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A DISCRETE MATHEMATICAL MODELING FOR DRINKING ALCOHOL MODEL RESULTING IN ROAD ACCIDENTS AND VIOLENCE: AN OPTIMAL CONTROL APPROACH

LAHCEN EL YOUSSOUFI[1,*], BOUCHAIB KHAJJI[1], OMAR BALATIF[2], MOSTAFA RACHIK[1]

[1]Laboratory of LAMS, Department of Mathematics and Computer Science, Faculty of Sciences Ben M'Sik, Hassan II University, Casablanca, Morocco

[2]Laboratory of Dynamical Systems, Mathematical Engineering Team (INMA), Department of Mathematics, Faculty of Sciences El Jadida, Chouaib Doukkali University, El Jadida, Morocco

Abstract. In this paper, we propose a discrete mathematical model that describes the interaction between the classes of drinkers, namely, potential drinkers (P), moderate drinkers (M), heavy drinkers(H), heavy drinkers that practice violence (V), the individuals that practice accidents(A) and recovred and quitters of drinking (Q). We also focus on the importance of awareness programs, media and education of drinkers to aiming to find the optimal strategies to minimize the number of drinkers practice violence and accidents and maximize the number of the individuals who recovred and quitters of drinking. We use three controls which represent awareness programs and traitment through media and education for the heavy drinkers, awareness programs and security campaigns for heavy drinkers that practice violence and heavy drinkers that practice accidents and follow-up and the psychological support for temporary quitters of drinking. We use Pontryagin's maximum principle in discrete time to characterize these optimal controls. The resulting optimality system is solved numerically by Matlab. Consequently, the obtained results confirm the performance of the optimization strategy.

Keywords: mathematical model; alcohol drinking; optimal control.

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*Corresponding author

E-mail address: elyoussoufilahcen5@gmail.com

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1. Introduction

Today, different societies are facing a real and suffocating crisis, which is marked by an increase in traffic accidents and violence that claim the lives of many people. This crisis has led to the loss of a large number of segments of society, especially young people, and an increase in the number of wounded and disabled. This crisis in our country shows a clear and dangerous situation. Hundreds of young Moroccans are exposed to traffic accidents and fatal violence caused by drinking alcohol, and the lack of solutions to control drivers who are under the influence of alcohol increases this crisis. Effective interventions include the design of safer infrastructures, the integration of road safety features into land-use planning and transport, improved vehicle safety features, improved post-accident care for victims, the development and enforcement of key risk laws, and awareness-raising through the media and programs. Awareness and education [3] .

According to the latest WHO report on global alcohol consumption data for 2016, 3 million people died as a result of alcohol abuse. This represents one in 20 deaths worldwide and more than 75% of these deaths are in men. The most affected are young adults aged 20 to 29 (13.5% of deaths). Alcohol abuse and addiction causes multiple disorders such as violence, trauma, physical health problems (cirrhosis, cancer, fatty liver, stroke, diabetes)[2]. Among all these 3 million deaths, there are: 28% of injuries (road accidents, violence, suicide), 21% to pathologies affecting the digestive system, 19% to cardiovascular diseases, 32% related to infectious diseases, cancers, mental disorders or other conditions[2].

Harmful use of alcohol caused some 1.7 million deaths from noncommunicable diseases in 2016, including some 1.2 million deaths from digestive and cardiovascular diseases (0.6 million for each condition) and 0.4 million deaths from cancers. Globally an estimated 0.9 million injury deaths were attributable to alcohol, including around 370000 deaths due to road injuries, 150000 due to self-harm and around 90000 due to interpersonal violence. Of the road traffic injuries, 187000 alcohol-attributable deaths were among people other than drivers [2] WHO2016.

Some researchers in mathematics draw a comparison between the spread of the drinking phenomenon and the spread of infectious diseases. Accordingly, several mathematicians did a lot of work in order to understand the dynamics of drinking and reduce its harm on the drinker and society as well as minimizing the number of addicted drinkers. For example [18, 19, 1, 4,5,6, 7, 14]: Khajji et al [18] introduced a discrete modeling of drinking for the purpose to minimize the number of drinkers and maximize the number of the rich and poor heavy drinkers who join private and public treatment centers of alcohol addiction and subsequently the number of quitters of drinking. He had taken into account the impact of private and public addiction treatment centers on alcoholics results showed that those centers have substantial influence on the dynamics of alcoholism and can greatly impact the spread of drinking. Thus, it is crucial to urge people to know and join private and public addiction treatment centers to quit drinking. He also presented three controls which, respectively, represent awareness programs, encouragement, and follow-up. He applied the results of the control theory and he managed to obtain the characterizations of the optimal controls. Khajji et al [19] presented a continuous mathematical model $PMHT_rT_pQ$ of alcohol drinking with the influence of private and public addiction treatment centers and the dynamical behavior of the model is studied. He also studied the sensitivity analysis of model parameters to know the parameters that have a high impact on the reproduction number R0. We used the stability analysis theory for nonlinear systems to analyze the mathematical drinking model and to study both the local and global behavior of drinking dynamics. Local asymptotic stability for the drinking-free equilibrium E0 can be obtained, if the threshold quantity $\mathbf{R0} \leq \mathbf{1}$. On the other hand, if $\mathbf{R0} > \mathbf{1}$, then the alcohol present equilibrium $\mathbf{E}^*$ is locally asymptotically stable. A Lyapunov function was used to show global stability of $\mathbf{E^0}$. $\mathbf{E}^0$ is globally asymptotically stable if $\mathbf{R0} \leq \mathbf{1}$. Also a Lyapunov function was used to show global stability of $\mathbf{E}^*$. $\mathbf{E}^*$ is globally asymptotically stable if $\mathbf{R0} > \mathbf{1}$. H. F. Huo and N. Song[5] divided heavy drinkers in thier study into two types: those who confess drinking and those who do not and they proposed a two-stage model for binge drinking problem taking into consideration the transition of drinkers from the class of susceptible individuals towards the class of admitting drinkers. H. F. Huo, and Q. Wang [7] developed a non-linear mathematical model with the effect of awareness programs on the binge drinking where they show that awareness programs are an effective measure in reducing alcohol problems. H. F. Huo et al [6] proposed a new social epidemic model to depict alcoholism with media coverage which was proven to be an effective way in pushing people to quit drinking. S. H. Ma et al [14] modeled alcoholism as a contagious disease and used an optimal control to study their mathematical model with awareness programs and time delay. Wang et al [4] proposed and analyzed a non-linear alcoholism model and used optimal control for the purpose of hindering interaction between susceptible individuals and infected individuals. I. k. Adu et al [1] used a non-linear $SHTR$ mathematical model to study the dynamics of drinking epidemic, they divided their population into four classes: non-drinkers (S), heavy drinkers (H), drinkers receiving treatment (T) and recovered drinkers (R). They discussed the existence and stability of drinking-free and endemic equilibrium. Other mathematical models has also been widely used to study this phenomenon (For example, [10,12,13,20.....]).

