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GEOGRAPHICALLY WEIGHTED BIVARIATE POISSON INVERSE GAUSSIAN REGRESSION MODELING WITH THE BERNDT-HALL-HALL-HAUSMAN ALGORITHM ON MATERNAL AND NEONATAL MORTALITY DATA

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Abstract: Maternal and neonatal mortality rates in South Sulawesi remain higher than the national average across all provinces in Indonesia. This study aims to identify significant variables for each district/city in South Sulawesi, Indonesia. The data used was overdispersed in the two cases which correlated and distributed Poisson. The Gaussian Poisson Inverse Bivariate Regression can be used to solve the problem but cannot solve the problem of spatial heterogeneity. Spatial heterogeneity causes bias in the interpretation of results. The method to overcome this problem is the Geographycally Weighted Bivariate Poisson Inverse Gaussian method. The Berndt-Hall-Hall-Hausman algorithm is used in the parameter estimation of the GWBPIGR model. The Kernel functions used are Adaptive Bisquare, Adaptive Tricube, and Fixed Gaussian. Generalized Cross Validation (GCV) is used to select the optimal bandwidth. The results of this study show that the Akaike Information Criterion (AIC) value in the GWBPIGR model with the Berndt-Hall-Hall-Hausman algorithm is better than that of BPIGR with the same algorithm.

Keywords: GWBPIGR; overdispersion; heterogeneity; Berndt-Hall-Hall-Hausman algorithm; maternal death; neonatal death.

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1. INTRODUCTION

One technique for examining the connection between response variables is poisson regression and predictor variables distributed by Poisson. Poisson regression requires the assumption that the mean and variance of the response variables must be the same (equidispersion). However, in reality, there is often a violation of assumptions, namely variance greater than the mean (overdispersion) [1-2].

Combining the poisson distribution with multiple discrete and continuous distributions (mixed Poisson distribution) is one way to get around the overdispersion issue. The Poisson Inverse Gaussian (PIG) distribution is one of the mixed Poisson distributions that is frequently employed in studies to address overdispersion situations. Poisson and inverse Gaussian distributions are combined to create PIG. PIG is used because it is more sensitive to overcome overdispersion than the Binomial Negative method [3-5].

In the PIG method, there is a stage of parameter stimulation using the Maximum Likelihood Estimation (MLE) technique which functions to maximize the likelihood function. In this process, not everything can be solved by analytical means. If implicit and non-linear forms are obtained, it can be solved using the Berndt-Hall-Hall-Hausman (BerndHallMan) algorithm. BerndHallMan's algorithm is a development of Newton Raphson' s algorithm.Newton Raphson's algorithm has a Hessian matrix whose content is second derived so that the iteration is more complex. Therefore, it was developed into the BerndHallMan algorithm which requires only the first derivative in a simpler Hessian matrix [6]. Therefore, it was developed into a BerndHallMan algorithm that requires only the first derivative of its Hessian matrix. One study found BerndHallMan's algorithm to be better than Newton Raphson's algorithm [7].

In fact, there are response variables that are interrelated with other response variables. The application of two regressions to the number of paired co-events results in inconsistent and inefficient estimators. So it is better to estimate together compared to separately [8]. One example of data fragmentation in the health environment is the case of maternal mortality and neonatal death. Maternal mortality and neonatal death are two things that are interconnected because nutritional status and maternal health are closely related to the health of the baby in the womb. The 2019 Indonesia Health Demographic Survey indicates that the maternal mortality rate was 359 per 100,000 live births, while the infant mortality rate was 34 per 1,000 live births, showing that these figures are still well below the MDGs target. Half of the infant deaths occurred during the neonatal period. The age of babies 0-28 days (neonatal period) is the most spanning period to be affected by various health problems. 80% occur in the first six days of life [9]. Based on this, the results of data exploration show that the cases are correlated with each other and overdispersed, so the analysis used is called Poisson Inverse Gaussian Bivariate Regression (BPIG) [10]. One of the researchers found that the BPIGR method for maternal mortality and neonatal mortality cases in South Sulawesi produced a model that only represented all districts/cities in South Sulawesi and did not explain the influence of predictor variables in each district/city in South Sulawesi [11]. Thus, the previous method was developed into a method that can overcome the spatial effects that occur on the data, namely the Geographically Weighted Bivarite Poisson Inverse Regression (GWBPIGR) method. Based on this background, the author will examine the application of the Geographycally Weighted Bivariate Poisson Inverse Gaussian Regression (GWBPIGR) method on data that is overdispersed using the BerndHallMan algorithm in maternal and neonatal mortality cases in South Sulawesi in 2021.

