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COST BENEFIT ANALYSIS OF A TWO NON-IDENTICAL UNIT STANDBY SYSTEM WITH TWO REPAIRMEN AND REPAIR MACHINE FAILURE

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Abstract: The paper analyses a two non-identical unit (unit-1 and unit-2) cold standby system model in respect of reliability and cost effectiveness indices. Unit-1 gets priority in operation and has two modes of operation-Normal and Quasi-normal (not as good as new). Two repairmen ordinary and skilled are always available with the system to repair and complete overhaul respectively. A repair machine is used to repair the unit-1 when it fails first time and after repair, it goes into quasi-normal mode. Again, when unit-1 fails from quasi-normal mode, it goes for complete overhaul and then it becomes as good as new. The unit-2 always needs complete overhaul upon its failure and becomes as good as new after this maintenance action. The repair time distribution of unit-1 (failed first time) is taken as general whereas the distributions of other random variables denoting failure and repair /complete overhaul of a unit or RM are taken exponential with different parameters.

Keywords: Transition Probability, Regenerative-point, Absorbing state, Pre-emptive repeat repair, Mean sojourn time, Reliability.

2010 AMS Subject Classification: 68M15.

1. Introduction

The incorporation of redundancy is one of the measures to enhance the effectiveness of a system. Various authors analyzed the system models in the field of reliability theory by considering active and passive redundancies. The performance of repair maintenance is an

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additional aspect to improve the reliability of a redundant system. Numerous authors have analyzed the redundant systems under different repair policies. Naidu and Gopalan [8] provided a cost benefit analysis of a two unit repairable system when an operating unit is subject to two types of failure and accordingly two types of repair were considered. Goel et al. [1] analyzed a redundant system comprising of two identical units in cold standby configuration in which each unit can be in either of three modes- normal, partial failure and total failure. They considered a single repairman who repairs the partially failed unit and minor and major repair of totally failed unit with fixed known probabilities p and q. Later, Goel et al. [3] analyzed a two unit cold standby system with two types of repair r_1 and r_2. The repair facility r_1 is cheaper but not immediately available at the time of failure of a unit while r_2 is instantaneously available always at a higher payment. Whenever a unit fails, the repair facility r_1 is intimated and the failed unit waits for a certain amount of time for getting the repairman available. Further, if r_1 is not available up to a maximum waiting time then the costlier repair facility r_2 starts the repair of failed unit. Whenever either type of repair facility is engaged and other unit also fails, it will be repaired by the same repair facility. In view of this both types of repairmen r_1 and r_2 can't be engaged simultaneously in the system. Rander et al. [4] studied a two unit cold standby system with a perfect master repairman and an imperfect assistant repairman. First the failed unit is attended by assistant repairman and if he fails to repair during a prescribed period master repairman undertakes the failed unit for repair immediately. Both repairmen are instantaneously available. Later on, Tuteja and Taneja [7] studied a two server two unit warm standby system assuming that an operative unit can fail completely either directly from the normal mode or via a partial failure mode, while the warm standby unit fails completely due to even minor faults.. A regular repairman always remains available and an expert repairman is called from outside only whenever needed. Goel et al. [2] analyzed a stochastic model related to a two unit chargeable standby system with interchangeable units. The system can fail due to power fluctuations or due to operators inefficiency. A single repair facility is available to decide the type of failure so that appropriate type of repair may be performed. Recently Sridharan and Mohanavadivu [9] analysed the stochastic behaviour of a two-unit standby system with two types of repairmen and patience time. Gupta and Bansal [6] introduced the concept of quasi normal mode in a three unit redundant system when shocks occur randomly on an operating unit i.e. due to occurrence of a shock its effect on the operating unit may be either of following types- (i) operating unit is not affected at all, (ii) the failure rate of the unit may increase and unit is said to work in quasi-normal mode and (iii) operating unit fails completely. Gupta and Chaudhary [5] analyzed the reliability characteristics of a two unit cold standby system under very practical assumption that a repair machine is needed to repair a failed unit which can also breakdown during its working.

The present paper analyses a two non-identical unit standby system model with the concept of repair machine failure, quasi-normal mode and two types of repairman who can work simultaneously. The following measures of system effectiveness, useful to system designers and operations managers, are obtained by using regenerative point technique.

i). Steady state transition probabilities and mean sojourn times.

ii). Reliability of the system and Mean time to system failure (MTSF).

iii). Point-wise and steady state availabilities of the system.

iv). Expected busy period of the repairman in time interval (0, t].

v). Net expected profit earned by the system in time interval (0, t] and in steady state.

2. System Description and Assumptions

Following are the assumptions of the system:

i). The system consists of two non-identical units: unit-1 and unit-2 with a repair machine (RM). Initially unit-1 is operative and unit-2 is kept into cold standby. The repair machine is initially good and is used to repair the unit-1 failed first time. RM cannot fail unless it becomes operative.

ii). The unit-2 of the system and Repair Machine has two modes: Normal (N) and total failure (F). The unit-1 upon its first time repair enters into quasi-normal mode (not as good as new) with increased failure rate.

iii). The unit-2 operates only when unit-1 fails completely i.e. unit-1 gets priority in operation in any operative mode.

iv). A switching device is used to put the standby unit into operation whenever the priority unit (unit-1) fails. The switching device is found always perfect and instantaneous whenever required.

v). Two repairmen ordinary and skilled are always available with the system to do their jobs of repair and complete overhaul respectively. The repair of RM and repair of unit-1 after first failure are performed by ordinary repairman whereas the skilled repairman performs as complete overhauling of the failed unit-2 as well as the failed unit-1 from quasi-normal mode.

vi). During the repair of failed unit-1, the RM may also fail. In this case the repair of RM is started superseding the repair of unit-1 as single ordinary repairman is available for these dual jobs. The re-started repair of unit-1 is of pre-emptive repeat type.

vii). The failure and repair/complete overhauling time distributions of RM/unit-2 are taken as exponential with different parameters whereas the failure time distribution of unit-1 is taken as exponential and its repair time (failed from N-mode) is taken as general. The failure time distributions of unit-1 working in quasi-normal mode and its complete over hauling time distributions are also taken exponentials.

3. Notations and States of the System

3.1 Notations:

E	: Set of regenerative states i.e. S_0 to S_8.
θ	: Constant failure rate of unit-1 working in normal mode.
$\alpha_1 > \theta$	: Constant failure rate of unit-1 working in quasi-normal mode.
β_1	: Constant rate of complete over hauling of unit-1.
α_2 / β_2	: Constant failure/complete over hauling rate of unit-2.
μ / η	: Constant failure/repair rate of Repair Machine.
$G(\cdot)/g(\cdot)$	: Cdf/pdf of repair time distribution of unit-1(failed from N-mode).
$Q_{ij}(\cdot)/q_{ij}(\cdot)$	: Cdf/pdf of transition time from state S_i to S_j.

p_{ij} : Steady state probability that the system transits from state S_i to S_j.

ψ_i : Mean sojourn time in state S_i.

3.2 Symbols for the states of the system:

N_{iO} : Unit-i (i=1, 2) is in Normal mode and operative.

N'_{1O} : Unit-1 is in quasi-normal mode and operative.

N_{2S} : Unit-2 is in Normal mode and kept into cold standby.

F_{1r} / F_{1w} : Unit-1 is in failure mode and under repair/waiting for repair.

F_{1R} / F_{1wR} : Unit-1 is under complete over hauling /waiting for complete over hauling.

$RM_g / RM_O / RM_r$: Repair Machine is good/operative/under repair.

F_{2R} : Unit-2 is under complete over hauling.

Using these symbols and keeping in view the above assumptions, the possible states of the system are shown in the transition diagram shown in fig.1.

