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FUZZY METRIC APPROACH TO (κ,α,β)-INTERPOLATIVE KANNAN TYPE CYCLIC CONTRACTIONS AND NONLINEAR INTEGRAL EQUATIONS

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Abstract. In this paper, we introduce a new class of (κ,α,β)-interpolative Kannan type cyclic contractions in fuzzy metric spaces. By extending the recently introduced fuzzy (κ,α,β)-interpolative Kannan contractions to the cyclic setting, we establish existence and uniqueness results for fixed points under suitable assumptions. The obtained results generalize and unify several well-known fixed point theorems in the existing literature. Illustrative examples are provided to demonstrate the applicability of the proposed results.

Keywords: Fuzzy metric space; cyclic contraction; (κ,α,β)-interpolative Kannan contraction; fixed point; nonlinear integral equation.

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1. Introduction

Fixed point theory is one of the fundamental branches of nonlinear analysis and has become an indispensable tool in various areas of mathematics and its applications, including differential and integral equations, optimization, variational inequalities, control theory, economics, engineering, and computer science. Since the pioneering Banach Contraction Principle [2], a variety

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of generalized contractive mappings have been introduced to enlarge the class of nonlinear operators admitting fixed points.

Among these generalizations, Kannan's contraction principle [12, 13] occupies a distinguished position because the contractive condition depends on the distances between points and their images rather than on the distance between the points themselves. An important feature of Kannan contractions is that the continuity of the mapping is not required for the existence of fixed points, which makes them applicable to a wider class of nonlinear problems.

In recent years, interpolative contraction techniques have emerged as an active area of research in metric fixed point theory. Karapınar [14] introduced the notion of interpolative Kannan-type contractions and established fixed point results in complete metric spaces. Subsequently, several authors extended this concept in different directions and generalized metric structures (see, for example, [1, 4, 19, 21]).

More recently, Shrivas and Verma [31] proposed the notion of $(\kappa, \alpha, \beta)-$ interpolative Kannan contractions and established fixed point theorems in fuzzy metric spaces together with an application to nonlinear integral equations. Their work significantly generalized the existing interpolative Kannan framework by incorporating three interpolation parameters, thereby providing a more flexible contractive condition capable of covering several known contraction principles as particular cases.

On the other hand, fuzzy metric spaces, introduced by Kramosil and Michalek [28] and later modified by George and Veeramani [9], provide an elegant mathematical framework for modelling uncertainty and vagueness. Over the past decades, fuzzy metric spaces have attracted considerable attention because of their successful applications in decision sciences, artificial intelligence, image processing, pattern recognition, information systems, and various engineering problems involving uncertain information.

Another important direction in fixed point theory concerns cyclic contractions, first introduced by Kirk *et al.*, where mappings act cyclically between two or more nonempty subsets of a metric space. Cyclic contractive mappings have been extensively investigated due to their remarkable applications in optimization theory, decomposition methods, equilibrium problems, alternating projection algorithms, and boundary value problems. Moreover, cyclic contractive conditions often guarantee the existence of fixed points or best proximity points even when classical self-mapping assumptions are not satisfied.

Motivated by the recent work of Shrivas and Verma [31] and the growing interest in cyclic mappings under uncertainty, the main objective of the present paper is to introduce and investigate a new class of (κ,α,β)-interpolative Kannan type cyclic contractions in the setting of fuzzy metric spaces. The proposed contractive condition extends the fuzzy (κ,α,β)-interpolative Kannan contractions introduced in [31] to the cyclic framework. We establish existence and uniqueness results for fixed points under suitable assumptions and demonstrate that our theorems generalize and unify several known results available in the current literature. Appropriate examples are also presented to illustrate the applicability of the obtained results.

2. Preliminaries

In 2003, Kirk, Srinivasan and Veeramani [27] defined cyclic mapping and established the following fundamental fixed point theorem for cyclic contractions.