2. MATERIALS AND METHODS

Spatial Heterogeneity Test

The spatial heterogeneity test aims to find out whether the observation location has different characteristics or not. Spatial heterogeneity can be identified using the Breusch Pagan test with the following hypotheses:

$H_0 : \sigma^2(u_i, v_i) = \sigma^2$ (No spatial heterogeneity occurs)

$H_1 : \sigma^2(u_i, v_i) \neq \sigma^2$ (Spatial heterogeneity occurs)

Test statistics

$$\text{BP} = \frac{1}{2}\mathbf{f}^{\mathbf{T}}\mathbf{Z}(\mathbf{Z}^{\mathbf{T}}\mathbf{Z})^{-\mathbf{1}}\mathbf{Z}^{\mathbf{T}}$$

With vector $\mathbf{f}$ Sized $n \times 1$

$$\mathbf{f} = \left[\left(\frac{e_1^2}{\sigma^2}\right) - 1, \dots, \left(\frac{e_n^2}{\sigma^2}\right) - 1\right]^T$$

To

Z : Dimensional matrix n x (p + 1) which contains independent variables that have been standardized for each observation

σ^2: residual variety (e_i) from Poisson's regression model

With rejection criteria, rejected H_0 if the value $BP > \chi^2_{(\alpha;p)}$ or p-value $< \alpha$ [12].

Overdispersion Test

In the Poisson regression model, there are a number of assumptions that must be met. One of them is the assumption of the same mean and variance called equidispersion. However, in statistical data analysis, data conditions are often found with variances sometimes greater than the mean (overdispersion). Overdispersion can lead to inefficient estimation of the parameters obtained. Incorrect use of Poisson's regression model (which is overdispersed) can be fatal in model interpretation, especially in estimating model parameters because it can estimate too low a standard error and can convey incorrect conclusions about the significance or not. Overdispersion can be written as Var(Y)>E(Y). In detecting cases of overexpression in the data, it can be seen that the statistical value of deviation and Pearson chi-square divided by the degree of freedom. If both values are more than 1, there is an overdispersion in the data [13].

Poisson Inverse Gaussian Regression

One solution to model hash data is to use Poisson mixed distributions. One of the distributions of mixed Poisson is the distribution of Poisson Inverse Gaussian (PIG). The PIG distribution is a combination of the Poisson and Inverse Gaussian distributions. Suppose Y is a response variable distributed by the Gaussian Poisson Inverse, then the opportunity density function for Y is [14]

$$P(Y = y|\mu, \tau) = \int_0^\infty f(y|\mu, v) g(v, \tau)\, dv$$

with

$$f(y|\mu, v) = \frac{e^{-v\mu}(\mu v)^y}{y!}$$

$g(v,\tau) \quad = (2\pi v^3)^{-\frac{1}{2}} e^{-(v-1)^2/2\tau v}$

v = Random effects distributed in the Gaussian Inverse

Based on the parameters, the PIG distribution consists of two parameters, namely the average parameter (μ) and dispersion parameters (τ). If Y is a PIG distributed response variable, then the PIG distribution can be notated with $Y \sim PIG(\mu, \tau)$. So the opportunity density function can be written in the following equation [15]:

$$P(y|\,\mu,\tau) = e^{\frac{1}{\tau}} K_s(z) \left(\frac{2}{\pi\tau}\right)^{\frac{1}{2}} (1+2\tau\mu)^{-\frac{\left(y-\frac{1}{2}\right)}{2}} \frac{\mu^y}{y\,!}\ ; y = 0,1,2,\ldots$$