In addition, most of these previous researches have focused on continuous-time modeling. In this research, we will adopt the discrete-time modeling as the statistical data are collected at discrete time (day, week, month and year) as well as the treatment and vaccination of some patients are done in discrete-time. So, it is more direct, more convenient, and more accurate to describe the phenomena by using the discrete-time modeling than the continuous-time modeling and the use of discrete time models may avoid some mathematical complexities such as the choice of a function space and regularity of the solution. Hence, difference equations appear as a more natural way to describe the epidemic models and discrete problems. Moreover, numerical solutions of differential equations use discretization and this encourages us to employ difference equations directly. The numerical exploration of discrete-time models is rather straightforward and therefore can be easily implemented by non mathematicians.

Besides these works, we will study the dynamics of a mathematical alcohol model $PMHVAQ$ which contains the following additions:

$\lozenge$ A discret mathematical modling.

$\lozenge$ A compartment V that represents the number of the violent heavy drinkers.

$\lozenge$ A compartment A that represents the number of the heavy drinkers who cause traffic accidents.

$\lozenge$ The death rate induced by the heavy drinkers δ_1.

$\lozenge$ The death rate induced by traffic accidents due to drinking alcohol δ_2.

$\Diamond$ The death rate induced by practice violence due to drinking alcohol δ_3.

The drinkers classes of this model are divided into sixe compartments: Potential drinkers (P), Moderate drinkers (M), Heavy drinkers (H), violent heavy drinkers (V), heavy drinkers who cause traffic accidents(A) and recovered and quitters of drinking(Q). Throughout this research, we seek to find the optimal strategies to minimize the number of heavy drinkers, violent heavy drinkers and heavy drinkers who cause traffic accidents and maximize the number of recovered and quitters of drinking (Q).

In order to achieve this purpose, we use optimal control strategies associated with three types of controls: the first represents awareness programs and traitment for heavy drinkers, the second is the effort of awareness programs and security campaigns to reduce and decrease the number of heavy drinkers to engage in violence and accidents .The third control represents the follow-up and the psychological support for temporary quitters of drinking.

The paper is organized as follows. In Section 2, we present our *PMHVAQ* mathematical model that describes the classes of drinkers. In Section 3 and 4, we present the optimal control problem for the proposed model where we give some results concerning the existence of the optimal controls and we characterize these optimal controls using Pontryagin's Maximum Principle in discrete time. Numerical simulations are given in Section 5. Finally, we conclude the paper in Section 6. We propose a discrete model *PMHVAQ* to describe the dynamics of population and the transmission of drinking. The population is divided into six compartments denoted by P, M,H,V,A and Q.

The graphical representation of the proposed model is shown in Figure1.

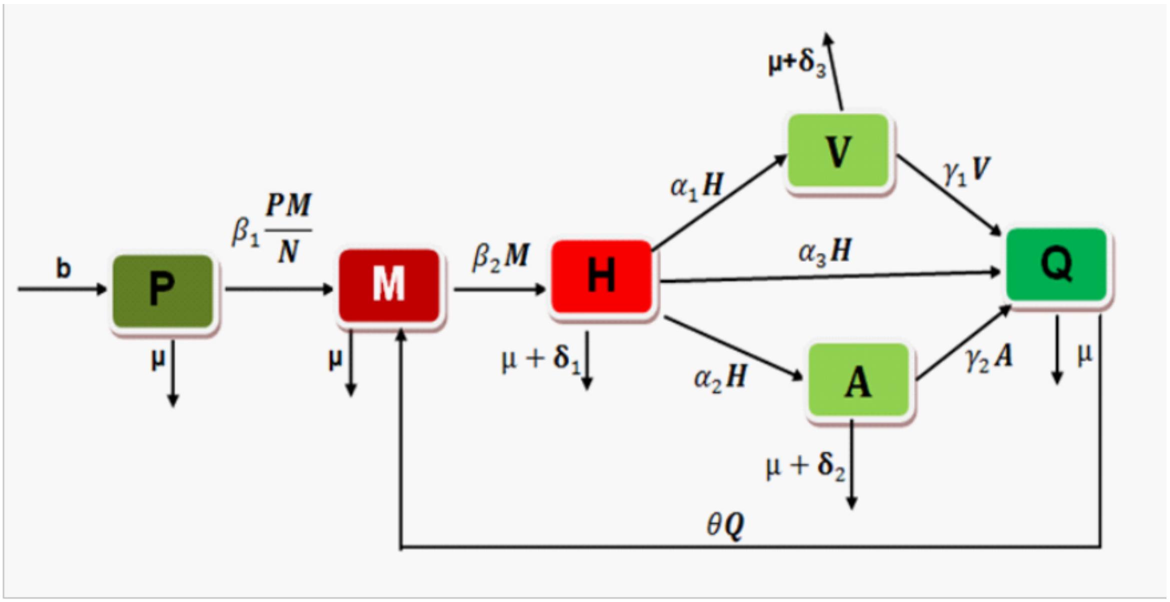

FIGURE 1. Schematic diagram of the six drinking classes in the model

The mathematical representation of the model consists of a system of non-linear difference equations:

