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INTUITIONISTIC FUZZY AUTOMATON AND THE MINIMAL MACHINE

A. UMA*, M. RAJASEKAR

Mathematics Section, Faculty of Engineering and Technology, Annamalai University, Annamalainagar, chidambaram, Tamil Nadu, India - 608 002.

Abstract: This paper presents a contribution to minimization of intuitionistic fuzzy automata. In this paper the concept of substitution property (SP) partition for a finite intuitionistic fuzzy automaton is formulated, and the quotient automaton with respect to an SP partition is defined.

Keywords: finite intuitionistic fuzzy automaton, SP partition, quotient machines.

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1. Introduction

The concept of fuzzy set is introduced by Zadeh [7]. Lee [5] introduced the concept of fuzzy automata and Lee [4] generalized the classical notion of languages to the concept of fuzzy languages. The concept of intuitionistic fuzzy set was introduced by [1] as a generalization of the notion of fuzzy set. Using the notion of intuitionistic fuzzy sets [1], it is possible to obtain intuitionistic fuzzy language [6]. The concept of (SP) partition for an ffa is formulated by [3] in analogy with that of stochastic automaton given by [2]. In [3] the finite fuzzy automata is minimized by using (SP) partition. This minimized quotient machine is shown to be behaviourally equivalent to the given machine.

In this paper the concept of substitution property (SP) partition for an iffa is formulated and the quotient iffa

*Corresponding author

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with respect to an (SP) partition that refines the set of final states is defined. The quotient machine is shown to be behaviourally equivalent to the given machine.

2. Preliminaries

For convenience, the operations of max and min will be denoted by $\vee$ and $\wedge$ respectively, *i.e.* for any two nonnegative numbers a,b.

$a \vee b = \max\{a,b\}, a \wedge b = \min\{a,b\}$.

For a finite number of nonnegative numbers $a_i (i = 1,2,\cdots,n)$

$\bigvee_{i=1}^{n} a_i = \max\{a_i : i = 1,2,\cdots,n\}$,

$\bigwedge_{i=1}^{n} a_i = \min\{a_i : i = 1,2,\cdots,n\}$.

Definition 2.1. (Intuitionistic Fuzzy Matrices) An intuitionistic fuzzy matrix is a pair of $m \times n$ matrices $A = (A_1, A_2)$ where $A_1 = (a_{1ij})$ and $A_2 = (a_{2ij})$ such that $0 \leq a_{1ij} + a_{2ij} \leq 1$.

Multiplication of two intuitionistic fuzzy matrices is defines as follows. Let $A = (A_1, A_2)$ be an $m \times n$ intuitionistic fuzzy matrix and $B = (B_1, B_2)$ be an $n \times p$intuitionistic fuzzy matrix then the product AB is an $m \times p$ intuitionistic fuzzy matrix defined by

$AB = (c_{1ik}, c_{2ik})_{m \times p}$ where for all i,k

$c_{1ik} = \bigvee_{j} (a_{1ij} \wedge b_{1jk})$

and $c_{2ik} = \bigwedge_{j} (a_{2ij} \vee b_{2jk})$.

Lemma 2.1. *Let $A = (A_1, A_2)$ be an $m \times n$ intuitionistic fuzzy matrix having identical row max a_1 in A_1 and identical row min a_2 in A_2 and $B = (B_1, B_2)$ be an $n \times p$ intuitionistic fuzzy matrix having identical row max b_1 in B_1 and identical row min b_2 in B_2. Then AB has identical row max $a_1 \wedge b_1$ in A_1B_1, and identical row min $a_2 \vee b_2$ in A_2B_2.*

Proof. Given

$$\forall i, \quad \bigvee_{j} a_{1ij} = a_1, \quad \forall j, \bigvee_{k} b_{1jk} = b_1,$$
$$\forall i, \quad \bigwedge_{j} a_{2ij} = a_2, \quad \forall j, \bigwedge_{k} b_{2jk} = b_2.$$