With

$z \quad = \sqrt{\frac{1}{\tau^2} + \frac{2\mu}{\tau}}$

$s \quad = y - \frac{1}{2}$

$K_s(z) = K_{y-\frac{1}{2}} \sqrt{\frac{1}{\tau^2} + \frac{2\mu}{\tau}}$ as a third type of modified Bessel function.

To

y = Response variables

τ = Dispersion Parameters

μ = Average.

Bivariate Poisson Inverse Gaussian Regression

If there are two random variables Y_1 and Y_2 that follow a Poisson distribution but are not independent, with means $v\mu_1$ and $v\mu_2$, where v is a random variable following an Inverse Gaussian distribution, this indicates that Y_1 and Y_2 follow a mixed Poisson distribution, specifically a Bivariate Poisson Inverse Gaussian (BPIG) distribution. The BPIG distribution is characterized by the following joint density functions [16].

$$P\left(y_j|j=1,2\right) = e^{\frac{1}{\tau}} K_s(z) \left(\frac{2}{\pi\tau}\right)^{\frac{1}{2}} \left(1 + 2\tau \sum_{j=1}^{2} \mu_j\right)^{-\frac{\left(2\sum_{j=1}^{2} y_j - 1\right)}{4}} \prod_{j=1}^{2} \frac{\mu_j^{\,y_j}}{y_j\,!}$$

With

$z \quad = \sqrt{\frac{1}{\tau^2} + \frac{2(\mu_1+\mu_2)}{\tau}}$

$s \quad = y_1 + y_2 - \frac{1}{2}$

$K_s(z) = K_{y_1+y_2-\frac{1}{2}}\left(\frac{1}{\tau}\sqrt{2\tau(\mu_1+\mu_2)+1}\right)$ as a third type of modified Bessel function

Suppose as a response variable for $y_{ij}$the i observation and the j response variable with a random sample $Y_{1i}Y_{2i} \sim BPIG(\mu_{ij}, \tau)$ where $i = 1,2,..,n$ and $j = 1,2,$. The natural log linkage function (ln) is required in BPIG modeling. The ln connecting function is used to connect parameters μ_{ij} with explanatory variables. So the BPIG regression model in the following equation [16].

$$ln(\mu_{ij}) = \boldsymbol{X}_i^T \boldsymbol{\beta}_j + \varepsilon_{ij}$$

$$\mu_{ij} = exp(\boldsymbol{X}_i^T \boldsymbol{\beta}_j + \varepsilon_{ij})$$

with

$\boldsymbol{X}_i^T = [1\ X_{i1}\ X_{i2}\ \dots\ X_{ip}\ ]_{1\times(p+1)}$ as a vector variable predictor $k = 1,2,\dots,p$ On the observation of the $i = 1,2,\dots,n$

$\boldsymbol{\beta}_j = [\beta_{j0}\ \beta_{j1}\ \beta_{j2}\ \cdots\ \beta_{jp}\ ]^T_{1\times(p+1)}$ as a regression coefficient vector

$\varepsilon_{ij} =$ error

Geographically Weighted Bivariate Poisson Inverse Gaussian Regression

Geographically Weighted Bivarite Poisson Inverse Regression (GWBPIGR) is a statistical method that is a development of the Gaussian Bivariate Poisson Inverse Regression, but the difference is that in this method, it pays attention to weights in the form of latitude and longitude of the observed observation points. GWBPIG distribution has the following combined density functions