$$(1)\quad \begin{cases} P_{k+1} = b - \beta_1 \frac{P_k M_k}{N_k} + (1-\mu) P_k \\ M_{k+1} = \beta_1 \frac{P_k M_k}{N_k} + \theta Q_k + (1-\mu-\beta_2) M_k \\ H_{k+1} = \beta_2 M_k + (1-\mu-\delta_1-\alpha_1-\alpha_2-\alpha_3) H_k \\ V_{k+1} = \alpha_1 H_k + (1-\gamma_1-\mu-\delta_3) V_k \\ A_{k+1} = \alpha_2 H_k + (1-\mu-\gamma_2-\delta_2) A_k \\ Q_{k+1} = \alpha_3 H_k + \gamma_1 V_k + \gamma_2 A_k + (1-\mu-\theta) Q_k \end{cases}$$

where $P_0 \geq 0, M_0 \geq 0, H_0 \geq 0, V_0 \geq 0, A_0 \geq 0$, and $Q_0 \geq 0$.

The compartment P: contains the potential drinkers who represent individuals whose age is over adolescence and adulthood and may become drinkers. This compartment is increased by the recruitment rate denoted by b and decreased by an effective contact with the moderate drinkers at a rate β_1 and natural death μ. It is assumed that potential drinkers can acquire drinking behavior and can become moderate drinkers through effective contact with moderate drinkers in some social occasions like weddings, celebrating graduation ceremonies, week-end parties and end of the year celebration. In other words, it is assumed that the acquisition of a drinking behavior is analogous to acquiring disease infection.

The compartment M: is composed of the moderate drinkers who drink alcohol in a controlled manner during some events and occasions or in a way that is unapparent to their social environment. It is increased by potential drinkers who turn to be moderate drinkers at a rate $\beta_1$and individuals who quit and recovred by drinking at a rate θ. This compartment is decreased when moderate drinkers become heavy drinkers at a rate β_2, and by natural death at rate μ.

The compartment H: comprises the heavy drinkers. This compartment becomes larger as the number of heavy drinkers increases by the rate β_2 and decreases when some of them give up drinking who quit drinking at a rate α_3 and outher becomes heavy drinkers with violence (V) at rate $\alpha_1(\alpha_1$ is a rate of heavy drinkers becomes heavy drinkers with violence) and some becomes heavy drinkers with accidents (A) at rate $\alpha_2(\alpha_2$ is a rate of heavy drinkers becomes individuals with accidents). In addition, this compartment decreases by natural death μ and due to deaths caused by diseases resulted from excessive alcohol intake at a rate δ_1.

The compartment V: represents the number of the violent heavy drinkers that commits various types of violence due to drinking alcohol. This compartment is increased by the rate α_1 and decreased by the rates γ_1 and $(\mu+\delta_3)$, where $\gamma_1$is the individiuals violent heavy drinkers who quit and recovred of drinking and $\delta_3$is the death rate induced by the heavy drinkers with violence.

The compartment A: contains the number of individuals of the heavy drinkers who cause traffic accidents that commits various types of traffic accidents due to drinking alcohol.This compartment is increased by the rates α_2 and decreased by the rates γ_2 and $(\mu+\delta_2)$, where γ_2 is a rate of the individuals heavy drinkers with accidents who quit and recovred of drinking.

The compartment Q: encompasses the individuals who recovered from violence and accidents and quitters of drinking. It is increased with the recruitment of individuals who have been treated at rates $\gamma_1$and γ_2. It also increases at the rate α_3 of those who quit and recovred alcohol and decreases by the rates μ and θ (θ is a rate of quitters of drinking that who becomes moderate drinkers).

The total population size at time k is denoted by N_k with $N_k = P_k + M_k + H_k + V_k + A_k + Q_k$ and it is supposed as constant.

2. THE OPTIMAL CONTROL PROBLEM

The objective of the proposed control strategy is to minimize the number of heavy drinkers, violent heavy drinkers and heavy drinkers who cause traffic accidents and thus we will maximize the number of recovered and quitters of drinking in the community during the time step $k = 0$ to $k = T$. The cost spent in the awareness programs and treatment are also to be minimized.

In order to achieve these objectives, we introduce three control variables. The first control u_1 represents the effort of the awareness programs and traitment to protect the heavy drinkers not to be drinkers. The second control u_2 measures the effort the awereness programs and security campaigns effort (education programs, media...) applied on the heavy drinkers for decrease the number of the violent heavy drinkers and heavy drinkers who cause traffic accidents. Finaly, u_3 measures the effort of follow-up and the psychological support for temporary quitters of drinking.

So, the controlled mathematical system is given by the following system of difference equations:

$$\left\{\begin{array}{l} P_{k+1}=b-\beta_1\frac{P_kM_k}{N_k}+(1-\mu)P_k \\ M_{k+1}=\beta_1\frac{P_kM_k}{N_k}-\beta_2M_k+(1-\mu)M_k+(1-u_{3,k})\theta Q_k+\varepsilon u_{1,k}H_k \\ H_{k+1}=\beta_2M_k+(1-\mu-\delta_1)H_k-\alpha_1(1-u_{2,k})H_k-\alpha_2(1-u_{2,k})H_k-\alpha_3H_k-u_{1,k}H_k \\ V_{k+1}=\alpha_1(1-u_{2,k})H_k+(1-\mu-\gamma_1-\delta_3)V_k \\ A_{k+1}=\alpha_2(1-u_{2,k})H_k+(1-\mu-\gamma_2-\delta_2)A_k \\ Q_{k+1}=\gamma_1V_k+\gamma_2A_k+(1-\mu)Q_k+\alpha_3H_k-(1-u_{3,k})\theta Q_k+(1-\varepsilon)u_{1,k}H_k \end{array}\right. \tag{2}$$

where $P_0 \geq 0, M_0 \geq 0, H_0 \geq 0, V_0 \geq 0, A_0 \geq 0$ and $Q_0 \geq 0$.

