Available online at http://scik.org
Eng. Math. Lett. 2014, 2014:17
ISSN: 2049-9337

WAVE ATTENUATION OVER A SUBMERGED POROUS MEDIA

I. MAGDALENA

Industrial and Financial Mathematics Research Group,
Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Indonesia,
Jalan Ganesha 10, Bandung, West Java, Indonesia

Abstract. In this paper, a numerical modeling for free surface flows over submerged porous media is outlined. The governing equations used are two layers linear shallow water model. For flow in a submerged porous medium, linear friction resistance is included in momentum equation. Solving the linearized full governing equations analytically, we can obtain dispersion relation that holds for gravity waves over submerged porous media. This dispersion relation explain diffusive mechanism of the porous structure. Numerically, finite volume on a staggered grid is implemented to solve these equations. Numerical simulation of incoming waves over porous media shows the wave amplitude is attenuated because of the porous structure. The validity of this numerical model is confirmed with analytical result.

Keywords: submerged porous media; two layers linear shallow water equation; linear friction resistance; staggered finite volume method.

2010 AMS Subject Classification: 76S05, 35L02.

1. Introduction

An array of artificial reefs can be used as a submerged breakwater for offshore protection. This type of breakwater provides environmental enhancement and aesthetics that are not found

Received June 12, 2014

in conventional breakwaters. An array of reefballs is assumed to be a submerged porous layer with certain dimension and diffusive parameters. This research study about wave amplitude reduction due to wave interaction with a submerged porous breakwater.

The literatures that are discussed about wave over a submerged breakwater is in the following. S.R. Pudjaprasetya [7] investigate the optimal dimension of submerged rigid structure as a wave reflector. Kobayashi [4] study wave transmission over rigid breakwater. Chao-Lung Ting et.al. [12] and K. Mizumura [6] study wave propagation with current over permeable ripple beds and consider Bragg resonance phenomenon. For the submerged porous structure, Armono [1] study the performance of submerged breakwater by using Multiple Regression Analysis. Chin-Piao Tsai *et al.* [13] use potential wave theory for formulating an approximate linear equation for monochromatic waves over submerged permeable breakwater. Pudjaprasetya and Magdalena [8] study the wave energy dissipation within porous breakwater by using potential theory and solve the approximate equation numerically by using Lax Wendroff Method. Wiryanto [14] explained the behavior monochromatic waves passing over submerged porous breakwater by using potential theory. Gu [3] study numerical modeling for wave energy dissipation within porous submerged breakwaters, and solve the equation numerically by using boundary integral element method.

In this paper, the effectiveness of a submerged porous permeable breakwater on dissipating wave amplitude is studied. We used the modified linear shallow water equation type for submerged porous media. The linearized Forchheimer resistance is included in the momentum equation for porous medium by means of extra friction terms. Here, we implement the staggered conservative scheme to solve the Linear Shallow Water Equations (LSWE) numerically. Numerical simulations were conducted to show wave damping of a submerged porous breakwater. To validate our model, we compare wave amplitude reduction with the analytical result.

2. Governing equations

The governing equation of two layer model is formulated below. Consider an upper layer Ω_1 which is fluid over a porous structure and lower layer Ω_2 is fluid in a porous medium.

For flow in a porous media with porosity n, the rate of change of free surface η depends on the filtered horizontal momentum with filtered velocity $\frac{U}{n}$, where $0 < n \leq 1$. The momentum

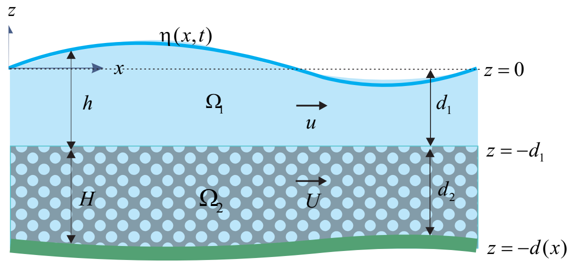

FIGURE 1. Sketch of two layer fluid domain.

