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THE MATHEMATICAL STUDY FOR MORTALITY COEFFICIENTS OF SMALL PELAGIC SPECIES

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Abstract. In the past few years, the parapenaeus longirostris population stock has seen a sharp reduction. In this work, we propose a bioeconomic model that represents the biomass evolution of this marine population in two moroccan maritime patches: protected area and unprotected area. In the model construction, we take in consideration the predation interaction between the parapenaeus longirostris population and the small pelagic species of moroccan coastal zones. We suppose the existence of coastal trawlers that exploit both the predator and prey populations. Our objective is to study the influence of the predator mortality rate variation on the evolution of prey biomass and the profit of coastal trawlers. It should be underlined that, coastal trawlers are constrained by the conservation of marine biodiversity. One of the key consequences of this is that the increase in the mortality rate of small pelagics leads to an evolution in the parapenaeus longirostris stock, and consequently to an increase in the profit of coastal trawlers after exploitation of this species. On the other hand, the level of fishing effort and catches of small pelagics is decreasing, which leads to a reduction in the profit of coastal trawlers after exploiting small pelagics.

Keywords: bioeconomic model; parapenaeus longirostris population; small pelagic species; mortality rate; Nash equilibrium problem.

2010 AMS Subject Classification: 92B05, 91A06, 91B02, 91B50.

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1. Introduction

The Moroccan coastline stretches for about 3,500 kilometers. Its waters are among the world's richest sources of fish. The FAO considers Morocco's production potential at nearly 1.5 million tonnes per year. Morocco is the Africa's largest producer of fish (FAO, 2001). It represents 1.2% of world production and ranks 18th in the world.

There are four fishing zones in Morocco whose relative importance in terms of activity has undergone a great change over time and the pace of exploitation (Figure 1). Mediterranean and North Atlantic zone to ElJadida (35°45'-32°N), zone A of Safi to Sidi Ifni (32°N-29°N), zone B of Sidi Ifni to Cape Boujdor (29°N-26°N) and zone C from Boujdor to Lagouira (26 °N to the South). Which allows the kingdom to be the first in the Arab world and in Africa for fishing for fish and seafood, including small pelagics species and shrimps. The shrimps fishing has several advantages both economically by being a source of foreign exchange and socially by the labor it generates. However, shrimp fishing in Moroccan fishing zones confronts several challenges and issues with regard to the nature of the resource itself and the context of exploitation. Mostly on the , it remains among the most fragile stocks. This resource has not yet covered its optimal state, it is a very important fishery for both the local market and for export.

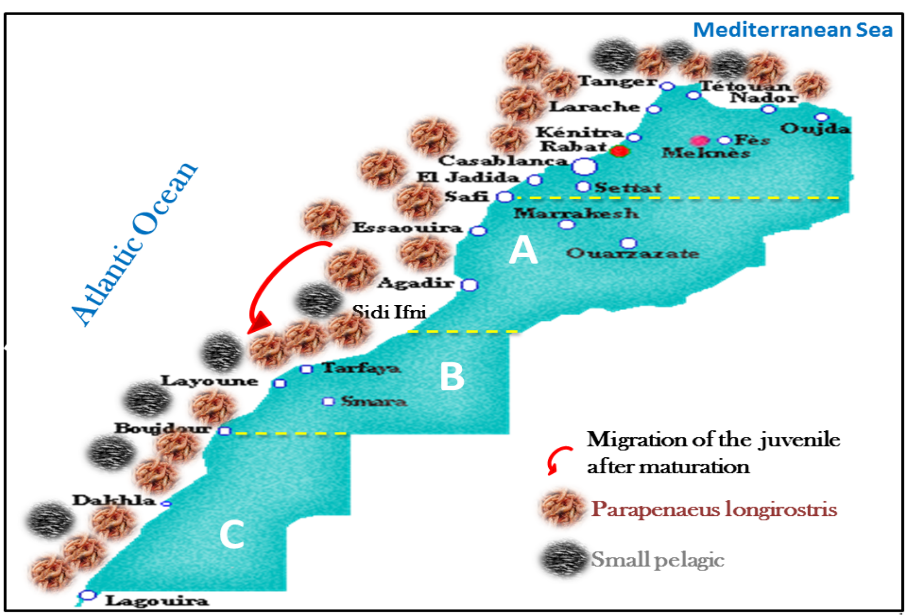

FIGURE 1. The distribution of the parapenaeus longirostris and small pelagic species stocks

The exploitation of parapenaeus longirostris offshore by Moroccan fleets developed from the 1980s. Its geographical distribution is quite wide. In Morocco, it is found in abundance in the plateau and the Atlantic continental slope, from Cap Spartel in the North to Lagouira in the South and in the Moroccan Mediterranean from Ceuta to Saidia. This deep species lives on muddy or sandy bottom. Its bathymetric distribution is wide, from 20 to 700 m and generally from 100 to 400 m depth (Heldt, 1954, Holthuis, 1987, Ardizzone et al, 1990).

Overexploitation, especially of growth (of juveniles) remains one of the main causes causing the state of more or less advanced degradation of certain resources. Excessive fishing pressure on juveniles (especially for parapenaeus longirostris) is leading to a significant shortfall for the fishery as a whole in terms of production per recruit and hence exploitable biomass. To this end, it is recommended to continue the effort already put in place for the management of parapenaeus longirostris stocks and to reinforce the current management measures, in particular those relating to the reduction and control of fishing mortality.

Thus, adequate scientific monitoring and appropriate adaptive management alone guarantee the sustainability of these short-lived resources. Let us add that understanding the biological mechanisms of parapenaeus longirostris modulating key environmental and ecosystem indicators of stock health, such as water temperature and predator abundance: small pelagics such as sardine, horse mackerel, anchovy, sardinella, etc., is an integral part of the preparation and enhancement of stock ecosystem assessments [1, 2, 3, 4]. This will make it possible to decide on the state of the resource and recommend recommendations for better management of these fisheries.

In this context, many mathematical models have been developed to describe the dynamics of fisheries, we can see for example [5, 6, 7, 8, 9, 10, 11, 12, 13]. Also, we can refer to Y. El foutayeni et al. [14] in their work, the autors have defined a bioeconomic equilibrium model for several coastal trawlers who catch two fish species. The authors have studied the existence of the steady states and its stability using eigenvalue analysis; they have solved two mathematical problems to determine the equilibrium point that maximizes the profit of each coastal trawler. Finally the authors have given some numerical simulations to illustrate the results.

An other important example in this context is also that Y. El foutayeni et al. [15], in this work the authors have defined a bioeconomic equilibrium model for several coastal trawlers exploiting three species, these species compete with each other for space or food; the natural growth of each species is modeled using a logistic law; the objective of their work is to calculate the fishing effort that maximizes the profit of each coastal trawler at biological equilibrium by using the generalized Nash equilibrium problem.

In the work of I. Agmour et all. [16], the authors have sought to highlight that the increase of the carrying capacity of marine species does not always lead to an increase on the catch levels and on the incomes. To effectively support the theoretical outcomes, we have considered a bioeconomic model of several seiners exploiting Sardina pilchardus, Engraulis encrasicolus and Xiphias gladius marine species in the Atlantic coast of Morocco based on the parameters given by 'Institut National de Recherche Halieutique'.

Recently, the parapenaeus longirostris resource stock was marked by a fall yields of the different fleets operating moroccan maritime zones, as well as a decrease in catches in this species. In addition, abundance indices decreased. The drop in parapenaeus longirostris stocks could be caused by over-fishing and the predation between this population and small pelagics. In this work, we consider the biomass evolution model of the parapenaeus longirostris population in the presence of predators (the five small pelagics) in two areas: protected area and unprotected area. In one side, the model introduces the small pelagic fish populations and parapenaeus longirostris fishery into free access fishing zone. The different parameters and variables used in the biological model are cited in tables 1 and 2. On the other hand, we search to study the impact of the variation of the predator mortality rate on the stock evolution of the prey population. It also seeks to interpret the best fishing situations, which allow seiners to have the maximum income by preserving stocks of small pelagic and parapenaeus longirostris populations.

Variables	Description
$P_J(t)$	The stock of parapenaeus longirostris juveniles into patch A
$P_A(t)$	The stock of parapenaeus longirostris adults into patch B
$S(t)$	The density of sardine
$A(t)$	The density of anchovy
$M(t)$	The density of mackerel
$H(t)$	The density of horse mackerel
$L(t)$	The density of sardinella

Table. 1. The description of variables.

Parameters	Description
κK	The carrying capacity in the reserved area
$(1-\kappa)K$	The carrying capacity in the unreserved area
r	The growth rate of parapenaeus longirostris
μ_s	The maximum per capita consumption rates of sardine
μ_a	The maximum per capita consumption rates of anchovy
μ_m	The maximum per capita consumption rates of mackerel
μ_h	The maximum per capita consumption rates of h. mackerel
μ_l	The maximum per capita consumption rates of sardinella
η	The maximal carrying of parapenaeus longirostris
β	The mobility coefficient
d_s	The natural death rates of sardine
d_a	The natural death rates of anchovy
d_m	The natural death rates of mackerel
d_h	The natural death rates of horse mackerel
d_l	The natural death rates of sardinella
α_s	The amount of the P.L required to support sardine
α_a	The amount of the P.L required to support anchovy
α_m	The amount of the P.L required to support mackerel
α_h	The amount of the P.L required to support h. mackerel
α_l	The amount of the P.L required to support sardinella

Table. 2. The description of parameters

The paper is organized as follows. In the next section, we present the biological model of the juveniles and adults of the parapenaeus longirostris evolutions with the presence of the predators; in other word, the five small pelagic species: Sardines, Anchovy, Mackerel, The horse mackerel, Sardinella; which consist in a system of seven ordinary differential equations, the first equation describes the natural growth of the juveniles of Parapenaeus longirostris fish population a prey of the small pelagic fish population, the second equation describes the natural growth of Parapenaeus longirostris fish population a prey of the small pelagic fish population, the third to seven equations describe the natural growth of the small pelagic fish population as a predators of the juveniles and adults of the parapenaeus longirostris. The existence of the steady states of this system and its stability are studied using eigenvalue analysis and we define a bioeconomic equilibrium model for all of this fish populations exploited by a fishing fleet. In section 3, We compute some numerical simulations to determine the optimal conditions under which the biological steady state can be attained and to draw some important conclusions regarding reserve designs. In section 4 we give a numerical simulation of the mathematical model and discussion of the results. Finally we give a conclusion and some potential perspectives in section 5.

2. BIOLOGICAL MODEL

Our study is based on a prey–predator system, the parapenaeus longirostris and the small pelagic fish (Sardine, Anchovy, Mackerel, The horse mackerel, Sardinella) in two patches (as shown in figure 1): a fishing protected area and free access fishing zone. Let us assume that $P_J(t)$ is the stock of parapenaeus longirostris juveniles into patch A, the reserve area, and $P_A(t)$ is the stock of parapenaeus longirostris adults into patch B, unreserved area, at time t. Assuming total region under consideration is unit and $0 < \kappa < 1$ is the reserved area, consequently $(1-\kappa)$ is the unreserved area.

It is assumed that the juveniles of parapenaeus longirostris fish population grows according to a logistic equation with growth rate r and and its carrying capacity is κK. The functions $\mu_s P_J(t) S(t) / \eta$, $\mu_a P_J(t) A(t) / \eta$, $\mu_m P_J(t) M(t) / \eta$, $\mu_h P_J(t) H(t) / \eta$ and $\mu_l P_J(t) L(t) / \eta$ are the function responses, where μ_s, μ_a, μ_m, μ_h, μ_l are respectively the maximum per capita consumption rates of sardine, anchovy, mackerel, horse mackerel and sardinella, i.e. the maximum rate at which the juvenile and adults of the parapenaeus longirostris population can be eaten by a sardine, anchovy, mackerel, the horse mackerel, sardinella per unit time. And η is the maximal carrying of the juvenile and adults of the parapenaeus longirostris. $\beta P_J(t) P_A(t) \diagup \kappa(1-\kappa)K^2$ represent the net transfer rate or migration, where β is a mobility coefficient. The net transfer from the protected area to the unprotected area is assumed to be the positive direction for migration.

