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MODELLING THE TRANSMISSION DYNAMICS OF COVID-19 UNDER LIMITED RESOURCES

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Abstract. Since the novel corona virus has been identified, the virus has rapidly spread to more than 200 countries. In Indonesia, as of 25 August, 2020, the total number of confirmed cases is 157,859 and there has been an increasing trend in the number of new cases. Therefore, it is urgently needed to study the disease transmission dynamics and the efficacy of its intervention strategies. In this paper, a compartment-based mathematical model has been formulated to study the COVID-19 disease transmission dynamics under limited resources. The model has been validated against data of Bali Province, Indonesia. We analyze the three different scenarios, which are 15%, 35% and 60% budget reduction. An optimal control approach has been used to examine the effects of control strategies and limited budget on the disease transmission dynamics. We found that the control strategy to detect COVID-19 infections in the population is crucial to reduce the number of infections. The results are corroborated by the results of sensitivity analysis. Furthermore, a higher reduction in the alloted budget would contribute to an increase in the number of COVID-19 infections.

Keywords: mathematical model; optimal control; limited resources; COVID-19.

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1. INTRODUCTION

A novel coronavirus pandemic is an ongoing pandemic of coronavirus disease 2019 (COVID-19). The virus has rapidly spread all over the world including Indonesia. The World Health Organization (WHO) has declared the epidemic as a public health emergency and on 11 March 2020 the WHO declared the outbreak a pandemic [1]. As of 25 August 2020, around 24,050,731 COVID-19 infections occurs worldwide [2] including 157,859 confirmed cases in Indonesia [3]. An outbreak leads to the need for better strategies and sufficient resources to mitigate the diseases since with no vaccine available near future, it has been predicted that the virus would exist in the population for longer period.

Understanding the transmission mechanism of the virus is important to study the best mitigation strategies against the virus. Studies showed the important feature of transmission including the infectiousness of asymptomatic and presymptomatic [4, 5]. The presymptomatic and asymptomatic individuals carry the virus and can transmit the virus to others. Therefore, it is important to consider these features in the analysis. Strategies to minimise the number of infections center around reducing the transmission rate [6] such as social distancing, mask use, and wash hands, and detecting infected invididuals [7, 8, 9] such as the use of Polymerase chain reaction (PCR) or quantitative Polymerase chain reaction (qPCR) and hence they can get treatment and/or quarantined. An early detection plays an important role in minimizing the burden of the disease. However, it should be noted that the lack of resources may be a challenge in minimising the number of infections.

The use of mathematical model to understand the disease transmission dynamics is common. A matematical model is generally formulated and studied to understand disease transmission dynamics and the effectiveness of intervention strategies [10, 11, 12, 13, 14] and an optimal control approach is commonly used to get insights of optimal control strategy that can minimize the number of infections at a low cost [15, 16]. To date, various mathematical models have been formulated to understand the COVID-19 disease transmission dynamics and the effectiveness of its intervention strategies [17, 18, 19, 20]. The models have been formulated to examine the disease transmission dynamics [21], to determine the effectiveness of the interventions [18, 20, 22], to calculate the reproduction number [23, 24]. Although many mathematical models have been formulated to study the COVID-19 transmission dynamics, studies of COVID-19 transmission in Indonesia are limited. Using a mathematical model, Aldila *et al.* [18] analyzed the potential transmission with different scenarios of relaxing the lockdown. They used the data of Jakarta to estimate the parameter values. They found that if the lockdown strategy would be relaxed, it is better to implement detection strategy such as rapid testing. Ndii *et al.* [25] formulated deterministic and stochastic models to determine the probability of disease extinction and found that a 70% reduction in the transmission rate may result in higher probability of disease transmission. However, none of these work analyzed the effects of optimal controls under limited resources on the transmission dynamics of the disease.

In this paper, we have formulated the mathematical model for COVID-19 disease transmission and have estimated the value of the transmission rate against incidence data of Bali Province, Indonesia. We have used an optimal control approach to investigate the effects of controls on disease transmission dynamics and take into account the limited budget for implementing the control. We study three different scenarios of budget reduction: 15%, 35% and 60% budget reductions.

The remainder of the paper is organized as follows. Section 2 presents material and methods which consist of mathematical model, the analysis of the model, disease free equilibrium and reproduction number, optimal control without and with budget constraint. Section 3 presents the results which consists of data and parameter estimation, numerical simulations, sensitivity analysis, optimal control without and with budget constraints. Finally, discussion and conclusions are presented.

2. Material and Methods

2.1. Mathematical Model.

We have formulated a deterministic mathematical model for COVID-19 transmission, where the population is divided into Susceptible (S), Exposed (E), Pre-symptomatic (P), Asymptomatic (A), Infected (I), Confirmed (C) and Recovered (R). We take into account the control to reduce the transmission rate (u_1), and to detect pre-symptomatic, asymptomatic and infected individuals (u_2).

In the model, it is assumed that only presymptomatic, asymptomatic and infected individuals can transmit the virus. The confirmed individuals do not transmit the virus since they are treated in the hospital or quarantined. Furthermore, the transmission rate of presymptomatic and asymptomatic is lower than that of infected individuals. Susceptible individuals get infected when they have contacted with the presymptomatic, asymptomatic and infected individuals. The transmission rate for infected individuals is higher than that of presymptomatic and asymptomatic. Hence, the parameter α is the proportion of reduction in the transmission rate of presymptomatic and asymptomatic individuals. Due to the implementation of control such as social distancing and mask use, the transmission rate would be reduced by $(1-u_1)$ where u_1 is the control rate. If individuals strictly implement the control such as mask use and social distancing, the control rate u_1 becomes higher and therefore $(1-u_1)$ gets smaller, and therefore, reduce the transmission rate. After the latent period, the exposed individuals become presymptomatic. A proportion, r, of presymptomatic individuals becomes infected and the rest become asymptomatic. The control rate, u_2, is implemented to detect the presymptomatic, asymptomatic and infected individuals. They are then moved to the confirmed class. The descriptions of the parameters and their values are given in Table 1 and the flowchart of the model is given in Figure 1.