$$P(y_j|j=1,2) = e^{\frac{1}{\tau}} K_s(z(u,v)) \left(\frac{2}{\pi\tau}\right)^{\frac{1}{2}} \left(1 + 2\tau \sum_{j=1}^{2} \mu_j(u,v)\right)^{-\frac{\left(2\sum_{j=1}^{2} y_j - 1\right)}{4}} \prod_{j=1}^{2} \frac{\mu_j(u,v)^{y_j}}{y_j\,!}$$

with

$z \quad = \sqrt{\frac{1}{\tau^2} + \frac{2\sum_{j=1}^{2}\mu_j(u,v)}{\tau}}$

$s \quad = y_1 + y_2 - \frac{1}{2}$

$K_s(z) = K_{y_1+y_2-\frac{1}{2}}\left(\frac{1}{\tau}\sqrt{2\tau\sum_{j=1}^{2}\mu_j(u,v)+1}\right)$ as a third type of modified Bessel function

The GWBPIGR equation model can be expressed in the following equation [15].

$$ln\left(\mu_{ij}\right) = \boldsymbol{X}_i^T\boldsymbol{\beta}_j(u_i,v_i)$$

$$\mu_{ij} = exp\left(\boldsymbol{X}_i^T\boldsymbol{\beta}_j(u_i,v_i)\right)$$

With

$\boldsymbol{X}_i^T = \left[1\ X_{i1}\ X_{i2}\ \dots\ X_{ip}\right]_{1\times(p+1)}$ as a vector variable predictor $k = 1,2,\dots,p$ On the observation of the $i = 1,2,\dots,n$

$\boldsymbol{\beta}_j(u_i,v_i) = \left[\beta_{j0}(u_i,v_i)\ \ \beta_{j1}(u_i,v_i)\ \ \cdots\ \beta_{jp}(u_i,v_i)\right]_{1\times(p+1)}^T$ as a regression coefficient vector with a spatial weighting matrix

Berndt-Hall-Hall-Hausman Algorithm

The Berndt-Hall-Hall-Hausman algorithm (BerndHallMan) is an extension of Newton Raphson's algorithm used in statistics to solve the Maximum Likelihood equation. The only distinction between Newton Raphson's algorithm and BerndHallMan's is that the latter does not call for a second derivative. The steps to evaluate parameters using the BerndHallMan algorithm are as follows [6].

a. Establish the starting approximation value.

b. Gradient vectors $\boldsymbol{D}\left(\hat{\theta}_{(t)}\right)$ are defined. Equation below is created by taking the first derivative of the ln likelihood function against the parameter that has to be estimated.:

$$\boldsymbol{D}\left(\hat{\theta}_{(t)}\right) = \left[\frac{\partial l(\boldsymbol{\theta})}{\partial \boldsymbol{\beta}_1}\quad \frac{\partial l(\boldsymbol{\theta})}{\partial \boldsymbol{\beta}_2}\quad \frac{\partial l(\boldsymbol{\theta})}{\partial \tau}\right]$$

c. Determine the Hessian matrix i.e.

$$\boldsymbol{H}\left(\hat{\theta}_{(t)}\right) = -\sum_{i=1}^{n}\boldsymbol{D}\left(\hat{\theta}_{(t)}\right)\boldsymbol{D}^T\left(\hat{\theta}_{(t)}\right).$$

d. Iterating starts at the following equation:$t = 0$

$$\hat{\boldsymbol{\theta}}_{(t+1)} = \hat{\boldsymbol{\theta}}_{(t)} - \left(\boldsymbol{H}\left(\hat{\theta}_{(t)}\right)\right)^{-\mathbf{1}}\boldsymbol{D}\left(\hat{\theta}_{(t)}\right)$$

e. The iteration will stop if the value $\parallel \hat{\boldsymbol{\theta}}_{(t+1)} - \hat{\boldsymbol{\theta}}_{(t)} \parallel \leq \varepsilon$, by being $\varepsilon = 10^{-3}$.