The population of parapenaeus longirostris adults grows according to a logistic equation with growth rate r, its carrying capacity is equal to $(1-\kappa)K$. We note that it is a prey of the small pelagic species.

Let $S(t)$, $A(t)$, $M(t)$, $H(t)$ and $L(t)$ be the densities of sardine, anchovy, mackerel, horse mackerel and sardinella, respectively. These populations are predators of parapenaeus longirostris juveniles and adults. The natural death rates of predators: sardine, anchovy, mackerel, horse mackerel and sardinella are denoted by d_s, d_a, d_m, d_h and d_l, respectively. The parameters α_s, α_a, α_m, α_h and α_l are the amount of the juvenile and adults of the parapenaeus longirostris required to support sardine, anchovy, mackerel, horse mackerel and sardinella at equilibrium, respectively.

Following the previous assumptions, the biological model is represented as follow

$$
(1) \qquad \begin{cases}
\dot{P}_J(t) = & rP_J(t)\left(1-\frac{P_J(t)}{\kappa K}\right)-\frac{\beta P_J(t)P_A(t)}{\kappa(1-\kappa)K^2}-\frac{\mu_s P_J(t)S(t)}{\eta}-\frac{\mu_a P_J(t)A(t)}{\eta} \\
& -\frac{\mu_m P_J(t)M(t)}{\eta}-\frac{\mu_h P_J(t)H(t)}{\eta}-\frac{\mu_l P_J(t)L(t)}{\eta} \\
\dot{P}_A(t) = & rP_A(t)\left(1-\frac{P_A(t)}{(1-\kappa)K}\right)+\frac{\beta P_J(t)P_A(t)}{\kappa(1-\kappa)K^2}-\frac{\mu_s P_A(t)S(t)}{\eta}-\frac{\mu_a P_A(t)A(t)}{\eta} \\
& -\frac{\mu_m P_A(t)M(t)}{\eta}-\frac{\mu_h P_A(t)H(t)}{\eta}-\frac{\mu_l P_A(t)L(t)}{\eta} \\
\dot{S}(t) = & -d_s S(t)+\alpha_s S(t)P_J(t)+\alpha_s S(t)P_A(t) \\
\dot{A}(t) = & -d_a A(t)+\alpha_a A(t)P_J(t)+\alpha_a A(t)P_A(t) \\
\dot{M}(t) = & -d_m M(t)+\alpha_m M(t)P_J(t)+\alpha_m M(t)P_A(t) \\
\dot{H}(t) = & -d_h H(t)+\alpha_h H(t)P_J(t)+\alpha_h H(t)P_A(t) \\
\dot{L}(t) = & -d_l L(t)+\alpha_l L(t)P_J(t)+\alpha_l L(t)P_A(t)
\end{cases}
$$

with positive initial conditions.

3. BIOLOGICAL MODEL ANALYSIS

Let $X(t) = (P_J(t), P_A(t), S(t), A(t), M(t), H(t), L(t))$ be the solution of the system (1) at the biological equilibrium. Then all the solutions of the system (1) are nonnegative. To demonstrate that we must recall that by [17], the system of equation (1) is a positive system.

Theorem 1. *All the solutions of system* (1) *which start in* $\mathbb{R}^7_+$ *are uniformly bounded.*

Proof. We define the function

$$\begin{aligned} f(t) = \quad & \alpha_s\alpha_a\alpha_m\alpha_h\alpha_l\left(P_J(t)+P_A(t)\right)+\tfrac{\mu_s\alpha_a\alpha_m\alpha_h\alpha_l}{\eta}S(t)+\tfrac{\mu_a\alpha_s\alpha_m\alpha_h\alpha_l}{\eta}A(t) \\ & +\tfrac{\mu_m\alpha_s\alpha_a\alpha_h\alpha_l}{\eta}M(t)+\tfrac{\mu_h\alpha_s\alpha_a\alpha_m\alpha_l}{\eta}H(t)+\tfrac{\mu_l\alpha_s\alpha_a\alpha_m\alpha_h}{\eta}L(t) \end{aligned}$$

Therefore, the time derivative along a solution of (1) is

$$\begin{aligned} \tfrac{df}{dt} = \quad & \alpha_s\alpha_a\alpha_m\alpha_h\alpha_l rP_J(t)\left[\left(1-\tfrac{P_J(t)}{\kappa K}\right)+P_A(t)\left(1-\tfrac{P_A(t)}{(1-\kappa)K}\right)\right] \\ & -\tfrac{\mu_a\alpha_s\alpha_m\alpha_h\alpha_l d_a}{\eta}A(t)-\tfrac{\mu_s\alpha_a\alpha_m\alpha_h\alpha_l d_s}{\eta}S(t)-\tfrac{\mu_m\alpha_s\alpha_a\alpha_h\alpha_l d_m}{\eta}M(t) \\ & -\tfrac{\mu_h\alpha_s\alpha_a\alpha_m\alpha_l d_h}{\eta}H(t)-\tfrac{\mu_l\alpha_s\alpha_a\alpha_m\alpha_h d_l}{\eta}L(t) \end{aligned}$$

For each $\vartheta > 0$, we have

$$\begin{aligned} \tfrac{df}{dt}+\vartheta f \leq \quad & \alpha_s\alpha_a\alpha_m\alpha_h\alpha_l\tfrac{K}{4r}(r+\vartheta)^2+\tfrac{\vartheta}{\eta}\left[\mu_s\alpha_a\alpha_m\alpha_h\alpha_l S(t)+\mu_a\alpha_s\alpha_m\alpha_h\alpha_l A(t)\right] \\ & +\tfrac{\vartheta}{\eta}\left[\mu_m\alpha_s\alpha_a\alpha_h\alpha_l M(t)+\mu_h\alpha_s\alpha_a\alpha_m\alpha_l H(t)+\mu_l\alpha_s\alpha_a\alpha_m\alpha_h L(t)\right] \end{aligned}$$

So, the right-hand side is positive and it is bounded for all $(P_J, P_A, S, A, M, H, l) \in \mathbb{R}^7_+$. Therefore, we find a $\theta > 0$ with $\frac{df}{dt}+\vartheta f < \theta$. Using the theory of differential inequality [18], we obtain

$$0 \leq f \leq \frac{\theta}{\vartheta}+\left[f(P_J(0), P_A(0), S(0), A(0), M(0), H(0), L(0))-\frac{\theta}{\vartheta}\right]e^{-\vartheta t}$$

which upon letting $t \longrightarrow \infty$, yields $0 \leq f \leq \frac{\theta}{\vartheta}$.

Then, we have

$$B = \left\{(P_J, P_A, S, A, M, H, L) \in \mathbb{R}^7_+ : f < \frac{\theta}{\vartheta}+\varepsilon,\ \forall\varepsilon > 0\right\}$$

where B is the region in which all the solutions of system of equation (1) that start in $\mathbb{R}^7_+$ are confined.

$\square$

For system (1), we can see that the equilibrium points of system (1) satisfies the following equations

$$(2)\qquad \begin{cases} r(1-\frac{P_J}{\kappa K})-\frac{\beta P_A}{\kappa(1-\kappa)K^2}-\frac{\mu_s}{\eta}S+\frac{\mu_a}{\eta}A+\frac{\mu_m}{\eta}M+\frac{\mu_h}{\eta}H+\frac{\mu_l}{\eta}L=0 \\ r(1-\frac{P_A}{(1-\kappa)K})+\frac{\beta P_J}{\kappa(1-\kappa)K^2}-\frac{\mu_s}{\eta}S-\frac{\mu_a}{\eta}A-\frac{\mu_m}{\eta}M-\frac{\mu_h}{\eta}H-\frac{\mu_l}{\eta}L=0 \\ -d_s+\alpha_s P_J+\alpha_s P_A=0 \\ -d_a+\alpha_a P_J+\alpha_a P_A=0 \\ -d_m+\alpha_m P_J+\alpha_m P_A=0 \\ -d_h+\alpha_h P_J+\alpha_h P_A=0 \\ -d_l+\alpha_l P_J+\alpha_l P_A s=0 \end{cases}$$

After calculation, it is obvious that system.(2) has a 128 equilibrium points (Annexe 1), but it has real positive solution is $X^*=[P_J^*,P_A^*,S^*,A^*,M^*,H^*,L^*]^T$, where

$$\begin{cases} P_J^*=\frac{d_s(Kr\kappa-\beta)}{Kr\alpha_s} \\ P_A^*=\frac{d_s(\beta+Kr(1-\kappa))}{Kr\alpha_s} \\ S^*=\frac{\eta d_s\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)(\mu_s-1)+K^3r^2\kappa\alpha_s(\eta-\mu_s)(\kappa-1)}{K^3r\kappa\alpha_s\mu_s(\kappa-1)(\mu_s-1)} \\ A^*=\frac{2K^2r^2\kappa\eta d_a(\kappa-1)-2\beta^2\eta d_a-K^3r\kappa\alpha_a(\kappa-1)(2r\eta-\mu_a)}{K^3r\kappa\alpha_a(\mu_a-1)(\kappa-1)} \\ M^*=\frac{M_1+M_2+M_3+M_4+M_5}{2K^3r\kappa\mu_m\alpha_s^2(\mu_s-1)(\mu_a-1)(\kappa-1)} \\ H^*=\frac{H_1+H_2+H_3+H_4}{K^3r\kappa\mu_h\alpha_h^2(\mu_m-1)(\mu_a-1)(\kappa-1)} \\ L^*=\frac{r\mu_l(1-\eta)}{2\mu_h(\mu_a-1)} \end{cases}$$

with

$$\begin{aligned} M_1 = {} & 2Kr\beta\eta d_m(\mu_s-1)(\mu_a-1)(d_m+2\kappa\alpha_m-4\kappa d_s) \\ M_2 = {} & K^3r\kappa\alpha_m^2(\kappa-1)(2\mu_m(\mu_a+2r\eta)-r(8\eta-\mu_m(9\eta-1))) \\ & -K^3r\kappa\alpha_m^2(\kappa-1)\mu_m^2(-r+2\mu_a+5r\eta) \\ M_3 = {} & 4\beta^2\eta d_m(\mu_m-1)(d_m(\mu_m-1)-\alpha_m(\mu_m-2)) \\ M_4 = {} & 2K^2r^2\kappa\eta d_m[\alpha_m(2\kappa+\mu_a-3)-\mu_m(2\kappa\alpha_s-d_m)] \\ M_5 = {} & 6K^2r^2\kappa\eta d_m^3\mu_m^2\alpha_m(2/3d_m-(2\kappa+\mu_m(2\kappa-3\alpha_s))) \\ & -6K^2r^2\kappa\eta d_m^3\mu_m^3\alpha_m(2\kappa\mu_m-3\alpha_m(2\kappa+\mu_a)) \end{aligned}$$

$$
\begin{aligned}
H_1 = &\ 4K^3r^2\kappa\alpha_h^2\eta\left(\mu_h-1\right)^2(\kappa-1)\\
H_2 = &\ K^2r^2\kappa d_h\eta\left[\alpha_h\left(\mu_h-1\right)(2\kappa-3)+\mu_h\left(2\kappa\alpha_h^2-d_h\right)2\kappa\left(3d_h+1\right)\right]\\
H_3 = &\ 3K^2r^2\kappa d_h^2\eta\mu_h^2\left(d_h-2\kappa\alpha_h^2\right)\left[2\kappa\left(\mu_h-1\right)+3\alpha_h-3\alpha_h\left(2\kappa+\mu_h\right)\right]\\
H_4 = &\ \beta\eta d_h\left[2\beta\left(\mu_h-1\right)^2\left(2\alpha_h-d_h\right)-Kr\left(\mu_h-1\right)^2\left(d_h+2\kappa\alpha_h-4\kappa d_h\right)\right]
\end{aligned}
$$

It must be noted that we only concentrate on the interior equilibrium point of the system (1), since the biological meaning of the interior equilibrium indicates that the parapenaeus longirostris and the small pelagic fish populations all exist. So in this paper, we assume that $Kr\kappa-\beta>0$, $d_s>K\alpha_s$, $d_a>K\alpha_a$, $d_m>K\alpha_m$, $d_h>K\alpha_h$ and $d_l>K\alpha_l$ with $K,r_s,r_a,r_m,r_h,r_l,\beta,\alpha>0$.