The optimal control problem to minimize the objective functional is given by:

$$\begin{aligned} J(u_1,u_2,u_3) &= H_T+V_T+A_T \\ &\quad +\sum_{k=0}^{T-1}\left(H_k+V_k+A_k+\frac{A_{1,k}u_{1,k}^2}{2}+\frac{A_{2,k}u_{2,k}^2}{2}+\frac{A_{3,k}u_{3,k}^2}{2}\right) \end{aligned} \tag{3}$$

Where the parameters $A_{1,k} > 0$, $A_{2,k} > 0$ and $A_{3,k} > 0$ are selected to weigh the relative importance of the cost of awareness programs and treatment respectively.

The aim is to find an optimal control u_1^*, u_2^* and u_3^* such that :

$$J(u_1^*, u_2^*, u_3^*) = \min_{(u_1,u_2,u_2)\in U_{ad}^3} J(u_1, u_2, u_3) \tag{4}$$

where U_{ad} is the set of admissible controls defined by

$$U_{ad} = \left\{(u_{1,k}, u_{2,k}, u_{3,k}) / \ 0 \leq u_{j\min} \leq u_{j,k} \leq u_{j\max} \leq 1;\ j = 1,2,3 \text{ and } k = 0,1,2,...,T-1\right\} \tag{5}$$

The sufficient condition for the existence of an optimal control (u_1^*, u_2^*, u_3^*) for problem (2) and (3) comes from the following theorem.

Theorem 2.1. *There exists the optimal control* (u_1^*, u_2^*, u_3^*) *such that*

$$J(u_1^*, u_2^*, u_3^*) = \min_{(u_1,u_2,u_3)\in U_{ad}^3} J(u_1, u_2, u_3) \tag{6}$$

subject to the control system (2) *with initial conditions.*

Proof. Since the coefficients of the state equations are bounded and there are a finite number of time steps, $P = (P_0, P_1, P_2, ..., P_T)$, $M = (M_0, M_1, M_2, ..., M_T)$, $H = (H_0, H_1, H_2, ..., H_T)$, $V = (V_0, V_1, V_2, ..., V_T)$, $A = (A_0, A_1, A_2, ..., A_T)$ and $Q_2 = (Q_0, Q_1, Q_2, ..., Q_T)$ are uniformly bounded for all (u_1, u_2, u_3) in the control set U_{ad};thus $J(u_1, u_2, u_3)$ is bounded for all $(u_1, u_2, u_3) \in U_{ad}^3$. Since $J(u_1, u_2, u_3)$ is bounded, $\inf_{(u_1,u_2,u_3)\in U_{ad}^3} J(u_1, u_2, u_3)$ is finite, and there exists a sequence $(u_1^j, u_2^j, u_3^j) \in U_{ad}^3$ such that $\lim_{j\to+\infty} (u_1^j, u_2^j, u_3^j) = \inf_{(u_1,u_2,u_3)\in U_{ad}^3} J(u_1, u_2, u_3)$ and corresponding sequences of states P^j, M^j, H^j, V^j, A^j and Q^j.Since there is a finite number of uniformly bounded sequences, there exist $(u_1^*, u_2^*, u_3^*) \in U_{ad}^3$ and P^*, M^*, H^*, V^*, A^* and $Q^* \in IR^{T+1}$ such that, on a subsequence, $\lim_{j\to+\infty} (u_1^j, u_2^j, u_3^j) = (u_1^*, u_2^*, u_3^*)$, $\lim_{j\to+\infty} P^j = P^*$, $\lim_{j\to+\infty} M^j = M^*$, $\lim_{j\to+\infty} H^j = H^*$, $\lim_{j\to+\infty} V^j = V^*$, $\lim_{j\to+\infty} A^j = A^*$ and $\lim_{j\to+\infty} Q^j = Q^*$.Finally, due to the finite dimensional structure of system (2) and the objective function $J(u_1, u_2, u_3)$, (u_1^*, u_2^*, u_3^*) is an optimal control with corresponding states P^*, M^*, H^*, V^*, $A^*$and Q^*.Therefore $\inf_{(u_1,u_2,u_3)\in U_{ad}^3} J(u_1, u_2, u_3)$ is achieved. □

3. Characterization of the Optimal Controls

We apply the discrete version of Pontryagin's Maximum Principle [2,3,8,11,16,28]. The key idea is introducing the adjoint function to attach the system of difference equations to the objective functional resulting in the formation of a function called the Hamiltonian. This principle converts the problem of finding the control to optimize the objective functional subject to the state difference equation with initial condition to find the control to optimize Hamiltonian pointwise (with respect to the control).

Now we have the Hamiltonian $\hat{H}$ at time step k, defined by:

$$\hat{H}_k = H_k + V_k + A_k + \frac{A_{1,k}u_{1,k}^2}{2} + \frac{A_{2,k}u_{2,k}^2}{2} + \frac{A_{3,k}u_{3,k}^2}{2} + \sum_{i=1}^{6} \lambda_{i,k+1} f_{i;k+1} \tag{7}$$

where $f_{i;k+1}$ the right-hand side of the system of difference equations (2) of the i^{th} state variable at time step $k+1$.