Then for every i, we have

$$\bigvee_{k} c_{1ik} = \bigvee_{k} (\bigvee_{j} (a_{1ij} \bigwedge b_{1jk})) = a_1 \wedge b_1,$$
$$\bigwedge_{k} c_{2ik} = \bigwedge_{k} (\bigwedge_{j} (a_{2ij} \bigvee b_{2jk})) = a_2 \vee b2$$

since for all $i,k, c_{1ik} \leq a_1 \wedge b_1$ and $c_{2ik} \geq a_2 \vee b_2$ but $\forall i, \exists j'$ such that $a_{1ij'} = a_1, a_{2ij'} = a_2$ and $\exists k'$ such that $b_{ij'k'} = b_1, b_{2j'k'} = b_2$.

3. Basic concepts and notations

Definition 3.1. (Intuitionistic Fuzzy Automata) A finite intuitionistic fuzzy automaton over an input alphabet X is defined to be an algebraic system.

$$M = \left(Q, X, \{A(a) = (A_1(a), A_2(a)) : a \in X\}, \alpha = (\alpha_1, \alpha_2), F, \eta^F = (\eta_1^F, \eta_2^F)\right)$$

where

(1) $Q = \{q_1, q_2, \cdots, q_n\}$ is a finite set of states.

(2) X is finite alphabet.

(3) $A(a) = (A_1(a), A_2(a))$ is an $n \times n$ intuitionistic fuzzy matrix.

(4) $\alpha_1 = (\alpha_{11}, \alpha_{12}, \cdots, \alpha_{1n})$ and $\alpha_2 = (\alpha_{21}, \alpha_{22}, \cdots, \alpha_{2n})$ is an intuitionistic fuzzy row vector, called the initial state designator.

(5) F is a subset of Q, called the set of final states.

(6) $\eta_1^F = (\eta_{11}, \eta_{12}, \cdots, \eta_{1n})^T$ where η_{1i} has value 1 if $q_i \in F$ or 0 if $q_i \in Q - F$.
$\eta_2^F = (\eta_{21}, \eta_{22}, \cdots, \eta_{2n})^T$ where η_{2i} has value 0 if $q_i \in F$ or 1 if $q_i \in Q - F$.
η_1^F and η_2^F is called the final state designator.

Example 3.1. Consider the intuitionistic fuzzy automaton

$$M = \left(Q, X, \{A(a) = (A_1(a), A_2(a)) : a \in X\}, \alpha = (\alpha_1, \alpha_2), F, \eta^F = (\eta_1^F, \eta_2^F)\right)$$

where $Q = \{q_1, q_2, q_3\}, F = \{q_3\}$

$\alpha_1 = (0.3\ 0.8\ 0.9)$, $\alpha_2 = (0.5\ 0.2\ 0.1)$ and

$$A_1(a) = \begin{pmatrix} 0.8 & 0.6 & 0.3 \\ 0.7 & 0.9 & 0.5 \\ 0.4 & 1 & 0.2 \end{pmatrix} A_1(b) = \begin{pmatrix} 0.4 & 0.9 & 0.6 \\ 0.1 & 0.7 & 0.5 \\ 1 & 0.3 & 0.8 \end{pmatrix}$$

$$A_2(a) = \begin{pmatrix} 0.2 & 0.3 & 0.7 \\ 0.1 & 0 & 0.4 \\ 0.5 & 0 & 0.8 \end{pmatrix} A_2(b) = \begin{pmatrix} 0.3 & 0.1 & 0.4 \\ 0.7 & 0.2 & 0.4 \\ 0 & 0.6 & 0.1 \end{pmatrix}$$

$\eta_1^F = (0\ 0\ 1)^T$

$\eta_2^F = (1\ 1\ 0)^T$

Note that For any word $x \in X^*$, the intuitionistic fuzzy matrix

$A = (A_1(x), A_2(x); x \in X^*)$

(1) $A_1(\lambda) = \{a_{1ii} = 1$ and $a_{1ij} = 0$ for $i \neq j$
$A_2(\lambda) = \{a_{2ii} = 0$ and $a_{2ij} = 1$ for $i \neq j$
λ being the null word.