To evaluate the variational matrix at this point and analyze its local stability we use the Routh Hurwitz criterion, as is shown in annexe 2.

The characteristic polynomial associated to the variational matrix $J\left(X^*\right)$ is written as $P(\lambda)=\sum_{k=0}^{7}\rho_k\lambda^k$. Note that the coefficients ρ_k are written according to all the parameters mentioned in the biological model (1) (*Annexe* 2). It is obvious to show that the ρ_k are positive for all $k=\{1,...,6\}$, and likewise for ρ_{ij} with $i=\{1,2,3\}$ and $j=\{1,...,6\}$. So, according to the Routh Hurwitz stability criterion, we can conclude that the equilibrium point X^* is stable.

4. Bioeconomic model

The exploitation scheme of the shrimp includes directed fishing on these species by the freezing, offshore and coastal fishing segments, and multi-species by the fleet of fresh coastal trawlers targeting shrimp and other groups of fish such as small pelagics.

At the Moroccan fishing zones level, catches of fresh-fishing coastal trawlers having landed parapenaeus longirostris are composed of more than 80 species of fish (according to ONP statistics). The top 20 species landed in these areas account for more than 84% of the total catch. parapenaeus longirostris dominates shrimp catches of the fresh coastal fishery segment at both elevations and accounts for 88% of catch volumes. Hence the importance of introducing the catches of fishing coastal trawlers into our biological model.

The proposed bioeconomic model (3) in the presence of harvesting includes three parts: a biological part connecting the catch to the biomass stock, an exploitation part connecting the catch to the fishing effort, and an economic part connecting the fishing effort to the profit.

We denote by H_{ij} the catches of fish population j by the coastal trawler i and it is given by the equation

$$H_{ij} = q_j E_{ij} x_j,$$

where E_{ij} is the fishing effort of the coastal trawler i to exploit the fish population j and q_j is the is the catchability coefficient of fish population j. Let us add that $H_j = H_{1j} + H_{2j}$ is the total catches of fish population j by the two coastal trawler, and $E_j = E_{1j} + E_{2j}$ the total fishing effort dedicated to fish population j by all coastal trawler, and we denote by $E^i = (E_{i1}, E_{i2}, E_{i3}, E_{i4}, E_{i5}, E_{i6}, E_{i7})^T$ the vector fishing effort which must be provided by the coastal trawler i to catch the fish populations.

$$(3) \qquad \begin{cases} \dot{P}_J(t) = & rP_J(t)\left(1 - \frac{P_J(t)}{\kappa K}\right) - \frac{\beta P_J(t) P_A(t)}{\kappa(1-\kappa)K^2} - \frac{\mu_s P_J(t) S(t)}{\eta} - \frac{\mu_a P_J(t) A(t)}{\eta} \\ & -\frac{\mu_m P_J(t) M(t)}{\eta} - \frac{\mu_h P_J(t) H(t)}{\eta} - \frac{\mu_l P_J(t) L(t)}{\eta} \\ \dot{P}_A(t) = & rP_A(t)\left(1 - \frac{P_A(t)}{(1-\kappa)K}\right) + \frac{\beta P_J(t) P_A(t)}{\kappa(1-\kappa)K^2} - \frac{\mu_s P_A(t) S(t)}{\eta} - \frac{\mu_a P_A(t) A(t)}{\eta} \\ & -\frac{\mu_m P_A(t) M(t)}{\eta} - \frac{\mu_h P_A(t) H(t)}{\eta} - \frac{\mu_l P_A(t) L(t)}{\eta} - H_1 \\ \dot{S}(t) = & -d_s S(t) + \alpha_s S(t) P_J(t) + \alpha_s S(t) P_A(t) - H_2 \\ \dot{A}(t) = & -d_a A(t) + \alpha_a A(t) P_J(t) + \alpha_a A(t) P_A(t) - H_3 \\ \dot{M}(t) = & -d_m M(t) + \alpha_m M(t) P_J(t) + \alpha_m M(t) P_A(t) - H_4 \\ \dot{H}(t) = & -d_h H(t) + \alpha_h H(t) P_J(t) + \alpha_h H(t) P_A(t) - H_5 \\ \dot{L}(t) = & -d_l L(t) + \alpha_l L(t) P_J(t) + \alpha_l L(t) P_A(t) - H_6 \end{cases}$$

In what follows of this paper, the product, the scalar product and the division of two vectors is similar to that in Y. El foutayeni et al. [10], also the product of vector and matrix.

For system (4), we can see that the expression of biomass as a function of fishing effort is given by the matrix form $X = -AE + X^*$ (the identification of each vector and matrix is cited in annexe 3.

An algebraic equation is also included due to the consideration of the economic profit of harvesting. According to Gordon's economic theory [19]: The profit (π) = Total Revenue (TR)$-$Total Cost(TC), where the Total Revenue (TR) and Total Cost (TC) are given by

The total revenue of the coastal trawler i is $(TR)_i = \sum_{k=1}^{6} p_k H_{k1}$, where $(p_k)_{1 \leqslant k \leqslant 6}$ are the prices per unit harvested biomasses of the parapenaeus longirostris adults and the five small pelagic fish populations. After calculation, it is obvious that $(TR)_i = \left\langle E^{(i)}, -qpAE^{(i)} \right\rangle + \left\langle E^{(i)}, qpX^* - qpA \sum_{k=1, k\neq i}^{n} E^{(k)} \right\rangle$.

The total cost is equal to $(TC)_i = \left\langle c^{(i)}, E^{(i)} \right\rangle$, where $c^{(i)} = (c_{i1}, c_{i2})^T$ is the vector of the cost per unit of harvesting.

While the economic profit π_i of each coastal trawler i is equal to

$$\pi_i\left(E^i\right) = \left\langle E^{(i)}, -qpAE^{(i)} pqX^* - c^{(i)} - pqA \sum_{k=1, k\neq i}^{n} E^{(k)} \right\rangle$$

As constraint of the bioeconomic model we should have a strictly positive biomass of all the marine fish populations, in mathematical words, we must have the following inequality

$$X = -AE + X^* \geq X_0 > 0 \Leftrightarrow AE^{(i)} \leq -AE^{(j)} + X^* \text{ (for coastal trawler } i\text{)}$$

4.1. Fishing effort: Mathematical formula. To determine the mathematical expression of fishing effort that maximizes each coastal trawler profit, we use the generelized Nash equilibrium problem. By definition a Nash equilibrium exists when no coastal trawler would take a different action as long as every other coastal trawler remains the same. This problem can be translated into mathematical problems of maximization. By applying the essential conditions of Karush-Kuhn-Tucker these mathematical problems can be translated to a Linear Complementarity Problem (*LCP*). One can prove that this last problem has one and only one solution [20, 21]. So, we can deduce that the generalized Nash equilibrium problem admits one and only one solution. This equilibrium solution is given by

$$\begin{cases} E^{(1)} = \dfrac{1}{3} A^{-1} \left(X^* - \dfrac{c^{(1)}}{pq} \right) \\ E^{(2)} = \dfrac{1}{3} A^{-1} \left(X^* - \dfrac{c^{(2)}}{pq} \right) \end{cases}$$

Then, the fishing effort that maximizes the profit of the first coastal trawler for catching the adults of parapenaeus longirostris species is

$$\begin{aligned}E_{11} = \; & \frac{K^2r^2\kappa(1-\kappa)+\beta^2}{3K^3r\kappa(\kappa-1)^2q_1}\left(\frac{c_{11}}{p_1q_1}+\frac{(d_s+d_a+d_m+d_h+d_l)(\beta-Kr(\kappa-1))}{Kr(\alpha_s+\alpha_a+\alpha_m+\alpha_h+\alpha_l)}\right)\\ & +\frac{(\beta+Kr(1-\kappa))}{3K\eta r(1-\kappa)q_1}\left(\frac{\mu_sc_{12}}{p_2q_2}+\frac{\mu_ac_{13}}{p_3q_3}+\frac{\mu_mc_{14}}{p_4q_4}+\frac{\mu_hc_{15}}{p_5q_5}+\frac{\mu_lc_{16}}{p_6q_6}\right)\\ & +\frac{(\beta+Kr(1-\kappa))\eta\left(5r+\left(\beta^2-K^2r^2\kappa(\kappa-1)\right)(\mu_sds+\mu_ad_a+\mu_md_m+\mu_hd_h+\mu_ld_l)\right)}{3K\eta r(1-\kappa)q_1}\end{aligned}$$

the fishing effort that maximizes the profit of the first coastal trawler for catching the sardine species is

$$\begin{aligned}E_{12} = \; & \frac{\beta\alpha_s-Kr\alpha_s+Kr\kappa\alpha_s}{3Kr(1-\kappa)q_2}\left(\frac{c_{11}}{p_1q_1}+\frac{(d_s+d_a+d_m+d_h+d_l)\beta+Kr-Kr\kappa}{Kr(\alpha_s+\alpha_a+\alpha_m+\alpha_h+\alpha_l)}\right)\\ & +\frac{K\kappa\alpha_s}{3rnq_2}\left(\frac{\mu_sc_{12}}{p_2q_2}+\frac{\mu_ac_{13}}{p_3q_3}+\frac{\mu_mc_{14}}{p_4q_4}+\frac{\mu_hc_{15}}{p_5q_5}+\frac{\mu_lc_{16}}{p_6q_6}\right)\\ & +\frac{K\kappa\alpha_s\eta\left(5r+\left(\beta^2-K^2r^2\kappa(\kappa-1)\right)(\mu_sds+\mu_ad_a+\mu_md_m+\mu_hd_h+\mu_ld_l)\right)}{3rnq_2}\end{aligned}$$

The fishing effort that maximizes the profit of the first coastal trawler for catching the anchovy species is

$$\begin{aligned}E_{13} = \; & \frac{\beta\alpha_a-Kr\alpha_a+Kr\kappa\alpha_a}{3Kr(1-\kappa)q_3}\left(\frac{c_{11}}{p_1q_1}+\frac{(d_s+d_a+d_m+d_h+d_l)\beta+Kr-Kr\kappa}{Kr(\alpha_s+\alpha_a+\alpha_m+\alpha_h+\alpha_l)}\right)\\ & +\frac{K\kappa\alpha_a}{3rnq_3}\left(\frac{\mu_sc_{12}}{p_2q_2}+\frac{\mu_ac_{13}}{p_3q_3}+\frac{\mu_mc_{14}}{p_4q_4}+\frac{\mu_hc_{15}}{p_5q_5}+\frac{\mu_lc_{16}}{p_6q_6}\right)\\ & +\frac{K\kappa\alpha_a\eta\left(5r+\left(\beta^2-K^2r^2\kappa(\kappa-1)\right)(\mu_sds+\mu_ad_a+\mu_md_m+\mu_hd_h+\mu_ld_l)\right)}{3rnq_3}\end{aligned}$$

the fishing effort that maximizes the profit of the first coastal trawler for catching the mackerel species is

$$\begin{aligned}E_{14} = \; & \frac{\beta\alpha_m+Kr\alpha_m(\kappa-1)}{3Kr(1-\kappa)q_4}\left(\frac{c_{11}}{p_1q_1}+\frac{(d_s+d_a+d_m+d_h+d_l)(\beta+Kr-Kr\kappa)}{Kr(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5)}\right)\\ & +\frac{K\kappa\alpha_m}{3rnq_4}\left(\frac{\mu_sc_{12}}{p_2q_2}+\frac{\mu_ac_{13}}{p_3q_3}+\frac{\mu_mc_{14}}{p_4q_4}+\frac{\mu_hc_{15}}{p_5q_5}+\frac{\mu_lc_{16}}{p_6q_6}\right)\\ & +\frac{K\kappa\alpha_m\eta\left(5r+\left(\beta^2-K^2r^2\kappa(\kappa-1)\right)(\mu_sds+\mu_ad_a+\mu_md_m+\mu_hd_h+\mu_ld_l)\right)}{3rnq_4}\end{aligned}$$