TRANSITION DIAGRAM

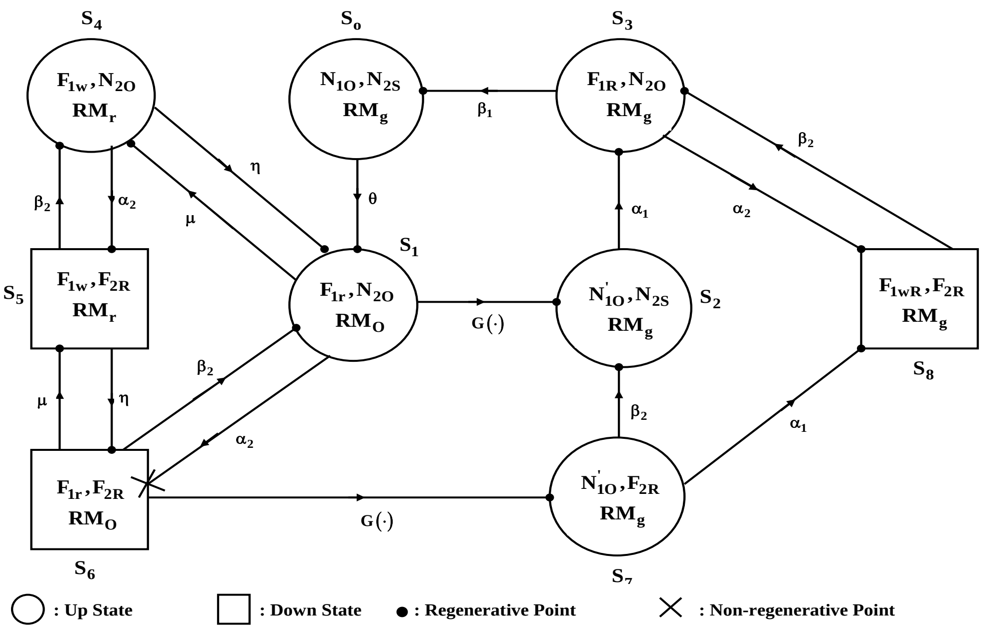

Fig. 1

4. TRANSITION PROBABILITIES

Let $X(t)$ be the state of the system at epoch t, then $\{X(t); t \geq 0\}$ constitutes a Markov-chain with state space E. The transition probability matrix (t.p.m.) of the embedded Markov Chain is

$$P = \begin{pmatrix} p_{00} & p_{01} & p_{02} & p_{03} & p_{04} & p_{05} & p_{06} & p_{07} & p_{08} \\ p_{10} & p_{11}^{(6)} & p_{12} & p_{13} & p_{14} & p_{15}^{(6)} & p_{16} & p_{17}^{(6)} & p_{18} \\ p_{20} & p_{21} & p_{22} & p_{23} & p_{24} & p_{25} & p_{26} & p_{27} & p_{28} \\ p_{30} & p_{31} & p_{32} & p_{33} & p_{34} & p_{35} & p_{36} & p_{37} & p_{38} \\ p_{40} & p_{41} & p_{42} & p_{43} & p_{44} & p_{45} & p_{46} & p_{47} & p_{48} \\ p_{50} & p_{51} & p_{52} & p_{53} & p_{54} & p_{55} & p_{56} & p_{57} & p_{58} \\ p_{60} & p_{61} & p_{62} & p_{63} & p_{64} & p_{65} & p_{66} & p_{67} & p_{68} \\ p_{70} & p_{71} & p_{72} & p_{73} & p_{74} & p_{75} & p_{76} & p_{77} & p_{78} \\ p_{80} & p_{81} & p_{82} & p_{83} & p_{84} & p_{85} & p_{86} & p_{87} & p_{88} \end{pmatrix}$$

with non-zero elements-

$$p_{01} = p_{23} = p_{83} = 1$$

$$p_{12} = \tilde{G}(\mu + \alpha_2), \qquad p_{14} = \frac{\mu}{(\mu + \alpha_2)}\left[\tilde{G}(\mu + \alpha_2)\right]$$

$$p_{11}^{(6)} = \frac{\alpha_2 \beta_2}{(\alpha_2 - \beta_2)}\left[\frac{\left(1 - \tilde{G}(\mu + \beta_2)\right)}{(\mu + \beta_2)} - \frac{\left(1 - \tilde{G}(\mu + \alpha_2)\right)}{(\mu + \alpha_2)}\right]$$

$$p_{15}^{(6)} = \frac{\alpha_2 \mu}{(\alpha_2 - \beta_2)}\left[\frac{\left(1 - \tilde{G}(\mu + \beta_2)\right)}{(\mu + \beta_2)} - \frac{\left(1 - \tilde{G}(\mu + \alpha_2)\right)}{(\mu + \alpha_2)}\right]$$

$$p_{17}^{(6)} = \frac{\alpha_2}{(\alpha_2 - \beta_2)}\left[\tilde{G}(\mu + \beta_2) - \tilde{G}(\mu + \alpha_2)\right]$$

$$p_{30} = \beta_1/(\beta_1 + \alpha_2), \qquad p_{38} = \alpha_2/(\beta_1 + \alpha_2), \qquad p_{41} = \eta/(\eta + \alpha_2),$$

$$p_{45} = \alpha_2/(\eta + \alpha_2), \qquad p_{54} = \beta_2/(\eta + \beta_2), \qquad p_{56} = \eta/(\eta + \beta_2)$$

$$p_{61} = \frac{\beta_2}{(\mu + \beta_2)}\left[1 - \tilde{G}(\mu + \beta_2)\right], \qquad p_{65} = \frac{\mu}{(\mu + \beta_2)}\left[1 - \tilde{G}(\mu + \beta_2)\right]$$

$$p_{67} = \tilde{G}(\mu + \beta_2), \qquad p_{72} = \beta_2/(\alpha_1 + \beta_2) \qquad p_{78} = \alpha_1/(\alpha_1 + \beta_2) \qquad (1\text{-}17)$$

The other elements of t.p.m will be zero.

It can be easily verified that

$$p_{12} + p_{11}^{(6)} + p_{14} + p_{15}^{(6)} + p_{17}^{(6)} = 1, \qquad p_{30} + p_{38} = 1, \qquad p_{41} + p_{45} = 1,$$

$$p_{54}+p_{56}=1 \qquad p_{61}+p_{65}+p_{67}=1, \qquad p_{72}+p_{78}=1 \qquad (18\text{-}23)$$

The mean sojourn time ψ_i in state S_i is defined as the expected time taken by the system in state S_i before transitioning into any other state. If random variable U_i denotes the sojourn time in state S_i then

$$\psi_i=\int_0^\infty P\left[U_i>t\right]dt$$

Therefore, its values for various regenerative states are as follows:

$$\psi_0=1/\theta \qquad \psi_1=\left[1-\tilde{G}\left(\mu+\alpha_2\right)\right]/\left(\mu+\alpha_2\right)$$

$$\psi_2=1/\alpha_1 \qquad \psi_3=1/\left(\alpha_2+\beta_1\right)$$

$$\psi_4=1/\left(\alpha_2+\eta\right) \qquad \psi_5=1/\left(\beta_2+\eta\right)$$

$$\psi_6=\left[1-\tilde{G}\left(\mu+\beta_2\right)\right]/\left(\mu+\beta_2\right) \qquad \psi_7=1/\left(\alpha_1+\beta_2\right)$$

$$\psi_8=1/\beta_2 \qquad (24\text{-}32)$$

5. ANALYSIS OF CHARACTERSTICS

5.1 Reliability of the system and MTSF

Let $R_i(t)$ be the probability that the system is operative during (0, t] given that at t=0 system starts from state $S_i\in E$. To obtain it we assume the failed states S_5, S_6 and S_8 as absorbing. By simple probabilistic arguments, the value of $R_0(t)$ in terms of its Laplace Transform is given by

$$R_0^*(s)=\frac{\left(1-q_{14}^*q_{41}^*\right)Z_0^*+q_{01}^*\left(Z_1^*+q_{12}^*Z_2^*+q_{12}^*q_{23}^*Z_3^*+q_{14}^*Z_4^*\right)}{1-q_{14}^*q_{41}^*-q_{01}^*q_{12}^*q_{23}^*q_{30}^*} \qquad (33)$$

$$Z_0(t)=\exp(-\theta t), \qquad Z_1(t)=\exp\left[-\left(\mu+\alpha_2\right)t\right]\overline{G}(t)dt,$$

$$Z_2(t)=\exp\left(-\alpha_1 t\right), \qquad Z_3(t)=\exp\left[-\left(\alpha_2+\beta_1\right)t\right],$$

$$Z_4(t)=\exp\left[-\left(\eta+\alpha_2\right)t\right]$$

Taking the Inverse Laplace of (33) we can get the reliability of the system when it starts from state S_0.

MTSF is given by

$$E(T_0)=\int R_0(t)dt=\lim_{s\to 0} R_0^*(s)=\frac{(1-p_{14}p_{41})\psi_0+\psi_1+p_{12}(\psi_2+\psi_3)+p_{14}\psi_4}{1-p_{14}p_{41}-p_{12}p_{30}} \tag{34}$$

5.2 Availability Analysis

Let $A_i^{1n}(t)$, $A_i^{1n'}(t)$ and $A_i^2(t)$ be the respective probabilities that the system is operative at epoch t due to the operation of unit-1 in normal mode, quasi-normal mode and unit-2 when system initially starts from $S_i \in E$. Using the regenerative point technique and the tools of L.T., one can obtain the values of $A_0^{1n}(t)$, $A_0^{1n'}(t)$ and $A_0^2(t)$ in terms of their L.T. i.e. $A_0^{1n*}(s)$, $A_0^{1n'*}(s)$ and $A_0^{2*}(s)$.