(2) $\forall x \in X^*, \forall a \in X, A_1(xa) = A_1(x)A_1(a)$

$$A_2(xa) = A_2(x)A_2(a).$$

The behavior of an iffa M is defined to be an intuitionistic fuzzy subset of X^*, denoted by $\beta_M = (\mu_{\beta_M}, \gamma_{\beta_M})$ whose membership and non membership function is given by, $\forall x \in X^*, \mu_{\beta_M}(x) = \alpha_1 A_1(x)\eta_1^F = \alpha_1 \eta_1^F(x)$ and $\gamma_{\beta_M}(x) = \alpha_2 A_2(x)\eta_2^F = \alpha_2 \eta_2^F(x)$, where we write $\eta_1^F(x) = A_1(x)\eta_1^F$ and $\eta_2^F(x) = A_2(x)\eta_2^F$. Two iffa's M and M' are said to be behaviourally equivalent, written $M \equiv M'$ if $\beta_M = \beta_{M'}$.

4. Quotient intuitionistic fuzzy automata

Definition 4.1. (Substitution Property) A partition π of Q for an iffa

$$M = \Big(Q, X, \{A(a) = (A_1(a), A_2(a)) : a \in X\}, \alpha = (\alpha_1, \alpha_2), F, \eta^F = (\eta_1^F, \eta_2^F)\Big)$$

is said to have the substitution property (SP) if for every $a \in X$, each of the intuitionistic fuzzy submatrices into which the matrix $A_1(a)$ is partitioned by the blocks of π has identical row max and $A_2(a)$ is partitioned by the blocks of π has identical row min. A partition π of Q is said to refine the set F of final states if every block of π is contained either in F or in $Q - F$.

Example 4.1. Consider an iffa $M = \Big(Q, X, \{A(a) = (A_1(a), A_2(a)) : a \in X\}, \alpha = (\alpha_1, \alpha_2), F, \eta^F = (\eta_1^F, \eta_2^F)\Big)$, where $Q = \{q_1, q_2, q_3, q_4, q_5\}$, $X = \{a, b\}$, $F = \{q_4, q_5\}$, $\alpha_1 = (0.5\ 0.6\ 0.1\ 0.7\ 0.4)$, $\alpha_2 = (0.4\ 0.2\ 0.8\ 0.3\ 0.5)$,

$$A_1(a) = \begin{pmatrix} 0.3 & 0.6 & 0.2 & 0.3 & 0.2 \\ 0.6 & 0.4 & 0.2 & 0.1 & 0.3 \\ 0.7 & 0.1 & 0.7 & 0.8 & 0.4 \\ 0.6 & 0.2 & 0.5 & 0.1 & 0.6 \\ 0.3 & 0.6 & 0.5 & 0.6 & 0.5 \end{pmatrix} A_2(a) = \begin{pmatrix} 0.6 & 0.2 & 0.7 & 0.4 & 0.5 \\ 0.2 & 0.5 & 0.7 & 0.8 & 0.4 \\ 0.2 & 0.8 & 0.1 & 0.2 & 0.5 \\ 0.4 & 0.6 & 0.5 & 0.7 & 0.3 \\ 0.7 & 0.4 & 0.5 & 0.3 & 0.4 \end{pmatrix}$$

$$A_1(b) = \begin{pmatrix} 0.7 & 0.9 & 0.2 & 0.4 & 0.3 \\ 0.9 & 0.7 & 0.2 & 0.1 & 0.4 \\ 0.8 & 0.2 & 0.8 & 0.9 & 0.5 \\ 0.6 & 0.3 & 0.3 & 0.2 & 0.7 \\ 0.4 & 0.6 & 0.3 & 0.7 & 0.6 \end{pmatrix} A_2(b) = \begin{pmatrix} 0.3 & 0.1 & 0.7 & 0.5 & 0.6 \\ 0.1 & 0.2 & 0.7 & 0.9 & 0.5 \\ 0.2 & 0.6 & 0.1 & 0.1 & 0.4 \\ 0.3 & 0.5 & 0.4 & 0.6 & 0.2 \\ 0.5 & 0.3 & 0.4 & 0.2 & 0.4 \end{pmatrix}$$

Hence the partition satisfies substitution property $\pi = \{[q_1, q_2], [q_3], [q_4, q_5]\}$.