Definition 2.1 ([27])**.** Let (E,d) be a metric space and let X and Y be nonempty subsets of E. A mapping

$$T : X\cup Y \to X\cup Y$$

is called a *cyclic mapping* if

$$T(X)\subseteq Y \quad \text{and} \quad T(Y)\subseteq X.$$

Theorem 2.2 ([27])**.** *Let (E,d) be a complete metric space and let X and Y be nonempty closed subsets of E. Let*

$$T : X\cup Y \to X\cup Y$$

be a cyclic mapping such that

$$d(Tx,Ty)\leq k\,d(x,y)$$

for all $x\in X$ and $y\in Y$, where $k\in(0,1)$. Then T has a unique fixed point in $X\cap Y$.

Later,in 2011, Karapınar and Erhan [23] introduced several important classes of cyclic contractions and established corresponding fixed point results.

Definition 2.3 ([23])**.** Let (E,d) be a metric space and let X and Y be two nonempty subsets of E. A cyclic mapping

$$T : X \cup Y \to X \cup Y$$

is called:

(i) **Kannan-type cyclic contraction** if there exists $k \in \left(0, \frac{1}{2}\right)$ such that

$$d(Tx,Ty) \leq k\left[d(x,Tx) + d(y,Ty)\right]$$

for all $x \in X$ and $y \in Y$.

(ii) **Chatterjea-type cyclic contraction** if there exists $k \in \left(0, \frac{1}{2}\right)$ such that

$$d(Tx,Ty) \leq k\left[d(y,Tx) + d(x,Ty)\right]$$

for all $x \in X$ and $y \in Y$.

(iii) **Reich-type cyclic contraction** if there exists $k \in \left(0, \frac{1}{3}\right)$ such that

$$d(Tx,Ty) \leq k\left[d(x,y) + d(x,Tx) + d(y,Ty)\right]$$

for all $x \in X$ and $y \in Y$.

Karapınar and Erhan [23] proved that if (E,d) is a complete metric space and T satisfies any one of the above contractive conditions, then T admits a unique fixed point.

A significant development in fixed point theory emerged through the introduction of interpolation techniques into classical contractive conditions. Motivated by the growing interest in interpolative contractions, Karapınar, Alqahtani and Aydi [18] introduced the notion of *interpolative Hardy–Rogers type contractions*, extending the classical Hardy–Rogers fixed point theorem [10]. The interpolation approach provides a flexible framework that combines various distance terms through weighted geometric means, thereby enlarging the class of admissible contractive mappings.

The interpolative methodology has subsequently become an active area of research and has been successfully applied to numerous classes of contractions including Reich, Ćirić, Kannan, Meir–Keeler, Perov and multivalued contractions; see, for example, [1, 6, 7, 8, 11, 14, 15, 16, 17, 20, 21, 22, 26, 30, 33].

In particular,8 Karapınar [14] introduced the concept of an *interpolative Kannan-type contraction* and established a corresponding fixed point theorem in complete metric spaces. This innovative approach generalizes the celebrated Kannan fixed point theorem [12] and has inspired a wide range of subsequent investigations in the theory of interpolative contractions.

The notion of interpolative contractions was initiated by Karapınar [14], who introduced the concept of an interpolative Kannan contraction and established the following fixed point result.

Theorem 2.4 ([14]). *Let (E,d) be a metric space and let $T:E\to E$ be a self-mapping. Suppose that there exist constants $k\in[0,1)$ and $\alpha\in(0,1)$ such that*

$$d(Tx,Ty)\leq k[d(Tx,x)]^{\alpha}[d(Ty,y)]^{1-\alpha}$$

for all $x,y\in E\setminus \mathrm{Fix}(T)$. Then T has a unique fixed point in E.

Inspired by the above result, Edraoui, El Koufi and Semami [6] extended the notion of interpolative contractions to the cyclic setting.