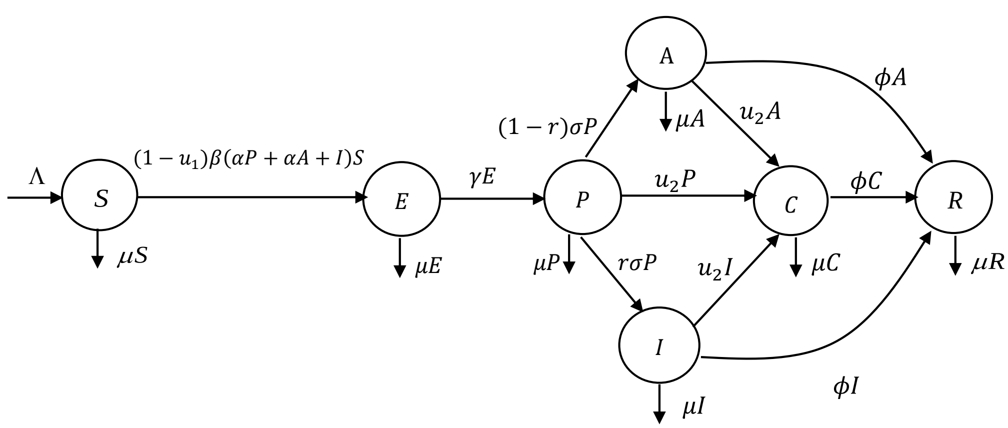


FIGURE 1. Flowchart of the model where the population is divided into Susceptible (S), Exposed (E), Presymptomatic (P), Asymptomatic (A), Infected (I), Confirmed (C) and Recovered (R).

Based on the above explanation, the mathematical model for COVID-19 transmission is governed by the following system of differential equations

$$
\begin{aligned}
\frac{dS}{dt} &= \Lambda - (1-u_1)\beta\,(\alpha P + \alpha A + I)\,S - \mu S\\
\frac{dE}{dt} &= (1-u_1)\beta\,(\alpha P + \alpha A + I)\,S - \gamma E - \mu E,\\
\frac{dP}{dt} &= \gamma E - \sigma P - \mu P - u_2 P,\\
\frac{dA}{dt} &= (1-r)\sigma P - \phi A - \mu A - u_2 A,\\
\frac{dI}{dt} &= r\sigma P - \phi I - \mu I - u_2 I,\\
\frac{dC}{dt} &= u_2(P + A + I) - \phi C - \mu C,\\
\frac{dR}{dt} &= \phi(A + I + C) - \mu R.
\end{aligned}
\tag{1}
$$

The descriptions of the parameters and their values are given in Table 1.

TABLE 1. Parameter descriptions and range of value for parameters of Model (1).

Parameters	Description	value/interval	Source
Λ	Human recruitment rate	$\frac{10^7}{70\times 365}$	Estimated
μ	natural death rate	$\frac{1}{70\times 365}$	[26]
β	The transmission rate	see text	Fitted
α	Proportion of reduction in the transmission rate for pre-symptomatic and asymptomatic individuals	see text	Fitted
γ^{-1}	latent period	5.2	[21]
σ^{-1}	Period of presymptomatic	2.3	[27]
r	Proportion of presymptomatic individuals become infected individuals	0.23	[27]
ϕ	Recovery rate of exposed individuals	0.05	[18]
u_2	Control rate to detect presymptomatic, asymptomatic and infected individuals	0.05	[18]
u_1	Control rate to reduce disease transmission	0.1	[18]

2.2. Mathematical Analysis.

Lemma 1. *For nonnegative initial conditions for the system* (1)

$$S(0)\geq 0, E(0)\geq 0, P(0)\geq 0, A(0)\geq 0, I(0)\geq 0, C(0)\geq 0, R(0)\geq 0,$$

the solutions of

$$S(t), E(t), P(t), A(t), I(t), C(t), R(t)$$

are nonnegative for all time $t>0$

Proof. Assume that $T = \sup\{t > 0, S > 0, E > 0, P > 0, A > 0, I > 0, C > 0, R > 0\} \in (0,t]$. Clearly, $T > 0$. From the first equation of the system (1), we obtain

$$\frac{d}{dt}\left(S(t)\exp\left(\int_0^t (1-u_1)\beta(\alpha P(\tau)+\alpha A(\tau)+I(\tau))d\tau+\mu t\right)\right)$$
$$=\Lambda\exp\left(\int_0^t \beta(\alpha P(\tau)+\alpha A(\tau)+I(\tau))d\tau+\mu t\right).$$

By integrating (2.2) from 0 to T to obtain

$$S(T)\exp\left(\int_0^t (1-u_1)\beta(\alpha P(\tau)+\alpha A(\tau)+I(\tau))d\tau+\mu t\right)-S(0)$$
$$=\int_0^T \Lambda\exp\left(\int_0^t (1-u_1)\beta(\alpha P(\tau)+\alpha A(\tau)+I(\tau))d\tau+\mu t\right),$$

$$s(T)=S(0)\exp\left\{-\left(\int_0^t (1-u_1)\beta(\alpha P(\tau)+\alpha A(\tau)+I(\tau))d\tau+\mu t\right)\right\}$$
$$+\exp\left\{-\left(\int_0^t (1-u_1)\beta(\alpha P(\tau)+\alpha A(\tau)+I(\tau))d\tau+\mu t\right)\right\}$$
$$\int_0^T \Lambda\exp\left(\int_0^t (1-u_1)\beta(\alpha P(\tau)+\alpha A(\tau)+I(\tau))d\tau+\mu t\right)ds,>0.$$