The steady-state values of above three availabilities are given by

$$A_0^{1n}=\lim_{s\to 0} s A_0^{1n*}(s)=U_0\psi_0 / D_1'(0)$$

Similarly,

$$A_0^{1n'}=(U_2\psi_2+U_7\psi_7)/D_1'(0)$$

$$A_0^2=(U_1\psi_1+U_3\psi_3+U_4\psi_4)/D_1'(0) \tag{35-37}$$

Where,

$$D_1'(0)=U_0\psi_0+U_1 n+\sum_{i=2}^{8} U_i\psi_i$$

and

$$U_0=p_{30}\left[(1-p_{56}p_{65}-p_{45}p_{54})\left(p_{12}+p_{17}^{(6)}\right)+p_{56}p_{67}\left(p_{15}^{(6)}+p_{14}p_{45}\right)\right]$$

$$U_1=p_{30}(1-p_{56}p_{65}-p_{45}p_{54})$$

$$U_2=p_{30}\left[(1-p_{56}p_{65}-p_{45}p_{54})\left(p_{12}+p_{17}^{(6)}p_{72}\right)+p_{56}p_{67}p_{72}\left(p_{15}^{(6)}+p_{14}p_{45}\right)\right]$$

$$U_3=(1-p_{56}p_{65}-p_{45}p_{54})\left(p_{12}+p_{17}^{(6)}\right)+p_{56}p_{67}\left(p_{15}^{(6)}+p_{14}p_{45}\right)$$

$$U_4=p_{30}\left[p_{15}^{(6)}p_{54}+p_{14}(1-p_{56}p_{65})\right]$$

$$U_5 = p_{30}\left(p_{15}^{(6)} + p_{14}p_{45}\right)$$

$$U_6 = p_{30}p_{56}\left(p_{15}^{(6)} + p_{14}p_{45}\right)$$

$$U_7 = p_{30}\left[p_{17}^{(6)}\left(1-p_{56}p_{65}-p_{45}p_{54}\right)+p_{56}p_{67}\left(p_{15}^{(6)}+p_{14}p_{45}\right)\right]$$

$$U_8 = p_{38}\left[\left(1-p_{11}^{(6)}\right)\left(1-p_{56}p_{65}-p_{45}p_{54}\right)-p_{14}\left\{p_{41}\left(1-p_{56}p_{65}\right)+p_{45}p_{61}p_{56}\right\}-p_{15}^{(6)}\left(p_{41}p_{54}+p_{61}p_{56}\right)\right]$$

$$+p_{30}p_{78}\left[p_{17}^{(6)}\left(1-p_{56}p_{65}-p_{45}p_{54}\right)+p_{56}p_{67}\left(p_{15}^{(6)}+p_{14}p_{45}\right)\right]$$

$$n = \frac{1}{(\alpha_2-\beta_2)}\left[\frac{\alpha_2\left\{1-\tilde{G}(\mu+\beta_2)\right\}}{(\mu+\beta_2)} - \frac{\beta_2\left\{1-\tilde{G}(\mu+\alpha_2)\right\}}{(\mu+\alpha_2)}\right] \tag{38}$$

The expected up (operative) time of the system during (0, t] due to the operation of unit-1 and unit-2 are given by

$$\mu_{up}^{1n}(t) = \int_0^t A_0^{1n}(u)\,du\,, \qquad \mu_{up}^{1n'}(t) = \int_0^t A_0^{1n'}(u)\,du \qquad \mu_{up}^{2}(t) = \int_0^t A_0^{2}(u)\,du$$

so that

$$\mu_{up}^{1n*}(s) = A_0^{1n*}(s)\big/s\,, \qquad \mu_{up}^{1n'*}(s) = A_0^{1n'*}(s)\big/s \qquad \mu_{up}^{2*}(s) = A_0^{2*}(s)\big/s$$

5.3 Busy Period Analysis-

Let $B_i^{1r}(t)$ and $B_i^{mr}(t)$ be the respective probabilities that the ordinary repairman will be busy in repair of unit-1 and in the repair of repair machine at time t and $B_i^{1R}(t)$ and $B_i^{2R}(t)$ be the respective probabilities that the skilled repairman will be busy in complete overhauling of unit-1 and unit-2, when system initially starts from $S_i \in E$. Using the regenerative point technique and the tools of L.T., one can obtain the values of above four probabilities in terms of their L.T. i.e. $B_0^{1r*}(s)$, $B_0^{1R*}(s)$, $B_0^{2R*}(s)$ and $B_0^{mr*}(s)$.

The steady state results for the above four probabilities are given by

$$B_0^{1r} = \lim_{s\to 0} s\,B_0^{1r*}(s) = \left[nU_1 + \psi_6 U_6\right]\Big/D_1'(0)$$

Similarly,

$$B_0^{mr} = \left(\psi_4 U_4 + \psi_5 U_5\right)\big/D_1'(0),$$

$$B_0^{1R} = \psi_3 U_3 \Big/ D_1^{'}(0),$$

$$B_0^{2R} = \sum_{j=5}^{8} \psi_j U_j \Big/ D_1^{'}(0) \qquad (39\text{-}42)$$

The expected busy periods of repairman due to repair, complete over hauling of either unit and RM during time (0, t] are respectively given by

$$\mu_b^{1r}(t) = \int_0^t B_0^{1r}(u)\,du, \qquad \mu_b^{mr}(t) = \int_0^t B_0^{mr}(u)\,du$$

$$\mu_b^{1R}(t) = \int_0^t B_0^{1R}(u)\,du, \qquad \mu_b^{2R}(t) = \int_0^t B_0^{2R}(u)\,du$$

So that

$$\mu_b^{1r*}(s) = B_0^{1r*}(s)\big/s, \qquad \mu_b^{mr*}(s) = B_0^{mr*}(s)\big/s,$$

$$\mu_b^{1R*}(s) = B_0^{1R*}(s)\big/s, \qquad \mu_b^{2R*}(s) = B_0^{2R*}(s)\big/s$$

6. PROFIT FUNCTION ANALYSIS

We are now in the position to obtain the net expected profit earned by the system during (0, t] on considering the characteristics obtained in sections 5.2 and 5.3. Let us suppose,

$K_0, K_1 =$ revenue per unit up-time by the system due to the operation of unit-1 in N and $N^{'}$ mode respectively.

$K_2 =$ revenue per unit up-time by the system due to the operation of unit-2.

$K_3, K_4 =$ payment to ordinary repairman per unit time when he is busy in the repair of unit-1 and repair machine respectively.

$K_5, K_6 =$ payment to skilled repairman per unit time when he is busy in complete overhauling of unit-1 and unit-2 respectively.

Then the expected profit incurred in time interval (0, t] is

$$C(t) = K_0\mu_{up}^{1n}(t) + K_1\mu_{up}^{1n^{'}}(t) + K_2\mu_{up}^{2}(t) - K_3\mu_b^{1r}(t) - K_4\mu_b^{mr}(t) - K_5\mu_b^{1R}(t) - K_6\mu_b^{2R}(t)$$

The net expected profit per unit time in steady state is given by

$$C = \lim_{t\to\infty}\frac{C(t)}{t} = \lim_{s\to 0} s^2 P^*(s) = K_0 A_0^{1n} + K_1 A_0^{1n^{'}} + K_2 A_0^{2} - K_3 B_0^{1r} - K_4 B_0^{mr} - K_5 B_0^{1R} - K_6 B_0^{2R}$$

7. PARTICULAR CASES

Case I: When repair time of unit-1 also follows **exponential distribution** i.e.

$$g(t)=\lambda e^{-\lambda t}\ ;t\geq 0 \quad \text{and} \quad g^*(s)=\tilde{G}(s)=\lambda/(s+\lambda).$$

In view of above, the changed values of transition probabilities and mean sojourn times will be as follows-

$$p_{11}^{(6)}=\frac{\alpha_2\beta_2}{(\lambda+\mu+\beta_2)(\lambda+\mu+\alpha_2)}, \qquad p_{12}=\frac{\lambda}{(\lambda+\mu+\alpha_2)}$$

$$p_{15}^{(6)}=\frac{\alpha_2\mu}{(\lambda+\mu+\beta_2)(\lambda+\mu+\alpha_2)}, \qquad p_{14}=\frac{\mu}{(\lambda+\mu+\alpha_2)}$$

$$p_{17}^{(6)}=\frac{\alpha_2\lambda}{(\lambda+\mu+\beta_2)(\lambda+\mu+\alpha_2)}, \qquad p_{61}=\frac{\beta_2}{(\lambda+\mu+\beta_2)},$$

$$p_{65}=\frac{\mu}{(\lambda+\mu+\beta_2)}, \qquad p_{67}=\frac{\lambda}{(\lambda+\mu+\beta_2)}$$

$$\psi_1=\frac{1}{(\lambda+\mu+\alpha_2)}, \qquad \psi_6=\frac{1}{(\lambda+\mu+\beta_2)}$$