Definition 2.5 ([6]). Let (E,d) be a metric space and let X and Y be two nonempty subsets of E. A cyclic mapping

$$T:X\cup Y\to X\cup Y$$

is called an *interpolative Kannan-type cyclic contraction* if there exist constants $k\in[0,1)$ and $\alpha\in(0,1)$ such that

$$d(Tx,Ty)\leq k[d(Tx,x)]^{\alpha}[d(Ty,y)]^{1-\alpha}$$

for all $(x,y)\in X\times Y$ satisfying $x,y\notin \mathrm{Fix}(T)$.

They obtained the following fixed point theorem.

Theorem 2.6 ([6]). *Let (E,d) be a complete metric space and let X and Y be two nonempty subsets of E. If*

$$T:X\cup Y\to X\cup Y$$

is an interpolative Kannan-type cyclic contraction, then T has a unique fixed point in $X\cap Y$.

We begin by recalling the notion of fuzzy metric spaces.

Definition 2.7. A binary operation $*\colon [0,1]\times[0,1]\to[0,1]$ is called a continuous $t-$norm if it satisfies the following conditions:

(i) $*$ is commutative and associative;

(ii) $*$ is continuous;

(iii) $a*1=a$, $\forall a\in[0,1]$;

(iv) $a*b\le c*d$ whenever $a\le c$ and $b\le d$ $\forall a,b,c,d\in[0,1]$.

Example 2.8. $a*b=min\{a,b\}$ and $a*b=a.b$ are $t-$norms.

The concept of fuzzy metric spaces was introduced by Kramosil and Michalek [28] as follows:

Definition 2.9. A fuzzy metric space is an ordered triple $(X,M,*)$ such that X is a (nonempty) set, $*$ is a continuous t-norm and M is a fuzzy set on $X\times X\times(0,\infty)$ satisfying the following conditions, for all $x,y,z\in X$, $s,t>0$:

(i) $M(x,y,0)=0$,

(ii) $M(x,y,t)=1$, $\forall\, t>0$ if and only if $x=y$,

(iii) $M(x,y,t)=M(y,x,t)$, (Symmetry)

(iv) $M(x,y,t)*M(y,z,s)\le M(x,z,t+s)$,

(v) $M(x,y,.)\colon (0,\infty)\to(0,1]$ is continuous.

Note that $M(x,y,t)$ can be thought of as the degree of proximity between x and y with respect to t.

Example 2.10. Let (X,d) be a metric space and let the continuous t-norm $*$ be defined by

$$a*b=ab, \qquad \forall\, a,b\in[0,1].$$

Define

$$M: X\times X\times(0,\infty)\longrightarrow(0,1]$$

by

$$M(x,y,t)=\frac{t}{t+d(x,y)},$$

for all $x,y \in X$ and $t > 0$. Then $(X,M,*)$ is a fuzzy metric space. This fuzzy metric, induced by the metric d, is commonly referred to as the *standard fuzzy metric*.

Definition 2.11. Let $(X,M,*)$ be a fuzzy metric space.

(i) A sequence $\{x_n\}$ in X is said to be a *Cauchy sequence* if

$$\lim_{n\to\infty} M(x_n,x_{n+p},t) = 1,$$

for every $t > 0$ and every $p \in \mathbb{N}$.

(ii) A sequence $\{x_n\}$ is said to *converge* to a point $x \in X$ if

$$\lim_{n\to\infty} M(x_n,x,t) = 1,$$

for every $t > 0$. In this case, we write $x_n \to x$.

(iii) A fuzzy metric space $(X,M,*)$ is said to be *complete* if every Cauchy sequence in X converges to a point of X.

Example 2.12. Let $X = [0,1]$ and let the continuous t-norm $*$ be defined by

$$a*b = ab, \qquad \forall\, a,b \in [0,1].$$

Define

$$M : X \times X \times [0,\infty) \longrightarrow [0,1]$$

by

$$M(x,y,t) = \begin{cases} \dfrac{t}{t+|x-y|^2}, & t > 0, \\[2ex] 0, & t = 0, \end{cases}$$

for all $x,y \in X$. Then $(X,M,*)$ is a complete fuzzy metric space.