the fishing effort that maximizes the profit of the first coastal trawler for catching the horse mackerel species is

$$\begin{aligned}E_{15} = \; & \frac{\beta\alpha_h-Kr\alpha_h+Kr\kappa\alpha_h}{3Kr(1-\kappa)q_5}\left(\frac{c_{11}}{p_1q_1}+\frac{(d_s+d_a+d_m+d_h+d_l)(\beta+Kr-Kr\kappa)}{Kr(\alpha_s+\alpha_a+\alpha_m+\alpha_h+\alpha_l)}\right)\\ & +\frac{K\kappa\alpha_h}{3rnq_5}\left(\frac{\mu_sc_{12}}{p_2q_2}+\frac{\mu_ac_{13}}{p_3q_3}+\frac{\mu_mc_{14}}{p_4q_4}+\frac{\mu_hc_{15}}{p_5q_5}+\frac{\mu_lc_{16}}{p_6q_6}\right)\\ & +\frac{K\kappa\alpha_h\eta\left(5r+\left(\beta^2-K^2r^2\kappa(\kappa-1)\right)(\mu_sds+\mu_ad_a+\mu_md_m+\mu_hd_h+\mu_ld_l)\right)}{3rnq_5}\end{aligned}$$

the fishing effort that maximizes the profit of the first coastal trawler for catching sardinella species is

$$\begin{aligned}E_{16} = \; & \frac{\beta\alpha_l-Kr\alpha_l+Kr\kappa\alpha_l}{3Kr(1-\kappa)q_6}\left(\frac{c_{11}}{p_1q_1}+\frac{(d_s+d_a+d_m+d_h+d_l)(\beta+Kr-Kr\kappa)}{Kr(\alpha_s+\alpha_a+\alpha_m+\alpha_h+\alpha_l)}\right)\\ & +\frac{K\kappa\alpha_l}{3rnq_6}\left(\frac{\mu_sc_{12}}{p_2q_2}+\frac{\mu_ac_{13}}{p_3q_3}+\frac{\mu_mc_{14}}{p_4q_4}+\frac{\mu_hc_{15}}{p_5q_5}+\frac{\mu_lc_{16}}{p_6q_6}\right)\\ & +\frac{K\kappa\alpha_l\eta\left(5r+\left(\beta^2-K^2r^2\kappa(\kappa-1)\right)(\mu_sds+\mu_ad_a+\mu_md_m+\mu_hd_h+\mu_ld_l)\right)}{3rnq_6}\end{aligned}$$

the fishing effort that maximizes the profit of the seconde coastal trawler for catching the adults of parapenaeus longirostris species is

$$
\begin{aligned}
E_{21} = \; & \frac{K^2r^2\kappa(1-\kappa)+\beta^2}{3K^3r\kappa(\kappa-1)^2q_1}\left(\frac{c_{21}}{p_1q_1}+\frac{(d_s+d_a+d_m+d_h+d_l)(\beta+Kr-Kr\kappa)}{Kr(\alpha_s+\alpha_a+\alpha_m+\alpha_h+\alpha_l)}\right)\\
&+\frac{(\beta+Kr(1-\kappa))}{3K\eta r(1-\kappa)q_1}\left(\frac{\mu_sc_{22}}{p_2q_2}+\frac{\mu_ac_{23}}{p_3q_3}+\frac{\mu_mc_{24}}{p_4q_4}+\frac{\mu_hc_{25}}{p_5q_5}+\frac{\mu_lc_{26}}{p_6q_6}\right)\\
&+\frac{(\beta+Kr(1-\kappa))\eta\left(5r+\left(\beta^2-K^2r^2\kappa(\kappa-1)\right)(\mu_sds+\mu_ad_a+\mu_md_m+\mu_hd_h+\mu_ld_l)\right)}{3K\eta r(1-\kappa)q_1}
\end{aligned}
$$

the fishing effort that maximizes the profit of the seconde coastal trawler for catching the sardine species is

$$
\begin{aligned}
E_{22} = \; & \frac{\beta\alpha_s-Kr\alpha_s+Kr\kappa\alpha_s}{3Kr(1-\kappa)q_2}\left(\frac{c_{21}}{p_1q_1}+\frac{(d_s+d_a+d_m+d_h+d_l)(\beta+Kr-Kr\kappa)}{Kr(\alpha_s+\alpha_a+\alpha_m+\alpha_h+\alpha_l)}\right)\\
&+\frac{K\kappa\alpha_s}{3rnq_2}\left(\frac{\mu_sc_{22}}{p_2q_2}+\frac{\mu_ac_{23}}{p_3q_3}+\frac{\mu_mc_{24}}{p_4q_4}+\frac{\mu_hc_{25}}{p_5q_5}+\frac{\mu_lc_{26}}{p_6q_6}\right)\\
&+\frac{K\kappa\alpha_s\eta\left(5r+\left(\beta^2-K^2r^2\kappa(\kappa-1)\right)(\mu_sds+\mu_ad_a+\mu_md_m+\mu_hd_h+\mu_ld_l)\right)}{3rnq_2}
\end{aligned}
$$

the fishing effort that maximizes the profit of the seconde coastal trawler for catching the anchovy species is

$$
\begin{aligned}
E_{23} = \; & \frac{\beta\alpha_a-Kr\alpha_a+Kr\kappa\alpha_a}{3Kr(1-\kappa)q_3}\left(\frac{c_{21}}{p_1q_1}+\frac{(d_s+d_a+d_m+d_h+d_l)(\beta+Kr-Kr\kappa)}{Kr(\alpha_s+\alpha_a+\alpha_m+\alpha_h+\alpha_l)}\right)\\
&+\frac{K\kappa\alpha_a}{3rnq_3}\left(\frac{\mu_sc_{22}}{p_2q_2}+\frac{\mu_ac_{23}}{p_3q_3}+\frac{\mu_mc_{24}}{p_4q_4}+\frac{\mu_hc_{25}}{p_5q_5}+\frac{\mu_lc_{26}}{p_6q_6}\right)\\
&+\frac{K\kappa\alpha_a\eta\left(5r+\left(\beta^2-K^2r^2\kappa(\kappa-1)\right)(\mu_sds+\mu_ad_a+\mu_md_m+\mu_hd_h+\mu_ld_l)\right)}{3rnq_3}
\end{aligned}
$$

the fishing effort that maximizes the profit of the seconde coastal trawler for catching the mackerel species is

$$
\begin{aligned}
E_{24} = \; & \frac{\beta\alpha_m-Kr\alpha_m+Kr\kappa\alpha_m}{3Kr(1-\kappa)q_4}\left(\frac{c_{21}}{p_1q_1}+\frac{(d_s+d_a+d_m+d_h+d_l)(\beta+Kr-Kr\kappa)}{Kr(\alpha_s+\alpha_a+\alpha_m+\alpha_h+\alpha_l)}\right)\\
&+\frac{K\kappa\alpha_m}{3rnq_4}\left(\frac{\mu_sc_{22}}{p_2q_2}+\frac{\mu_ac_{23}}{p_3q_3}+\frac{\mu_mc_{24}}{p_4q_4}+\frac{\mu_hc_{25}}{p_5q_5}+\frac{\mu_lc_{26}}{p_6q_6}\right)\\
&+\frac{K\kappa\alpha_m\eta\left(5r+\left(\beta^2-K^2r^2\kappa(\kappa-1)\right)(\mu_sds+\mu_ad_a+\mu_md_m+\mu_hd_h+\mu_ld_l)\right)}{3rnq_4}
\end{aligned}
$$

the fishing effort that maximizes the profit of the seconde coastal trawler for catching the horse mackerel species is

$$
\begin{aligned}
E_{25} = \; & \frac{\beta\alpha_h-Kr\alpha_h+Kr\kappa\alpha_h}{3Kr(1-\kappa)q_5}\left(\frac{c_{21}}{p_1q_1}+\frac{(d_s+d_a+d_m+d_h+d_l)(\beta+Kr-Kr\kappa)}{Kr(\alpha_s+\alpha_a+\alpha_m+\alpha_h+\alpha_l)}\right)\\
&+\left(\frac{\mu_sc_{22}}{p_2q_2}+\frac{\mu_ac_{23}}{p_3q_3}+\frac{\mu_mc_{24}}{p_4q_4}+\frac{\mu_hc_{25}}{p_5q_5}+\frac{\mu_lc_{26}}{p_6q_6}\right)\\
&+\frac{K\kappa\alpha_h\eta\left(5r+\left(\beta^2-K^2r^2\kappa(\kappa-1)\right)(\mu_sds+\mu_ad_a+\mu_md_m+\mu_hd_h+\mu_ld_l)\right)}{3rnq_5}
\end{aligned}
$$

the fishing effort that maximizes the profit of the seconde coastal trawler for catching the sardinella species is

$$
\begin{aligned}
E_{26} = \; & \frac{\beta\alpha_l-Kr\alpha_l+Kr\kappa\alpha_l}{3Kr(1-\kappa)q_6}\left(\frac{c_{21}}{p_1q_1}+\frac{(d_s+d_a+d_m+d_h+d_l)(\beta+Kr-Kr\kappa)}{Kr(\alpha_s+\alpha_a+\alpha_m+\alpha_h+\alpha_l)}\right)\\
&+\frac{K\kappa\alpha_l}{3rnq_6}\left(\frac{\mu_sc_{22}}{p_2q_2}+\frac{\mu_ac_{23}}{p_3q_3}+\frac{\mu_mc_{24}}{p_4q_4}+\frac{\mu_hc_{25}}{p_5q_5}+\frac{\mu_lc_{26}}{p_6q_6}\right)\\
&+\frac{K\kappa\alpha_l\eta\left(5r+\left(\beta^2-K^2r^2\kappa(\kappa-1)\right)(\mu_sds+\mu_ad_a+\mu_md_m+\mu_hd_h+\mu_ld_l)\right)}{3rnq_6}
\end{aligned}
$$

5. Numerical Simulations and Discussion of the Results

As can be seen from the figure 2, the mortality coefficient increase of small pelagics results an evolution in the stock of the parapenaeus longirostris population. Witch is justified by the absence of predators that feed on parapenaeus longirostris fish. And therefore, the level of small pelagic population stocks decrease figure 3.

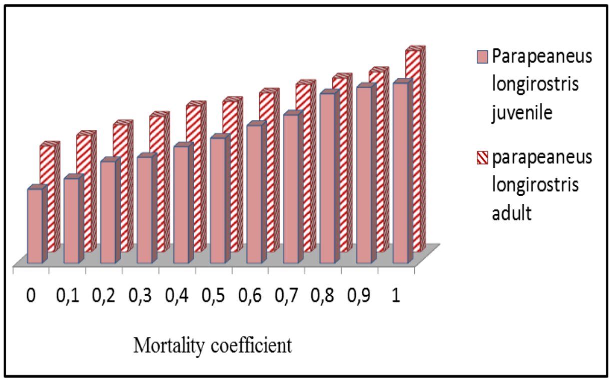

FIGURE 2. The influence of mortality coefficient on the arapenaeus longirostris population stocks

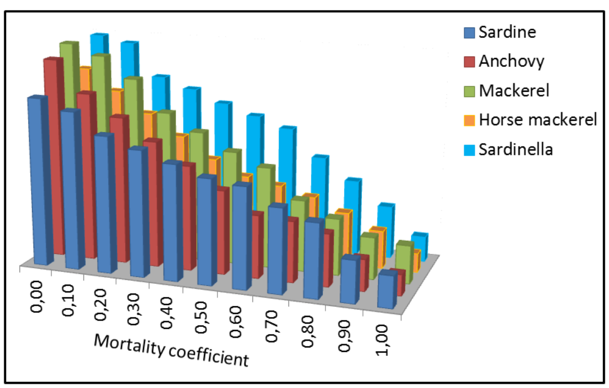

FIGURE 3. The influence of mortality coefficient on the small pelagic population stocks

In this situation, according to figure 4, the number of fishing trips, that must be made by coastal trawlers to harvest parapenaeus longirostris, increase. This increase enable them to make more catches, which allows them to get high economic returns taking into consideration the marine resources conservation.