The value of n becomes

$$n=\frac{1}{(\alpha_2-\beta_2)}\left(\frac{\alpha_2}{(\lambda+\mu+\beta_2)}-\frac{\beta_2}{(\lambda+\mu+\alpha_2)}\right)$$

Case II: When repair time of unit-1 follows **Lindley distribution** i.e. $g(t)=\frac{\lambda^2}{1+\lambda}(1+t)e^{-\lambda t}$

$$\text{and} \quad g^*(s)=\tilde{G}(s)=\frac{(s+\lambda+1)\lambda^2}{(1+\lambda)(s+\lambda)^2}.$$

In this regard, the changed expressions of transition probabilities and mean sojourn times will be as follows-

$$p_{12}=\left(1+\frac{\mu+\alpha_2}{1+\lambda}\right)\left(\frac{\lambda}{\lambda+\mu+\alpha_2}\right)^2, \qquad p_{14}=\frac{\mu}{(\mu+\alpha_2)}\left[1-\left(1+\frac{\mu+\alpha_2}{1+\lambda}\right)\left(\frac{\lambda}{\lambda+\mu+\alpha_2}\right)^2\right],$$

$$p_{11}^{(6)}=\frac{\alpha_2\beta_2}{(\alpha_2-\beta_2)}\left[\frac{1}{(\mu+\beta_2)}\left\{1-\left(1+\frac{(\mu+\beta_2)}{1+\lambda}\right)\left(\frac{\lambda}{\lambda+\mu+\beta_2}\right)^2\right\}-\frac{1}{(\mu+\alpha_2)}\left\{1-\left(1+\frac{(\mu+\alpha_2)}{1+\lambda}\right)\left(\frac{\lambda}{\lambda+\mu+\alpha_2}\right)^2\right\}\right]$$

$$P_{15}^{(6)}=\frac{\alpha_2\mu}{(\alpha_2-\beta_2)}\left[\frac{1}{(\mu+\beta_2)}\left\{1-\left(1+\frac{(\mu+\beta_2)}{1+\lambda}\right)\left(\frac{\lambda}{\lambda+\mu+\beta_2}\right)^2\right\}-\frac{1}{(\mu+\alpha_2)}\left\{1-\left(1+\frac{(\mu+\alpha_2)}{1+\lambda}\right)\left(\frac{\lambda}{\lambda+\mu+\alpha_2}\right)^2\right\}\right]$$

$$P_{17}^{(6)}=\frac{\alpha_2\mu}{(\alpha_2-\beta_2)}\left[\left(1+\frac{(\mu+\beta_2)}{1+\lambda}\right)\left(\frac{\lambda}{\lambda+\mu+\beta_2}\right)^2-\left(1+\frac{(\mu+\alpha_2)}{1+\lambda}\right)\left(\frac{\lambda}{\lambda+\mu+\alpha_2}\right)^2\right]$$

$$p_{61} = \frac{\beta_2}{(\mu+\beta_2)}\left[1-\left(1+\frac{\mu+\beta_2}{1+\lambda}\right)\left(\frac{\lambda}{\lambda+\mu+\beta_2}\right)^2\right], \quad p_{65} = \frac{\mu}{(\mu+\beta_2)}\left[1-\left(1+\frac{\mu+\beta_2}{1+\lambda}\right)\left(\frac{\lambda}{\lambda+\mu+\beta_2}\right)^2\right]$$

$$p_{67} = \left(1+\frac{\mu+\beta_2}{1+\lambda}\right)\left(\frac{\lambda}{\lambda+\mu+\beta_2}\right)^2, \quad \psi_1 = \frac{1}{(\mu+\alpha_2)}\left[1-\left(1+\frac{\mu+\alpha_2}{1+\lambda}\right)\left(\frac{\lambda}{\lambda+\mu+\alpha_2}\right)^2\right]$$

$$\psi_6 = \frac{1}{(\mu+\beta_2)}\left[1-\left(1+\frac{\mu+\beta_2}{1+\lambda}\right)\left(\frac{\lambda}{\lambda+\mu+\beta_2}\right)^2\right]$$

$$n = \frac{1}{(\alpha_2-\beta_2)}\left[\frac{\alpha_2}{(\mu+\beta_2)}\left\{1-\left(1+\frac{\mu+\beta_2}{1+\lambda}\right)\left(\frac{\lambda}{\lambda+\mu+\beta_2}\right)^2\right\}-\frac{\beta_2}{(\mu+\alpha_2)}\left\{1-\left(1+\frac{\mu+\alpha_2}{1+\lambda}\right)\left(\frac{\lambda}{\lambda+\mu+\alpha_2}\right)^2\right\}\right]$$

Case III: If repair time of unit-1 follows **gamma distribution** i.e. $g(t) = \frac{e^{-t}t^{\lambda-1}}{\Gamma\lambda}$ and

$$g^*(s) = \tilde{G}(s) = (1+s)^{-\lambda}$$

Thus the following changes are observed in $p_{ij}, p_{ij}^{(k)}, \psi_i$ and n.

$$p_{12} = (1+\mu+\alpha_2)^{-\lambda}, \quad p_{14} = \frac{\mu}{(\mu+\alpha_2)}\left[1-(1+\mu+\alpha_2)^{-\lambda}\right],$$

$$p_{11}^{(6)} = \frac{\alpha_2\beta_2}{(\alpha_2-\beta_2)}\left[\frac{1}{(\mu+\beta_2)}\left\{1-(1+\mu+\beta_2)^{-\lambda}\right\}-\frac{1}{(\mu+\alpha_2)}\left\{1-(1+\mu+\alpha_2)^{-\lambda}\right\}\right],$$

$$p_{15}^{(6)} = \frac{\alpha_2\mu}{(\alpha_2-\beta_2)}\left[\frac{1}{(\mu+\beta_2)}\left\{1-(1+\mu+\beta_2)^{-\lambda}\right\}-\frac{1}{(\mu+\alpha_2)}\left\{1-(1+\mu+\alpha_2)^{-\lambda}\right\}\right],$$

$$p_{17}^{(6)} = \frac{\alpha_2}{(\alpha_2-\beta_2)}\left[(1+\mu+\beta_2)^{-\lambda}-(1+\mu+\alpha_2)^{-\lambda}\right], \quad p_{61} = \frac{\beta_2}{(\mu+\beta_2)}\left[1-(1+\mu+\beta_2)^{-\lambda}\right]$$

$$p_{65} = \frac{\mu}{(\mu+\beta_2)}\left[1-(1+\mu+\beta_2)^{-\lambda}\right], \quad p_{67} = (1+\mu+\beta_2)^{-\lambda}$$

$$\psi_1 = \frac{1}{(\mu+\alpha_2)}\left[1-(1+\mu+\alpha_2)^{-\lambda}\right], \quad \psi_6 = \frac{1}{(\mu+\beta_2)}\left[1-(1+\mu+\beta_2)^{-\lambda}\right]$$

$$n = \frac{1}{(\alpha_2-\beta_2)}\left[\frac{\alpha_2}{(\mu+\beta_2)}\left\{1-(1+\mu+\beta_2)^{-\lambda}\right\}-\frac{\beta_2}{(\mu+\alpha_2)}\left\{1-(1+\mu+\alpha_2)^{-\lambda}\right\}\right]$$

Case IV: When repair time of unit-1 follows **Inverse Gaussian distribution** i.e.