3. MAIN RESULTS

Overdispersion Test

In the Poisson regression model, there are a number of assumptions that must be met. Equidispersion, or the requirement that the variance and mean be equal, is one of them. However, in statistical data analysis, data conditions are often found with variances sometimes greater than the mean (overdispersion). Overdispersion can be detected by conducting a deviation test. The results of the overdispersion test are shown in Table 1:

Table 1. Overdispersion test

Variable	Test Statistics
Y_1	1.298
Y_2	8.649

Based on the test statistic values presented in Table 1, the maternal mortality and neonatal mortality variables exhibit overdispersion, with test statistic values greater than 1.

Spasial Heterogeneity

The spatial heterogeneity test aims to find out whether the observation location has different characteristics or not. The Breusch Pagan test can be used to determine spatial heterogeneity. Table 6 displays the findings of the spatial heterogeneity test utilizing the Breusch Pagan test based on R-Studio program output.

Tabel 2. Spatial heterogeneity test

Variable	**Test Statistics**
Y_1	11,69
Y_2	11,86

Based on the test results, test statistical values were obtained for the response variables of maternal mortality and neonatal death respectively, namely $11{,}69$ and $11{,}86$ which is greater than the value $\mathcal{X}^2_{(0,05;5)}$ =11,07. This indicates that it is rejected H_0. Therefore, it can be concluded that cases of maternal mortality and neonatal mortality experience spatial heterogeneity.

Best Bandwidth

The first step in GWBPIGR modeling is to determine the kernel function to be used for bototoan, with the selection criterion being the kernel function with the optimal bandwidth that produces the smallest GCV value which is the best kernel function for GWBPIGR modeling.

Table 3. Bandwith model GWBPIGR

Kernel Function	Bandwith	GCV
Adaptif Bisquare	0.8108	2.3373
Adaptif Tricube	0.0001	3.7160
Fixed Gaussian	3.4386	3.7103

Table 3 shows that the Adaptive Bisquare Kernel function with a spatial bandwidth of 0.8108 produces the smallest GCV value compared to the GCV of other kernel functions. So the Bisquare Kernel Adaptive function was chosen for the weighting of the GWBPIGR model.

Geographycally Weighted Bivariate Poisson Inverse Gaussian Regression

The next step is to form a GWBPIGR model based on 24 districts/cities in South Sulawesi with estimates that have been obtained previously. Table 4 shows some of the GWBPIGR models obtained.

Table 4. GWBPIGR modeling results

Regency/City	GWBPIGR Model
Kepulauan Selayar	$\hat{y}_{1,1} = 0.577393 + 0.021906X_1 - 0.00441X_2$ $-0.00383X_3 + 0.003407X_4 - 36.21267X_5$
	$\hat{y}_{1,2} = 2.138123 + 0.032382X_1 - 0.00197X_2$ $-0.00464X_3 + 0.00261X_4 - 36.93524X_5$
Bulukumba	$\hat{y}_{2,1} = 0.577394 + 0.02191X_1 - 0.00436X_2$ $-0.00395X_3 + 0.003493X_4 - 36.2127X_5$
	$\hat{y}_{2,2} = 2.138113 + 0.032284X_1 - 0.00228X_2$ $-0.00381X_3 + 0.00326X_5 - 36.9352X_5$
Bantaeng	$\hat{y}_{3,1} = 0.577394 + 0.021919X_1 - 0.00432X_2$ $-0.00394X_3 + 0.00346X_4 - 36.2127X_5$
	$\hat{y}_{3,2} = 2.138127 + 0.032398X_1 - 0.00155X_2$ $-0.00501X_3 - 0.00268X_5 - 36.9352X_5$
⋮	⋮
Palopo	$\hat{y}_{24,1} = 0.577394 + 0.02187X_1 - 0.00434X_2$ $-0.00397X_3 + 0.00368X_4 - 36.2127X_5$
	$\hat{y}_{24,2} = 2.138135 + 0.032437X_1 - 0.00193X_2$ $-0.00543X_3 - 0.00149X_5 - 36.9352X_5$

Next, determine the simultaneous model testing to test the significance of all predictor variables. The hypothesis used in the GWBPIGR model simultaneously is as follows.

