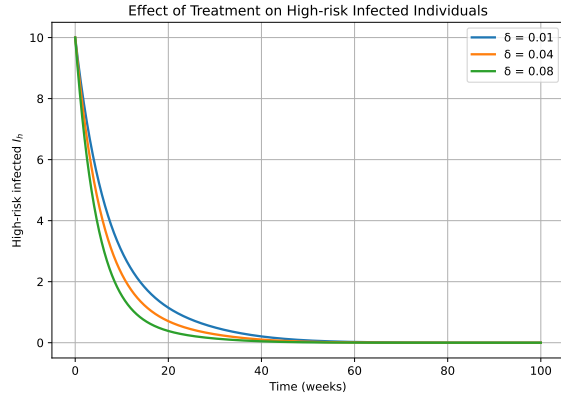
(a) High-risk infected individuals, $I_h(t)$.

(b) Low-risk infected individuals, $I_l(t)$.

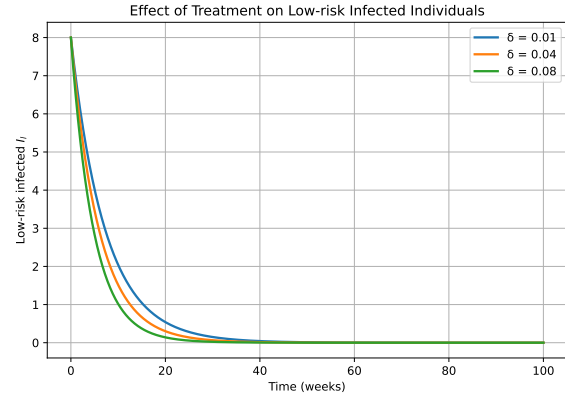


(c) Contaminated equipment, $E_i(t)$.

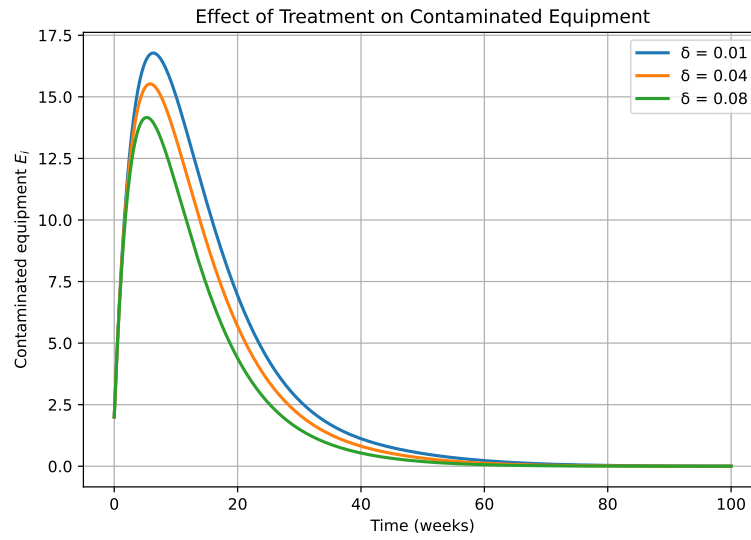
FIGURE 8. Effect of clean-equipment production rate, θ , on HCV dynamics among PWID. Three values of θ are shown: 0.01, 0.27, and 0.54. Lower θ corresponds to fewer clean needles, leading to more sharing and reuse of contaminated equipment, higher human infections (I_h , I_l), and more contaminated equipment (E_i). Higher θ reduces these risks.



(a) High-risk infected individuals, $I_h(t)$.

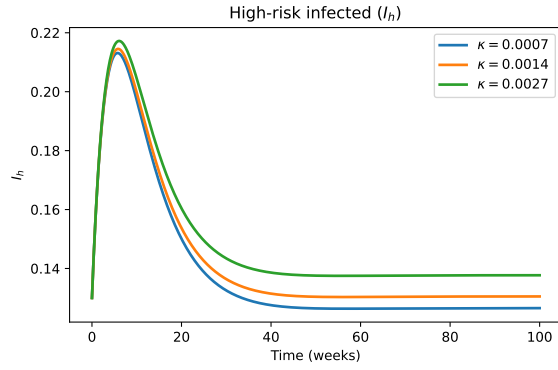


(b) Low-risk infected individuals, $I_l(t)$.