$g(t) = \left(2\pi t^3\right)^{-(1/2)} e^{-(t-\lambda)^2/2\lambda t}$ and

$$\tilde{G}(s) = g^*(s) = \exp\left[\lambda^{-1}\left\{1-\left(1+2s\lambda^2\right)^{1/2}\right\}\right]$$

In this case the changed expressions will be as follows-

$$p_{12} = \tilde{G}_\alpha(\mu+\alpha_2), \qquad p_{14} = \frac{\mu}{(\mu+\alpha_2)}\left[1-\tilde{G}_\alpha(\mu+\alpha_2)\right],$$

$$p_{11}^{(6)} = \frac{\alpha_2\beta_2}{(\alpha_2-\beta_2)}\left[\frac{1}{(\mu+\beta_2)}\left\{1-\tilde{G}_\beta(\mu+\beta_2)\right\} - \frac{1}{(\mu+\alpha_2)}\left\{1-\tilde{G}_\alpha(\mu+\alpha_2)\right\}\right],$$

$$p_{15}^{(6)} = \frac{\alpha_2\mu}{(\alpha_2-\beta_2)}\left[\frac{1}{(\mu+\beta_2)}\left\{1-\tilde{G}_\beta(\mu+\beta_2)\right\} - \frac{1}{(\mu+\alpha_2)}\left\{1-\tilde{G}_\alpha(\mu+\alpha_2)\right\}\right],$$

$$p_{17}^{(6)} = \frac{\alpha_2}{(\alpha_2-\beta_2)}\left[\tilde{G}_\beta(\mu+\beta_2)-\tilde{G}_\alpha(\mu+\alpha_2)\right], \qquad p_{61} = \frac{\beta_2}{(\mu+\beta_2)}\left[1-\tilde{G}_\beta(\mu+\beta_2)\right]$$

$$p_{65} = \frac{\mu}{(\mu+\beta_2)}\left[1-\tilde{G}_\beta(\mu+\beta_2)\right], \qquad p_{67} = \tilde{G}_\beta(\mu+\beta_2)$$

$$\psi_1 = \frac{1}{(\mu+\alpha_2)}\left[1-\tilde{G}_\alpha(\mu+\alpha_2)\right], \qquad \psi_6 = \frac{1}{(\mu+\beta_2)}\left[1-\tilde{G}_\beta(\mu+\beta_2)\right]$$

$$n = \frac{1}{(\alpha_2-\beta_2)}\left[\frac{\alpha_2}{(\mu+\beta_2)}\left\{1-\tilde{G}_\beta(\mu+\beta_2)\right\} - \frac{\beta_2}{(\mu+\alpha_2)}\left\{1-\tilde{G}_\alpha(\mu+\alpha_2)\right\}\right]$$

Where,

$$\tilde{G}_\alpha(\mu+\alpha_2) = \exp\left[\lambda^{-1}\left\{1-\left(1+2(\mu+\alpha_2)\lambda^2\right)^{1/2}\right\}\right]$$

$$\tilde{G}_\beta(\mu+\beta_2) = \exp\left[\lambda^{-1}\left\{1-\left(1+2(\mu+\beta_2)\lambda^2\right)^{1/2}\right\}\right]$$

Case V: Let the random variable X denotes the failure time of p-unit (from N-mode) and Y its repair time. Considering their joint distribution as **Bi-variate exponential** with pdf

$$f(x,y) = \theta\lambda(1-r)e^{-(\theta x+\lambda y)}\sum_{j=0}^{\infty}\frac{(\theta\lambda rxy)^j}{(j!)^2} \quad ; \quad x,y \geq 0;\ \theta,\lambda > 0 \text{ and } 0 \leq r < 1.$$

Where, r is the correlation coefficient between X and Y. The marginal distributions of random variables X and Y will be exponential having the pdf

$$h(x) = \theta(1-r)e^{-\theta(1-r)x} \quad \text{and} \quad k(y) = \lambda(1-r)e^{-\lambda(1-r)y}$$

As the repair time of the p-unit (from N-mode) depends upon its failure time, therefore, the conditional pdf of Y for given X is given by

$$g(y|x) = \lambda e^{-(\lambda y+\theta rx)}\sum_{j=0}^{\infty}\frac{(\theta\lambda rxy)^j}{(j!)^2}$$

So that the value of its Laplace Transform is

$$g^*(s|x) = \frac{\lambda}{s+\lambda}e^{-rs\theta x/(s+\lambda)}$$

The unconditional Laplace Transform of $g(y|x)$ will be

$$g^*(s) = \tilde{G}(s) = \int g^*(s \mid x).h(x)dx = \frac{\lambda(1-r)}{s+\lambda(1-r)}$$

Then the changed expressions will be as given under-

$$p_{12} = (1-r)\lambda' \Big/ \left(1-r\lambda'\right), \qquad p_{14} = \frac{\mu}{(\mu+\alpha_2)}\left[1-(1-r)\lambda' \Big/ \left(1-r\lambda'\right)\right],$$

$$p_{11}^{(6)} = \frac{\alpha_2\beta_2}{(\alpha_2-\beta_2)}\left[\frac{1}{(\mu+\beta_2)}\left\{1-\frac{(1-r)\lambda''}{1-r\lambda''}\right\}-\frac{1}{(\mu+\alpha_2)}\left\{1-\frac{(1-r)\lambda'}{1-r\lambda'}\right\}\right],$$

$$p_{15}^{(6)} = \frac{\alpha_2\mu}{(\alpha_2-\beta_2)}\left[\frac{1}{(\mu+\beta_2)}\left\{1-\frac{(1-r)\lambda''}{1-r\lambda''}\right\}-\frac{1}{(\mu+\alpha_2)}\left\{1-\frac{(1-r)\lambda'}{1-r\lambda'}\right\}\right],$$

$$p_{17}^{(6)} = \frac{\alpha_2}{(\alpha_2-\beta_2)}\left[\frac{(1-r)\lambda''}{1-r\lambda''}-\frac{(1-r)\lambda'}{1-r\lambda'}\right], \qquad p_{61} = \frac{\beta_2}{(\mu+\beta_2)}\left[1-\frac{(1-r)\lambda''}{1-r\lambda''}\right]$$

$$p_{65} = \frac{\mu}{(\mu+\beta_2)}\left[1-\frac{(1-r)\lambda''}{1-r\lambda''}\right], \qquad p_{67} = \frac{(1-r)\lambda''}{1-r\lambda''}$$

$$\psi_0 = \left[\theta(1-r)\right]^{-1}, \qquad \psi_1 = \left[\mu+\alpha_2+\lambda(1-r)\right]^{-1},$$

$$\psi_6 = \left[\mu+\beta_2+\lambda(1-r)\right]^{-1}$$

$$n = \frac{1}{(\alpha_2-\beta_2)}\left[\frac{\alpha_2}{\beta_2+\mu}\left\{1-\frac{(1-r)\lambda''}{1-r\lambda''}\right\}-\frac{\beta_2}{\alpha_2+\mu}\left\{1-\frac{(1-r)\lambda'}{1-r\lambda'}\right\}\right]$$

Where,

$$\lambda' = \lambda/(\lambda+\mu+\alpha_2) \qquad \lambda'' = \lambda/(\lambda+\mu+\beta_2)$$

8. CONCLUSIONS

The curves for MTSF and Profit function are drawn for all the five particular cases (i) to (v) discussed in section 7 in respect of different parameters. In case (i) when repair time follows exponential distribution, Fig. 1(a) and Fig. 1(b) depicts the variation in MTSF and steady state profit with respect to failure parameter θ for three different values of repair parameter λ(=0.40, 0.60, 0.80). We may clearly observe that MTSF decrease with the increase in θ and increases with increase in λ. The same trends are observed for the graph of steady state profit in respect of λ and θ. Further from fig. 1(b) we observed that system is profitable only if θ is less than 0.05, 0.065 and 0.0725 for λ= 0.40, 0.60 and 0.80 respectively for fixed values of α_1=0.09, β_1=0.80, α_2=0.25, β_2=0.40, η=0.60, μ=0.50 K_0=120, K_1=100, K_2=80, K_3=260, K_4=350, K_5=400 and K_6=300.

In case (ii) when repair time follows Lindley distribution, Fig. 2(a) and Fig. 2(b) depicts the variation in MTSF and steady state profit with respect to failure parameter θ for three different values of repair parameter λ(=0.40, 0.60, 0.80). We may clearly observe that MTSF decrease with the increase in θ and increases with increase in λ. The same trends are observed for the graph of steady state profit in respect of λ and θ.

In case (iii) when the repair time follows gamma distribution, Fig 3(a) and Fig 3(b) represent the curves for MTSF and Profit with respect to θ and varying values of failure rate α_2 (=0.30, 0.50, 0.80) instead of repair parameter λ=0.70. The trend is that both MTSF and profit decrease with the increase in α_2 and θ and other parameters are fixed as α_1=0.09, β_1=0.80, λ=0.70, β_2=0.40, η=0.60, and μ=0.50.

In case (iv) when the repair time follows Inverse Gaussian distribution, Fig. 4(a) and Fig. 4(b) represent the plot for MTSF and Profit function with respect to θ and varying values of repair parameter of repair machine say η. The trend reveals that MTSF and Profit function decreases with the increase in θ and increase with increase in η. From Fig. 4(b), we observe that our system is profitable if θ is less than 0.06, 0.12, and 0.24 when η=0.20, 0.45 and 1.00 respectively for fixed value of the other parameters as α_1=0.09, β_1=0.80, α_2=0.25, β_2=0.40, λ=0.70, μ=0.50 and K_0=120, K_1=100, K_2=80, K_3=260, K_4=350, K_5=400 and K_6=300 for all five cases.