Similarly, it can be shown for $E(t) > 0$, $P(t) > 0$, $A(t) > 0$, $I(t) > 0$, $C(t) > 0$ and $R(t) > 0$ for all $t > 0$. □

Lemma 2. *All solution of System* (1) *are bounded for all* $t \in [0,t_0]$

Proof. Since $N(t) = S(t)+E(t)+P(t)+A(t)+I(t)+C(t)+R(t)$, we get

$$\frac{dN}{dt}=\Lambda-\mu N.$$

Thus we have $0 \le \lim_{t\to\infty} \sup N(t) \le \frac{\Lambda}{\mu}$, so all solutions of (2) are ultimately bounded for all $t \in [0,t_0]$. □

2.3. Disease free equilibrium and Reproduction number. The equilibrium points are obtained by setting the right hand side of the Equation (1) to zero and do algebraic manipulation. We obtain the disease free equilibrium as follows

$$E_0 = (\frac{\Lambda}{\mu}, 0, 0, 0, 0, 0, 0).$$

By applying the procedure of finding the next-generation matrix described in [28], we have

$$\mathscr{F} = \begin{bmatrix} \beta(1-u_1)(\alpha P + \alpha A + I)S \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

and

$$\mathscr{V} = \begin{bmatrix} (\gamma+\mu)E \\ -\gamma E + (\sigma+\mu+u_2)P \\ -(1-r)\sigma P + (\phi+\mu+u_2)A \\ -r\sigma P + (\phi+\mu+u_2)I \end{bmatrix}.$$

The Jacobian matrices of $\mathscr{F}$ and $\mathscr{V}$ evaluated at the disease-free equilibrium point are

$$F = \begin{bmatrix} 0 & (1-u_1)\alpha\beta\frac{\Lambda}{\mu} & (1-u_1)\alpha\beta\frac{\Lambda}{\mu} & (1-u_1)\beta\frac{\Lambda}{\mu} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$V = \begin{bmatrix} \gamma+\mu & 0 & 0 & 0 \\ -\gamma & \sigma+\mu+u_2 & 0 & 0 \\ 0 & -(1-r)\sigma & \phi+\mu+u_2 & 0 \\ 0 & -r\sigma & 0 & \phi+\mu+u_2 \end{bmatrix}.$$

Therefore, we have

$$V^{-1} = \begin{bmatrix} \frac{1}{\gamma+\mu} & 0 & 0 & 0 \\ \frac{\gamma}{(\gamma+\mu)(\sigma+\mu+u_2)} & \frac{1}{\sigma+\mu+u_2} & 0 & 0 \\ \frac{\gamma(1-r)\sigma}{(\gamma+\mu)(\sigma+\mu+u_2)(\phi+\mu+u_2)} & \frac{(1-r)\sigma}{(\sigma+\mu+u_2)(\phi+\mu+u_2)} & \frac{1}{\phi+\mu+u_2} & 0 \\ \frac{\gamma\sigma r}{(\gamma+\mu)(\sigma+\mu+u_2)(\phi+\mu+u_2)} & \frac{r\sigma}{(\sigma+\mu+u_2)(\phi+\mu+u_2)} & 0 & \frac{1}{\phi+\mu+u_2} \end{bmatrix}.$$

Then, we have the next-generation matrix defined as FV^{-1} and the spectral radius of that matrix is the reproduction number, which is

$$R_0 = \frac{\Lambda\beta(1-u_1)\gamma(-\alpha\sigma(r-1)+\alpha(\phi+\mu+u_2)+r\sigma)}{\mu\,(\gamma+\mu)\,(\mu+\sigma+u_2)\,(\phi+\mu+u_2)} \tag{2}$$

Lemma 3. *The disease-free equilibrium point E_0 is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.*

Proof. The Jacobian matrix of Equations (1) is evaluated at the disease-free equilibrium point, $E_0 = (\frac{\Lambda}{\mu}, 0, 0, 0, 0, 0, 0)$, and we find a characteristic polynomial as

$$(3)\quad \begin{aligned}&(\lambda+\mu)^2(\lambda+a_1)(\lambda+a_2)(\lambda^3+(a+a_0+a_1)\lambda^2+\mu a a_0 a_1(1-R_0)\\ &+\frac{1}{a_1\mu}\left(a_1^2\mu(a+a_0)+a a_0 a_1\mu(1-R_0)+\Lambda\beta\gamma(1-u_1)\alpha\sigma(1-r)+r\sigma\right))=0\end{aligned}$$

where

$$a=\gamma+\mu, \quad a_0=\sigma+\mu+u_2, \quad a_1=\phi+\mu+u_2, \quad a_2=\mu+\phi.$$

Since $1-u_1>0$ and $r<1$, the equation (3)will has strictly negative real root if $R_0<1$. Hence, the disease-free equilibrium E_0 is locally asymptotically stable if $R_0<1$ and unstable if $R_0>1$. □

