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# GREGUS TYPE FIXED POINT RESULTS FOR TANGENTIAL MAPPINGS SATISFYING CONTRACTIVE CONDITION IN FUZZY METRIC SPACES

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**Abstract:** In this paper, we define the tangential property for a pair of set valued and single valued mappings in fuzzy metric spaces and using this new notion we prove some coupled coincidence and common fixed point theorems for hybrid pair of mappings without appeal to the completeness of the underlying space. Our results generalize, unify and extend the results of A.Djoudi ,A.Alioche , J. Math. Anal. Appl. 329(2007),31-45], Parin et.al, chaipunya et.al Advances in Difference Equations 2012,2012:83, W.Sintunavarat and P.Kumam, Int.J.Math. and Math Sci. Vol.2011, Article ID 923458, 12pages,Pathak and Shahazad (Bull. Belg .Math Soc. Simon Stevin 16,277-288,2009) and several known fixed point results.

**Keywords:** Tangential mappings, fixed point results, fuzzy metric spaces

**2000 AMS Subject Classification:** 47H10

## 1. Introduction :

The notion of fuzzy sets introduced by Zadeh [6] proved a turning point in the development of mathematics . Fuzzy set theory has applications in applied sciences such as neural network theory, stability theory, mathematical programming, modelling theory, engineering sciences, medical sciences (medical genetics, nervous system), image processing, control theory, communication etc. The study of fixed points for multi-valued contraction mappings using the Hausdorff metric was initiated by Nadler [7] and Markin [8] . Later on , an interesting and rich fixed point theory for such maps was developed which has found applications in control theory, convex optimization, differential inclusion and economics.

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Bhaskar and Lakshmikantham [11] introduced the concepts of coupled fixed points and mixed monotone property and illustrated these results by proving the existence and uniqueness of the solution for a periodic boundary value problem. Later on these results were extended and generalized by Sedghi et al. [10] , Fang [4] etc. Recently, Abbas et.al. [1] proved coupled common fixed point theorem for a hybrid pair of mappings satisfying w-compatible in complete metric space.

In 2009 ,Pathak and Shazad [3] introduced the new concept of weak tangent point and tangential property for single valued mappings in metric spaces. In 2011 ,Sintunavarat and Kumum [13-14] developed a tangential property for a pair of hybrid mappings and proved common fixed point for weakly compatible mappings satisfying tangential property .

In the setting of fuzzy-metric space ,the strict contractive condition do not insure the existence of a common fixed point unless the space is assumed complete or the strict conditions are replaced by strong conditions .The intent of this paper is to define the tangential property ( which is generalization of mappings satisfying (E.A) property and generalized coincidence property for single-valued and multi-valued mappings and prove fixed point results for coupled mappings in fuzzy metric spaces.

So, our improvement in this paper is four fold as

(i) Relaxed continuity of maps completely

(ii) Completeness of the whole space or any of its range space removed.

(iii) Minimal type contractive condition used.

(iv) The condition $\lim_{t\to\infty} M(x,y,t)$ is not used.

## 2. Definitions and Preliminaries :

To set up our results in the next section we recall some definitions and facts .

**Definition 2.1[10].** A fuzzy set A in X is a function with domain X and values in [0, 1].

**Definition 2.2 [10].** A binary operation * : [0,1] × [0,1]→ [0,1] is a continuous t-norm if

([0,1], *) is a topological abelian monoid with unit 1 s.t. a * b ≤ c * d whenever a ≤ c and b≤ d , ∀ a, b, c, d ∈ [0,1]. Some examples are below:

(i) *(a, b) = ab,

(ii) *(a, b) = min.{a, b}.