Theorem 3.1. *Given an optimal control $(u_1^*, u_2^*, u_3^*) \in U_{ad}^3$ and solutions P_k^* ,M_k^* ,H_k^* ,V_k^* ,$A_k^*$and Q_k^* of corresponding state system (2), there exist adjoint functions λ_1, λ_2, λ_3, λ_4, λ_5 and λ_6 satisfying the equations:*

$$\lambda_{1,k} = \frac{\partial \hat{H}_k}{\partial P_k} = (\lambda_{2,k+1} - \lambda_{1,k+1})\beta_1 \frac{M_k}{N_k} + \lambda_{1,k+1}(1-\mu).$$

$$\lambda_{2,k} = \frac{\partial \hat{H}_k}{\partial M_k} = (\lambda_{2,k+1} - \lambda_{1,k+1})\beta_1 \frac{P_k}{N_k} + \beta_2(\lambda_{3,k+1} - \lambda_{2,k+1}) + \lambda_{2,k+1}(1-\mu)$$

$$\lambda_{3,k} = \frac{\partial \hat{H}_k}{\partial H_k} = 1 + (1-u_{2,k})(\alpha_1\lambda_{4,k+1} + \alpha_2\lambda_{5,k+1} - \alpha_1\lambda_{3,k+1} - \alpha_2\lambda_{3,k+1}) + \tag{8}$$

$$\alpha_3(\lambda_{6,k+1} - \lambda_{3,k+1}) + (1-\mu-\delta_1)\lambda_{3,k+1} + u_{1,k}(\varepsilon\lambda_{2,k+1} - \lambda_{3,k+1} + (1-\varepsilon)\lambda_{6,k+1}) \tag{9}$$

$$\lambda_{4,k} = \frac{\partial \hat{H}_k}{\partial V_k} = 1 + \gamma_1(\lambda_{6,k+1} - \lambda_{4,k+1}) + (1-\mu-\delta_3)\lambda_{4,k+1}. \tag{10}$$

$$\lambda_{5,k} = \frac{\partial \hat{H}_k}{\partial A_k} = 1 + \gamma_2(\lambda_{6,k+1} - \lambda_{5,k+1}) + (1-\mu-\delta_2)\lambda_{5,k+1}.$$

$$\lambda_{6,k} = \frac{\partial \hat{H}_k}{\partial Q_k} = \theta(1-u_{3,k})(\lambda_{3,k+1} - \lambda_{6,k+1}) + \lambda_{6,k+1}(1-\mu).$$

with the transversality conditions at time T

$$\lambda_1(T) = 0, \lambda_2(T) = 0, \lambda_3(T) = 1, \lambda_4(T) = 1, \lambda_5(T) = 1 \text{ and } \lambda_6(T) = 0 \tag{11}$$

Furthermore, for $i = 0, 1, ..., T-1$ *we obtain the optimal control* $(u^*_{1,k}, u^*_{2,k}, u^*_{3,k})$ *as*

$$\begin{aligned} u^*_{1,k} &= \min\left\{\max(u_{1\min}, \frac{(\lambda_{3,k+1} - \varepsilon\lambda_{2,k+1} - (1-\varepsilon)\lambda_{6,k+1})H_k}{A_{1,k}}), u_{1\max}\right\} \\ u^*_{2,k} &= \min\left\{\max(u_{2\min}, \frac{\alpha_1(\lambda_{4,k+1} - \lambda_{3,k+1})H_k + \alpha_2(\lambda_{5,k+1} - \lambda_{3,k+1})H_k}{A_{2,k}}), u_{2\max}\right\} \\ u^*_{3,k} &= \min\left\{\max(u_{3\min}, \frac{(\lambda_{2,k+1} - \lambda_{6,k+1})\theta Q_k}{A_{3,k}}), u_{3\max}\right\} \end{aligned} \tag{12}$$

Proof. The Hamiltonian $\hat{H}_k$ at time step k is given by

$$\begin{aligned} \hat{H}_k = &\ H_k + V_k + A_k + \frac{A_{1,k}u^2_{1,k}}{2} + \frac{A_{2,k}u^2_{2,k}}{2} + \frac{A_{3,k}u^2_{3,k}}{2} \\ &+\lambda_{1,k+1}\left[b - \beta_1\frac{P_kM_k}{N_k} + (1-\mu)P_k\right] \\ &+\lambda_{2,k+1}\left[\beta_1\frac{P_kM_k}{N_k} + (1-\mu-\beta_2)M_k + (1-u_{3,k})\theta Q_k + \varepsilon u_{1,k}H_k\right] \\ &+\lambda_{3,k+1}\left[\beta_2M_k + (1-\mu-\delta_1)H_k - \alpha_1(1-u_{2,k})H_k - \alpha_2(1-u_{2,k})H_k - \alpha_3H_k - u_{1,k}H_k\right] \\ &+\lambda_{4,k+1}\left[\alpha_1(1-u_{2,k})H_k + (1-\mu-\gamma_1-\delta_3)V_k\right] \\ &+\lambda_{5,k+1}\left[\alpha_2(1-u_{2,k})H_k + (1-\mu-\gamma_2-\delta_2)A_k\right] \\ &+\lambda_{6,k+1}\left[\gamma_1V_k + \gamma_2A_k + (1-\mu)Q_k + \alpha_3H_k - (1-u_{3,k})\theta Q_k + (1-\varepsilon)u_{1,k}H_k\right]. \end{aligned} \tag{13}$$

For $k = 0, 1, ..., T-1$, the adjoint equations and transversality conditions can be obtained by using Pontryagin's Maximum Principle, in discrete time, given in [2,3,8,11,16] such that

$$\begin{aligned} \lambda_{1,k} &= \frac{\partial \hat{H}_k}{\partial P_k},\ \lambda_1(T) = 0 \\ \lambda_{2,k} &= \frac{\partial \hat{H}_k}{\partial M_k},\ \lambda_2(T) = 0 \\ \lambda_{3,k} &= \frac{\partial \hat{H}_k}{\partial H_k},\ \lambda_3(T) = 1 \\ \lambda_{4,k} &= \frac{\partial \hat{H}_k}{\partial V_k},\ \lambda_4(T) = 1 \\ \lambda_{5,k} &= \frac{\partial \hat{H}_k}{\partial A_k},\ \lambda_5(T) = 1 \\ \lambda_{6,k} &= \frac{\partial \hat{H}_k}{\partial Q_k},\ \lambda_6(T) = 0 \end{aligned} \tag{14}$$