Theorem 4.1. *If for an iffa*

$$M = \Big(Q, X, \{A(a) = (A_1(a), A_2(a)) : a \in X\}, \alpha = (\alpha_1, \alpha_2), F, \eta^F = (\eta_1^F, \eta_2^F)\Big).$$

π is an SP partition of Q, then for any $x \in X^*$, each of the submatrices into which the intuitionistic fuzzy matrix $A_1(x)$ is partitioned by the blocks of π has identical row max and $A_2(x)$ is partitioned by the blocks of π has identical row min.

Proof. Let $A_1(a) = a_{1ij}, A_2(a) = a_{2ij}$ and $x \in X^*$. We prove the result by induction on $\lg(x) = n$. If $n = 0$ then $x = \lambda,\ A_1(\lambda) = \{a_{1ii} = 1$ and $a_{1ij} = 0$ for $i \neq j,\ A_2(\lambda) = \{a_{2ii} = 0$ and $a_{2ij} = 1$ for $i \neq j$. Clearly, $A_1(\lambda)$ is partitioned by the blocks of π has identical row max and $A_2(\lambda)$ is partitioned by the blocks of π has identical row min.

Suppose now the result is true $\forall y \in X^*$ where length is $\leq n-1, n > 0$. Let $x = ya$ with $y \in X^*, a \in X$ and $A_1(ya) = A_1(y).A_1(a), A_2(ya) = A_2(y).A_2(a)$.

By lemma 2.1, $A_1(x)$ is partitioned by the blocks of π has identical row max and $A_2(x)$ is partitioned by the blocks of π has identical row min.

Definition 4.2. (Quotient Intuitionistic Fuzzy Automata) Let $M = \Big(Q, X, \{A(a) = (A_1(a), A_2(a)) : a \in X\}, \alpha = (\alpha_1, \alpha_2), F, \eta^F = (\eta_1^F, \eta_2^F)\Big)$ be an iffa and π is an SP partition of Q which refines F. Then the quotient iffa $M/\pi = \Big(Q', X, \{A'(a) = (A'_1(a), A'_2(a)) : a \in X\}, \alpha' = (\alpha'_1, \alpha'_2), F', \eta^{F'} = (\eta_1^{F'}, \eta_2^{F'})\Big)$ is an iffa where $Q' = \{B_1, B_2, \cdots, B_m\}$ is the set of blocks of $\pi (m = \text{rank } \pi)$, for every $a \in X, A'_1(a)$ is the $m \times m$ fuzzy matrix obtained by replacing each of the submatrices into which $A_1(a)$ is partitioned by the blocks of π by its constant row max and $A'_2(a)$ is the $m \times m$ fuzzy matrix obtained by replacing each of the submatrices into which $A_2(a)$ is partitioned by the blocks of π by its constant row min, α'_1 is the fuzzy row m-vector obtained by replacing each of the subvectors into which α_1 is partitioned by the blocks of π by its max and α'_2 is the fuzzy row m-vector obtained by replacing each of the subvectors into which α_2 is partitioned by the blocks of π by its min and F' is the set of blocks of π partitioning F.

Definition 4.3.