**Definition 2.3[10].**The 3-tuple (X, M, *) is called a fuzzy metric space if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set on $X^2 \times [0,\infty)$ satisfying the following conditions:

(FM-1) M(x, y, t) > 0 and M(x, y, 0) = 0

(FM-2) M(x, y, t) = 1 iff x=y,

(FM-3) M(x, y, t) = M(y, x, t),

(FM-4) M(x, y, t) * M(y, z, s) ≤ M(x, z, t + s),

(FM-5) M(x, y, . ) : (0,∞) → [0,1] is continuous, for all x, y, z ∈ X and s, t > 0.

We note that M(x, y, . ) is non-decreasing for all x, y ∈ X.

**Definition 2.4** Let $(X, M, *)$ be a fuzzy metric space.

(i) A sequence $\{x_n\}$ is said to be convergent to a point x ∈ X if $\lim_{n\to\infty} M(x_n, x, t) = 1$ for all t > 0 .

(ii) A subset A⊆X is said to be closed if each convergent sequence $\{x_n\}$ with $x_n \in A$ and $x_n \to x$ , we have x ∈ A .

(iii) A subset A⊆X is said to be compact if each sequence in A has a convergent subsequence .

Through out the paper X will represent the fuzzy metric space (X,M,∗) and $\kappa(X)$ , the set of compact subsets of X. For A, B ∈ $\kappa(X)$ and for every t >0 , denote

$$M_{\nabla}(A,B,t) = \min\{\min_{a\in A}(a,B,t), \min_{b\in B}(A,b,t)\}$$

$$M^{\Delta}(A,y,t) = \max\{M(x,y,t); x, y \in A\}$$

Remark : Obviously , $M_{\nabla}(A,B,t) \le M^{\Delta}(a,B,t)$ whenever $a \in A$ and $M_{\nabla}(A,B,t) = 1 \Leftrightarrow A = B.$ Also $M^{\Delta}(A,y,t) = 1$ if $y \in A$.

**Definition 2.5** Let X be a nonempty set with $f : X \to X$ and $A : X \times X \to \kappa(X)$. A point (x , y) ∈ X is called

(i) Coupled coincidence point of the pair { f, A } if fx∈ A( x ,y ) and fy∈ A( y ,x )

(ii) Coupled fixed point of the pair { f, A } if x = fx∈ A( x ,y ) and y = fy∈ A( y ,x )

We denote the set of coupled coincidence points of mappings $f : X \to X$ and $A : X \times X \to \kappa(X)$ by C(A, f) . Note that if (x , y) ∈ C (A,f) then (y,x) ∈ C (A,f).

Pathak and Shahzad [3] , Sintunavarat and Kumum [ 13] defined the concept of tangential property for single –valued and multi-valued mappings in metric spaces as follows :

**Definition 2.6.**Let ( X , d ) be a metric space and f , g : X→X . Then z ∈X is called weak tangent point to the pair ( f, g ) if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = z \in X .$$

**Definition 2.7.**Let ( X , d ) be a metric space . Let f : X→X and $g : X \to \kappa(X)$. Then f is called tangential with respect to the mapping g if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} fy_n = z$ , for some z ∈X , then

$$z \in \lim_{n\to\infty} gx_n = \lim_{n\to\infty} gy_n = C \in \kappa(X)$$

**Definition 2.8.**Let ( X , d ) be a metric space . Let f , g : X→X and $A, B : X \to \kappa(X)$. Then the pair ( f, g) is called tangential with respect to the pair ( A , B ) if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = z$ , for some z ∈X , then

$z \in \lim_{n\to\infty} Ax_n = \lim_{n\to\infty} By_n = C \in \kappa(X)$.

Now , we fuzzify the above notions and define the tangential property for single-valued and multi-valued mappings for coupled maps as follows :

**Definition 2.9.**Let f : X→X and $A: X \times X \to \kappa(X)$. Then f is called tangential with respect to the mapping A if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\begin{aligned} \lim_{n\to\infty} fx_n = z_1 \in \lim_{n\to\infty} A(x_n, y_n) = C \in \kappa(X) \\ \lim_{n\to\infty} fy_n = z_2 \in \lim_{n\to\infty} A(y_n, x_n) = D \in \kappa(X) \end{aligned},$$

for some $z_1, z_2 \in X$ . And ( $z_1, z_2$ ) is called a coupled weak tangent point to the mapping f.