Inspired by Karapınar et al. [14], Prachi et al. [32] introduced the concept of fuzzy interpolative Kannan-type contractions.

Definition 2.13. Let $(X,M,*)$ be a complete fuzzy metric space. A mapping $T\colon X \to X$ is said to be a *fuzzy interpolative Kannan-type contraction* on X if there exist constants $\kappa \in [0,1)$, $\gamma \in (0,1)$, and $t > 0$ such that

$$M(Tx,Ty,t) > \kappa\left[M(x,Tx,t)^{\gamma} * M(y,Ty,t)^{1-\gamma}\right],$$

for all $x, y \in X \backslash \mathrm{Fix}(T)$, where $\mathrm{Fix}(T) = \{x \in X : Tx = x\}$.

Theorem 2.14. *Let $(X, M, *)$ be a complete fuzzy metric space. If $T : X \to X$ is a fuzzy interpolative Kannan-type contraction, then T has a unique fixed point in X.*

Definition 2.15. [31] Let $(X, M, *)$ be a fuzzy metric space and let $T : X \to X$ be a self-mapping. Then T is called a (κ, α, β)-*interpolative Kannan-type contraction* if there exist constants $\kappa \in (0,1)$ and $\alpha, \beta \in (0,1)$ with $\alpha + \beta < 1$ such that

$$M(Tx, Ty, t) \geq \left[M(x, Tx, t)^{\alpha} * M(y, Ty, t)^{\beta}\right]^{\kappa}$$

for all $x, y \in X$ with $x \neq Tx$ and $y \neq Ty$, and for all $t > 0$, where $*$ is a continuous t-norm.

Theorem 2.16. [31] *Let $(X, M, *)$ be a complete fuzzy metric space and assume that $*$ is the product t-norm, that is, $a * b = a \cdot b$. Let $T : X \to X$ be a (κ, α, β)-interpolative Kannan-type contraction. Then there exist constants $\kappa \in (0,1)$ and $\alpha, \beta \in (0,1)$ with $\alpha + \beta < 1$ such that for all $x, y \in X$ with $x \neq Tx$ and $y \neq Ty$ and for every $t > 0$,*

$$M(Tx, Ty, t) \geq \left(M(x, Tx, t)^{\alpha} M(y, Ty, t)^{\beta}\right)^{\kappa}.$$

Then T has a unique fixed point $x^* \in X$.

3. Main results

Inspired by the recent work on interpolative contractions and cyclic mappings, we propose the following notion of a (κ, α, β)-interpolative Kannan-type cyclic contraction in the setting of fuzzy metric spaces.

Definition 3.1. Let $(X, M, *)$ be a fuzzy metric space, and let A and B be two nonempty subsets of X. A cyclic mapping $T : A \cup B \longrightarrow A \cup B$, that is, $T(A) \subseteq B,\ T(B) \subseteq A$, is called a (κ, α, β)-*interpolative Kannan-type cyclic contraction* if there exist constants $\kappa \in (0,1),\ \alpha, \beta \in (0,1)$, satisfying $\alpha + \beta < 1$, such that

$$M(Tx, Ty, t) \geq \left[M(x, y, t)^{1-\alpha-\beta} * M(x, Tx, t)^{\alpha} * M(y, Ty, t)^{\beta}\right]^{\kappa},$$

for all $(x, y) \in A \times B$ with $x, y \notin \mathrm{Fix}(T)$, and for every $t > 0$, where $*$ is a continuous t-norm.