(1) M/π is well defined.
(2) For any $x \in X^*, A'(x) = (A'_1(x), A'_2(x))$ where $A'_1(x)$ is an $m \times m$ fuzzy matrix obtained by replacing each of the submatrices into which $A_1(x)$ is partitioned by the blocks of π by its constant row max and $A'_2(x)$ is an $m \times m$ fuzzy matrix obtained by replacing each of the submatrices into which $A_2(x)$ is partitioned by the blocks of π by its constant row min.
(3) For any $x \in X^*, \eta^F(x) = (\eta_1^F(x), \eta_2^F(x))$ has identitcal entries corresponding to each block of π and $\eta^{F'}(x) = (\eta_1^{F'}(x), \eta_2^{F'}(x))$ where $\eta_1^{F'}(x)$ is a fuzzy column m-vector obtained by replacing each of the column subvectors into which $\eta_1^F(x)$ is partitioned by the blocks of π by its constant entry and $\eta_2^{F'}(x)$ is a fuzzy column m-vector obtained by replacing each of the column subvectors into which $\eta_2^F(x)$ is partitioned by the blocks of π by its constant entry.

Example 4.2. From example 4.1, consider a quotient iffa
$M/\pi = \left(Q', X, \{A'(a) = (A'_1(a), A'_2(a)) : a \in X\}, \alpha' = (\alpha'_1, \alpha'_2), F', \eta^{F'} = (\eta_1^{F'}, \eta_2^{F'})\right)$, where $Q' = \{B_1, B_2, B_3\}$, $X = \{a, b\}$, $F' = \{B_3\}$, $\alpha'_1 = (0.6\ 0.1\ 0.7)$, $\alpha'_2 = (0.2\ 0.8\ 0.3)$,

$$A'_1(a) = \begin{pmatrix} 0.6 & 0.2 & 0.3 \\ 0.7 & 0.7 & 0.8 \\ 0.6 & 0.5 & 0.6 \end{pmatrix} A'_2(a) = \begin{pmatrix} 0.2 & 0.7 & 0.4 \\ 0.2 & 0.1 & 0.2 \\ 0.4 & 0.5 & 0.3 \end{pmatrix}$$

$$A'_1(b) = \begin{pmatrix} 0.9 & 0.2 & 0.4 \\ 0.8 & 0.8 & 0.9 \\ 0.6 & 0.3 & 0.7 \end{pmatrix} A'_2(b) = \begin{pmatrix} 0.1 & 0.7 & 0.5 \\ 0.2 & 0.1 & 0.1 \\ 0.3 & 0.4 & 0.2 \end{pmatrix}$$

$\eta_1^{F'} = (0\ 0\ 1)^T$, $\eta_2^{F'} = (1\ 1\ 0)^T$.

5. State equivalence, induced SP partition and minimal machine

Definition 5.1. (State Equivalence) Let $M = \left(Q, X, \{A(a) = (A_1(a), A_2(a)) : a \in X\}, \alpha = (\alpha_1, \alpha_2), F, \eta^F = (\eta_1^F, \eta_2^F)\right)$ be given iffa. Two states q_i and q_j of M are said to be equivalent, written $q_i \equiv q_j$, iff for every word $x \in X^*, \eta_1^F(x)$ has identical i^{th} and j^{th} entries and $\eta_2^F(x)$ has identical i^{th} and j^{th} entries. The partition of Q induced by this equivalence relation will be denoted by π_F and called the induced partition for M.

Theorem 5.1.

(1) π_F refines F.
(2) π_F is an SP partition.

Proof.

(1) $\eta_1^F(\lambda) = \eta_1^F$ which has constant entries 1 on F and 0 on $Q - F$.
$\eta_2^F(\lambda) = \eta_2^F$ which has constant entries 0 on F and 1 on $Q - F$
and hence follows (1).
(2) Let $a \in X, x \in X^*$, then if $\pi_F = \{B_k : k = 1, \cdots, m\}, \eta_1^F(ax) = A_1(a)\eta_1^F(x)$ and $\eta_2^F(ax) = A_2(a)\eta_2^F(x)$ so that

$$\begin{aligned} \eta_{1i}^F(ax) &= \bigvee_j (a_{1ij}(a) \wedge \eta_{1j}^F(x)) \\ &= \bigvee_k \bigvee_{q_j \in B_k} (a_{1ij}(a) \wedge \eta_{1j}^F(x)) \\ \eta_{2i}^F(ax) &= \bigwedge_j (a_{2ij}(a) \vee \eta_{2j}^F(x)) \\ &= \bigwedge_k \bigwedge_{q_j \in B_k} (a_{2ij}(a) \vee \eta_{2j}^F(x)) \end{aligned}$$