**Remark :** If $z_1, z_2 \in f(X)$ then ( $z_1, z_2$ ) is called contained coupled weak tangent point to the mapping f.

**Example 2.1** Let X = R and $a * b = ab$ and $M(x, y, t) = \dfrac{t}{t + d(x, y)}$, then $(X, M, *)$ is a fuzzy metric space. Define f : X→X and $A: X \times X \to \kappa(X)$ by setting $fx = x$ and $A(x, y) = [x-2, y+3]$. Consider the sequences $\{x_n\} = 1 + \dfrac{1}{n}$ and $\{y_n\} = 2 - \dfrac{1}{n}$, then

$$\begin{aligned} \lim_{n\to\infty} fx_n \to 1 \in \lim_{n\to\infty} A(x_n, y_n) = [-1, 5] \\ \lim_{n\to\infty} fy_n \to 2 \in \lim_{n\to\infty} A(y_n, x_n) = [0, 4] \end{aligned}$$

Hence the mapping f is tangential w.r.t. the mapping A. Also $f(1) = 1$ and $f(2) = 2$ so

(1,2) ∈ f(X) and hence (1,2) is a contained weak tangent point to the mapping f.

**Definition 2.10.**Let f : X→X and $A: X \times X \to X$ . Then f is called tangential with respect to the mapping A if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\begin{aligned} \lim_{n\to\infty} fx_n = z_1 = \lim_{n\to\infty} A(x_n, y_n) \\ \lim_{n\to\infty} fy_n = z_2 = \lim_{n\to\infty} A(y_n, x_n) \end{aligned}$$

for some $z_1, z_2 \in X$ . And ( $z_1, z_2$ ) is called a coupled weak tangent point to the mapping f.

**Example 2.2.**Let X = R and $a*b=ab$ and $M(x,y,t)=\dfrac{t}{t+d(x,y)}$, then $(X,M,*)$ is a fuzzy metric space. Define f : X→X and $A:X\times X\to X$ by setting $fx=3-2x$ and $A(x,y)=y-x$. Consider the sequences $\{x_n\}=1-\dfrac{1}{n}$ and $\{y_n\}=2+\dfrac{1}{n}$, then

$\lim_{n\to\infty} fx_n \to 1=\lim_{n\to\infty} A(x_n,y_n)$ and $\lim_{n\to\infty} fy_n \to -1=\lim_{n\to\infty} A(y_n,x_n)$ . Also $f(1)=1$ and $f(2)=-1$ and hence (1,-1) is a contained weak tangent point to the mapping f.

## 3. Main Results

**Theorem 3.1.**Let A , B : X→X and $S,T:X\times X\to\kappa(X)$ be single and set-valued mappings satisfying the following conditions :

(3.1) there exist contained coupled weak tangential points $(z_1,z_2)$ to the mappings A and B .

(3.2) (A,B) is tangential w.r.t ( S,T).

(3.3) $\int_0^{M_\nabla(S(x,y),T(u,v),t)}\psi(s)\,ds \ge \int_0^{m(x,y,u,v,t)}\psi(s)\,ds$ where

$$m(x,y,u,v,t)=\phi\left[\begin{array}{l} a\left(M^\Delta\left(S(x,y),Ax,t\right)*M^\Delta\left(T(u,v),Bu,t\right)\right) \\ +(1-a)\max\left\{\begin{array}{l} M^\Delta\left(S(x,y),Ax,t\right),M^\Delta\left(T(u,v),Bu,t\right), \\ M_\nabla\left(S(x,y),Ax,t\right),M_\nabla\left(T(u,v),Bu,t\right) \end{array}\right\}\end{array}\right]$$

(3.4) $AAa=Aa, BBc=Bc, S(Aa,Ab)=T(Bc,Bd)$ and

$AAb=Ab, BBd=Bd, S(Ab,Aa)=T(Bd,Bc)$ for $(a,b)\varepsilon C(A,S)$ and $(c,d)\varepsilon C(B,T)$

(3.5) the pair (A,S) is weakly compatible ,

for all $x,y,u,v\varepsilon X$ , $0\le a<1$ and $\phi:[0,1]\to[0,1]$ be a non-decreasing map such that $\phi(t)>t, t\ge 0$ Then A,B,S and T have a common coupled fixed point in X .