In case (v), when failure and repair times are correlated random variables and follow bi-variate exponential distribution, in this situation, the failure parameter θ change to θ(1-r) where r is the correlation coefficient. We draw the curves for MTSF and Profit with respect to the correlation coefficient r for varying values of repair parameter λ. From Fig. 5(a), we see that MTSF increases slowly with r and λ while, when r>0.80, it increases rapidly. From Fig. 5(b), its clear that the profit increases with increase in r and λ for fixed values of other parameters.

Conflict of Interests

The author declares that there is no conflict of interests.

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Graphs for MTSF and Profit with respect to θ for particular Case (a) when α_1=0.09, β_1=0.80, α_2=0.25, β_2=0.40, η=0.60, μ=0.50 and $K_0 = 120, K_1 = 100, K_2 = 80,$ $K_3 = 260, K_4 = 350, K_5 = 400,\ K_6 = 300$.

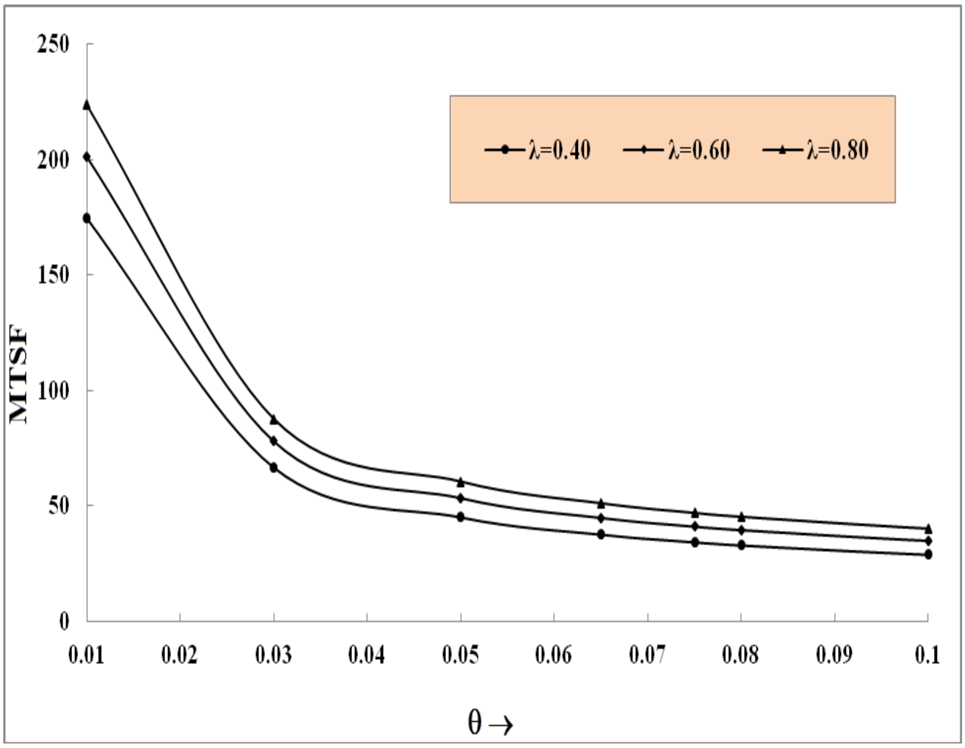

Fig. 1(a)

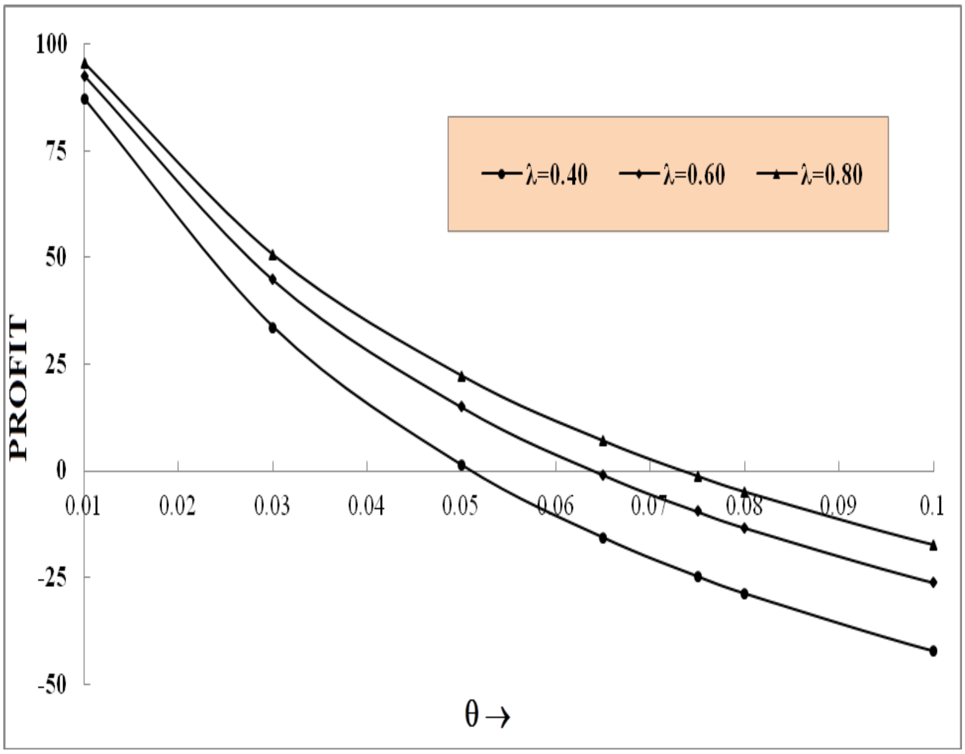

Fig. 1(b)

Graphs for MTSF and Profit with respect to θ for particular Case (b) when α_1=0.09, β_1=0.80, α_2=0.25, β_2=0.40, η=0.60, μ=0.50 and $K_0 = 120, K_1 = 100, K_2 = 80$, $K_3 = 260, K_4 = 350, K_5 = 400,\quad K_6 = 300$.

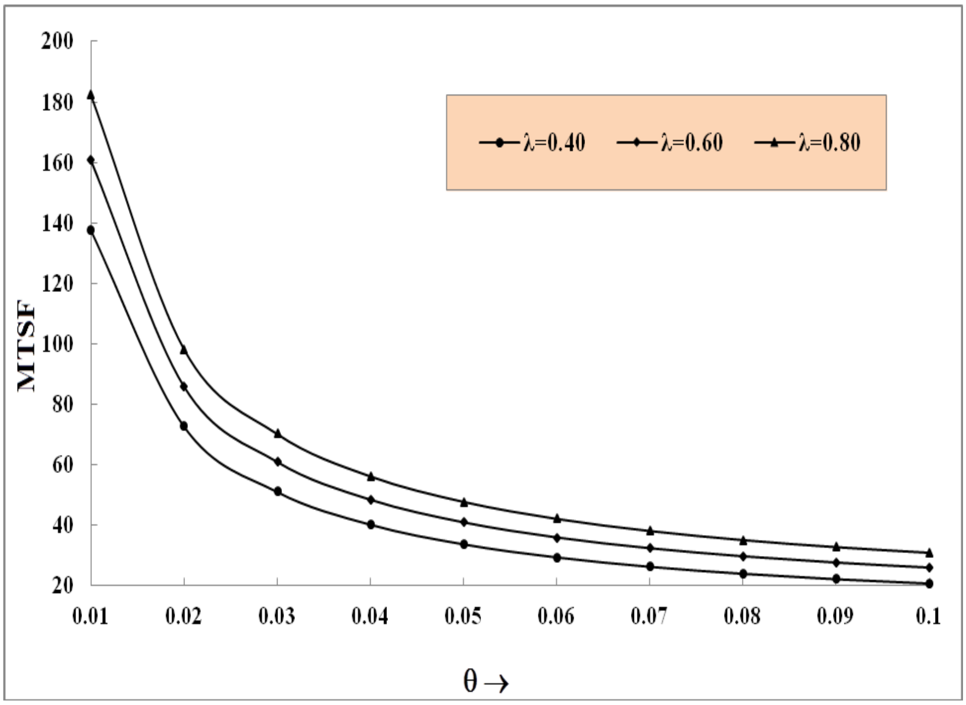

Fig. 2(a)