equation (2) also gets an additional resistance by porous structure is denoted by $f\frac{\omega}{n}U$. The full governing equations are

$$\eta_t = -(hu)_x - \frac{1}{n}(HU)_x \text{ in } \Omega_1 \cup \Omega_2, \tag{1}$$

$$u_t = -g\eta_x - uu_x \text{ in } \Omega_1, \tag{2}$$

$$\frac{1}{n}U_t = -g\eta_x - \frac{1}{n^2}UU_x - f\frac{\omega}{n}U \text{ in } \Omega_2, \tag{3}$$

where ω wave frequency, f friction coefficient, and g gravitational acceleration. Note that capital notation is denoted information in lower layer and small letter is for upper layer. Notation η denotes surface elevation, u, U horizontal fluid velocity, and $h = d_1(x) + \eta(x,t), H = d_2(x)$ are the layer thickness, where d_1 is undisturbed thickness for upper layer and d_2 is for lower layer.

3. Dispersion relation

In this section, we derived the dispersion relation for porous flow. We assume that our observation is in shallow area then we neglect the nonlinear term and approximate $h \simeq d_1$ and $H \simeq d_2$. The linearized governing equations for flat bottom are:

$$\eta_t = -d_1 u_x - d_2\frac{1}{n}U_x \text{ in } \Omega_1 \cup \Omega_2, \tag{4}$$

$$u_t = -g\eta_x \text{ in } \Omega_1, \tag{5}$$

$$\frac{1}{n}U_t = -g\eta_x - f\frac{\omega}{n}U \text{ in } \Omega_2, \tag{6}$$

In this section we derived dispersion relation for the full linearized governing equations (4-6). We solve the full linearized governing equations using separation variables and assume periodic in time. They are expressed as

$$\eta(x,t) = F(x)e^{i\omega t}, \tag{7}$$

$$u(x,t) = G(x)e^{i\omega t}, \tag{8}$$

$$U(x,t) = E(x)e^{i\omega t}. \tag{9}$$

Introducing the equations (7, 8, 9) into equations (1-3) yields

$$i\omega F = -d_1 G_x - \frac{d_2}{n}E_x, \tag{10}$$

$$G = \frac{-g}{i\omega}F_x, \tag{11}$$

$$E = \frac{-ng}{i\omega + f\omega}F_x. \tag{12}$$

By substituting (11,12) into equation (10), we get the general solution for ordinary differential equation $F_x x + k^2 F = 0$:

$$F(x) = Ae^{ikx} + Be^{-ikx}, \tag{13}$$

where k as a complex wave number follows this dispersion relation

$$k^2 = \frac{\omega^2(1-if)}{gd_1(1-if)+gd_2}. \tag{14}$$

We test the condition of no porous $f = 0$ then dispersion relation (14) become the well-known dispersion relation for shallow water:

$$k^2 = \frac{\omega^2}{g(d_1+d_2)}.$$

Then, we test for an emerged porous structure $d_1 = 0$ we have dispersion relation (14) become

$$k^2 = \frac{\omega^2(1-if)}{gd_2}$$

and it confirms the dispersion relation for surface waves pass through an emerged porous breakwater [5].

Taking parameter values $\omega = 6\pi$, $d_1 = 10$, $d_2 = 10$, $g = 9.81$, $n = 0.7, f = 0.25$, dispersion relation (14) will give us a complex value wave number $k = 1.358501600 - 0.0820332133\ i$. A monochromatic wave $\exp^{-i(\kappa x - \omega t)}$ with negative imaginary part $\Im(k)$ will undergo amplitude reduction, see Figure 2.