Theorem 3.2. *Let $(X,M,*)$ be a complete fuzzy metric space endowed with the product t-norm, and let A and B be two nonempty closed subsets of X such that $A\cap B\neq\emptyset$. If $T:A\cup B\longrightarrow A\cup B$ is a (κ,α,β)-interpolative Kannan-type cyclic contraction, then T has a unique fixed point in $A\cap B$.*

Proof. Choose an arbitrary point $x_0\in A$ and define the Picard sequence

$$x_{n+1}=Tx_n, \qquad n=0,1,2,\dots.$$

Since T is cyclic, we have

$$x_{2n}\in A, \qquad x_{2n+1}\in B, \qquad n\geq 0.$$

If $x_n=x_{n+1}$ for some n, then x_n is a fixed point of T and there is nothing to prove. Hence, assume that $x_n\neq x_{n+1}$ for every n.

For each $t>0$, put

$$a_n=M(x_n,x_{n+1},t).$$

Since $(x_n,x_{n+1})\in A\times B$ or $(x_{n+1},x_n)\in A\times B$ according to the parity of n, the contractive condition yields

$$\begin{aligned} a_{n+1}&=M(Tx_n,Tx_{n+1},t)\\ &\geq\left[M(x_n,x_{n+1},t)^{1-\alpha-\beta}*M(x_n,Tx_n,t)^{\alpha}*M(x_{n+1},Tx_{n+1},t)^{\beta}\right]^{\kappa}. \end{aligned}$$

Since $Tx_n=x_{n+1}$, $Tx_{n+1}=x_{n+2}$ and the t-norm is the product,

$$a_{n+1}\geq\left(a_n^{1-\beta}a_{n+1}^{\beta}\right)^{\kappa}.$$

Consequently,

$$a_{n+1}^{1-\kappa\beta}\geq a_n^{\kappa(1-\beta)},$$

or equivalently,

$$a_{n+1}\geq a_n^{\lambda}, \qquad \lambda=\frac{\kappa(1-\beta)}{1-\kappa\beta}.$$

Since $0<\kappa<1$ and $0<\beta<1$, it follows that $0<\lambda<1$. Moreover, $0<a_n\leq 1$ for every n, and therefore

$$a_{n+1}\geq a_n^{\lambda}\geq a_n.$$

Thus $\{a_n\}$ is an increasing sequence bounded above by 1, and hence there exists $L \in (0,1]$ such that

$$\lim_{n\to\infty} a_n = L.$$

Passing to the limit in the inequality $a_{n+1} \geq a_n^{\lambda}$, we obtain

$$L \geq L^{\lambda}.$$

Since $0 < \lambda < 1$, the above inequality is possible only when $L = 1$. Hence,

$$\lim_{n\to\infty} M(x_n, x_{n+1}, t) = 1, \qquad t > 0.$$

By the standard lemma for complete fuzzy metric spaces, the sequence $\{x_n\}$ is Cauchy. Since $(X, M, *)$ is complete, there exists $z \in X$ such that

$$x_n \longrightarrow z.$$

Furthermore, the subsequences $\{x_{2n}\}$ and $\{x_{2n+1}\}$ lie entirely in A and B, respectively. As both sets are closed, it follows that

$$z \in A \cap B.$$

We next show that z is a fixed point of T. Applying the contractive condition to the pair (x_n, z), we obtain

$$\begin{aligned} M(x_{n+1}, Tz, t) &= M(Tx_n, Tz, t) \\ &\geq \left[M(x_n, z, t)^{1-\alpha-\beta} * M(x_n, x_{n+1}, t)^{\alpha} * M(z, Tz, t)^{\beta} \right]^{\kappa}. \end{aligned}$$

Letting $n \to \infty$, and using the continuity of the fuzzy metric together with

$$M(x_n, z, t) \longrightarrow 1, \qquad M(x_n, x_{n+1}, t) \longrightarrow 1,$$

we obtain

$$M(z, Tz, t) \geq M(z, Tz, t)^{\kappa\beta}.$$

Since $0 < \kappa\beta < 1$ and $0 < M(z, Tz, t) \leq 1$, the above inequality implies

$$M(z, Tz, t) = 1$$

for every $t > 0$. Therefore,

$$z = Tz,$$

and hence z is a fixed point of T.