Since $\eta^F_{1j}(x)$ and $\eta^F_{2j}(x)$ has a constant value for all j for which $q_j \in B_k$, and writing

$\eta^F_{1j}(x) = \lambda_k(x)$ for $q_j \in B_k (k = 1, \cdots, m)$,

$\eta^F_{1i}(ax) = \bigvee_k (\lambda_k(x) \wedge \bigvee_{q_j \in B_k} a_{1ij}(a))$

If $q_i \equiv q_l$, then $\eta^F_{1i}(ax) = \eta^F_{1l}(ax)$

or $\eta^F_{1i}(ax) = \bigvee_k (\lambda_k(x) \wedge \bigvee_{q_j \in B_k} a_{1ij}(a))$

$= \bigvee_k (\lambda_k(x) \wedge \bigvee_{q_j \in B_k} a_{1lj}(a))$

and $\eta^F_{2j}(x) = \beta_k(x)$ for$q_j \in B_k (k = 1, \cdots, m)$,

$\eta^F_{2i}(ax) = \bigwedge_k (\beta_k(x) \vee \bigwedge_{q_j \in B_k} a_{2ij}(a))$

If $q_i \equiv q_l$, then $\eta^F_{2i}(ax) = \eta^F_{2l}(ax)$

or $\eta^F_{2i}(ax) = \bigwedge_k (\beta_k(x) \vee \bigwedge_{q_j \in B_k} a_{2ij}(a))$

$= \bigwedge_k (\beta_k(x) \vee \bigwedge_{q_j \in B_k} a_{2lj}(a))$

Since this holds for $\lambda_k(x)$ and $\beta_k(x)$ $(k = 1, \cdots, m)$ $\forall x \in X^*$, we conclude

$\bigvee_{q_j \in B_k} a_{1ij}(a) = \bigvee_{q_j \in B_k} a_{1lj}(a)$

and $\bigwedge_{q_j \in B_k} a_{2ij}(a) = \bigwedge_{q_j \in B_k} a_{2lj}(a)$

for all $a \in X$ which proves (2).

Theorem 5.2.π_F *is the largest SP partition for A which refines F.*

Definition 5.2. If π_F is the induced partition for an iffa M, then the quotient iffa M/π_F will be called minimal machine associated with M and denoted by M^M.

Theorem 5.3.

(1) M^M is well defined.
(2) $M^M \equiv M$.

6. Finite procedure for computing the minimal machine

For obtaining a finite procedure for computing the minimal machine of a given iffa, i.e., for computing the induced partition, we introduce the concept of k-equivalences.

Definition 6.1. Let k be a nonnegative integer and M a given iffa. A state q_i is said to be k-equivalent to a state q_j iff for any word $x \in X^*$ such that $\lg(x) \leq k, \eta^F(x) = (\eta^F_1(x), \eta^F_2(x))$ in which $\eta^F_1(x)$ and $\eta^F_2(x)$ has the same i^{th} and j^{th} entries. And the partition induced by this k-equivalence will be denoted by π_k.

We are now in a position to give an algorithm for computation of the induced partition.

Algorithm 6.1. For a given iffa

$$M = \Big(Q, X, \{A(a) = (A_1(a), A_2(a)) : a \in X\}, \alpha = (\alpha_1, \alpha_2), F, \eta^F = (\eta^F_1, \eta^F_2)\Big),$$

to compute the induced partition π_F, we proceed along the following steps.

(1) Compute $\pi_0 = \{F, S-F\}$.