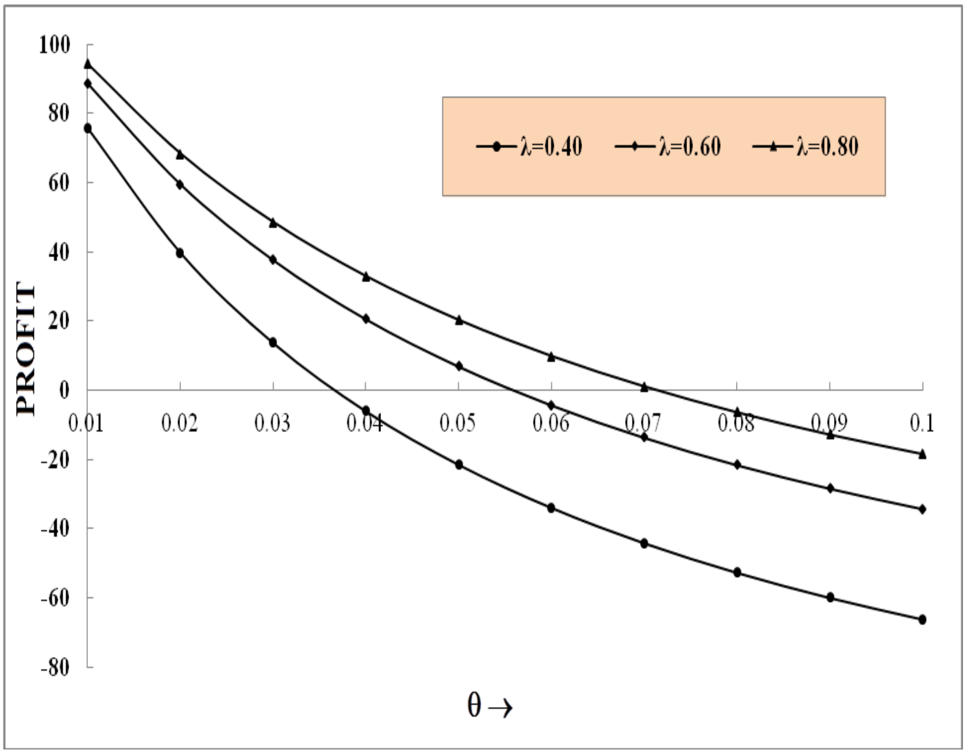

Fig. 2(b)

Graphs for MTSF and Profit with respect to θ for particular Case (c) when α_1=0.09, β_1=0.80, λ=0.70, β_2=0.40, η=0.60, μ=0.50 and $K_0=120, K_1=100, K_2=80, K_3=260, K_4=350, K_5=400, \quad K_6=300$.

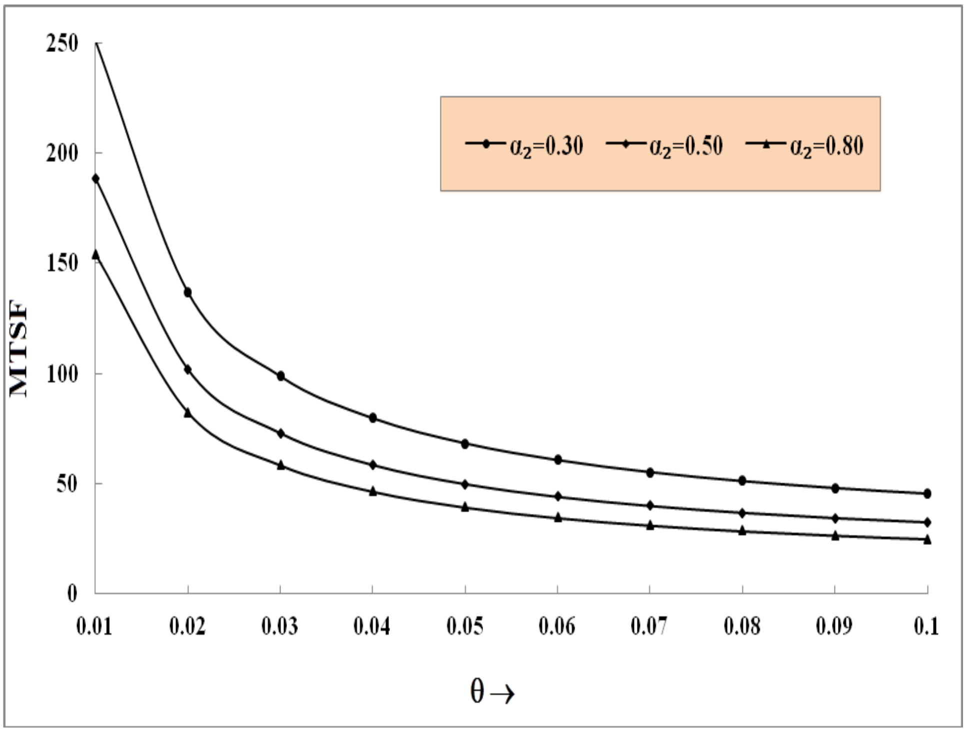

Fig. 3(a)

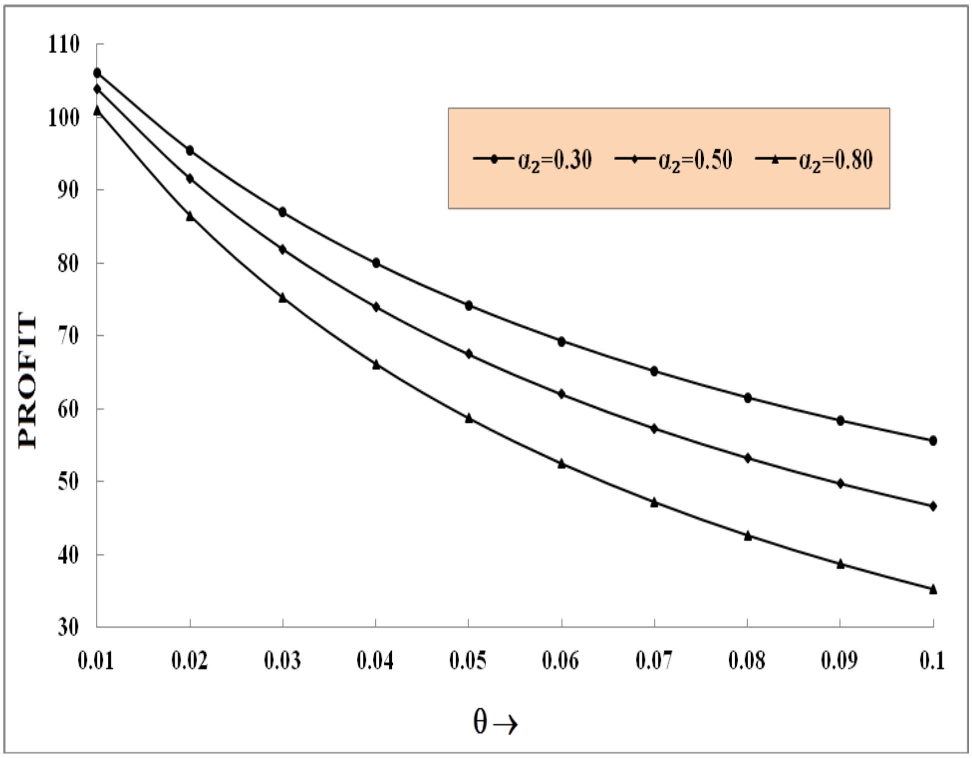

Fig. 3(b)

Graphs for MTSF and Profit with respect to θ for particular Case (d) when α_1=0.09, β_1=0.80, α_2=0.25, β_2=0.40, λ=0.70, μ=0.50 and $K_0 = 120, K_1 = 100, K_2 = 80, K_3 = 260, K_4 = 350, K_5 = 400, K_6 = 300$.