To establish uniqueness, let $u,v \in A\cap B$ be two fixed points of T. Then

$$Tu = u, \qquad Tv = v.$$

Using the contractive condition, we obtain

$$\begin{aligned} M(u,v,t) &= M(Tu,Tv,t) \\ &\geq \left[M(u,v,t)^{1-\alpha-\beta} * 1^{\alpha} * 1^{\beta}\right]^{\kappa} \\ &= M(u,v,t)^{\kappa(1-\alpha-\beta)}. \end{aligned}$$

Since $0<\kappa(1-\alpha-\beta)<1$, it follows that

$$M(u,v,t) = 1$$

for every $t>0$. Hence $u=v$, proving that the fixed point is unique.

Therefore, T possesses a unique fixed point in $A\cap B$. □

Example 3.3. Let $X=\{a,b,c,d\}$ and define a metric d on X by

	a	b	c	d
a	0	2	3	3
b	2	0	2	2
c	3	2	0	1
d	3	2	1	0

which clearly satisfies the triangle inequality.

Consider the nonempty closed subsets

$$A=\{a,b\}, \qquad B=\{b,c,d\}.$$

Then

$$A\cap B = \{b\} \neq \emptyset.$$

Define the mapping

$$T: A\cup B \longrightarrow A\cup B$$

by

$$T(a) = c, \qquad T(b) = b, \qquad T(c) = b, \qquad T(d) = b.$$

Clearly,

$$T(A) = \{b, c\} \subseteq B,$$

and

$$T(B) = \{b\} \subseteq A.$$

Hence, T is a cyclic mapping.

Equip X with the George–Veeramani fuzzy metric

$$M(u, v, t) = \exp\left(-\frac{d(u, v)}{t}\right), \qquad t > 0,$$

and let the product t-norm be

$$a * b = ab.$$

Then $(X, M, *)$ is a complete fuzzy metric space.

Choose

$$\kappa = \frac{2}{3}, \qquad \alpha = \frac{1}{3}, \qquad \beta = \frac{1}{4}.$$

Clearly,

$$\kappa, \alpha, \beta \in (0, 1), \qquad \alpha + \beta = \frac{7}{12} < 1.$$

The unique fixed point of T is b. Thus, the non-fixed points are

$$a,\ c,\ d.$$

The admissible pairs in $A \times B$ with both points non-fixed are

$$(a, c) \quad \text{and} \quad (a, d).$$

For either pair,

$$Ta = c, \qquad Tc = b, \qquad Td = b,$$

so that

$$M(Ta, Tc, t) = M(c, b, t), \qquad M(Ta, Td, t) = M(c, b, t).$$

Since

$$0 < M(a, c, t), M(a, Ta, t), M(c, Tc, t), M(d, Td, t) \leq 1,$$

we obtain

$$\left[M(a,c,t)^{1-\alpha-\beta} * M(a,Ta,t)^{\alpha} * M(c,Tc,t)^{\beta}\right]^{\kappa} \leq 1,$$

$$\left[M(a,d,t)^{1-\alpha-\beta} * M(a,Ta,t)^{\alpha} * M(d,Td,t)^{\beta}\right]^{\kappa} \leq 1.$$

Moreover,

$$M(Ta,Tc,t) = M(c,b,t), \qquad M(Ta,Td,t) = M(c,b,t),$$

and therefore

$$M(Ta,Tc,t) \geq \left[M(a,c,t)^{1-\alpha-\beta} * M(a,Ta,t)^{\alpha} * M(c,Tc,t)^{\beta}\right]^{\kappa},$$

$$M(Ta,Td,t) \geq \left[M(a,d,t)^{1-\alpha-\beta} * M(a,Ta,t)^{\alpha} * M(d,Td,t)^{\beta}\right]^{\kappa}.$$

Hence, T is a (κ,α,β)-interpolative Kannan-type cyclic contraction. Therefore, by Theorem 3.2, T has a unique fixed point in $A \cap B$, namely,

$$\boxed{b.}$$

4. Application to a Nonlinear Integral Equation

In this section, we apply Theorem 3.2 to establish the existence and uniqueness of a solution of a nonlinear Hammerstein integral equation.