2.4. Optimal control without budget constraint.

In this paper, we perform optimal control approach. The use of optimal control approach in the epidemic model can be found in [15, 16, 29]. In this work, there are two controls to be considered: the control to reduce the transmission rate (u_1) such as social distancing and mask use, and the control to detect pre-symptomatic, asymptomatic, and infected individuals (u_2) such as the use of PCR/qPCR test. The aim is to minimise the number of COVID-19 infections and the costs associated with the infections and the implementation of controls. The objective functional is written as

$$(4)\quad J(u_1,u_2)=\int_0^T (W_1E(t)+W_2P(t)+W_3A(t)+W_4I(t)+W_5u_1^2+W_6u_2^2)dt,$$

where W_1, W_2, W_3, W_4, W_5, W_6 are balancing coefficient relating the costs associated with exposed, presymptomatic, asymptomatic, infected, the control to reduce the disease transmission and the control to detect presymptomatic, asymptomatic and infected individuals, respectively. Let set the admissible control as

$$(5)\quad \mathscr{U}=\left\{u_i\in(L^\infty(0,T))^2 | 0\le u_i(t)\le 1; u_i \text{ for } i=1,2 \text{ are Lebesgue measurable}\right\}.$$

We use the quadratic functions on the control to account for the societal effects due the implementation of controls. The necessary conditions that an optimal control must satisfy is from the Pontryagin's Maximum Principle [35]. The Pontryagin's Maximum Principle converts the Equations (1) and (4) into a problem of minimizing pointwise a Hamiltonian H, with respect to the controls, u_1 and u_2. We formulate the Hamiltonian function as

$$
\begin{aligned}
H = {} & W_1E(t) + W_2P(t) + W_3A(t) + W_4I(t) + W_5u_1^2 + W_6u_2^2 \\
& + \lambda_S\left(\Lambda - (1-u_1)\beta\left(\alpha P + \alpha A + I\right)S - \mu S\right) \\
& + \lambda_E\left((1-u_1)\beta\left(\alpha P + \alpha A + I\right)S - \gamma E - \mu E\right) \\
& + \lambda_P\left(\gamma E - \sigma P - \mu P - u_2P\right) \\
& + \lambda_A\left((1-r)\sigma P - \phi A - \mu A - u_2A\right) \\
& + \lambda_I\left(r\sigma P - \phi I - \mu I - u_2I\right) \\
& + \lambda_C\left(u_2(P+A+I) - \phi C - \mu C\right) \\
& + \lambda_R\left(\phi(A+I+C) - \mu R\right),
\end{aligned} \tag{6}
$$

where λ_S, λ_E,λ_P, λ_A,λ_I, λ_C,λ_R are the associated adjoints for the states S, E, P, A, I, C, and R. We can find the system of adjoint equations by taking the partial derivatives of the Hamiltonian with respect to the associated state and control variables.

Theorem 1. *There exists optimal controls, u_1^* and u_2^* and state solutions of the corresponding system that minimize $J(u_1,u_2)$ over the set U. Then there exists adjoint variables λ_L satisfying*

$$\frac{d\lambda_L}{dt} = -\frac{\partial H}{\partial L}$$

where $L = S,E,P,A,I,C,R$ and with transversility condition $\lambda_L(t_f) = 0$ The optimality conditions are given as

$$\frac{\partial H}{\partial u_i} = 0, \quad i = 1,2.$$

Furthermore, the controls u_1^* and u_2^* are given as

$$u_1^*(t) = \min\{1, \max[0, \hat{u_1}]\}$$
$$u_2^*(t) = \min\{1, \max[0, \hat{u_2}]\}$$
$$\text{where} \quad \hat{u}_1 = \frac{\beta S((A\alpha + P\alpha + I)(\lambda_E - \lambda_S)))}{2W_5} \tag{7}$$
$$\hat{u_2} = \frac{A(\lambda_A - \lambda_C) + P(\lambda_P - \lambda_C) + I(\lambda_I - \lambda_C)}{2W_6}$$

Proof. The sufficient condition to determine the optimal control u_i^* for $i = 1,2$ in $\mathscr{U}$ such that

$$J(u_i^*) = \min_{\mathscr{U}} J(u_i). \tag{8}$$

The differential equations of the adjoint variables are obtained by taking the diffential of the Hamiltonian function. The adjoint system can be written as

$$\begin{aligned}
\frac{d\lambda_S}{dt} &= -\frac{\partial H}{\partial S}, \quad \lambda_S(t_f) = 0,\\
\frac{d\lambda_E}{dt} &= -\frac{\partial H}{\partial E}, \quad \lambda_E(t_f) = 0,\\
\frac{d\lambda_P}{dt} &= -\frac{\partial H}{\partial P}, \quad \lambda_P(t_f) = 0,\\
\frac{d\lambda_A}{dt} &= -\frac{\partial H}{\partial A}, \quad \lambda_A(t_f) = 0,\\
\frac{d\lambda_I}{dt} &= -\frac{\partial H}{\partial I}, \quad \lambda_I(t_f) = 0,\\
\frac{d\lambda_C}{dt} &= -\frac{\partial H}{\partial C}, \quad \lambda_C(t_f) = 0,\\
\frac{d\lambda_R}{dt} &= -\frac{\partial H}{\partial R}, \quad \lambda_R(t_f) = 0.
\end{aligned} \tag{9}$$