**Proof:** Since $z_1, z_2 \varepsilon A(x) \cap B(X)$ so there exist points $w_1, w_2, w_1^{/}, w_2^{/} \varepsilon X$ such that $z_1 = Aw_1 = Bw_1^{/}, z_2 = Aw_2 = Bw_2^{/}$. Again (A,B) is tangential w.r.t ( S,T) so there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = z_1 = \lim_{n\to\infty} By_n \varepsilon C \varepsilon \kappa(X) = \lim_{n\to\infty} S(x_n, y_n) \cap \lim_{n\to\infty} T(y_n, x_n)$$
$$\lim_{n\to\infty} Ay_n = z_2 = \lim_{n\to\infty} Bx_n \varepsilon D \varepsilon \kappa(X) = \lim_{n\to\infty} S(y_n, x_n) \cap \lim_{n\to\infty} T(x_n, y_n)$$

Now , we shall prove that $Aw_1 \varepsilon S(w_1, w_2), Aw_2 \varepsilon S(w_2, w_1), Bw_1^{/} \varepsilon T(w_1^{/}, w_2^{/})$ and $Bw_2^{/} \varepsilon T(w_2^{/}, w_1^{/})$ If not , putting $x = x_n, y = y_n, u = w_1^{/}$ and $v = w_2^{/}$ in (3.3) , we get

$$m(x_n, y_n, w_1^{/}, w_2^{/}, t) = \phi\left[\begin{array}{l} a\left(M^{\Delta}\left(S\left(x_n, y_n\right), Ax_n, t\right) * M^{\Delta}\left(T\left(w_1^{/}, w_2^{/}\right), Bw_1^{/}, t\right)\right) \\ +(1-a)\max\left\{\begin{array}{l} M^{\Delta}\left(S\left(x_n, y_n\right), Ax_n, t\right), M^{\Delta}\left(T\left(w_1^{/}, w_2^{/}\right), Bw_1^{/}, t\right), \\ M_{\nabla}\left(S\left(x_n, y_n\right), Ax_n, t\right), M_{\nabla}\left(T\left(w_1^{/}, w_2^{/}\right), Bw_1^{/}, t\right) \end{array}\right\} \end{array}\right]$$

$$\int_0^{M_{\nabla}\left(S(x_n, y_n), T\left(w_1^{/}, w_2^{/}\right), t\right)} \psi(s)\, ds \geq \int_0^{m(x_n, y_n, w_1^{/}, w_2^{/}, t)} \psi(s) ds$$

Letting n →∞ , we have

$$\int_0^{\lim_{n\to\infty} M_{\nabla}\left(S(x_n, y_n), T\left(w_1^{/}, w_2^{/}\right), t\right)} \psi(s)\, ds \geq \int_0^{\lim_{n\to\infty} m(x_n, y_n, w_1^{/}, w_2^{/}, t)} \psi(s) ds, \quad \text{but}$$

$$\lim_{n\to\infty} m\left(x_n, y_n, w_1^{/}, w_2^{/}, t\right) = \phi\left[a\left(1 * M^{\Delta}\left(T\left(w_1^{/}, w_2^{/}\right), Bw_1^{/}, t\right)\right) + (1-a)\max\left\{\begin{array}{l} 1, M^{\Delta}\left(T\left(w_1^{/}, w_2^{/}\right), Bw_1^{/}, t\right), \\ 1, M_{\nabla}\left(T\left(w_1^{/}, w_2^{/}\right), Bw_1^{/}, t\right) \end{array}\right\}\right]$$