(2) Having obtained π_h, compute π_{h+1} as follows:
Write down $\eta^F(x) = (\eta_1^F(x), \eta_2^F(x))$ for all $x \in X^*$ such that $\lg(x) = h$. Calculate $A_1(a)\eta_1^F(x)$ and $A_2(a)\eta_2^F(x)$ for all $a \in X$ and for all $x \in X^*$, such that $\lg(x) = h$. Denote by τ_h the partition of Q over the blocks of which the entries of $A_1(a)\eta_1^F(x)$ and $A_2(a)\eta_2^F(x)$ are constant.
Then $\tau_{h+1} = \pi_h \tau_h$.

(3) If $\pi_h = \pi_{h+1}$, then go to step 5, otherwise goto step 4.

(4) Replace h by $h+1$ and repeat step 2.

(5) Halt and set $\pi_F = \pi_h$.

Example 6.1. Consider an iffa

$$M = \left(Q, X, \{A(a) = (A_1(a), A_2(a)) : a \in X\}, \alpha = (\alpha_1, \alpha_2), F, \eta^F = (\eta_1^F, \eta_2^F)\right),$$

where $Q = \{q_1, q_2, \cdots, q_9\}, X = \{a, b\}, F = \{q_6, q_7, q_8, q_9\}$
$\alpha_1 = (0.3\ 0.7\ 0.8\ 0.2\ 1\ 0.4\ 0\ 0.5\ 0.9)$ and $\alpha_2 = (0.4\ 0.2\ 0.2\ 0.6\ 0\ 0.5\ 0.7\ 0.3\ 0.1)$

$$A_1(a) = \begin{pmatrix} 1 & 0.3 & 0.5 & 0.8 & 0.7 & 0.9 & 0.7 & 0.9 & 0.3 \\ 0.3 & 1 & 0.5 & 0.4 & 0.8 & 0.9 & 0.5 & 0.4 & 0.9 \\ 0.5 & 0.2 & 1 & 0.3 & 0.8 & 0.1 & 0.9 & 0.9 & 0.7 \\ 0.8 & 0.4 & 0.8 & 1 & 0.2 & 0 & 0.5 & 0.3 & 0.2 \\ 0.7 & 0.8 & 0.8 & 0.2 & 1 & 0.5 & 0.4 & 0.5 & 0.5 \\ 0.9 & 0.9 & 0.7 & 0 & 0.5 & 1 & 0.8 & 0.7 & 0.4 \\ 0.7 & 0.9 & 0.7 & 0.5 & 0.4 & 0.8 & 1 & 0.6 & 0.7 \\ 0.9 & 0.4 & 0.7 & 0.3 & 0.5 & 0.7 & 0.6 & 1 & 0.2 \\ 0.3 & 0.9 & 0.7 & 0.2 & 0.5 & 0.4 & 0.7 & 0.2 & 1 \end{pmatrix}$$

$$A_2(a) = \begin{pmatrix} 0 & 0.7 & 0.4 & 0.1 & 0.2 & 0.1 & 0.3 & 0.1 & 0.6 \\ 0.5 & 0 & 0.4 & 0.5 & 0.1 & 0.1 & 0.4 & 0.5 & 0.1 \\ 0.4 & 0.6 & 0 & 0.7 & 0.2 & 0.9 & 0.1 & 0.07 & 0.2 \\ 0.2 & 0.6 & 0.2 & 0 & 0.8 & 1 & 0.4 & 0.6 & 0.7 \\ 0.3 & 0.2 & 0.2 & 0.7 & 0 & 0.4 & 0.6 & 0.4 & 0.5 \\ 0.1 & 0.1 & 0.3 & 0.9 & 0.4 & 0 & 0.2 & 0.1 & 0.3 \\ 0.2 & 0.1 & 0.3 & 0.4 & 0.5 & 0.2 & 0 & 0.4 & 0.2 \\ 0.1 & 0.3 & 0.3 & 0.7 & 0.4 & 0.3 & 0.4 & 0 & 0.8 \\ 0.4 & 0.1 & 0.3 & 0.4 & 0.5 & 0.5 & 0.3 & 0.7 & 0 \end{pmatrix}$$