The results of the adjoint system is given in the following Equation

$$\begin{aligned}
\frac{d\lambda_S}{dt} &= -\lambda_E(1-u_1)\beta(A\alpha + P\alpha + I) - \lambda_S(-(1-u_1)\beta(A\alpha + P\alpha + I) - \mu),\\
\frac{d\lambda_E}{dt} &= -W_1 - \lambda_E(-\gamma - \mu) - \lambda_P\gamma,\\
\frac{d\lambda_P}{dt} &= -W_2 - \lambda_A(1-r)\sigma - \lambda_C u_2 - \lambda_E(1-u_1)\beta\alpha S - \lambda_I r\sigma - \lambda_P(-\mu - \sigma - u_2)\\
&+ \lambda_S(1-u_1)\beta\alpha S,
\end{aligned} \tag{10}$$

$$
\begin{aligned}
\frac{d\lambda_A}{dt} &= -W_3-\lambda_A(-\phi-\mu-u_2)-\lambda_C u_2-\lambda_E(1-u_1)\beta\alpha S-\lambda_R\phi+\lambda_S(1-u_1)\beta\alpha S,\\
\frac{d\lambda_I}{dt} &= -W_4-\lambda_C u_2-\lambda_E(1-u_1)\beta S-\lambda_I(-\phi-\mu-u_2)-\lambda_R\phi+\lambda_S(1-u_1)\beta S,\\
\frac{d\lambda_C}{dt} &= -\lambda_C(-\mu-\phi)-\lambda_R\phi,\\
\frac{d\lambda_R}{dt} &= \lambda_R\mu,
\end{aligned}
$$

and the transversality condition $\lambda_L(T_f)=0$ for $L=S,E,P,A,I,C,R$. To find the optimal controls, we take the partial derivative of the Hamiltonian H with respect to each control u_i for $i=1,2$ and we obtain

$$
\begin{aligned}
\frac{\partial H}{\partial u_1} &= 2W_5u_1-\lambda_E\beta(A\alpha+P\alpha+I)S+\lambda_S\beta(A\alpha+P\alpha+I)S,\\
\frac{\partial H}{\partial u_2} &= 2W_6u_2-\lambda_A A+\lambda_C(P+A+I)-\lambda[I]I-\lambda_P P,
\end{aligned}
\tag{11}
$$

and by solving for u_1 and u_2, and taking the bounds, we obtain the optimal control as given in Equation (7). □

2.5. Optimal control with budget constraint. In this section, we present an optimal control with limited budget. The use of optimal control with limited resources in epidemic problem can also be found in [31, 32, 36]. To the best of our knowledge, none of the work has been applied to study COVID-19 disease transmission. We assume that the control to reduce COVID-19 transmission such as social distancing and mask use is affordable and easy to be implemented. Due to limited budget, the capacity of the control to detect COVID-19 infection is limited. Let denote X be the total budget allocated to implement the control for detection of COVID-19 infections (presymptomatic, asymptomatic and infected individuals), u_2. Hence we have integral constraint

$$
\int_0^T W_6u_2(t)dt = X. \tag{12}
$$

Such constraint is called isoperimetric. We can handle this problem by creating another state variables such that

$$\frac{dZ}{dt} = W_6 u_2(t), \quad Z(0) = 0 \quad \text{and} \quad Z(t) = X. \tag{13}$$

The goal is to minimise the number of COVID-19 infections and the the cost associated with infections and the implementation of control. The objective function is

$$J(u_1, u_2) = \int_0^T (W_1 E + W_2 P + W_3 A + W_4 I + W_5 u_1^2 + W_6 u_2^2) dt \tag{14}$$

subject to Equations (1), and (13). The Hamiltonian function can be written as

$$\begin{aligned} H = {} & W_1 E(t) + W_2 P(t) + W_3 A(t) + W_4 I(t) + W_5 u_1^2 + W_6 u_2^2 \\ & + \lambda_S \left(\Lambda - (1-u_1)\beta \left(\alpha P + \alpha A + I\right) S - \mu S\right) \\ & + \lambda_E \left((1-u_1)\beta \left(\alpha P + \alpha A + I\right) S - \gamma E - \mu E\right) \\ & + \lambda_P \left(\gamma E - \sigma P - \mu P - u_2 P\right) \\ & + \lambda_A \left((1-r)\sigma P - \phi A - \mu A - u_2 A\right) \\ & + \lambda_I \left(r\sigma P - \phi I - \mu I - u_2 I\right) \\ & + \lambda_C \left(u_2(P + A + I) - \phi C - \mu C\right) \\ & + \lambda_R \left(\phi(A + I + C) - \mu R\right) \\ & + \lambda_Z \left(W_6 u_2\right). \end{aligned} \tag{15}$$

The differential equations for associated adjoints for S, E, P, A, I, C, R are the same as in Equation (11) and that for Z

$$\frac{d\lambda_Z}{dt} = -\frac{\partial H}{\partial Z} = 0, \tag{16}$$

and the transversality conditions $\lambda_S(t_f) = \lambda_S(t_f) = \lambda_E(t_f) = \lambda_P(t_f) = \lambda_A(t_f) = \lambda_I(t_f) = \lambda_R(t_f) = 0$. There is no end point for $\lambda_Z(t_f)$ because its corresponding state variable, $Z(t)$, has two conditions (see Equation (13)). From Equation (16), it is clear that $\lambda_Z(t) = k$ with $k \in \mathbb{R}$. This means that a constant gain would be expected in the objective value when (12) is relaxed by one unit [30, 29].

Taking the derivative of the Hamitonian with respect to controls, u_1 and u_2, to obtain

$$\begin{aligned}\frac{\partial H}{\partial u_1} &= 2W_5u_1 - \lambda_E\beta(A\alpha+P\alpha+I)S+\lambda_S\beta(A\alpha+P\alpha+I)S \\ \frac{\partial H}{\partial u_2} &= 2W_6u_2 - \lambda_A A+\lambda_C(P+A+I)-\lambda_I I-\lambda_P P+\lambda_Z W_6.\end{aligned} \tag{17}$$

We then solve for u_i, $i=1,2$ to obtain

$$\begin{aligned}\hat{u_1} &= \frac{\beta S\left((A\alpha+P\alpha+I)(\lambda_E-\lambda_S)\right))}{2W_5}, \\ \hat{u_2} &= \frac{A(\lambda_A-\lambda_C)+P(\lambda_P-\lambda_C)+I(\lambda_I-\lambda_C)-W_6\lambda_Z}{2W_6}\end{aligned} \tag{18}$$

Taking the bounds, the controls u_1^* and u_2^* are given as

$$\begin{aligned}u_1^*(t) &= \min\{1,\max[0,\hat{u_1}]\}, \\ u_2^*(t) &= \min\{1,\max[0,\hat{u_2}]\}.\end{aligned} \tag{19}$$

3. Results

3.1. Data and Parameter estimation. We estimated the parameters β and α against data of Bali, Indonesia. The data are obtained from the website `infocorona.baliprov.go.id`. We estimate the parameter values by minimizing the sum of squared error. The parameters β and α are estimated against the data for the first 30 days. It is sufficient since the aim is to obtain the general insights of the values of parameters β and α in the early period of the outbreak. The other parameter values are obtained from literature and are given in Table 1.