$$= \phi\left[aM^{\Delta}\left(T\left(w_1^{/}, w_2^{/}\right), Bw_1^{/}, t\right) + (1-a)M^{\Delta}\left(T\left(w_1^{/}, w_2^{/}\right), Bw_1^{/}, t\right)\right]$$

$$= \phi\left(M^{\Delta}\left(T\left(w_1^{/}, w_2^{/}\right), Bw_1^{/}, t\right)\right)$$

$$\int_0^{M_{\nabla}\left(C, T\left(w_1^{/}, w_2^{/}\right), t\right)} \psi(s)\, ds \geq \int_0^{=\phi\left(M^{\Delta}\left(T\left(w_1^{/}, w_2^{/}\right), Bw_1^{/}, t\right)\right)} \psi(s) ds .$$

Since , $z_1 = Bw_1^{/} \in C$ , we have

$$\int_0^{M^{\Delta}\left(Bw_1^{/},T\left(w_1^{/},w_2^{/}\right),t\right)}\psi\left(s\right)ds \geq \int_0^{M_{\nabla}\left(C,T\left(w_1^{/},w_2^{/}\right),kt\right)}\psi\left(s\right)ds \geq \int_0^{\phi\left(M^{\Delta}\left(T\left(w_1^{/},w_2^{/}\right),Bw_1^{/},t\right)\right)}\psi\left(s\right)ds$$

$$\Rightarrow \int_0^{M^{\Delta}\left(Bw_1^{/},T\left(w_1^{/},w_2^{/}\right),t\right)}\psi\left(s\right)ds \geq \int_0^{\phi\left(M^{\Delta}\left(Bw_1^{/},T\left(w_1^{/},w_2^{/}\right),t\right)\right)}\psi\left(s\right)ds \geq \int_0^{M^{\Delta}\left(Bw_1^{/},T\left(w_1^{/},w_2^{/}\right),t\right)}\psi\left(s\right)ds$$

which is a contradiction. Hence $Bw_1^{/} \in T(w_1^{/}, w_2^{/})$ .

Similarly , by putting $x = y_n, y = x_n, u = w_2^{/}$ and $v = w_1^{/}$ in (3.3) , we get $Bw_2^{/} \varepsilon T\left(w_2^{/}, w_1^{/}\right)$.

Again by taking $x = w_1, y = w_2, u = y_n$ and $v = x_n$ in (3.3) , we get

$$m\left(w_1, w_2, y_n, x_n, t\right) = \phi\left[\begin{array}{l} a\left(M^{\Delta}\left(S\left(w_1, w_2\right), Aw_1, t\right) * M^{\Delta}\left(T\left(y_n, x_n\right), By_n, t\right)\right) \\ +\left(1-a\right)\max\left\{\begin{array}{l} M^{\Delta}\left(S\left(w_1, w_2\right), Aw_1, t\right), M^{\Delta}\left(T\left(y_n, x_n\right), By_n, t\right), \\ M_{\nabla}\left(S\left(w_1, w_2\right), Aw_1, t\right), M_{\nabla}\left(T\left(y_n, x_n\right), By_n, t\right) \end{array}\right\} \end{array}\right]$$

$\int_0^{M_{\nabla}\left(S\left(w_1,w_2\right),T\left(y_n,x_n\right),t\right)}\psi\left(s\right)ds \geq \int_0^{m\left(w_1,w_2,y_n,x_n,t\right)}\psi\left(s\right)ds$, hence

$$\int_0^{\lim_{n\to\infty} M_{\nabla}\left(S\left(w_1,w_2\right),C,t\right)}\psi\left(s\right)ds \geq \int_0^{\lim_{n\to\infty} m\left(w_1,w_2,y_n,x_n,t\right)}\psi\left(s\right)ds = \int_0^{\phi\left(M^{\Delta}\left(S\left(w_1,w_2\right),Aw_1,t\right)\right)}\psi\left(s\right)ds$$