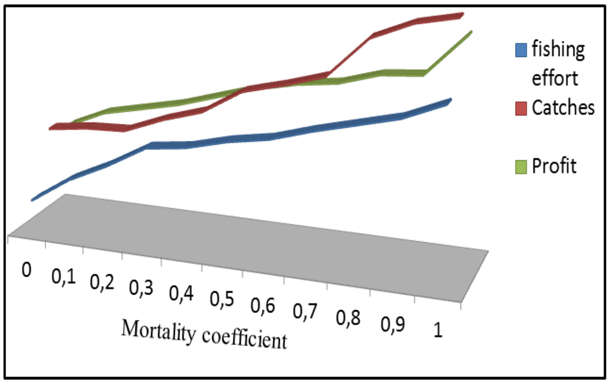

FIGURE 4. The influence of mortality coefficient on the fishing trips

However, when the biomass of small pelagic fish decreases, the number of fishing trips dedicated to these species decreases, because the carrying capacity of each small pelagic species does not contain the enough biomass that allows the coastal trawlers to catch a greater amount of fish, note that each coastal trawler is constrained by the sustainability of marine species. In this situation, coastal trawlers do not have the opportunity to catch more small pelagic fish, and as a result, their profit related to the exploitation of these resources decreases, as shown in figure 5.

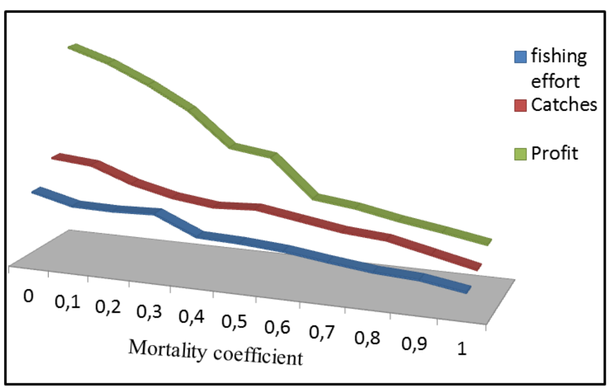

FIGURE 5. The influence of mortality coefficient on profit

If the mortality rate is sufficiently high, which means the almost total absence of small pelagics, then the coastal trawlers will be forced to exploit only parapenaeus longirostris. In this situation, their catch level is equal to 300 tons and the profit is equal to 34500000.

In the opposite situation, the mortality rate approaches zero, which means a total abandonment of small pelagics, and which leads to a very high predation of parapenaeus longirostris. In this situation, the profit of coastal trawlers is equal to 1332000 after the exploitation of 61000 tons of small pelagics.

But whereas these two situations do not ensure the sustainability and abundance of stocks of all the marine populations considered in this work.

However, if we consider the normal mortality rate, assumed equal to 0.27, 0.19, 0.21, 0.17, 0.37 for sardine, anchovy, mackerel, horse mackerel, sardinella, respectively. In this case the abundance of prey and predator stocks is ensured. Coastal trawlers capture 890 tons of small pelagic stocks and 552 tons of parapenaeus longirostris stock, to have the maximum economic return equal to 17629200 for small pelagics and 92316000 for parapenaeus longirostris. We can notice that their total profit equal to 109945200 higher than 34500000 and 1332000 cited in the other situations.

CONCLUSION

In this paper, we presents a contribution to the modeling of parapenaeus longirostris fishing on Moroccan coasts. The modeling is based on the knowledge and available data on its dynamics and its harvest. We study the interaction between parapenaeus longirostris and small pelagic fish on two different spatial zones connected by migration. For that, we propose to define a bioeconomic model of prey-predator (parapenaeus longirostris-small pelagic fish) on two patch with homogeneous environments. One of these patches is considered to be a fishing protected area and the remaining adjacent patch is a free access fishing area. We first outline the basic theoretical model describing the biological dynamics of marine species stocks and then fisheries is introduced to the system.

ANNEX 1: EQUILIBRIUM POINTS

The equilibrium points are:

$P_1\left(0,0,0,0,0,0,0\right)$, $P_2\left(K\kappa,0,0,0,0,0,0\right)$, $P_3\left(0,K\left(1-\kappa\right),0,0,0,0,0\right)$,

$$P_4\left(\frac{Krd_m\left(\kappa-1\right)}{\beta\alpha_m},0,\frac{r\eta\left(K\kappa\alpha_m-d_m\right)}{K\kappa\alpha_m\mu_s},0,0,0,0\right),$$

$P_5\left(\dfrac{Krd_a\left(\kappa-1\right)}{\beta\alpha_a},0,0,\dfrac{r\eta\left(K\kappa\alpha_a-d_a\right)}{K\kappa\alpha_a\mu_a},0,0,0\right)$,
$P_6\left(\dfrac{d_m-2K\kappa\alpha_m}{\alpha_m},0,0,0,\dfrac{r\eta\left(K\kappa\alpha_m-d_m\right)}{K\kappa\alpha_m\mu_m},0,0\right)$,
$P_7\left(P_{71},P_{72},0,0,0,0,0\right)$, with

$$\begin{cases} P_{71}=\dfrac{Kr\kappa\left(d_h+K\alpha_h\left(\kappa-1\right)\right)}{\alpha_h\left(\beta+Kr\kappa\right)} \\ P_{72}=\dfrac{Kr\left(d_a+K\kappa\alpha_a\right)\left(\kappa-1\right)}{\alpha_a\left(\beta+Kr\left(\kappa-1\right)\right)} \end{cases}$$

$P_8\left(P_{81},0,0,0,0,P_{82},0\right)$, with

$$\begin{cases} P_{81}=\dfrac{2K^2r\kappa\eta\alpha_h\left(\kappa-1\right)-\beta\mu_hd_h}{Kr\eta\alpha_h\left(\kappa-1\right)} \\ P_{82}=\dfrac{\beta\mu_hd_h-K^2r\kappa\eta\alpha_h\left(\kappa-1\right)}{K^2\kappa\alpha_h\mu_h\left(\kappa-1\right)} \end{cases}$$

$P_9\left(P_{91},0,0,0,0,0,P_{92}\right)$, with

$$\begin{cases} P_{91}=\dfrac{Kr\mu_ld_l\left(\kappa-1\right)}{\beta\alpha_l\mu_l} \\ P_{92}=\dfrac{r\eta\left(-\mu_ld_l+K\kappa\alpha_l\mu_l\right)}{K\kappa\mu_l\alpha_l} \end{cases}$$

$P_{10}\left(0,P_{101},P_{102},0,0,0,0\right)$, with

$$\begin{cases} P_{101}=\dfrac{\mu_s^2d_s-K\alpha_s\left(\eta^2-\mu_s^2\right)\left(\kappa-1\right)}{\eta^2\alpha_s} \\ P_{102}=\dfrac{r\mu_s\left(d_s+K\alpha_s\left(\kappa-1\right)\right)}{K\eta\alpha_s\left(\kappa-1\right)} \end{cases}$$

$P_{11}\left(0,P_{111},0,P_{112},0,0,0\right)$, with

$$\begin{cases} P_{111}=\dfrac{\beta\mu_a^2d_a+K^2r\kappa\alpha_a\left(\mu_a-1\right)\left(\mu_a+1\right)\left(\kappa-1\right)}{Kr\kappa\alpha_a} \\ P_{112}=\dfrac{\eta\mu_a\left(\beta d_a+K^2r\kappa\alpha_a\left(\kappa-1\right)\right)}{K^2\kappa\alpha_a\left(\kappa-1\right)} \end{cases}$$

$P_{12}\left(0,P_{121},0,0,P_{122},0,0\right)$, with

$$\begin{cases} P_{121}=\dfrac{d_m+2K\alpha_m\left(\kappa-1\right)}{\alpha_m} \\ P_{122}=r\eta\dfrac{d_m+K\alpha_m\left(\kappa-1\right)}{K\alpha_m\mu_m\left(\kappa-1\right)} \end{cases}$$

$P_{13}\left(0,P_{131},0,0,0,P_{132},0\right)$, with

$$\begin{cases} P_{131} = Kr\kappa \dfrac{d_h}{\beta \alpha_h} \\ P_{132} = r\eta \dfrac{d_h + K\alpha_h(\kappa - 1)}{K\alpha_h \mu_h(\kappa - 1)} \end{cases}$$

$P_{14}(0, P_{141}, 0, 0, 0, 0, P_{141})$, with

$$\begin{cases} P_{141} = \dfrac{\beta d_l - 2K^2 r\kappa\alpha_l(\kappa - 1)}{Kr\kappa\alpha_l} \\ P_{142} = \eta \dfrac{\beta d_l - K^2 r\kappa\alpha_l(\kappa - 1)}{K^2\kappa\alpha_l\mu_l(\kappa - 1)} \end{cases}$$

$P_{15}(P_{151}, P_{152}, P_{153}, 0, 0, 0, 0)$, with

$$\begin{cases} P_{151} = \dfrac{Kr\kappa(K\alpha_s(\kappa - 1)(S - 1) + Sd_s) - \beta d_s}{\beta\alpha_s(S - 1) + Kr\alpha_s(-\kappa + S\kappa + 1)} \\ P_{152} = \dfrac{S\beta d_s - Kr(\kappa - 1)(d_s + K\kappa\alpha_s(S - 1))}{\beta\alpha_s(S - 1) + Kr\alpha_s(-\kappa + S\kappa + 1)} \\ P_{153} = \dfrac{K^2 r^2 \kappa\eta(\kappa - 1)(d_s - K\alpha_s) - \beta^2 \eta d_s}{K^2\kappa\alpha_s\mu_s(\kappa - 1)(\beta - S\beta + Kr(-\kappa(S - 1) - 1))} \end{cases}$$

$P_{16}(P_{161}, P_{162}, 0, P_{163}, 0, 0, 0)$, with

$$\begin{cases} P_{161} = \dfrac{(Kr\kappa - \beta) d_a}{Kr\alpha_a} \\ P_{162} = \dfrac{(\beta - Kr(\kappa - 1)) d_a}{Kr\alpha_a} \\ P_{163} = \dfrac{K^2 r^2 \kappa\eta(\kappa - 1)(K\alpha_a - d_a) - \beta^2\eta d_a}{-K^3 r\kappa\alpha_a\mu_a(\kappa - 1)} \end{cases}$$

$P_{17}(P_{171}, P_{172}, 0, 0, P_{173}, 0, 0)$, with

$$\begin{cases} P_{171} = \dfrac{\beta\mu_a d_m + Kr\kappa(K\alpha_m(\mu_a - \mu_m)(\kappa - 1) - \mu_m d_m)}{\beta\alpha_m(\mu_a - \mu_m) - Kr\alpha_m(\mu_a - \kappa\mu_a + \kappa\mu_m)} \\ P_{172} = \dfrac{\beta\mu_m d_m + Kr(\kappa - 1)(K\kappa\alpha_m(\mu_a - \mu_m) - \mu_a d_m)}{Kr\alpha_m(\mu_a - \kappa\mu_a + \kappa\mu_m) - \beta\alpha_m(\mu_a - \mu_m)} \\ P_{173} = \dfrac{+K^2 r^2\kappa\eta(\kappa - 1)(d_m - K\alpha_m) - \beta^2\eta d_m}{K^2\kappa\alpha_m(\kappa - 1)(\beta\mu_a - \beta\mu_m + Kr(-\mu_a + \kappa\mu_a - \kappa\mu_m))} \end{cases}$$

$P_{18}(P_{181}, P_{182}, 0, 0, 0, P_{183}, 0)$, with

$$\begin{cases} P_{181} = \dfrac{(Kr\kappa - \beta)\, d_h}{Kr\alpha_h} \\ P_{182} = \dfrac{(\beta + Kr(1-\kappa))\, d_h}{Kr\alpha_h} \\ P_{183} = \dfrac{K^2 r^2 \kappa\eta\,(\kappa-1)\,(K\alpha_h - d_h) - \beta^2\eta d_h}{K^3 r\kappa\alpha_h\mu_h\,(1-\kappa)} \end{cases}$$