The lsqnonlin built-in function in MATLAB is used for te parameter estimation. The initial conditions used are $S(0)=4,360,000$, $E(0)=10$, $P(0)=10$, $A(0)=10$, $I(0)=10$, $C(0)=25$, $R(0)=0$. The initial conditions for susceptible individuals are an approximate total population in Bali. The fitted values of β and α are 0.46240×10^{-6} and $0.000000022216805\times10^{-6}$. The values are then used in the numerical simulation. The plot of model's solution and data is given in Figure 3. With these parameter values, the reproduction number for Bali $\mathscr{R}_0=2.4939$. This means that an outbreak happens and the control needs to be implemented to minimize the risk of infections.

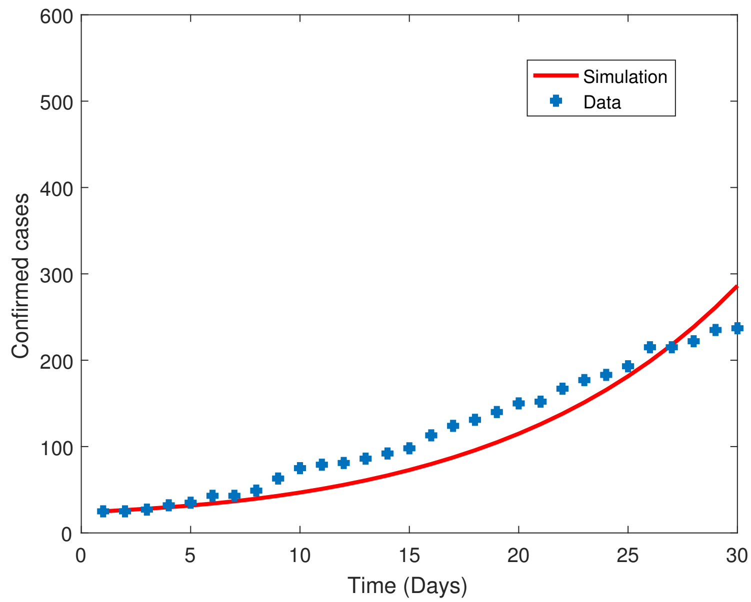


FIGURE 2

FIGURE 3. Plot of simulations and data.

3.2. Numerical results. In this section, we present sensitivity analysis, optimal control with and without budget constraint. The parameter values used are given in Table 1. The weight constant $W_1 = W_2 = W3 = W4 = 1$, and $W_5 = 150,000$ is an approximated price in Rupiah that is needed to implement the control u_1 and the value $W_6 = 1,500,000$ is approximated price in Rupiah for implementing control u_2 such as PCR test in Indonesia. Note that the weights used in the simulations are only of theoretical sense to illustrate the control strategies. For optimal control approach, we use the forward-backward sweep method implemented in MATLAB [30].

3.3. Sensitivity analysis. A global sensitivity analysis has been performed to assess the influential parameters on the reproduction number. We use the combination of Latin Hypercube Sampling and Partial Rank Correlation Coefficient (PRCC) to perform sensitivity analysis [33, 34]. Over 2000 samples are used and then PRCC values are calculated, and the result is given in Figure 4.

Figure 4 shows that the control rates, u_1 and u_2, and the transmission rate, β, are the most important parameters on the basic reproduction number. The control rates have negative relationship and the transmission rate has positive relationship, which means that when the control

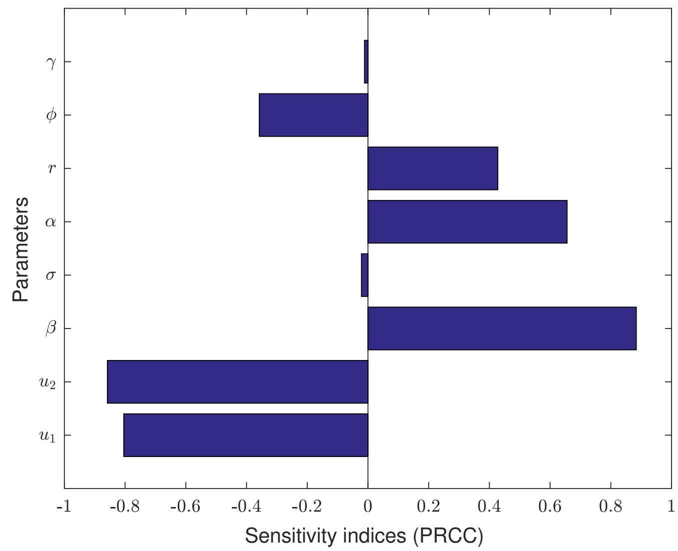


FIGURE 4. Sensitivity indices (PRCC) for the reproduction number

rates, u_1 and u_2, increase, the reproduction number decreases. The results suggest that in order to reduce the transmission, it is better to implement increase the control to reduce the transmission rate and to detect COVID-19 infections. Furthermore, reducing the transmission rate plays an important role in decreasing the reproduction number and hence reduce the probability of an outbreak. The results of sensitivity analysis suggest that implementation of both strategies are crucial to minimize the risk of infections.