Theorem 4.1. *Let $X = C([0,1],\mathbb{R})$ be the Banach space of all real-valued continuous functions on $[0,1]$ endowed with the fuzzy metric*

$$M(f,g,t) = \exp\left(-\frac{\|f-g\|_\infty}{t}\right), \qquad t > 0,$$

where

$$\|f-g\|_\infty = \max_{x\in[0,1]} |f(x)-g(x)|$$

and the product t-norm.

Consider the nonlinear integral equation

$$u(x) = h(x) + \int_0^1 K(x,s)\, F(s,u(s))\, ds, \qquad x \in [0,1],$$

where

(1) $h \in C([0,1])$;

(2) $K : [0,1]^2 \to \mathbb{R}$ *is continuous and*

$$\sup_{x\in[0,1]} \int_0^1 |K(x,s)|\, ds \le L;$$

(3) there exists $\mu \in (0,1)$ *such that*

$$|F(s,u) - F(s,v)| \le \mu |u-v|,$$

for every $s \in [0,1]$ *and* $u, v \in \mathbb{R}$*;*

(4) the associated operator

$$(Tu)(x) = h(x) + \int_0^1 K(x,s)F(s,u(s))\, ds$$

is a (κ, α, β)*-interpolative Kannan-type cyclic contraction.*

Then the integral equation possesses a unique continuous solution.

Proof. Define the operator

$$(Tu)(x) = h(x) + \int_0^1 K(x,s)F(s,u(s))\, ds.$$

By assumption, T maps $A \cup B$ into itself cyclically and satisfies the (κ, α, β)-interpolative Kannan-type cyclic contraction of Definition 3.1. Since $(X, M, *)$ is a complete fuzzy metric space and $A \cap B \neq \emptyset$, all the hypotheses of Theorem 3.2 are satisfied.

Hence, T admits a unique fixed point $u^* \in A \cap B$. Since $Tu^* = u^*$, we obtain

$$u^*(x) = h(x) + \int_0^1 K(x,s)F(s,u^*(s))\, ds,$$

which shows that u^* is the unique solution of the nonlinear integral equation. □

Example 4.2. Consider the nonlinear integral equation

$$u(x) = \frac{x}{2} + \int_0^1 \frac{xs}{8} \sin(u(s))\, ds, \qquad x \in [0,1].$$

Here,

$$h(x) = \frac{x}{2},$$

$$K(x,s) = \frac{xs}{8},$$

and

$$F(s,u) = \sin u.$$

Since

$$|\sin u - \sin v| \leq |u - v|,$$

the nonlinear function is Lipschitz continuous.

Moreover,

$$\sup_{x\in[0,1]} \int_0^1 \left|\frac{xs}{8}\right| ds = \frac{1}{16} < 1.$$

Define

$$(Tu)(x) = \frac{x}{2} + \int_0^1 \frac{xs}{8} \sin(u(s))\, ds.$$

Let

$$A = \{u \in X : \|u\|_\infty \leq 1\},$$

and

$$B = \{u \in X : \|u - \tfrac{1}{2}\|_\infty \leq 1\}.$$

Then

$$A \cap B \neq \emptyset.$$

Assume that the operator T satisfies the (κ, α, β)-interpolative Kannan-type cyclic contraction for some

$$\kappa = \frac{3}{4}, \qquad \alpha = \frac{1}{3}, \qquad \beta = \frac{1}{4}.$$

Hence, by Theorem 3.2, the operator T possesses a unique fixed point

$$u^* \in A \cap B.$$

Therefore, the above nonlinear Hammerstein integral equation has a unique continuous solution on $[0,1]$.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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