$P_{19}\,(P_{191}, P_{192}, 0, 0, 0, 0, P_{193})$, with

$$\begin{cases} P_{191} = \dfrac{d_l\,(Kr\kappa - \beta)}{Kr\alpha_l} \\ P_{192} = \dfrac{(\beta - Kr(\kappa-1))\, d_l}{Kr\alpha_l} \\ P_{193} = \dfrac{K^2 r^2 \kappa\eta\,(\kappa-1)\,(-d_l + K\alpha_l) - \beta^2\eta d_l}{K^3 r\kappa\alpha_l\mu_l\,(1-\kappa)} \end{cases}$$

$P_{20}\,(P_{201}, 0, P_{202}, A, 0, 0, 0)$, with

$$\begin{cases} P_{201} = \dfrac{d_s}{\alpha_s} \\ P_{202} = \dfrac{K\kappa\alpha_s\,(A\mu_a - r\eta) - r\eta d_s}{K\kappa\alpha_s\mu_s} \end{cases}$$

$P_{21}\,(P_{211}, 0, P_{212}, 0, 0, H, 0)$, with

$$\begin{cases} P_{211} = \dfrac{d_h}{\alpha_h} \\ P_{212} = \dfrac{K\kappa\alpha_h\,(r\eta - H\mu_h) - r\eta d_h}{K\kappa\alpha_h\mu_h} \end{cases}$$

$P_{22}\,(P_{221}, 0, 0, P_{222}, M, 0, 0)$, with

$$\begin{cases} P_{221} = \dfrac{d_m}{\alpha_m} \\ P_{222} = \dfrac{\beta\eta\,(d_m - K\kappa\alpha_m) + K^2 M\kappa\alpha_m\mu_m\,(\kappa-1)}{K^2\alpha_m\mu_a\kappa\,(1-\kappa)} \end{cases}$$

$P_{23}\,(P_{231}, 0, 0, P_{232}, 0, H, 0)$, with

$$\begin{cases} P_{231} = \dfrac{d_h}{\alpha_h} \\ P_{232} = \dfrac{r\eta d_h + K\kappa\alpha_h\,(H\mu_h - \eta r)}{K\kappa\alpha_h\mu_a} \end{cases}$$

$P_{24}\,(P_{241}, 0, 0, P_{242}, 0, 0, L)$, with

$$\begin{cases} P_{241} = \dfrac{d_l}{\alpha_l} \\ P_{242} = \dfrac{K\kappa\alpha_l\left(\eta r - L\mu_l\right) - r\eta d_l}{K\kappa\alpha_l\mu_a} \end{cases}$$

$P_{25}\left(P_{251},0,0,0,P_{252},H,0\right)$, with

$$\begin{cases} P_{251} = \dfrac{d_m}{\alpha_m} \\ P_{252} = \dfrac{K\kappa\alpha_m\left(r\eta - H\mu_h\right) - r\eta d_m}{K\kappa\alpha_m\mu_m} \end{cases}$$

$P_{26}\left(P_{261},0,0,0,P_{262},0,L\right)$, with

$$\begin{cases} P_{261} = \dfrac{d_l}{\alpha_l} \\ P_{262} = \dfrac{\left(r\eta - L\mu_l\right)K\kappa\alpha_l - r\eta d_l}{K\kappa\alpha_l\mu_m} \end{cases}$$

$P_{27}\left(P_{271},0,P_{272},A,M,0,0\right)$, with

$$\begin{cases} P_{271} = \dfrac{d_a}{\alpha_a} \\ P_{272} = \dfrac{\beta\eta\left(d_a - K\kappa\alpha_a\right) - K^2\kappa\alpha_a\left(A\mu_a + \mu_m M\right)(1-\kappa)}{K^2\kappa\alpha_a\mu_s(1-\kappa)} \end{cases}$$

$P_{28}\left(P_{281},0,P_{282},A,0,H,0\right)$, with

$$\begin{cases} P_{281} = \dfrac{d_s}{\alpha_s} \\ P_{282} = \dfrac{K\kappa\alpha_s\left(r\eta - A\mu_a - \mu_h H\right) - r\eta d_s}{K\kappa\alpha_s\mu_s} \end{cases}$$

P_{29} $\left(P_{291},0,,A,0,0,L\right)$, with

$$\begin{cases} P_{291} = \dfrac{d_l}{\alpha_l} \\ P_{292} = \dfrac{K\kappa\alpha_l\left(r\eta - A\mu_a - \mu_l L\right) - r\eta d_l}{K\kappa\alpha_l\mu_s} \end{cases}$$

$P_{30}\left(P_{301},0,P_{302},0,M,H,0\right)$, with

$$\begin{cases} P_{301} = \dfrac{d_s}{\alpha_s} \\ P_{302} = \dfrac{K\alpha_s\kappa\left(r\eta - M\mu_m - \mu_h H\right) - r\eta d_s}{K\kappa\alpha_s\mu_s} \end{cases}$$

$P_{31}\left(P_{311},0,P_{312},0,M,0,L\right)$, with

$$\begin{cases} P_{311} = \dfrac{d_l}{\alpha_l} \\ P_{312} = \dfrac{K\kappa\alpha_l\left(\eta r - M\mu_m - \mu_l L\right) - r\eta d_l}{K\kappa\alpha_l\mu_s} \end{cases}$$

$P_{32}\left(P_{321}, 0, P_{322}, 0, 0, 0, L\right)$, with

$$\begin{cases} P_{321} = \dfrac{d_l}{\alpha_l} \\ P_{322} = \dfrac{K\kappa\alpha_l\left(r\eta - L\mu_l\right) - r\eta d_l}{K\kappa\alpha_l\mu_l} \end{cases}$$

$P_{33}\left(P_{331}, 0, 0, P_{332}, M, H, 0\right)$, with

$$\begin{cases} P_{331} = \dfrac{d_a}{\alpha_a} \\ P_{332} = \dfrac{K\kappa\alpha_a\left(r\eta - M\mu_m - \mu_h H\right) - r\eta d_a}{K\kappa\alpha_a\mu_a} \end{cases}$$

$P_{34}\left(P_{341}, 0, 0, P_{342}, M, 0, L\right)$, with

$$\begin{cases} P_{341} = \dfrac{d_a}{\alpha_a} \\ P_{342} = \dfrac{K\kappa\alpha_a\left(r\eta - M\mu_m - \mu_l L\right) - r\eta d_a}{K\kappa\alpha_a\mu_a} \end{cases}$$

$P_{35}\left(P_{351}, 0, 0, P_{352}, 0, H, L\right)$, with

$$\begin{cases} P_{351} = \dfrac{d_a}{\alpha_a} \\ P_{352} = \dfrac{K\kappa\alpha_l\left(r\eta - H\mu_h - \mu_l L\right) - r\eta d_l}{K\kappa\alpha_a\mu_a} \end{cases}$$

$p_{36}\left(\dfrac{d_1}{\alpha_1}, 0, P_{362}, A, M, 0, 0\right)$, with

$$\begin{cases} P_{361} = \dfrac{d_s}{\alpha_s} \\ P_{362} = \dfrac{K\kappa\alpha_s\left(r\eta - A\mu_a - \mu_m M\right) - r\eta d_s}{K\kappa\alpha_s\mu_a} \end{cases}$$

$P_{37}\left(P_{371}, 0, P_{372}, A, 0, H, 0\right)$, with

$$\begin{cases} P_{371} = \dfrac{d_a}{\alpha_a} \\ P_{372} = \dfrac{K\kappa\alpha_s\left(r\eta - A\mu_a - \mu_h H\right) - r\eta d_a}{K\kappa\alpha_a\mu_a} \end{cases}$$

$P_{38}\left(P_{381}, 0, P_{382}, A, 0, 0, L\right)$, with

$$\begin{cases} P_{381} = \dfrac{d_l}{\alpha_l} \\ P_{382} = \dfrac{K\kappa\alpha_l\left(r\eta - A\mu_a - \mu_l L\right) - r\eta d_l}{K\kappa\alpha_l\mu_a} \end{cases}$$

$P_{39}\left(0, P_{391}, P_{392}, 0, M, H, 0\right)$, with

$$\begin{cases} P_{391} = \dfrac{d_s}{\alpha_s} \\ P_{392} = \dfrac{K\alpha_s\left(\kappa - 1\right)\left(r\eta + \mu_m H + \mu_s M\right) - r\eta d_s}{K\alpha_s\mu_h\left(1-\kappa\right)} \end{cases}$$

$P_{40}\left(0, P_{401}, P_{402}, 0, M, 0, L\right)$, with

$$\begin{cases} P_{401} = \dfrac{d_l}{\alpha_l} \\ P_{402} = \dfrac{K\alpha_l\left(\kappa - 1\right)\left(r\eta + \mu_m L + \mu_s M\right) - r\eta d_l}{K\alpha_l\mu_l\left(1-\kappa\right)} \end{cases}$$

$P_{41}\ \left(0, P_{411}, P_{412}, 0, 0, H, L\right)$, with

$$\begin{cases} P_{411} = \dfrac{d_h}{\alpha_h} \\ P_{412} = \dfrac{K\alpha_h\left(\kappa - 1\right)\left(r\eta + \mu_h L + \mu_s H\right) - r\eta d_h}{K\alpha_h\mu_l\left(1-\kappa\right)} \end{cases}$$

$P_{42}\left(0, 0, \dfrac{r\eta - A\mu_a - M\mu_m - \mu_h H}{\mu_s}, A, M, H, 0\right)$,
$P_{43}\left(0, 0, \dfrac{r\eta - A\mu_a - M\mu_m - \mu_l L}{\mu_s}, A, M, 0, L\right)$,
$P_{44}\left(0, 0, \dfrac{r\eta - A\mu_a - H\mu_h - \mu_l L}{\mu_s}, A, 0, H, L\right)$,
$P_{45}\left(0, 0, 0, \dfrac{r\eta - M\mu_h - H\mu_m - \mu_l L}{\mu_a}, M, H, L\right)$,
$P_{46}\left(0, 0, \dfrac{r\eta - A\mu_a - M\mu_m - \mu_h H - \mu_l L}{\mu_s}, A, M, H, L\right)$,
$P_{47}\left(0, \dfrac{(1-\kappa)\left[K\beta\eta - K^2\kappa\left(\mu_h H + \mu_l L + A\mu_a + M\mu_m\right)\right]}{\beta\eta}, 0, A, M, H, L\right)$,
$P_{48}\left(0, P_{481}, S, 0, P_{482}, H, L\right)$, with

$$\begin{cases} P_{481} = \dfrac{d_s}{\alpha_s} \\ P_{482} = \dfrac{K\alpha_s\left(\kappa - 1\right)\left(r\eta + \mu_h H + \mu_l l\right) - r\eta d_s}{K\alpha_s\mu_m\left(1-\kappa\right)} \end{cases}$$

$P_{49}\left(0, P_{491}, P_{492}, A, 0, H, L\right)$, with

$$\begin{cases} P_{491} = \dfrac{d_m}{\alpha_m} \\ P_{492} = \dfrac{K\alpha_m(\kappa-1)(r\eta+\mu_a A+\mu_l l)-r\eta d_m}{K\alpha_m\mu_s(1-\kappa)} \end{cases}$$

$P_{50}\ (0, P_{501}, P_{502}, A, M, 0, L)$, with

$$\begin{cases} P_{501} = \dfrac{d_m}{\alpha_m} \\ P_{502} = \dfrac{K\alpha_m(\kappa-1)(r\eta+\mu_m M+\mu_l l)-r\eta d_m}{K\alpha_m\mu_s(1-\kappa)} \end{cases}$$

$P_{51}\,(0, P_{511}, P_{512}, A, M, H, L)$, with

$$\begin{cases} P_{511} = \dfrac{d_l}{\alpha_l} \\ P_{512} = \dfrac{K\alpha_l(\kappa-1)(r\eta+\mu_m M+\mu_a A)-r\eta d_l}{K\alpha_l\mu_s(1-\kappa)} \end{cases}$$