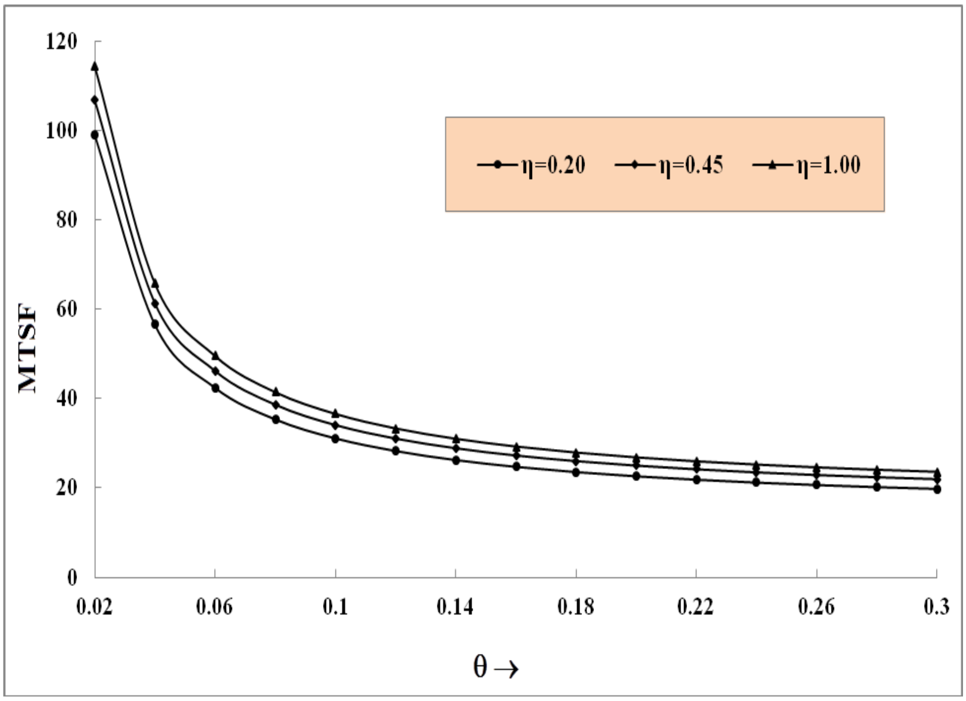

Fig. 4(a)

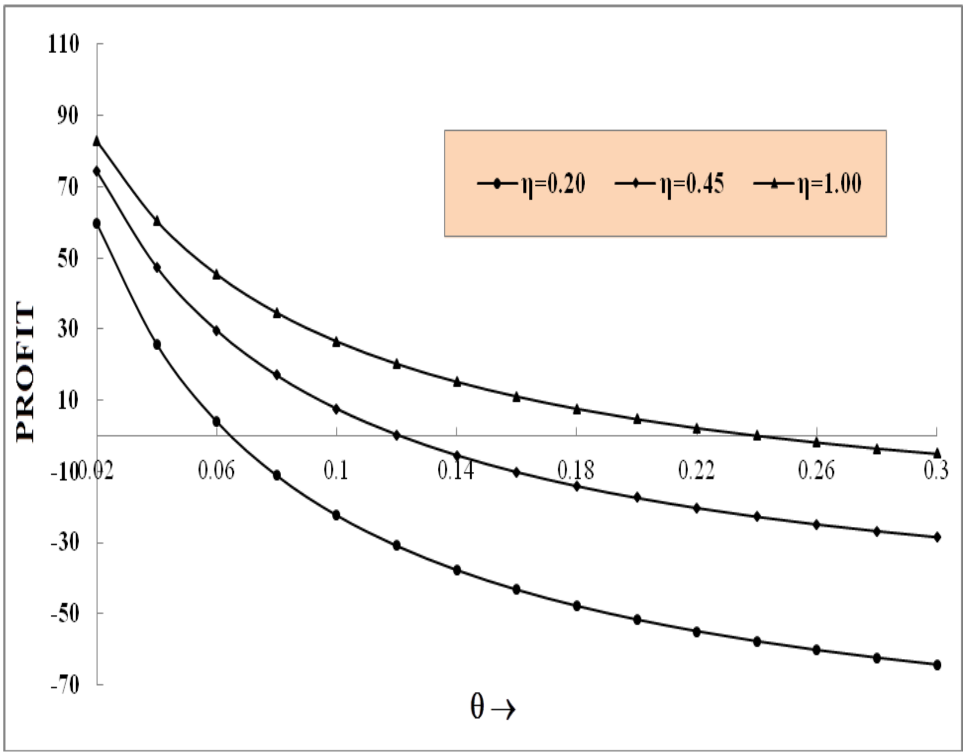

Fig. 4(b)

Graphs for MTSF and Profit with respect to r for particular Case (e) when α_1=0.09, β_1=0.80, α_2=0.25, β_2=0.40, η=0.60, μ=0.50, θ=0.05 and $K_0=120, K_1=100, K_2=80$, $K_3=260, K_4=350$, $K_5=400, K_6=300$.

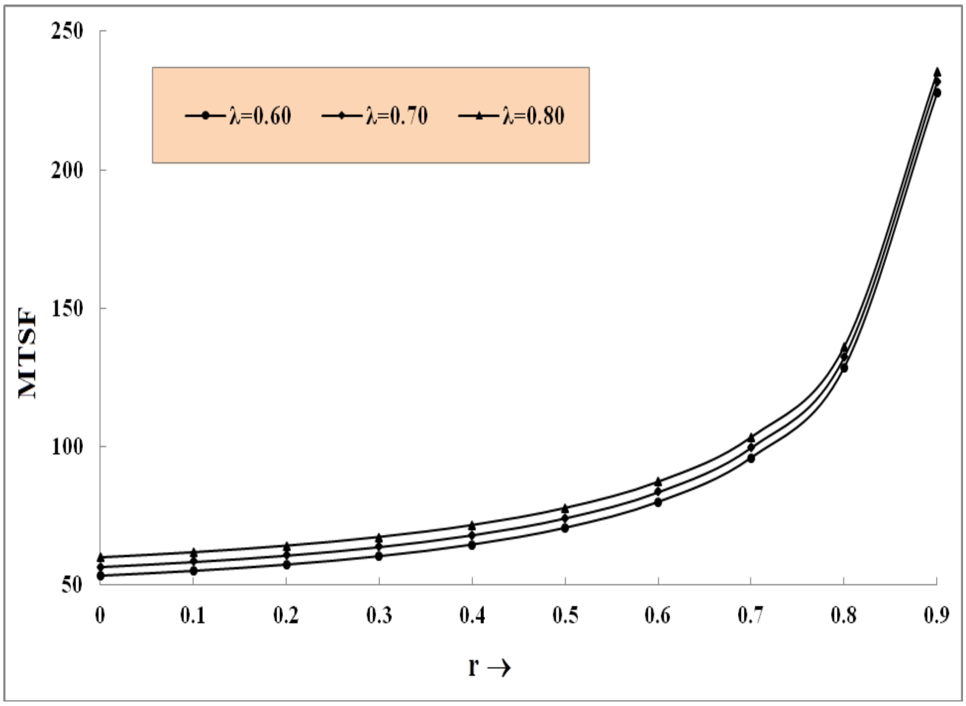

Fig. 5(a)

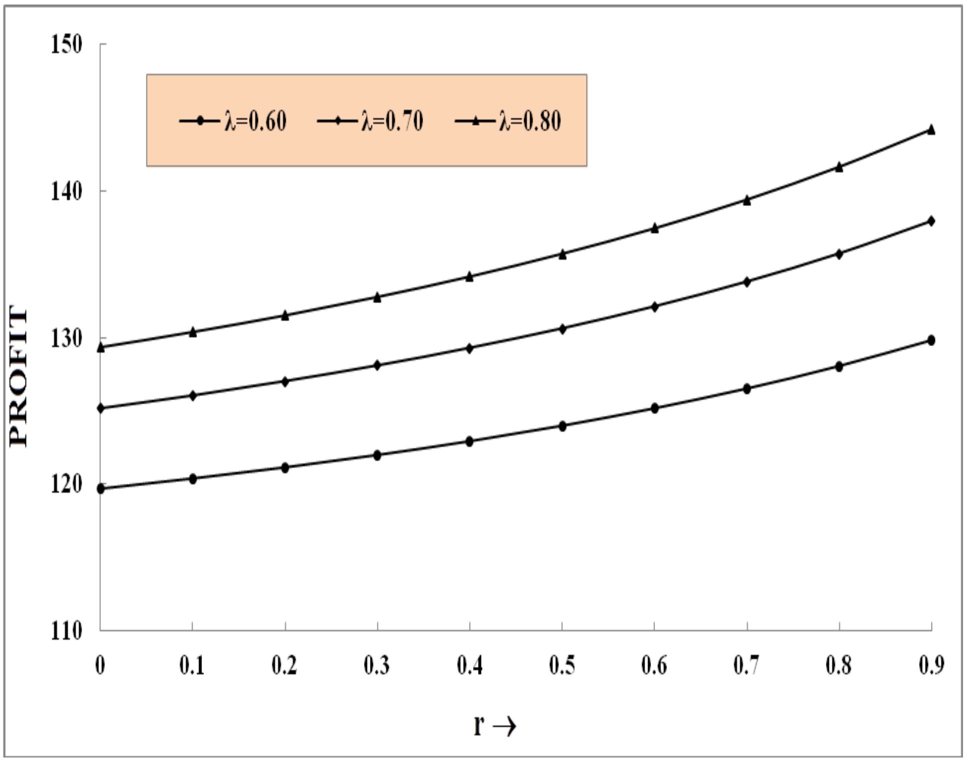

Fig. 5(b)