For $k = 0, 1, ..., T-1$, the optimal controls $u^*_{1,k}$, $u^*_{2,k}$ and $u^*_{3,k}$ can be solved from the optimality condition

$$\begin{aligned}
\frac{\partial \hat{H}_k}{\partial u_{1,k}} &= A_{1,k}u_{1,k} + \lambda_{2,k+1}\varepsilon H_k - \lambda_{3,k+1}H_k + \lambda_{6,k+1}(1-\varepsilon)H_k = 0 \\
\frac{\partial \hat{H}_k}{\partial u_{2,k}} &= A_{2,k}u_{2,k} + \alpha_1\lambda_{3,k+1}H_k - \alpha_2\lambda_{5,k+1}H_k + \alpha_2\lambda_{3,k+1}H_k - \alpha_1\lambda_{4,k+1}H_k = 0 \\
\frac{\partial \hat{H}_k}{\partial u_{3,k}} &= A_{3,k}u_{3k} + \lambda_{6,k+1}\theta H_k - \lambda_{2,k+1}\theta H_k = 0 \qquad (15)
\end{aligned}$$

So, we obtain:

$$\begin{aligned}
u_{1,k} &= \frac{\left[\lambda_{3,k+1} - \varepsilon\lambda_{2,k+1} - (1-\varepsilon)\lambda_{6,k+1}\right] H_k}{A_{1,k}} \\
u_{2,k} &= \frac{\alpha_1(\lambda_{4,k+1} - \lambda_{3,k+1})H_k + \alpha_2(\lambda_{5,k+1} - \lambda_{3,k+1})H_k}{A_{2,k}} \qquad (16) \\
u_{3,k} &= \frac{(\lambda_{2,k+1} - \lambda_{6,k+1})\theta Q_k}{A_{3,k}}
\end{aligned}$$

By the bounds in U_{ad} of the controls, it is easy to obtain $u^*_{1,k}$,$u^*_{2,k}$ and $u^*_{3,k}$ in the form of (12). □

4. Numerical Simulation

In this section, we shall solve numerically the optimal control problem for our *PMHVAQ* model. Here, we obtain the optimality system from the state and adjoint equations. The proposed optimal control strategy is obtained by solving the optimal system which consists of six difference equations and boundary conditions. The optimality system can be solved by using an iterative method. Using an initial guess for the control variables, $u_{1,k}$,$u_{2,k}$ and $u_{3,k}$, the state variables, P, M, H, V, A and Q are solved forward and the adjoint variables λ_i for $i = 1,2,3,4,5,6$ are solved backwards at times step $k = 0$ and $k = T$. If the new values of the state and adjoint variables differ from the previous values, the new values are used to update $u_{1,k}$, $u_{2,k}$ and $u_{3,k}$, and the process is repeated until the system converges.

The numerical solution of model (1) is executed using Matlab with the following parameter values and initial values of state variable: $P_0 = 500$, $M_0 = 300$, $H_0 = 100$, $V_0 = 60$, $A_0 = 30$, $Q_0 = 10$, $b = 100$, $N = 1000$, $\mu = 0.065$, $\beta_1 = 0.75$, $\beta_2 = 0.14$, $\alpha_1 = 0.03$, $\alpha_2 = 0.02$, $\alpha_3 = 0.02$, $\gamma_1 = 0.001$, $\gamma_2 = 0.001$, $\delta_1 = 0.02$, $\delta_2 = 0.002$, $\delta_3 = 0.001$, $\theta = 0.1$.

We begin by presenting the solution evolution of our model (1) with and without controls that are represented in Figures2.

The proposed control strategy in this work helps to achieve several objectives:

First objective: Traitment the heavy drinkers and encouragement them for quitters of drinking or return for moderate drinkers.

To realize this objective, we apply only the control u_1 i.e. the implementation of awareness, information and educational programs, traitment on heavy drinkers to make them know the risks of this phenomenon and the resulting health and social damages. Figure2(a) shows that the number of heavy drinkers decreases from 388.14 (without control) to 264.71 (with control) at the end of the proposed control strategy. Figure2(b) shows that the number of heavy drinkers with violence decreases from 173.57 (without control) to 118.26 (with control) at the end of the proposed control strategy. Also, we observe in Figure2(c) that the number of individuals heavy drinkers with accidents decreases and has reached the value 77.68 (with control) compared to the situation when there is no control 114.01 at the end of the proposed strategy. Figure2(d) shows that the number of recovred and quitters of drinking decreases from 48.78 (without control) to 225.76 (with control) at the end of the proposed control strategy. So, our objective has been achieved.

Second objective: Protecting and preventing heavy drinkers from falling into violence and accidents.

To achieve this objective, we only use the control u_2 i.e. the implementation of awareness programs and security campaigns on heavy drinkers to make them know the risks of this phenomenon and the resulting health and social damages caused by violence and accidents. In Figure3(a), it is observed that there is a significant decrease in the number of heavy drinkers with violence with control compared to a situation when there is no control where the decrease reaches 45% at the end of the proposed control strategy. Figure 3(b) shows that the number of the heavy drinkers with accidents decreased from 114.01 (without control) to 85.84 (with control) at the end of the proposed control. Figure 3(c) shows that the number of the heavy drinkers increased from 388.14 (without control) to 487.54 (with control) at the end of the proposed control. Figure 3(d) shows that the number of people who recovred and quitters of