$P_{52}\,(P_{521}, 0, P_{522}, A, 0, H, L)$, with

$$\begin{cases} P_{521} = \dfrac{K^2\kappa\mu_h(L-H)}{\eta(\beta+Kr)} \\ P_{522} = \dfrac{\beta(r\eta-A\mu_m-\mu_h H-\mu_l L)+Kr(\kappa-1)(r\eta-A\mu_m)}{\mu_a(\beta+Kr(1-\kappa))} \end{cases}$$

$P_{53}\,(P_{531}, 0, P_{532}, 0, M, H, L)$, with

$$\begin{cases} P_{531} = \dfrac{d_m}{\alpha_m} \\ P_{532} = \dfrac{K\kappa\alpha_m(r\eta-M\mu_m-\mu_h H-\mu_l L)-r\eta d_m}{K\alpha_m\mu_s\kappa} \end{cases}$$

$P_{54}\,(P_{541}, 0, P_{542}, A, M, 0, L)$, with

$$\begin{cases} P_{541} = \dfrac{d_h}{\alpha_h} \\ P_{542} = \dfrac{K\kappa\alpha_h(r\eta-M\mu_m-\mu_a A-\mu_l L)-r\eta d_h}{K\alpha_h\mu_s\kappa} \end{cases}$$

$P_{55}\left(\dfrac{d_s}{\alpha_s}, 0, -\dfrac{A\mu_a+\mu_h H+\mu_m M}{\mu_s}, A, M, H, 0\right)$,
$P_{56}\,(P_{561}, P_{562}, 0, 0, M, H, P_{563})$, with

$$\begin{cases} P_{561} = \dfrac{d_m\left(Kr\kappa - \beta\right)}{Kr\alpha_m} \\ P_{562} = \dfrac{\beta d_m + Krd_m\left(1-\kappa\right)}{Kr\alpha_m} \\ P_{563} = \dfrac{\left(\kappa-1\right)K^2r^2\kappa\eta\left(d_m+\alpha_m\right) - \beta^2\eta d_m}{K^3r\kappa\alpha_m\mu_l\left(1-\kappa\right)} \end{cases}$$

$P_{57}\left(P_{571}, P_{572}, 0, P_{573}, 0, H, L\right)$ with

$$\begin{cases} P_{571} = \dfrac{d_m\left(Kr\kappa - \beta\right)}{Kr\alpha_m} \\ P_{572} = \dfrac{\beta d_m + Krd_m\left(1-\kappa\right)}{Kr\alpha_m} \\ P_{573} = \dfrac{K^2r\kappa\left(\kappa-1\right)\left(r\eta d_m - K\alpha_m\left(r\eta+\mu_hH+\mu_lL\right)\right) - \beta^2\eta d_m}{K^3r\kappa\alpha_m\mu_a\left(1-\kappa\right)} \end{cases}$$

$P_{58}\left(P_{581}, P_{582}, 0, P_{583}, M, 0, L\right)$, with

$$\begin{cases} P_{581} = \dfrac{d_m\left(Kr\kappa - \beta\right)}{Kr\alpha_m} \\ P_{582} = \dfrac{\beta d_m + Krd_m\left(1-\kappa\right)}{Kr\alpha_m} \\ P_{583} = \dfrac{K^2r\kappa\left(\kappa-1\right)\left(r\eta d_m - K\alpha_m\left(r\eta+\mu_mM+\mu_lL\right)\right) - \beta^2\eta d_m}{K^3r\kappa\alpha_m\mu_a\left(1-\kappa\right)} \end{cases}$$

$P_{59}\left(P_{591}, P_{592}, 0, P_{593}, M, H, 0\right)$, with

$$\begin{cases} P_{591} = \dfrac{d_a\left(Kr\kappa - \beta\right)}{Kr\alpha_a} \\ P_{592} = \dfrac{\beta d_a + Krd_a\left(1-\kappa\right)}{Kr\alpha_a} \\ P_{593} = \dfrac{K^2r\kappa\left(\kappa-1\right)\left(r\eta d_a - K\alpha_a\left(r\eta+\mu_mM+\mu_hH\right)\right) - \beta^2\eta d_a}{K^3r\kappa\alpha_a\mu_a\left(1-\kappa\right)} \end{cases}$$

$P_{60}\left(P_{601}, P_{602}, P_{603}, A, 0, H, 0\right)$, with

$$\begin{cases} P_{601} = \dfrac{d_s\left(Kr\kappa - \beta\right)}{Kr\alpha_s} \\ P_{602} = \dfrac{\beta d_s + Krd_s\left(1-\kappa\right)}{Kr\alpha_s} \\ P_{603} = \dfrac{K^2r\kappa\left(\kappa-1\right)\left(r\eta d_s - K\alpha_s\left(r\eta+\mu_aA+\mu_hH\right)\right) - \beta^2\eta d_s}{K^3r\kappa\alpha_s\mu_a\left(1-\kappa\right)} \end{cases}$$

$P_{61}\left(P_{611}, P_{612}, P_{613}, A, 0, 0, L\right)$ with

$$\begin{cases} P_{611} = \dfrac{d_a\left(Kr\kappa-\beta\right)}{Kr\alpha_a} \\ P_{612} = \dfrac{\beta d_a + Krd_a\left(1-\kappa\right)}{Kr\alpha_a} \\ P_{613} = \dfrac{K^2r\kappa\left(\kappa-1\right)\left(r\eta d_a - K\alpha_a\left(r\eta+\mu_a A+\mu_l L\right)\right)-\beta^2\eta d_a}{K^3r\kappa\alpha_a\mu_s\left(1-\kappa\right)} \end{cases}$$

$P_{62}\left(P_{621},P_{622},P_{623},A,M,0,0\right)$, with

$$\begin{cases} P_{621} = \dfrac{d_m\left(Kr\kappa-\beta\right)}{Kr\alpha_m} \\ P_{622} = \dfrac{\beta d_m + Krd_m\left(1-\kappa\right)}{Kr\alpha_m} \\ P_{623} = \dfrac{K^2r\kappa\left(\kappa-1\right)\left(r\eta d_m - K\alpha_m\left(r\eta+\mu_a A+\mu_m M\right)\right)-\beta^2\eta d_m}{K^3r\kappa\alpha_m\mu_s\left(1-\kappa\right)} \end{cases}$$

$P_{63}\left(P_{631},P_{632},0,P_{633},M,H,L\right)$, with

$$\begin{cases} P_{631} = \dfrac{d_s\left(Kr\kappa-\beta\right)}{Kr\alpha_s} \\ P_{632} = \dfrac{\beta d_s + Krd_s\left(1-\kappa\right)}{Kr\alpha_s} \\ P_{633} = \dfrac{K^2r\kappa\left(\kappa-1\right)\left(r\eta d_s - K\alpha_s\left(r\eta+\mu_l L+\mu_m M\right)\right)-\beta^2\eta d_s}{K^3r\kappa\alpha_s\mu_a\left(1-\kappa\right)} \end{cases}$$

$P_{64}\left(P_{641},P_{642},P_{643},0,M,H,L\right)$, with

$$\begin{cases} P_{641} = \dfrac{d_a\left(Kr\kappa-\beta\right)}{Kr\alpha_a} \\ P_{642} = \dfrac{\beta d_a + Krd_a\left(1-\kappa\right)}{Kr\alpha_a} \\ P_{643} = \dfrac{K^2r\kappa\left(\kappa-1\right)\left(r\eta d_a - K\alpha_a\left(r\eta+\mu_l L+\mu_h H\right)\right)-\beta^2\eta d_s}{K^3r\kappa\alpha_a\mu_s\left(1-\kappa\right)} \end{cases}$$

$P_{65}\left(P_{651},0,P_{652},0,M,0,0\right)$, with

$$\begin{cases} P_{651} = \dfrac{d_s}{\alpha_s} \\ P_{652} = \dfrac{Kr\kappa\alpha_s\left(\mu_m M-\eta\right)-r\eta d_s}{K\kappa\alpha_s\mu_s} \end{cases}$$

$P_{66}\left(0,P_{661},P_{662},A,M,H,L\right)$, with

$$\begin{cases} P_{661} = \dfrac{d_s}{\alpha_s} \\ P_{662} = \dfrac{K\alpha_s\left(1-\kappa\right)\left(r\eta - A\mu_a-\mu_h H-\mu_m M-\mu_l L\right)-r\eta d_s}{K\alpha_s\mu_s\left(1-\kappa\right)} \end{cases}$$

$P_{67}\left(P_{671},0,P_{672},A,M,H,L\right)$, with

$$\begin{cases} P_{671}=\dfrac{d_l}{\alpha_l} \\ P_{672}=\dfrac{K\alpha_l\left(1-\kappa\right)\left(r\eta-A\mu_a-\mu_hH-\mu_mM-\mu_lL\right)-r\eta d_l}{K\kappa\alpha_l\mu_s} \end{cases}$$

$P_{68}\left(P_{681},P_{682},P_{683},A,0,H,L\right)$, with

$$\begin{cases} P_{681}=\dfrac{d_h\left(Kr\kappa-\beta\right)}{Kr\alpha_h} \\ P_{682}=\dfrac{\beta d_h+Krd_h\left(1-\kappa\right)}{Kr\alpha_h} \\ P_{683}=\dfrac{K^2r\kappa\left(\kappa-1\right)\left(r\eta d_h-K\alpha_h\left(r\eta+\mu_lL+\mu_hH+\mu_aA\right)\right)-\beta^2\eta d_h}{K^3r\kappa\alpha_h\mu_s\left(1-\kappa\right)} \end{cases}$$

$P_{69}\left(P_{691},P_{692},P_{693},A,M,0,L\right)$, with

$$\begin{cases} P_{691}=\dfrac{d_l\left(Kr\kappa-\beta\right)}{Kr\alpha_l} \\ P_{692}=\dfrac{\beta d_l+Krd_l\left(1-\kappa\right)}{Kr\alpha_l} \\ P_{693}=\dfrac{K^2r\kappa\left(\kappa-1\right)\left(r\eta d_l-K\alpha_l\left(r\eta+\mu_lL+\mu_mM+\mu_aA\right)\right)-\beta^2\eta d_l}{K^3r\kappa\alpha_l\mu_s\left(1-\kappa\right)} \end{cases}$$

$P_{70}\left(P_{701},P_{702},P_{703},A,M,0,L\right)$, with

$$\begin{cases} P_{701}=\dfrac{d_h\left(Kr\kappa-\beta\right)}{Kr\alpha_h} \\ P_{702}=\dfrac{\beta d_h+Krd_h\left(1-\kappa\right)}{Kr\alpha_h} \\ P_{703}=\dfrac{K^2r\kappa\left(\kappa-1\right)\left(r\eta d_h-K\alpha_h\left(r\eta+\mu_lL+\mu_mM+\mu_aA\right)\right)-\beta^2\eta d_h}{K^3r\kappa\alpha_h\mu_s\left(1-\kappa\right)} \end{cases}$$

$P_{71}\left(P_{711},P_{712},P_{713},A,M,H,0\right)$, with

$$\begin{cases} P_{711}=\dfrac{d_a\left(Kr\kappa-\beta\right)}{Kr\alpha_a} \\ P_{712}=\dfrac{\beta d_a+Krd_a\left(1-\kappa\right)}{Kr\alpha_a} \\ P_{713}=\dfrac{K^2r\kappa\left(\kappa-1\right)\left(r\eta d_a-K\alpha_a\left(r\eta+\mu_mM+\mu_hH+\mu_aA\right)\right)-\beta^2\eta d_a}{K^3r\kappa\alpha_a\mu_s\left(1-\kappa\right)} \end{cases}$$