As $z_1 = Aw_1 \in C \in C$ , we have

$$\int_0^{M^{\Delta}\left(S\left(w_1,w_2\right),Aw_1,t\right)}\psi\left(s\right)ds \geq \int_0^{M_{\nabla}\left(S\left(w_1,w_2\right),C,t\right)}\psi\left(s\right)ds \geq \int_0^{\phi\left(M^{\Delta}\left(S\left(w_1,w_2\right),Aw_1,t\right)\right)}\psi\left(s\right)ds \geq \int_0^{M^{\Delta}\left(S\left(w_1,w_2\right),Aw_1,t\right)}\psi\left(s\right)ds$$

which is a contradiction . Hence $Aw_1 \in S\left(w_1, w_2\right)$.

Similarly , by putting $x = w_2, y = w_1, u = x_n$ and $v = y_n$ in (3.3) , we get $z_2 = Aw_2 \varepsilon S\left(w_2, w_1\right)$.

Hence $\left(w_1, w_2\right)\varepsilon C(A,S)$ and $(w_1^{/}, w_2^{/})\varepsilon C(B,T)$ . Now (3.4) ,gives

$AAw_1 = Aw_1, BBw_1^{/} = Bw_1^{/}$ and $S(Aw_1, Aw_2) = T(Bw_1^{/}, Bw_2^{/})$

$AAw_2 = Aw_2, BBw_2^{/} = Bw_2^{/}$ and $S(Aw_2, Aw_1) = T(Bw_2^{/}, Bw_1^{/})$ .

But we have $z_1 = Aw_1 = Bw_1{}^{/}, z_2 = Aw_2 = Bw_2{}^{/}$.Which gives ,

$Az_1 = z_1 = Bz_1$ and $S(z_1, z_2) = T(z_1, z_2)$

$Az_2 = z_2 = Bz_2$ and $S(z_2, z_1) = T(z_2, z_1)$

Also , weak compatibility of ( A , S ) gives

$AS(w_1, w_2) \subseteq S(Aw_1, Aw_2) \Rightarrow z_1 = Bz_1 = Az_1 \varepsilon AS(w_1, w_2) \subseteq S(Aw_1, Aw_2) = S(z_1, z_2) = T(z_1, z_2)$ .Similarly , we can have $z_2 = Bz_2 = Az_2 \varepsilon S(z_2, z_1) = T(z_2, z_1)$ . Hence $(z_1, z_2)$ is a common coupled fixed point of the mappings A,B,S and T.

**Example 2.3.**Let X = R and $a*b = ab$ and $M(x, y, t) = \dfrac{t}{t + d(x, y)}$, then $(X, M, *)$ is a fuzzy metric space. Define A,B: X→X and $S, T : X \times X \to \kappa(X)$ by setting $Ax = \begin{Bmatrix} 2x-1, x \le 1 \\ 3, x > 1 \end{Bmatrix}, Bx = \begin{Bmatrix} 2-x, 1 \le x \le 2 \\ 3, otherwise \end{Bmatrix}$ and

$$S(x, y) = \begin{Bmatrix} [x+y-4, x+y+2], if x, y \in [0,3] \\ [x-1, y-1], otherwise \end{Bmatrix}.$$

$$T(x, y) = \begin{Bmatrix} [2x-y+1, 3x+y], x < y \\ [x-2y-1, x+3], x \ge y \end{Bmatrix}$$

Consider the sequences $\{x_n\} = 1 - \dfrac{1}{n}$ and $\{y_n\} = 1 + \dfrac{1}{n}$, then

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} By_n \to 1 \in \lim_{n\to\infty} S(x_n, y_n) \cap \lim_{n\to\infty} T(y_n, x_n)$$
$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n \to 3 \in \lim_{n\to\infty} S(y_n, x_n) \cap \lim_{n\to\infty} T(x_n, y_n)$$

This shows that (A,B) is tangential w.r.t ( S,T)

Also , $\begin{matrix} A1 = B1 = 1 \in [0,6] = S(1,3) \cap T(3,1) \\ A3 = B3 = 3 \in [0,6] = S(3,1) \cap T(1,3) \end{matrix}$

Hence all the conditions of above theorems are satisfied and (1,3) is a coupled fixed point of the maps A,B,S and T.