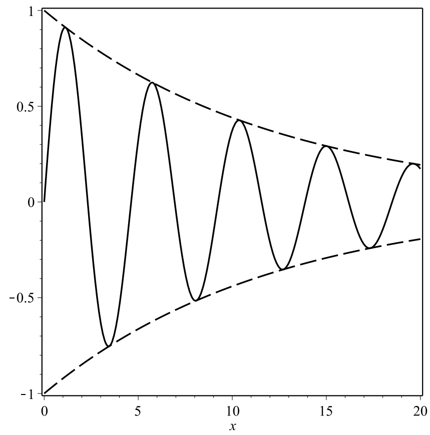

FIGURE 2. The curve $\eta(x,t) = \exp^{-i(kx-\omega t)}$ with $k = 1.358501600 - 0.0820332133\, i$ (solid line) and its envelope (dotted line) $K_T = |\eta(x,t)|$.

4. Staggered finite volume method

In this section, we solve equations (1, 2, 3) numerically using finite volume method on a staggered grid. This method but for free fluid flow is described extensively in G.S.Stelling [10, 11].

Equation (1) is approximated at a cell centered at full grid points x_i. Equations (2,3) are approximated at a cell centered at half grid points $x_{i+1/2}$. The values of η will be computed at every full grid points x_i, with $i = 1,2,...,Nx$ using mass conservation (1). Velocity u, U will be computed at every staggered grid points $x_{i+\frac{1}{2}}$, with $i = 1,2,...,Nx-1$ using momentum equation (2,3), see Figure (3).

Implement forward time center space to equations (4, 5, and 6) yield:

$$\frac{\eta_i^{n+1}-\eta_i^n}{\Delta t} + \frac{{}^*q|_{i+1/2}^n - {}^*q|_{i-1/2}^n}{\Delta x} + \frac{1}{n}\frac{{}^*Q|_{i+1/2}^n - {}^*Q|_{i-1/2}^n}{\Delta x} = 0 \tag{15}$$

$$\frac{u_{i+1/2}^{n+1}-u_{i+1/2}^n}{\Delta t} + g\frac{\eta_{i+1}^{n+1}-\eta_i^{n+1}}{\Delta x} = 0 \tag{16}$$

$$\frac{\frac{1}{n}U_{i+1/2}^{n+1}-\frac{1}{n}U_{i+1/2}^n}{\Delta t} + g\frac{\eta_{i+1}^{n+1}-\eta_i^{n+1}}{\Delta x} + f\frac{\omega}{n}U_{i+1/2}^{n+1} = 0, \tag{17}$$

where ${}^*q = {}^*d_1 u$ and $Q = {}^*d_2 U$.

In equation (15), the value of water thickness $d_{1,i+1/2}$ and $d_{2,i+1/2}$ in q, Q are not known, because they are calculated only at full point x_i. We approximate its value using first order

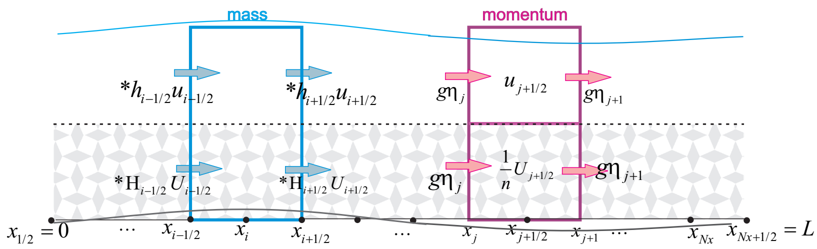

FIGURE 3. Illustration of staggered grid with cell $[x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}]$ for mass conservation and cell $[x_{i-1}, x_i]$ for momentum equation.

upwind method:

$$
^*d_{1,i+\frac{1}{2}} = \begin{cases} d_{1,i}, & \text{if } u_{i+1/2} \geq 0, \\ d_{1,i+1}, & \text{if } u_{i+1/2} < 0, \end{cases} \tag{18}
$$

$$
^*d_{2,i+\frac{1}{2}} = \begin{cases} d_{2,i}, & \text{if } U_{i+1/2} \geq 0, \\ d_{2,i+1}, & \text{if } U_{i+1/2} < 0. \end{cases} \tag{19}
$$

These choice mean when the flow is going to the right $u_{i+1/2} \geq 0$ the $x-$ direction flux across $i+1/2$ then we take information from the left $d_{1,i}u_{i+1/2}$. And when the flow is going to the left $u_{i+1/2} < 0$ the $x-$ direction flux across $i+1/2$ then we take information from the right $d_{1,i+1}u_{i+1/2}$. It is also hold for lower layer. That means we maintain mass conservation in each cell $[x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}]$.