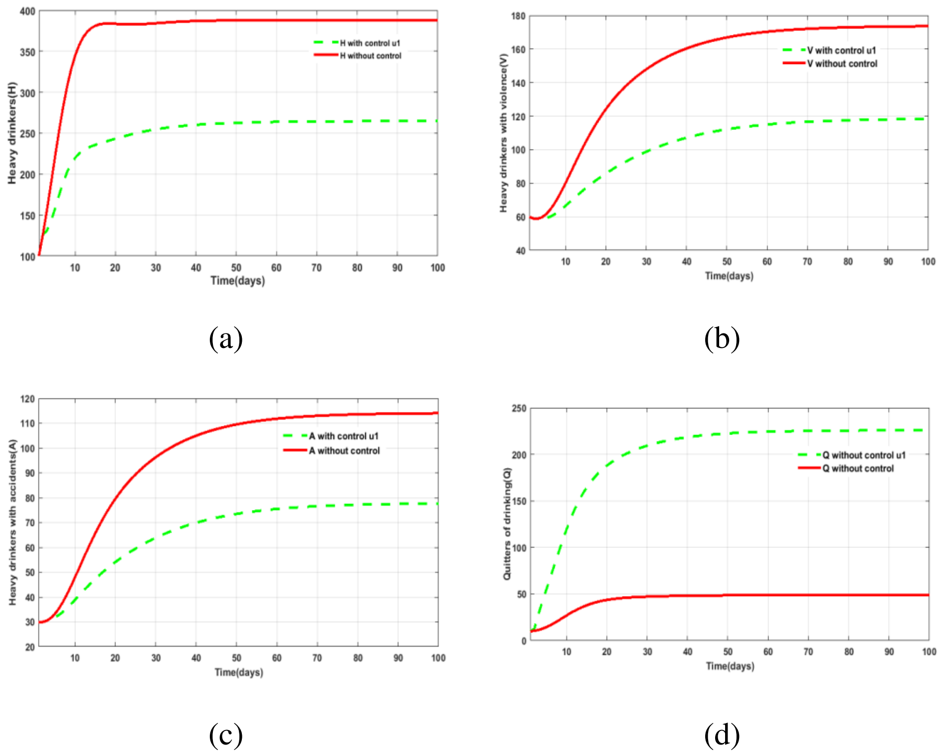

FIGURE 2. represents the drinkers class with and without control u_1

drinking without control increases and approaches a value of 120. It's increase appears with control u_2.

Third objective: Protecting, preventing and traitment the heavy drinkers with and without violence and accidents from falling into accidents and violence.

To meet this objective, we use the controls u_1 and u_2 i.e. the implementation of awareness programs and traitment on heavy drinkers to make them know the risks of this phenomenon and the resulting health and social damages caused by accidents and violence. Figure $4(a)$ shows that the number of the heavy drinkers decreases starting from the early days a value 388.14 (without controls) to 291.12 (with controls). Also, Figure $4(b)$ shows that the number of the heavy drinkers with violence decreases from 173.57 (without controls) to 78.02 (with controls).The number of the individuals with accidents decreases from 173.57 (without controls) to 78.02 (with controls) (See Figure $4(c)$). Figure $4(d)$ depicts clearly an increase in the number of the recoverd and quitters of drinking from 48.78 (without controls) to 247.74 (with controls). As a result, the objective set before has been achieved.