**Theorem 3.2.** Let A , B : X→X and $S,T : X\times X \to \kappa(X)$ be single and set-valued mappings satisfying the following conditions :

(3.6) there exist contained coupled weak tangential points $(z_1, z_2)$ to the mappings A and B .

(3.7) (A,B) is tangential w.r.t ( S,T).

(3.8) $\int_0^{M_\nabla(S(x,y),T(u,v),t)} \psi(s)\,ds \ge \int_0^{m(x,y,u,v,t)} \psi(s)\,ds$ where

$$m(x,y,u,v,t)=\phi\left[\begin{array}{l} a\left(M^\Delta\left(S(x,y),Ax,t\right),M^\Delta\left(T(u,v),Bu,t\right)\right) \\ +(1-a)\max\left\{\begin{array}{l} M^\Delta\left(S(x,y),Ax,t\right)*M^\Delta\left(T(u,v),Bu,t\right), \\ M_\nabla\left(S(x,y),Ax,t\right),M_\nabla\left(T(u,v),Bu,t\right) \end{array}\right\}\end{array}\right]$$

(3.9) $AAa = Aa = BBc, S(Aa,Ab) = T(Bc,Bd)$ and $AAb = Ab = BBd, S(Ab,Aa) = T(Bd,Bc)$ for $(a,b)\varepsilon C(A,S)$ and $(c,d)\varepsilon C(B,T)$

(3.10) the pair (A,S) is weakly compatible ,

for all $x,y,u,v\varepsilon X$ , $0\le a<1$ and $\phi:[0,1]\to[0,1]$ be a non-decreasing map such that $\phi(t)>t, t\ge 0$ . Then A , B , S and T have a common coupled fixed point in X .

Proof The result follows directly from theorem 3.1 .

**Theorem 3.3.** Let A , B : X→X and $S,T : X\times X \to \kappa(X)$ be single and set-valued mappings satisfying the following conditions :

(3.11) there exist contained coupled weak tangential points $(z_1, z_2)$ to the mappings A and B .

(3.12) (A,B) is tangential w.r.t ( S,T).

(3.13) $\int_0^{M_\nabla(S(x,y),T(u,v),t)} \psi(s)\,ds \ge \int_0^{m(x,y,u,v,t)} \psi(s)\,ds$ where

$$m(x,y,u,v,t)=\phi\left[\begin{array}{l} a\left(M^{\Delta}\left(S\left(x,y\right),Ax,t\right),M^{\Delta}\left(T\left(u,v\right),Bu,t\right)\right) \\ +\left(1-a\right)\max\left\{\begin{array}{l} M^{\Delta}\left(S\left(x,y\right),Ax,t\right),M^{\Delta}\left(T\left(u,v\right),Bu,t\right), \\ M_{\nabla}\left(S\left(x,y\right),Ax,t\right),M_{\nabla}\left(T\left(u,v\right),Bu,t\right) \end{array}\right\} \end{array}\right]$$

(3.14) $$M_{\nabla}\left(S\left(x,y\right),T\left(u,v\right),t\right)\geq\left[\begin{array}{l} a\left(M^{\Delta}\left(S\left(x,y\right),Ax,t\right),M^{\Delta}\left(T\left(u,v\right),Bu,t\right)\right) \\ +\left(1-a\right)\max\left\{\begin{array}{l} M^{\Delta}\left(S\left(x,y\right),Ax,t\right)*M^{\Delta}\left(T\left(u,v\right),Bu,t\right), \\ M_{\nabla}\left(S\left(x,y\right),Ax,t\right),M_{\nabla}\left(T\left(u,v\right),Bu,t\right) \end{array}\right\} \end{array}\right]$$