3.4. **Optimal Control without constraint.**

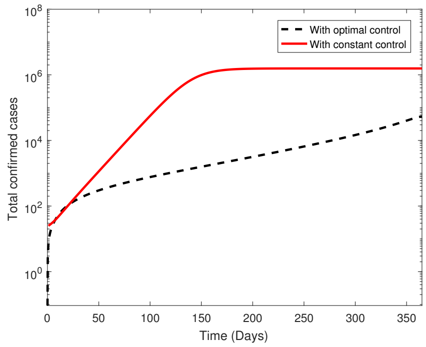
Here we present the optimal control without constraint. In our simulation, for better visibility, it is convenient to introduce the additional variable which is the cumulative number of confirmed cases as

$$C_{tot}(t) = C(0) + \int_0^t u_2((P)(s) + A(s) + I(s))ds. \tag{20}$$

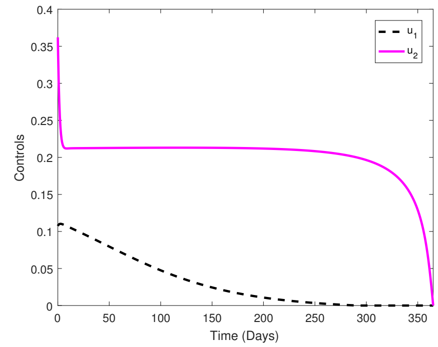
This represents the cumulative number of confirmed cases due to implementation of the control, u_2, during the period $[0 \quad T]$.

The optimal control problem

$$J(u_1, u_2) = \int_0^T (W_1 E(t) + W_2 P(t) + W_3 A(t) + W_4 I(t) + W_5 u_1^2 + W_6 u_2^2)dt \tag{21}$$



(A)



(B)

FIGURE 5. Top: Plot of the daily confirmed cases with constant and optimal control, and the control profile.

subject to Equation (1) and

$$\frac{dC_{tot}}{dt} = u_2(P + A + I) \tag{22}$$

where the variable C_{tot} has been added to the dynamical system. Equation (22) is equivalent to Equation (20). The result is given in Figure 5.

Figure 5 shows that in the implementation of optimal control, the cumulative number of confirmed cases is low in comparison to constant control which means that in the implementation of control, the infections are likely to be reduced. Furthermore, the control profiles reveal that the control rate for u_1 is lower compared to that for u_2 and therefore the implementation of control to detect COVID-19 infections should be higher to reach the optimal solutions. Table 2 presents The number of confirmed cases generated by optimal control and constant control.

Before we analyse the problem with budget constraint, we can estimate the marginal cost, X, due to implementation of control, u_2 by using the formula

$$\text{Cost of } u_2^*(t) = W_6 \int_0^T u_2^*(t)dt. \tag{23}$$

Without budget constraint, we obtain the marginal budget $X = 110,088,111.12$.

Figure 6 shows the total budget needed when applying constant control for 365 days and the cumulative number of presymptomatic cases. It shows that an increase in the constant control rate leads to a higher marginal budget. For example, if we apply constant control at a rate of 0.3,

TABLE 2. Comparison of the outcomes produced by optimal control and constant control in the number of confirmed COVID-19 infections.

	Confirmed
Optimal control	74,298
Constant control	1,562,431

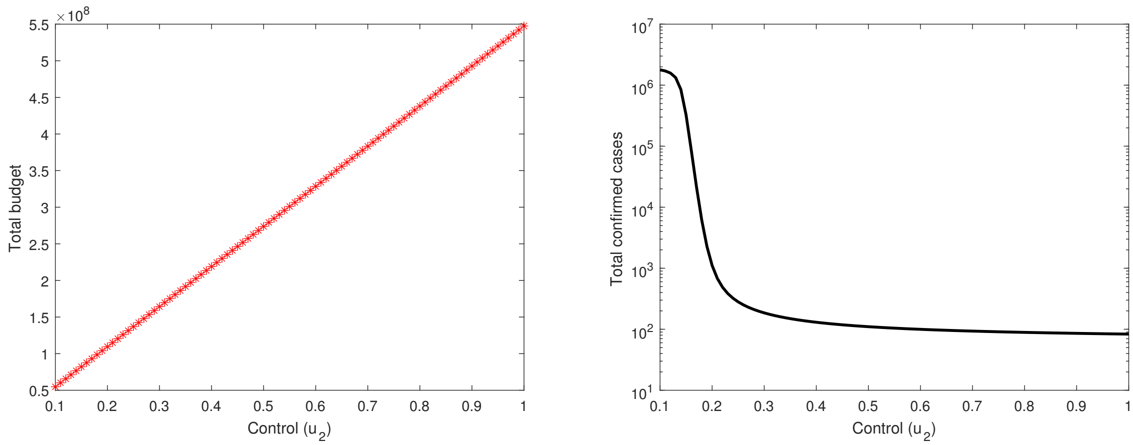


FIGURE 6. Left: Plot of total budget with constant control for 365 days. Right: Plot of total confirmed with constant control

the marginal budget is 164,350,000 which is higher than budget when applying optimal control. Although a higher constant control rate can reduce the number of confirmed cases, the budget required to reach the small infections is higher.