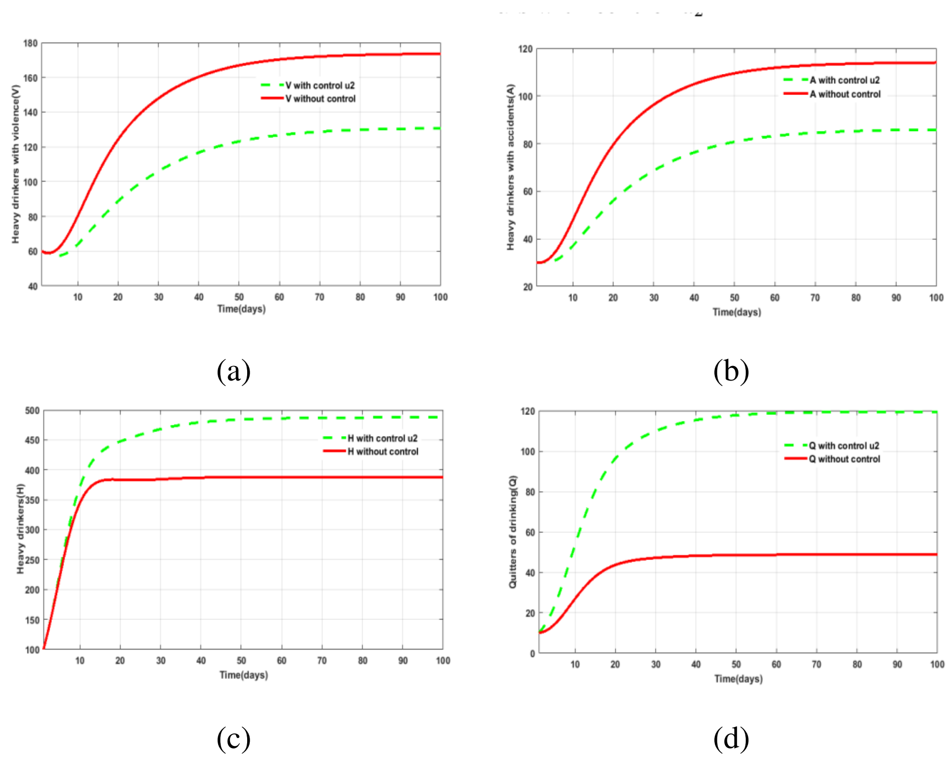

FIGURE 3. represents the drinkers class with and without control u_2

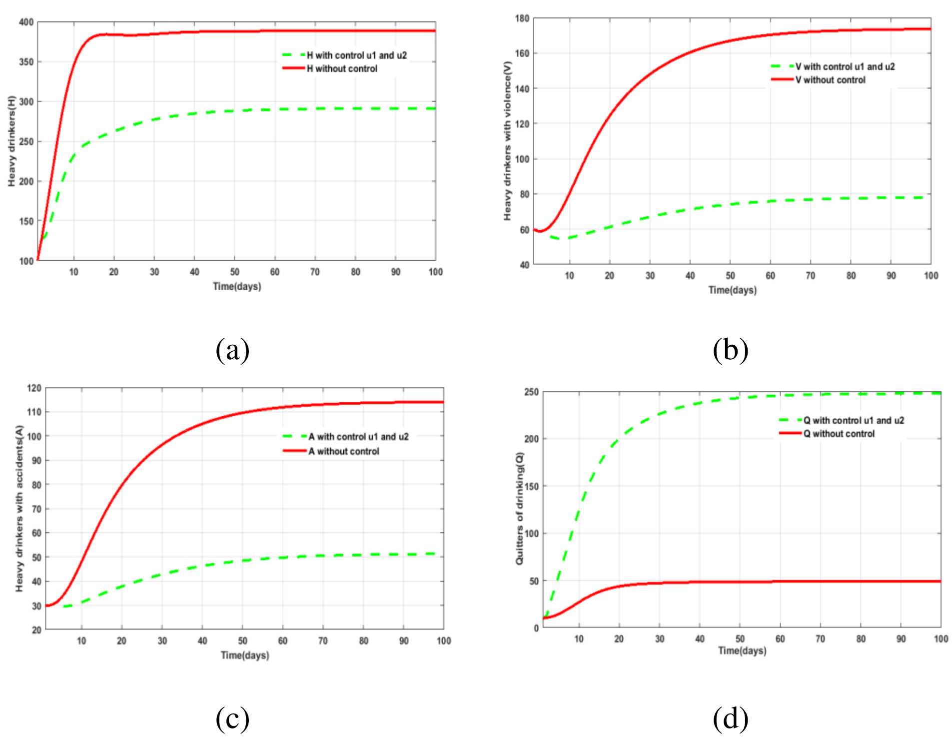

FIGURE 4. out optimal control $u_1$and u_2

Forth objective: Protecting and preventing heavy drinkers from falling into alcohol addiction, violence and accidents and follow-up the quit temporarly.

To meet this objective, we use the controls u_1, u_2 and u_3 i.e. the implementation of awareness programs and traitment on heavy drinkers to make them know the risks of this phenomenon and the resulting health and social damages caused by violence and accidents and follow-up the quit temporarly to not return drinking. Figure $5(a)$ shows that the number of the heavy drinkers decreases starting from the early days a value 388.14 (without controls) to 267.22 (with controls). Also, Figure $5(b)$ shows that the number of the heavy drinkers with violence decreases from 173.57 (without controls) to 71.64 (with controls).The number of the individuals with accidents decreases a value 114.01 (without controls) to 58.35 (with controls) (See Figure $5(c)$). Figure $5(d)$ depicts clearly an increase in the number of the recoverd and quitters of drinking from 48.78 (without controls) to 326.21(with controls). As a result, the objective set before has been achieved.

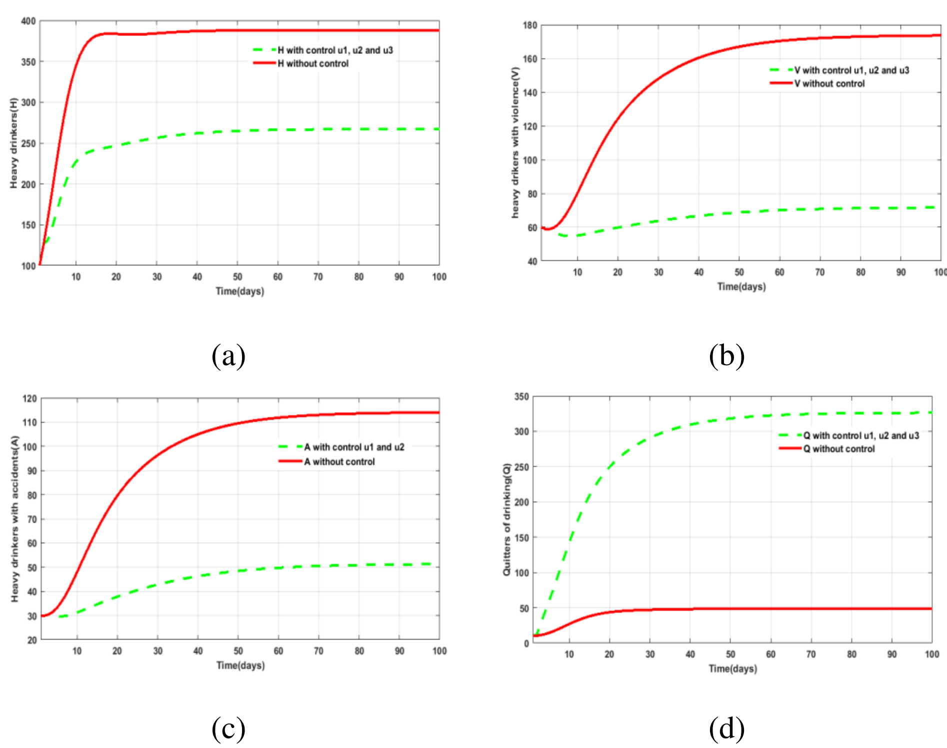

FIGURE 5. represents the drinkers class with and without optimal control u_1, u_2 and u_3

5. Conclusion

In this research paper, we introduced a discrete mathematical modeling of drinking alcohol resulting in roed accidents and violence for the purpose of minimizing the number of the heavy drinkers, violent heavy drinkers and heavy drinkers cause roed accidents and maximizing the number of recovered and quitters of drinking. Unlike some other previous models, we have taken into account the impact the awareness programs, media, education, traitment and security campaigns for on alcoholics and follow-up and the psychological support for temporary quitters of drinking. The results showed that awareness programs, media, education, traitment and security campaigns and follow-up and the psychological support haves substantial influence on the dynamics of alcoholism and can greatly impact the spread of the drinking, thus, it is crucial to urge people to know alcoholism complications and the consequences of traffic accidents and violence due to alcohol drinking. We also presented three controls which represent traitment and awareness programs for heavy drinkers, traitment and security campaigns for violent heavy drinkers and heavy drinkers cause roed accidents and follow-up and psychological support for temporarly quitters of drinking. We applied the results of the control theory and we managed to obtain the characterizations of the optimal controls. The numerical simulation of the obtained results showed the effectiveness of the proposed control strategies.

Conflict of Interests

The authors declare that there is no conflict of interests.

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