$H_0 : \beta_{jk}(u_i, v_i) = 0$ to $j = 1,2 ; k = 1,2, \ldots, p$

$H_1 :$ There is at least one $\beta_{jk}(u_i, v_i) \neq 0$ to $j = 1,2; \; k = 1,2, \ldots, p$

The results of the simultaneous test obtained were 133701.2 greater than the value of $\mathcal{X}^2_{(0,05;10)} = 18.307$ so H_0 it was rejected. This indicates that, at the very least, some predictor variables specifically, the number of maternal and neonatal deaths in South Sulawesi, Indonesia have a considerable impact on the number of response variables. As seen in Figures 1 and 2, partial testing is therefore required to determine which predictor factors significantly affect each district or city in South Sulawesi.

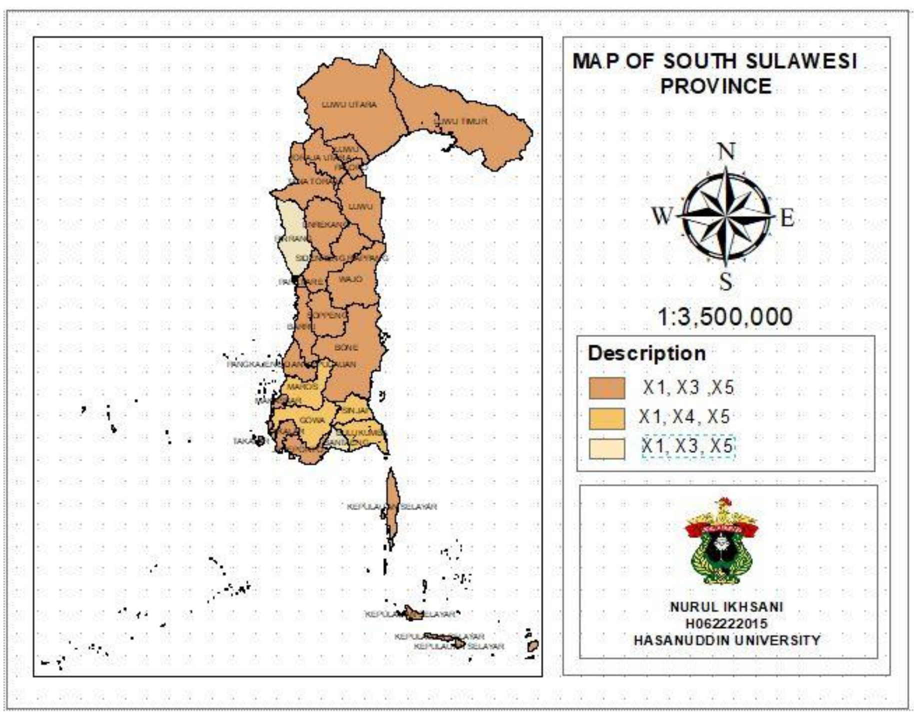

Figure 1. Distribution of the influence of maternal mortality variables on the GWBPIGR model

Based on Figure 1, it can be seen that there are three groups of districts/cities that have significant differences in predictor variables. The first group is the X_1, X_3, X_4, X_5 shows that brown areas have maternal mortality cases significantly influenced by the factors of poor population, active family planning participants, neonatal complications, and health centers. The second group is the X_1, X_4, X_5 It shows that the orange area has maternal mortality cases significantly influenced by the factors of poor population, neonatal complications, and health centers. The third group is the X_1, X_3, X_5 shows that the beige area has maternal mortality cases significantly influenced by the factors of the poor population, active family planning participants, and health centers.