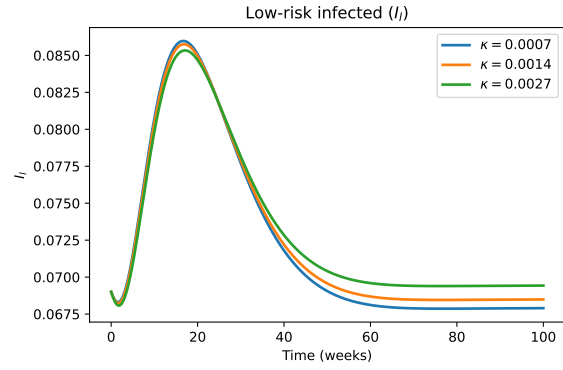


(c) Contaminated equipment, $E_i(t)$.

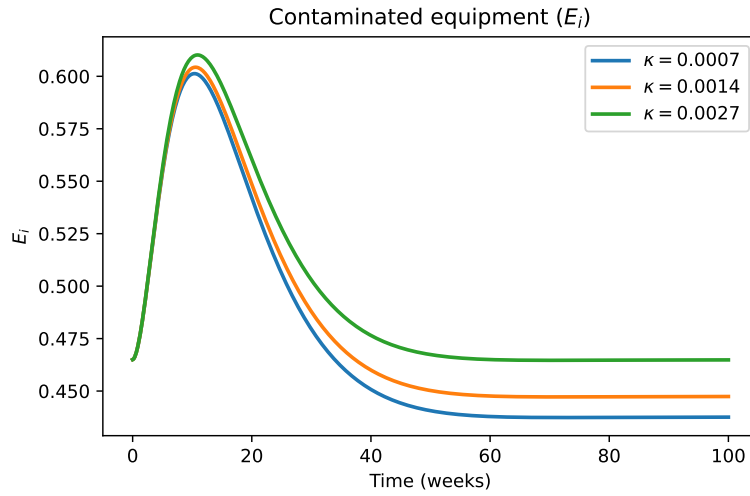
FIGURE 9. Effect of treatment rate δ on high-risk and low-risk infected populations and contaminated equipment. Higher treatment rates reduce infection prevalence and environmental contamination.



(a) High-risk infected individuals, $I_h(t)$.



(b) Low-risk infected individuals, $I_l(t)$.



(c) Contaminated equipment, $E_i(t)$.

FIGURE 10. The effect of relapse rate κ on infected persons at high and low risk, including contaminated injecting equipment

CONCLUSION

This study provides a comprehensive mathematical model that takes into account behaviour heterogeneity, equipment contamination, reinfection, and treatment dynamics in order to investigate the indirect transmission of HCV among PWID. Our analysis shows that disease dynamics are summarised by the basic reproduction number $\mathcal{R}_0$. When $\mathcal{R}_0 < 1$, the disease-free equilibrium is globally stable, whereas when $\mathcal{R}_0 > 1$, a distinct endemic equilibrium occurs. Bifurcation analyses indicate that the transmission rate of contaminated equipment among high-risk PWID (α_{wh}) and the behavioral shift from low- to high-risk (κ) are major drivers of disease-free and endemic states. The reinfection rate ψ does not alter $\mathcal{R}_0$, but it greatly increases endemic infection levels and environmental pollution.

Sensitivity analysis reveals that recruitment to the PWID population, particularly into the high-risk category, and contamination-related factors are the primary contributors to increasing $\mathcal{R}_0$, while mortality and equipment sterilisation play significant suppressive roles. Numerical simulations show that improved sterilisation, greater availability of clean equipment, and better treatment rates significantly lower both the infectious load and the levels of contaminated equipment.

Overall, our results highlight the critical role of environmental contamination and unsafe injecting practices in sustaining HCV transmission. Controls will thus be effective only through a combination of methods, including improved availability to sterile injection equipment, as well as greater sterilisation or disposal programs; higher coverage of antiviral therapy; and efforts to limit risk transitions. These findings offer evidence-based guidelines for the development of integrated public-health interventions aimed at lowering HCV transmission in PWID communities.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

- [1] D. Toma, L. Anghel, D. Patraş, A. Ciubără, Hepatitis C Virus: Epidemiological Challenges and Global Strategies for Elimination, *Viruses* 17 (2025), 1069. <https://doi.org/10.3390/v17081069>.