Implementing Von Neumann stability analysis, we obtain stability condition for the linear equations of (15,16,17) is $\sqrt{g(d_1+d_2)}\frac{\Delta t}{\Delta x} \leq 1$, where d_1+d_2 is the flat bottom depth. Furthermore, this discretization has an amplification matrix that has eigenvalues with norm one, that means this discretization is non-dissipative. Note the laminar friction term $f\frac{\omega}{n}U$ is calculated implicitly in order to avoid more restricted stability condition.

5. Numerical simulation

When wave propagate over a porous breakwater, damping effect is observed due to interaction between fluid and porous structure. Here we will simulate the effects of the porous structure on wave damping. For validating our numerical scheme, we compare wave attenuation with the analytical result.

5.1. Simulation of wave damping by a porous breakwater

For simulation of an incoming monochromatic wave passing over a submerged porous breakwater, we take a computational domain $0 < x < 20, 0 < t < 2$. We take $g = 9.81$ and a constant depth $d_1 = 10$, $d_2 = 10$. The initial condition is still water level $\eta(x,0) = u_1(x,0) = u_2(x,0) = 0$ and the left influx monochromatic wave of amplitude $a = 1$:

$$\eta(0,t) \quad = \quad a \sin 6\pi t. \tag{20}$$

Along the right boundary, we apply absorbing boundary by means of a sponge layer techniques. For computations we use $\Delta x = 0.05, \Delta t = \Delta x/\sqrt{g(d_1+d_2)} = 0.0036$.

We test the scheme for no porous condition: $n = 1$ and $f = 0$ equation (1, 2, and 3) reduces to the shallow water equations. Numerical simulation for flat bottom, using zero initial condition and left influx (20) will yield a monochromatic wave travels undisturbed in shape, as we expect.

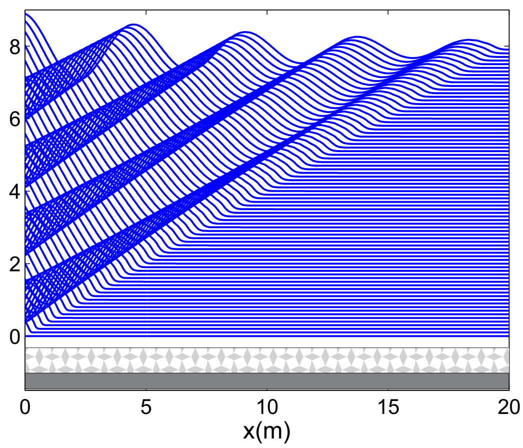

FIGURE 4. Amplitude of incoming monochromatic wave reduces over a submerged porous breakwater.

When the lower layer in whole domain $0 \le x \le 20$ is a porous media with parameters $n = 0.7$, $f = 0.25$ then the results are given in Figure . It shows that wave amplitude reduced by the porous media.

5.2. Comparison with Analytical Result

In this subsection, we show that our numerical surface profile reduces by the porous media with an envelope that confirms the analytical curve. For numerical computations, we use the same setup and parameters that we have used in Subsection . Using parameter values above, dispersion relation (14) results in a complex value wave number $k = 1.358501600 - 0.0820332133\, i$ that we have calculated in Subsection . We plot together the evolution of surface elevation along x direction with the envelope which is the curve of wave damping solution from analytic $|\exp^{-i(kx-\omega t)}|$. The surface profile in a porous media is plotted in Figure . Clearly, the numerical wave amplitude reduction confirms the analytical result.