3.5. Optimal control with budget constraint. This section presents numerical solutions with budget constraint. We consider three cases of budget reduction: 15% (low), 35% (medium) and 60% (severe). The optimal control problem

$$J(u_1,u_2) = \int_0^T (W_1E+W_2P+W_3A+W_4I+W_5u_1^2+W_6u_2^2)dt \tag{24}$$

subject to Equations (1), and (13). Let us re-write Equation (13)

$$\frac{dZ}{dt} = W_6u_2(t), \quad Z(0)=0 \quad \text{and} \quad Z(t)=X. \tag{25}$$

The X is the budget at the end of the period and the values are given in Table 3.

TABLE 3. The marginal budget at day 365 when the budget reduction has been reduced.

Budget Reduction	X
15%	$0.85 \times 110,088,111.12$
35%	$0.65 \times 110,088,111.12$
60%	$0.40 \times 110,088,111.12$

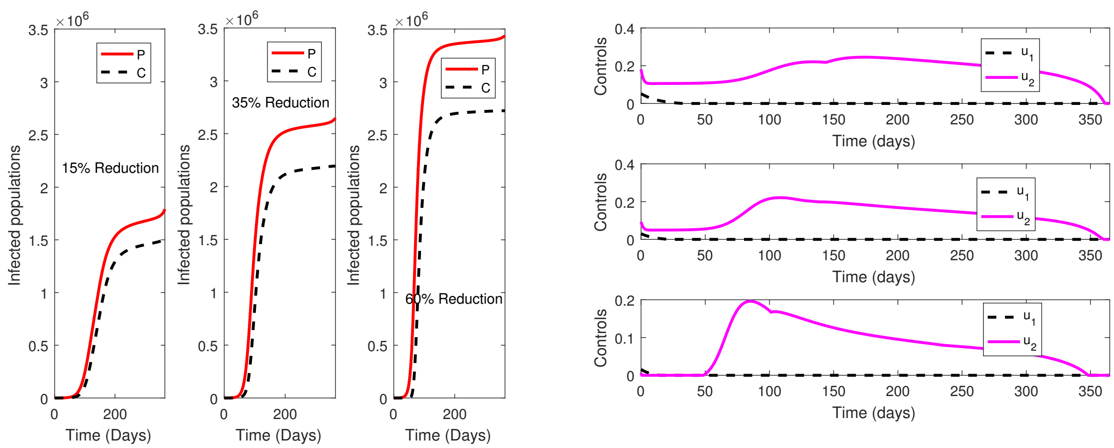


FIGURE 7. Left: Plot of cumulative number of presymptomatic and confirmed cases. Right: Plot of the control profiles for the case of 15% budget reduction (Top), 35% budget reduction (Middle), and 60% budget reduction (Bottom).

Figure 7 shows that when the budget reduction is higher, the number of infections is also higher. Furthermore, it shows that in order to reach optimal solution, the control strategy to detect infections (u_2) should be higher than that to reduce the transmission rate (u_1). A higher control rate to detect infections should be implemented around day 100 and 85 when budget reduction is 35% and 60%. It can be seen that when the budget is reduced by 60%, when the control rate reach the highest rate at around day 85, the control rate, u_2, rapidly decline.

Table 4 shows that in when the budget is reduced, the number of COVID-19 incidence increases. The result suggests that with the budget reduction, the number of undetected cases

TABLE 4. Cumulative number of confirmed and presymptomatic cases, difference and for different budget reduction

% Reduction	Cum. confirmed	Cum. presymptomatic	Difference
15%	1,493,931	1,791,337	297,406
35%	2,193,012	2,648,296	455,284
60%	2,724,976	3,437,464	712,488

increases. An increase in the number of undetected cases would contribute to further infections since they can transmit virus to the others.

4. DISCUSSION AND CONCLUSIONS

We have formulated a mathematical model for COVID-19 transmission and validated the model against data of Bali Province, Indonesia. The model is a compartment-based model in the form of system of differential equations, where the population is divided into Susceptible (S), Exposed (E), Presymptomatic (P), Asymptomatic (A), Infected (I), Confirmed (C) and Recovered (R). Using an optimal control approach, we assess the effects of the control strategies with and without budget constraint on disease transmission dynamics. There are two strategies considered: reducing the transmission rates such as mask use and social distancing, and detecting the infections such as the use of PCR or qPCR. To determine the most influential parameters on the reproduction number, a global sensitivity analysis has also been performed by using the combination of Latin Hypercube Sampling (LHS) and Partial Rank Correlation Coefficient (PRCC) multivariate analysis.

The results showed that the control rates (u_1 and u_2), and the transmission rate (β) are the influential parameters on the reproduction number where the first two have negative relationship and the latter has positive relationship. This means that an increase in the control rates and decrease in the transmission rate can minimize the reproduction number. The reduction in the reproduction number implies small probability for disease to take off. The similar result has also been found by Ndii *et al.* [25]. Furthermore, with the implementation of optimal control, the number of infections can be minimized. The results suggest the importance of the control strategy for the detection of infections. Aldila *et al.* [18] also found similar results. They showed that the implementation detection strategy such as the use of rapid testing should be done if the lockdown strategy would be relaxed. Although with the constant control implementation, the lower number of confirmed cases would be obtained, it requires higher marginal budget. This may be a problem when the budget is limited. Reduction in budget would also contribute to higher number of infections. However, the detection rate is higher. That is, higher number of infected individuals can be detected. The implementation of strategy to detect COVID-19 infections is higher than that to reduce the disease transmission to reach the optimal solutions. This may indicate the importance of the strategy to detect COVID-19 infections.

Overall, while applying the strategy to reduce the disease transmission such as the use of mask and social distancing, it is better to implement the control strategies to detect COVID-19 infections in the population. This would reduce the number of infected individuals and hence minimize the risk of infections. The limited resources for the implementation of control strategies to detect infections in the population are the challenges to be solved and this should be managed properly.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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