(3.15) $AAa=Aa=BBc, S(Aa,Ab)=T(Bc,Bd)$ and $AAb=Ab=BBd, S(Ab,Aa)=T(Bd,Bc)$

for $(a,b)\varepsilon C(A,S)$ and $(c,d)\varepsilon C(B,T)$

(3.16) the pair (A,S) is weakly compatible ,

for all $x,y,u,v\varepsilon X$ , $0\leq a<1$. Then A , B , S and T have a common coupled fixed point in X .

Proof The result follows directly from theorem 3.1 if we take $\phi(t)=t$ .

**Corollary 3.1.**Let A , B : X→X and $S,T: X\times X\to X$ be single and set-valued mappings satisfying the following conditions :

(3.17) there exist contained coupled weak tangential points $(z_1,z_2)$ to the mappings A and B .

(3.18) (A,B) is tangential w.r.t ( S,T).

(3.19) $\int_0^{M_{\nabla}\left(S(x,y),T(u,v),t\right)}\psi(s)\,ds\geq\int_0^{m(x,y,u,v,t)}\psi(s)\,ds$ where

$$m(x,y,u,v,t)=\phi\left[\begin{array}{l} a\left(M\left(S\left(x,y\right),Ax,t\right),M\left(T\left(u,v\right),Bu,t\right)\right) \\ +\left(1-a\right)\max\left\{\begin{array}{l} M\left(S\left(x,y\right),Ax,t\right)*M\left(T\left(u,v\right),Bu,t\right), \\ M\left(S\left(x,y\right),Ax,t\right),M\left(T\left(u,v\right),Bu,t\right) \end{array}\right\} \end{array}\right]$$

(3.20) $AAa=Aa=BBc, S(Aa,Ab)=T(Bc,Bd)$ and $AAb=Ab=BBd, S(Ab,Aa)=T(Bd,Bc)$

for $(a,b)\varepsilon C(A,S)$ and $(c,d)\varepsilon C(B,T)$

(3.21) the pair (A,S) is weakly compatible , for all $x,y,u,v\varepsilon X$ , $0\le a<1$ and $\phi:[0,1]\to[0,1]$ be a non-decreasing map such that $\phi(t)>t,t\ge 0$. Then A , B , S and T have a common coupled fixed point in X .

**Corollary 3.2.**Let A , B : X→X and $S,T:X\times X\to X$ be single and set-valued mappings satisfying the following conditions :

(3.22) there exist contained coupled weak tangential points $(z_1,z_2)$ to the mappings A and B .

(3.23) (A,B) is tangential w.r.t ( S,T).

(3.24) $\int_0^{M_\nabla(S(x,y),T(u,v),t)}\psi(s)\,ds\ge\int_0^{m(x,y,u,v,t)}\psi(s)\,ds$ where

$$m(x,y,u,v,t)=\left[\begin{array}{l} a\left(M\left(S(x,y),Ax,t\right),M\left(T(u,v),Bu,t\right)\right) \\ +(1-a)\max\left\{\begin{array}{l} M\left(S(x,y),Ax,t\right)*M\left(T(u,v),Bu,t\right), \\ M\left(S(x,y),Ax,t\right),M\left(T(u,v),Bu,t\right) \end{array}\right\}\end{array}\right]$$

(3.25) $AAa=Aa=BBc,S(Aa,Ab)=T(Bc,Bd)$ and $AAb=Ab=BBd,S(Ab,Aa)=T(Bd,Bc)$

for $(a,b)\varepsilon C(A,S)$ and $(c,d)\varepsilon C(B,T)$

(3.26) the pair (A,S) is weakly compatible ,

for all $x,y,u,v\varepsilon X$ , $0\le a<1$. Then A , B , S and T have a common coupled fixed point in X .

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