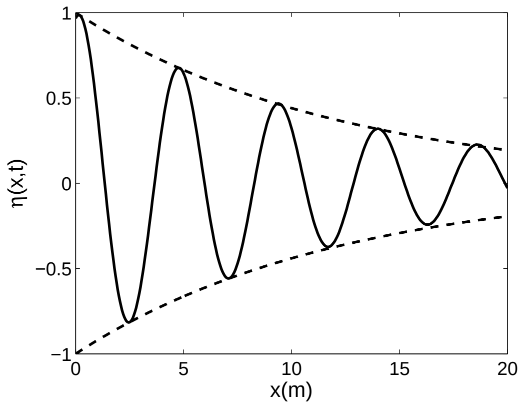

FIGURE 5. Comparison between numerical result (line) with analytical data (dash).

6. Conclusions

We have presented a nonlinear two layer shallow water equation for wave propagation over porous bottom. From the full governing equation, the dispersion relation for this problem is obtained. The dispersion relation give us a clear picture of the dissipative effect of gravity waves over a submerged porous breakwater with certain characteristics. We implemented numerical method for solving the nonlinear two layer model. Numerical simulation of wave dissipation over porous media has been conducted. The numerical wave attenuation has confirmed the analytical wave and experimental data from literature.

Conflict of Interests

The author declares that there is no conflict of interests.

References

[1] H.D. Armono, K.R. Hall, Wave transmission on submerged breakwater made of hollow hemispherical shape artificial reefs, Proceeding of The 1st Coastal, Estuary and Offshore Engineering Speciality Conference CSC-185-1:10, 2003. for Numerical Methods in Fluids 30 (1999), 425-440.

[2] H.J.S. Fernando, S.P. Samarawickrama, S. Balasubramanian, S.S.L. Hettiarachchi, S. Voropayev, Effects of porous barriers such as coral reefs on coastal wave propagation, J. Hydro-environment Res. 1 (2008), 187-194.

[3] Z. Gu, H. Wang, Numerical Modeling for Wave Energy Dissipation within Porous Submerged Breakwaters of Irregular Cross Section, Coastal Eng. 90 (1992), 1189-1198.

[4] N. Kobayashi, Wurjanto, Wave Transmission over submerged breakwaters, J. Waterways, Port, Coastal Engineering, ASCE 115 (1989), 662-680.

[5] I. Magdalena, S.R. Pudjaprasetya, L.H. Wiryanto, Wave interaction with an emerged porous media, Adv. Appl. Math. Mech. 6 (2014), 680-692.

[6] K. Mizumura, Free Surface Flow over Permeable Wavy Bed, J. Hydraulic Eng. 124 (1998), 955-962.

[7] S.R. Pudjaprasetya, H.D. Chendra, An optimal dimension of submerged parallel bars as a wave reflector, Bull. Malaysian Math. Sci. Soc. 32 (2009), 55-62.

[8] S.R. Pudjaprasetya, I. Magdalena, Wave energy dissipation over porous media, Appl. Math. Sci. 7 (2013), 2925-2937.

[9] S. Rojanakamthorn, M. Isobe, A. Watanabe, A mathematical model of wave transformation over a submerged breakwater, Coastal Engineering in Japan 31 (1989), 209-234.

[10] G.S. Stelling, S.P.A. Duinmeijer, A staggered conservative scheme for every Froude number in rapidly varied shallow water flows, Int. J. Numer. Meth. Fluids 43 (2003), 1329-1354.

[11] G.S. Stelling, M. Zijlema, An accurate and efficient finite-difference algorithm for non-hydrostatic free-surface flow woth application to wave propagation, Int. J. Numer. Meth. Fluids 2003.

[12] C.L. Ting, M.C. Lin, C.L. Kuo, Bragg scattering of surface waves over permeable rippled beds with current, Phy. Fluids 12 (2010), 1382-1388.

[13] C.P. Tsai, H.B. Chen, F.C. Lee, Wave transformation over submerged permeable breakwater on porous bottom, Ocean Eng. 33 (2006), 1623-1643.

[14] L.H. Wiryanto, Wave propagation passing over a submerged porous breakwater, J. Eng. Math. 70 (2011), 129-136.