- [2] P. Torre, M. Festa, T. Sarcina, M. Masarone, M. Persico, Elimination of HCV Infection: Recent Epidemiological Findings, Barriers, and Strategies for the Coming Years, *Viruses* 16 (2024), 1792. <https://doi.org/10.3390/v16111792>.
- [3] O. Alshuwaykh, P.Y. Kwo, Current and Future Strategies for the Treatment of Chronic Hepatitis C, *Clin. Mol. Hepatol.* 27 (2021), 246–256. <https://doi.org/10.3350/cmh.2020.0230>.
- [4] World Health Organization, Hepatitis C Fact Sheet, World Health Organization, 2022. <https://www.who.int/news-room/fact-sheets/detail/hepatitis-c> (accessed 2026-09-06).
- [5] K.M.E. Turner, S. Hutchinson, P. Vickerman, V. Hope, N. Craine, et al. The Impact of Needle and Syringe Provision and Opiate Substitution Therapy on the Incidence of Hepatitis C Virus in Injecting Drug Users: Pooling of UK Evidence, *Addiction* 106 (2011), 1978–1988. <https://doi.org/10.1111/j.1360-0443.2011.03515.x>.
- [6] L. Platt, S. Minozzi, J. Reed, P. Vickerman, H. Hagan, et al. Needle and Syringe Programmes and Opioid Substitution Therapy for Preventing HCV Transmission among People Who Inject Drugs: Findings from a Cochrane Review and Meta-Analysis, *Addiction* 113 (2018), 545–563. <https://doi.org/10.1111/add.14012>.
- [7] Z. Belrhiti, F.Z. Serghini, Perceived Barriers to Access to Hepatitis C for People Who Inject Drugs in Morocco: A Qualitative Explorative Study, *Drug Sci. Policy Law* 9 (2023), 20503245231199163. <https://doi.org/10.1177/20503245231199163>.
- [8] S.E. Habib, Understanding the Context of Risk Practices among Injecting Drug Users: Implications for Hepatitis C Prevention, *Aust. J. Soc. Issues* 38 (2003), 1–17. <https://doi.org/10.1002/j.1839-4655.2003.tb01132.x>.
- [9] C.-L. Liu, F.-R. Ren, Q.-S. Lü, J.-H. Liu, H. Zhuang, Stability of Hepatitis C Virus RNA in Various Processing and Storage Conditions (in Chinese), *Zhongguo Shi Yan Xue Ye Xue Za Zhi* 14 (2006), 1238–1243. <https://pubmed.ncbi.nlm.nih.gov/17204202/>.
- [10] Y. Guo, Y. Li, S. Mu, J. Zhang, Z. Yan, Evidence That Methylation of Hepatitis B Virus Covalently Closed Circular DNA in Liver Tissues of Patients with Chronic Hepatitis B Modulates HBV Replication, *J. Med. Virol.* 81 (2009), 1177–1183. <https://doi.org/10.1002/jmv.21525>.
- [11] H. Hagan, E.R. Pouget, D.C. Des Jarlais, A Systematic Review and Meta-Analysis of Interventions to Prevent Hepatitis C Virus Infection in People Who Inject Drugs, *J. Infect. Dis.* 204 (2011), 74–83. <https://doi.org/10.1093/infdis/jir196>.
- [12] H. Fraser, C. Mukandavire, N.K. Martin, D. Goldberg, N. Palmateer, A. Munro, A. Taylor, M. Hickman, S. Hutchinson, P. Vickerman, Modelling the Impact of a National Scale-up of Interventions on Hepatitis C Virus Transmission among People Who Inject Drugs in Scotland, *Addiction* 113 (2018), 2118–2131. <https://doi.org/10.1111/add.14267>.