ANNEX 2: THE LOCAL STABILITY OF THE POSITIVE EQUILIBRIUM POINT X^*

The variational matrix $J(X^*)$ evaluated at the positive equilibrium point X^* is given by

$$J = \begin{pmatrix} J_{11} & -\dfrac{\beta P_J}{\kappa(1-\kappa)K^2} & -\dfrac{\mu_s P_J}{\eta} & -\dfrac{\mu_a P_J}{\eta} & -\dfrac{\mu_m P_J}{\eta} & -\dfrac{\mu_h P_J}{\eta} & -\dfrac{\mu_l P_J}{\eta} \\ \dfrac{\beta P_A}{\kappa(1-\kappa)K^2} & J_{22} & -\dfrac{\mu_s P_A}{\eta} & -\dfrac{\mu_a P_A}{\eta} & -\dfrac{\mu_m P_A}{\eta} & -\dfrac{\mu_h P_A}{\eta} & -\dfrac{\mu_l P_A}{\eta} \\ \alpha_s S & \alpha_s S & J_{33} & 0 & 0 & 0 & 0 \\ \alpha_a A & \alpha_a A & 0 & J_{44} & 0 & 0 & 0 \\ \alpha_m M & \alpha_m M & 0 & 0 & J_{55} & 0 & 0 \\ \alpha_h H & \alpha_h H & 0 & 0 & 0 & J_{66} & 0 \\ \alpha_l L & \alpha_l z L & 0 & 0 & 0 & 0 & J_{77} \end{pmatrix}$$

where

$J_{11} = -\dfrac{rP_J}{\kappa K}$

$J_{22} = -\dfrac{rP_A}{(1-\kappa)K}$

$J_{33} = -d_s + \alpha_s P_J + \alpha_s P_A$

$J_{44} = -d_a + \alpha_a P_J + \alpha_a P_A$

$J_{55} = -d_m + \alpha_m P_J + \alpha_m P_A$

$J_{66} = -d_h + \alpha_h P_J + \alpha_h P_A$

$J_{77} = -d_l + \alpha_l P_J + \alpha_l P_A$

The characteristic polynomial associated to the variational matrix $(J(X^*))$ is written as

$$\rho_7\lambda^7 + \rho_6\lambda^6 + \rho_5\lambda^5 + \rho_4\lambda^4 + \rho_3\lambda^3 + \rho_2\lambda^2 + \rho_1\lambda + \rho_0,$$

where $\rho_7 = 1$, $\rho_6 = \left(\frac{1}{k} r \frac{y}{a-1} - \frac{1}{ak} rx\right)$ and the other coefficients are positive and are written according to all the parameters mentioned in the mathematical model, we avoided to integrate them in the article because their expressions are laborious and too long.

Since all the coefficients ρ_i exist and they are positive, then we move to form the following Routh array

$$
\begin{array}{c|cccc}
\lambda^7 & \rho_7 & \rho_5 & \rho_3 & \rho_1 \\
\lambda^6 & \rho_6 & \rho_4 & \rho_2 & \rho_0 \\
\lambda^5 & \rho_{11} & \rho_{12} & \rho_{13} & 0 \\
\lambda^4 & \rho_{21} & \rho_{22} & \rho_{23} & 0 \\
\lambda^3 & \rho_{31} & \rho_{32} & 0 & 0 \\
\lambda^2 & \rho_{41} & \rho_{42} & 0 & 0 \\
\lambda^1 & \rho_{51} & 0 & 0 & 0 \\
1 & \rho_{61} & 0 & 0 & 0
\end{array}
$$

with

$\rho_{11} = \dfrac{\rho_6\rho_5 - \rho_7\rho_4}{\rho_6} > 0$, $\rho_{12} = \dfrac{\rho_3\rho_6 - \rho_7\rho_2}{\rho_6}$, $\rho_{13} = \dfrac{\rho_6\rho_1 - \rho_7\rho_0}{\rho_6}$

$\rho_{21} = \dfrac{\rho_{11}\rho_4 - \rho_6\rho_{12}}{\rho_{11}} > 0$, $\rho_{22} = \dfrac{\rho_{11}\rho_2 - \rho_6\rho_{13}}{\rho_{11}}$, $\rho_{23} = \dfrac{\rho_{11}\rho_0}{\rho_{11}}$

$\rho_{31} = \dfrac{\rho_{12}\rho_{21} - \rho_{11}\rho_{22}}{\rho_{21}} > 0$, $\rho_{32} = \dfrac{\rho_{13}\rho_{21} - \rho_{11}\rho_{23}}{\rho_{21}}$, $\rho_{33} = \dfrac{\rho_{22} \times 0 - \rho_{11} \times 0}{\rho_{21}} = 0$

$\rho_{41} = \dfrac{\rho_{31}\rho_{22} - \rho_{21}\rho_{32}}{\rho_{31}} > 0$, $\rho_{42} = \dfrac{\rho_{31}\rho_{23}}{\rho_{31}}$

$\rho_{51} = \dfrac{\rho_{41}\rho_{32} - \rho_{42}\rho_{31}}{\rho_{41}} > 0$, $\rho_{52} = 0$

$\rho_{61} = \dfrac{\rho_{51}\rho_{42}}{\rho_{51}} > 0.$

From this array, we can clearly see that all of the signs of the first column are positive, there are no sign changes, and therefore the interior equilibrium point X^* is locally asymptotically stable.

ANNEX 3: SOLUTION OF THE BIOECONOMIC MODEL

The biomasses at biological equilibrium are the solutions of the system:

$$
\begin{cases}
r - \frac{rP_J}{\kappa K} - \frac{\beta P_A}{\kappa(1-\kappa)K^2} - \frac{\mu_s S}{\eta} - \frac{\mu_a A}{\eta} - \frac{\mu_m M}{\eta} - \frac{\mu_h H}{\eta} - \frac{\mu_l L}{\eta} = 0 \\
r - \frac{rP_A}{(1-\kappa)K} + \frac{\beta P_J}{\kappa(1-\kappa)K^2} - \frac{\mu_s S}{\eta} - \frac{\mu_a A}{\eta} - \frac{\mu_m M}{\eta} - \frac{\mu_h H}{\eta} - \frac{\mu_l L}{\eta} - q_1 E_1 = 0 \\
-d_s + \alpha_s P_J + \alpha_s P_A - q_2 E_2 = 0 \\
-d_a + \alpha_a P_J + \alpha_a P_A - q_3 E_3 = 0 \\
-d_m + \alpha_m P_J + \alpha_m P_A - q_4 E_4 = 0 \\
-d_h + \alpha_h P_J + \alpha_h P_A - q_5 E_5 = 0 \\
-d_l + \alpha_l P_J + \alpha_l P_A - q_6 E_6 = 0
\end{cases}
$$

The solution of the system is given by the matrix form $X = -AE + X^*$, where

$E = [E_1, E_2, E_3, E_4, E_5, E_6, 0]^T$, $X = [P_J, P_A, S, A, M, H, L]^T$ and $A = \left(-a_{ij}\right)_{1\le i,j\le 7}$ with

$a_{11} = \frac{q_1(\kappa-1)K^2\kappa}{Kr}$, $a_{12} = \frac{(\beta-Kr\kappa)q_2}{Kr(\alpha_s+\alpha_a+\alpha_m+\alpha_h+\alpha_l)}$, $a_{13} = \frac{(\beta-Kr\kappa)q_3}{Kr(\alpha_s+\alpha_a+\alpha_m+\alpha_h+\alpha_l)}$,

$a_{14} = \frac{(\beta-Kr\kappa)q_4}{Kr(\alpha_s+\alpha_a+\alpha_m+\alpha_h+\alpha_l)}$, $a_{15} = \frac{(\beta-Kr\kappa)q_5E_5}{Kr(\alpha_s+\alpha_a+\alpha_m+\alpha_h+\alpha_l)}$, $a_{16} = \frac{(\beta-Kr\kappa)q_6E_6}{Kr(\alpha_s+\alpha_a+\alpha_m+\alpha_h+\alpha_l)}$,

$a_{21} = \frac{q_1(1-\kappa)K^2\kappa}{Kr}$, $a_{22} = \frac{(Kr(\kappa-1)-\beta)q_2}{Kr(\alpha_s+\alpha_a+\alpha_m+\alpha_h+\alpha_l)}$, $a_{23} = \frac{(Kr(\kappa-1)-\beta)q_3}{Kr(\alpha_s+\alpha_a+\alpha_m+\alpha_h+\alpha_l)}$,

$a_{24} = \frac{(Kr(\kappa-1)-\beta)q_4}{Kr(\alpha_s+\alpha_a+\alpha_m+\alpha_h+\alpha_l)}$, $a_{25} = \frac{(Kr(\kappa-1)-\beta)q_5}{Kr(\alpha_s+\alpha_a+\alpha_m+\alpha_h+\alpha_l)}$, $a_{26} = \frac{(Kr(\kappa-1)-\beta)q_6}{Kr(\alpha_s+\alpha_a+\alpha_m+\alpha_h+\alpha_l)}$,

$a_{31} = \frac{(1-\kappa)(\beta+Kr(\kappa-1))K^2\eta\kappa\alpha_1q_1}{K^3\mu_sr\kappa\alpha_s(\kappa-1)}$, $a_{32} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_2}{K^3\mu_sr\kappa\alpha_s(\kappa-1)}$, $a_{33} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_3}{K^3\mu_sr\kappa\alpha_s(\kappa-1)}$,

$a_{34} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_4}{K^3\mu_sr\kappa\alpha_s(\kappa-1)}$, $a_{35} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_5}{K^3\mu_sr\kappa\alpha_s(\kappa-1)}$, $a_{36} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_6}{K^3\mu_sr\kappa\alpha_s(\kappa-1)}$,

$a_{41} = \frac{(1-\kappa)(\beta+Kr(\kappa-1))K^2\eta\kappa\alpha_2q_1}{K^3\mu_ar\kappa\alpha_a(\kappa-1)}$, $a_{42} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_2}{K^3\mu_ar\kappa\alpha_a(\kappa-1)}$, $a_{43} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_3}{K^3\mu_ar\kappa\alpha_a(\kappa-1)}$,

$a_{44} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_4}{K^3\mu_ar\kappa\alpha_a(\kappa-1)}$, $a_{45} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_5}{K^3\mu_ar\kappa\alpha_a(\kappa-1)}$, $a_{46} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_6}{K^3\mu_ar\kappa\alpha_a(\kappa-1)}$,

$a_{51} = \frac{(1-\kappa)(\beta+Kr(\kappa-1))K^2\eta\kappa\alpha_3q_1}{K^3\mu_mr\kappa\alpha_m(\kappa-1)}$, $a_{52} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_2}{K^3\mu_mr\kappa\alpha_m(\kappa-1)}$, $a_{53} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_3}{K^3\mu_mr\kappa\alpha_m(\kappa-1)}$,

$a_{54} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_4}{K^3\mu_mr\kappa\alpha_m(\kappa-1)}$, $a_{55} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_5}{K^3\mu_mr\kappa\alpha_m(\kappa-1)}$, $a_{56} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_6}{K^3\mu_mr\kappa\alpha_m(\kappa-1)}$,

$a_{61} = \frac{(1-\kappa)(\beta+Kr(\kappa-1))K^2\eta\kappa\alpha_4q_1}{K^3\mu_hr\kappa\alpha_h(\kappa-1)}$, $a_{62} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_2}{K^3\mu_hr\kappa\alpha_h(\kappa-1)}$, $a_{63} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_3}{K^3\mu_hr\kappa\alpha_h(\kappa-1)}$,

$a_{64} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_4}{K^3\mu_hr\kappa\alpha_h(\kappa-1)}$, $a_{65} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_5}{K^3\mu_hr\kappa\alpha_h(\kappa-1)}$, $a_{66} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_6}{K^3\mu_hr\kappa\alpha_h(\kappa-1)}$,

$a_{71} = \frac{(1-\kappa)(\beta+Kr(\kappa-1))K^2\eta\kappa\alpha_lq_1}{K^3\mu_lr\kappa\alpha_l(\kappa-1)}$, $a_{72} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_2}{K^3\mu_lr\kappa\alpha_l(\kappa-1)}$, $a_{73} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_3}{K^3\mu_lr\kappa\alpha_l(\kappa-1)}$,

$a_{74} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_4}{K^3\mu_lr\kappa\alpha_l(\kappa-1)}$, $a_{75} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_5}{K^3\mu_lr\kappa\alpha_l(\kappa-1)}$, $a_{76} = \frac{\left(K^2r^2\kappa(\kappa-1)-\beta^2\right)\eta q_6}{K^3\mu_lr\kappa\alpha_l(\kappa-1)}$

$a_{77} = -1$, $a_{i7} = 0$, $\forall i = \{1,\dots,6\}$

Conflict of Interests

The authors declare that there is no conflict of interests.

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