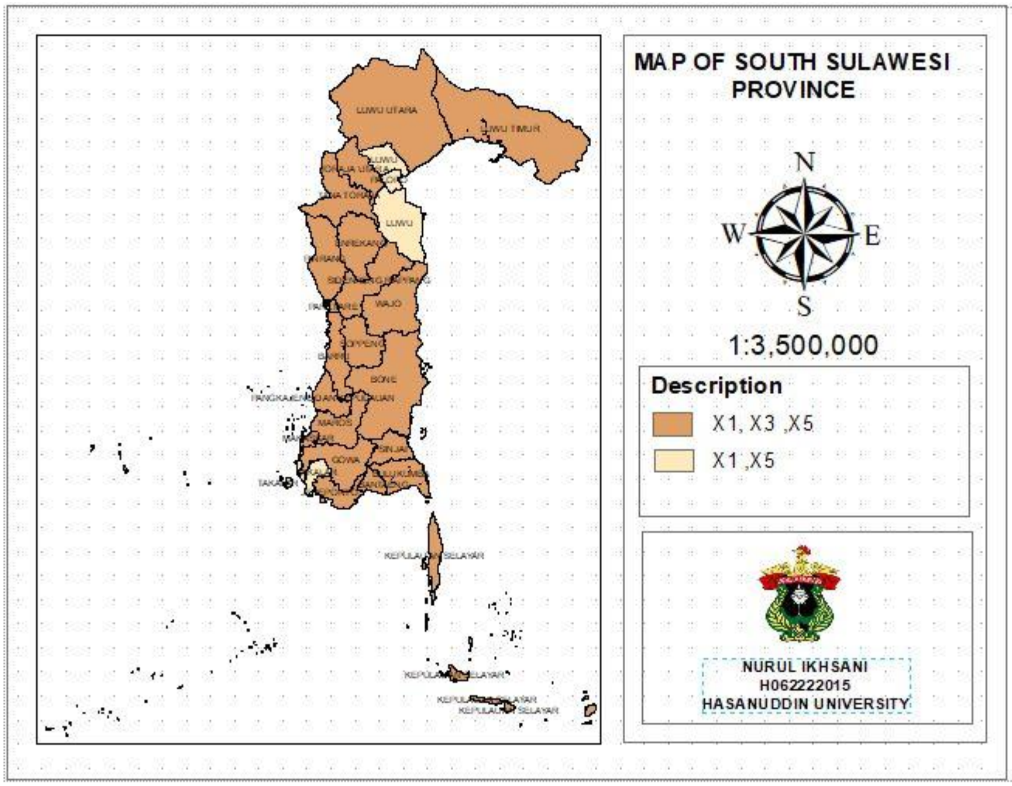

Figure 2. Distribution of the influence of neonatal mortality variables on the GWBPIGR model

Based on Figure 2, it can be seen that there are two groups of districts/cities that have significant differences in predictor variables. The first group is the X_1, X_3, X_5 shows that brown areas have neonatal death cases significantly influenced by the factors of poor population, active family planning participants, and health centers. The second group is the X_1, X_5 shows that the beige area has neonatal death cases significantly influenced by the factors of the poor population and health centers.

Selection of the Best Models

Comparison of GWBPIGR model with Newton Raphson and BerndtHallMann algorithms and BPIGR with BerndtHallMann algorithm. The purpose of this is to determine which model is more appropriate for simulating cases of newborn deaths and maternal mortality in each South Sulawesi district and city in 2021. The lowest AIC value was taken into consideration when choosing the optimum model for this investigation. The top models are displayed in Table 5 as follows.

Table 5. Selection of the best models

Models	AIC Values
BPIGR with BerndtHallMann algorithm	109.74
GWBPIGR with BerndtHallMann algorithm	102.84

Based on Table 5, it can be seen that the GWBPIGR model with the BerndtHallMann algorithm produces a smaller AIC value than other models. Therefore, it can be concluded that the GWBPIGR model with the BerndtHallMann algorithm is the best model to be used in estimating maternal mortality cases and neonatal deaths in each district/city of South Sulawesi, Indonesia in 2021 that experienced overdispersion and spatial diversity.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interest.

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