- [13] L. Platt, S. Minozzi, J. Reed, P. Vickerman, H. Hagan, et al. Needle Syringe Programmes and Opioid Substitution Therapy for Preventing Hepatitis C Transmission in People Who Inject Drugs, *Cochrane Database Syst. Rev.* 2017 (2017), CD012021. <https://doi.org/10.1002/14651858.CD012021.pub2>.
- [14] L.T. Tran, A. Peacock, S. Colledge, S. Memedovic, J. Grebely, et al. Injecting Risk Behaviours amongst People Who Inject Drugs: A Global Multi-Stage Systematic Review and Meta-Analysis, *Int. J. Drug Policy* 84 (2020), 102866. <https://doi.org/10.1016/j.drugpo.2020.102866>.
- [15] N. Scott, M. Hellard, E.S. McBryde, Modeling Hepatitis C Virus Transmission among People Who Inject Drugs: Assumptions, Limitations and Future Challenges, *Virulence* 7 (2016), 201–208. <https://doi.org/10.1080/21505594.2015.1085151>.
- [16] Z. Ward, L. Platt, S. Sweeney, V.D. Hope, L. Maher, et al. Impact of Current and Scaled-up Levels of Hepatitis C Prevention and Treatment Interventions for People Who Inject Drugs in Three UK Settings—What Is Required to Achieve the WHO’s HCV Elimination Targets?, *Addiction* 113 (2018), 1727–1738. <https://doi.org/10.1111/add.14217>.
- [17] M.D. Miller-Dickson, V.A. Meszaros, S. Almagro-Moreno, C.B. Ogbunugafor, Hepatitis C Virus Modelled as an Indirectly Transmitted Infection Highlights the Centrality of Injection Drug Equipment in Disease Dynamics, *J. R. Soc. Interface* 16 (2019), 20190334. <https://doi.org/10.1098/rsif.2019.0334>.
- [18] M. Makhoul, G.R. Mumtaz, H.H. Ayoub, M.S. Jamil, J.G. Hermez, A.S. Alaama, L.J. Abu-Raddad, Hepatitis C Virus Transmission among People Who Inject Drugs in the Middle East and North Africa: Mathematical Modeling Analyses of Incidence and Intervention Impact, *eClinicalMedicine* 80 (2025), 103040. <https://doi.org/10.1016/j.eclinm.2024.103040>.
- [19] O. Diekmann, J.A.P. Heesterbeek, J.A.J. Metz, On the Definition and the Computation of the Basic Reproduction Ratio R_0 in Models for Infectious Diseases in Heterogeneous Populations, *J. Math. Biol.* 28 (1990), 365–382. <https://doi.org/10.1007/BF00178324>.
- [20] O.J. Peter, S. Kumar, N. Kumari, F.A. Oguntolu, K. Oshinubi, R. Musa, Transmission Dynamics of Monkeypox Virus: A Mathematical Modelling Approach, *Model. Earth Syst. Environ.* 8 (2022), 3423–3434. <https://doi.org/10.1007/s40808-021-01313-2>.
- [21] O.J. Peter, H.S. Panigoro, M.A. Ibrahim, O.M. Otunuga, T.A. Ayoola, A.O. Oladapo, Analysis and Dynamics of Measles with Control Strategies: A Mathematical Modeling Approach, *Int. J. Dyn. Control* 11 (2023), 2538–2552. <https://doi.org/10.1007/s40435-022-01105-1>.
- [22] C. Castillo-Chavez, Z. Feng, W. Huang, On the Computation of R_0 and Its Role on Global Stability, in: C. Castillo-Chavez, S. Blower, P. van den Driessche, D. Kirschner, A.-A. Yakubu (Eds.), *Mathematical Approaches for Emerging and Reemerging Infectious Diseases: An Introduction*, Springer, New York, pp. 229–250, (2002). https://doi.org/10.1007/978-1-4757-3667-0_13.

- [23] R. Shi, Y. Cui, Global Analysis of a Mathematical Model for Hepatitis C Virus Transmissions, *Virus Res.* 217 (2016), 8–17. <https://doi.org/10.1016/j.virusres.2016.02.006>.
- [24] L. Boelen, S. Teutsch, D.P. Wilson, K. Dolan, G.J. Dore, A.R. Lloyd, F. Luciani, on behalf of the HITS Investigators, Per-Event Probability of Hepatitis C Infection during Sharing of Injecting Equipment, *PLoS One* 9 (2014), e100749. <https://doi.org/10.1371/journal.pone.0100749>.
- [25] L.E. Thorpe, L.J. Ouellet, R. Hershow, S.L. Bailey, I.T. Williams, J. Williamson, E.R. Monterroso, R.S. Garfein, Risk of Hepatitis C Virus Infection among Young Adult Injection Drug Users Who Share Injection Equipment, *Am. J. Epidemiol.* 155 (2002), 645–653. <https://doi.org/10.1093/aje/155.7.645>.
- [26] J. Grebely, G.J. Dore, F.L. Altice, B. Conway, A.H. Litwin, et al. Reinfection and Risk Behaviors after Treatment of Hepatitis C Virus Infection in Persons Receiving Opioid Agonist Therapy: A Cohort Study, *Ann. Intern. Med.* 175 (2022), 1221–1229. <https://doi.org/10.7326/M21-4119>.
- [27] H. Fraser, N.K. Martin, H. Brummer-Korvenkontio, P. Carrieri, O. Dalgard, et al. Model Projections on the Impact of HCV Treatment in the Prevention of HCV Transmission among People Who Inject Drugs in Europe, *J. Hepatol.* 68 (2018), 402–411. <https://doi.org/10.1016/j.jhep.2017.10.010>.
- [28] World Health Organization, UNAIDS, UNODC, Guide to Starting and Managing Needle and Syringe Programmes, World Health Organization, 2007. <https://iris.who.int/handle/10665/43816>.
- [29] M. Hellard, D.A. Rolls, R. Sacks-Davis, G. Robins, P. Pattison, P. Higgs, C. Aitken, E. McBryde, The Impact of Injecting Networks on Hepatitis C Transmission and Treatment in People Who Inject Drugs, *Hepatology* 60 (2014), 1861–1870. <https://doi.org/10.1002/hep.27403>.