$$A_1(b) = \begin{pmatrix} 1 & 0.6 & 0.7 & 0.2 & 0.5 & 0.8 & 0.7 & 0.2 & 0.4 \\ 0.6 & 1 & 0.7 & 0.5 & 0.3 & 0.4 & 0.8 & 0.1 & 0.4 \\ 0.7 & 0.2 & 1 & 0.3 & 0.5 & 0.8 & 0.1 & 0.8 & 0.8 \\ 0.2 & 0.5 & 0.3 & 1 & 0.7 & 0.5 & 0.6 & 0.6 & 0.5 \\ 0.5 & 0.3 & 0.3 & 0.7 & 1 & 0.6 & 0.2 & 0.5 & 0.6 \\ 0.8 & 0.4 & 0.8 & 0.5 & 0.6 & 1 & 0.3 & 0.2 & 0.1 \\ 0.7 & 0.8 & 0.8 & 0.6 & 0.2 & 0.3 & 1 & 0.1 & 0.2 \\ 0.2 & 0.8 & 0.8 & 0.6 & 0.5 & 0.2 & 0.1 & 1 & 0.7 \\ 0.8 & 0.4 & 0.8 & 0.5 & 0.6 & 0.1 & 0.2 & 0.7 & 1 \end{pmatrix}$$

$$A_2(b) = \begin{pmatrix} 0 & 0.3 & 0.2 & 0.5 & 0.3 & 0.09 & 0.3 & 0.5 & 0.6 \\ 0.4 & 0 & 0.2 & 0.3 & 0.5 & 0.5 & 0.2 & 0.7 & 0.09 \\ 0.3 & 0.7 & 0 & 0.5 & 0.3 & 0.2 & 0.9 & 0.2 & 0.1 \\ 0.8 & 0.4 & 0.6 & 0 & 0.3 & 0.4 & 0.3 & 0.2 & 0.5 \\ 0.4 & 0.7 & 0.6 & 0.2 & 0 & 0.4 & 0.6 & 0.4 & 0.2 \\ 0.1 & 0.5 & 0.2 & 0.1 & 0.2 & 0 & 0.7 & 0.6 & 0.9 \\ 0.3 & 0.1 & 0.2 & 0.2 & 0.6 & 0.6 & 0 & 0.9 & 0.7 \\ 0.6 & 0.1 & 0.2 & 0.4 & 0.2 & 0.8 & 0.7 & 0 & 0.3 \\ 0.1 & 0.5 & 0.2 & 0.2 & 0.3 & 0.7 & 0.4 & 0.2 & 0 \end{pmatrix}$$

$\eta_1^F = (0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1)^T$

$\eta_2^F = (1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0)^T$

$\pi_0 = \{[q_1\ q_2\ q_3\ q_4\ q_5], [q_6\ q_7\ q_8\ q_9]\}$

$\eta_1^F(a) = A_1(a)\eta_1^F = (0.9\ 0.9\ 0.9\ 0.5\ 0.5\ 1\ 1\ 1\ 1)^T$

$\eta_1^F(b) = A_1(b)\eta_1^F = (0.8\ 0.8\ 0.8\ 0.6\ 0.6\ 1\ 1\ 1\ 1)^T$

$\eta_2^F(a) = A_2(a)\eta_2^F = (0.1\ 0.1\ 0.07\ 0.4\ 0.4\ 0\ 0\ 0\ 0)^T$

$\eta_2^F(b) = A_2(b)\eta_2^F = (0.09\ 0.09\ 0.1\ 0.2\ 0.2\ 0\ 0\ 0\ 0)^T$

$\tau_0 = \{[q_1,\ q_2], [q_3], [q_4,\ q_5], [q_6,\ q_7,\ q_8,\ q_9]\}$

$\pi_1 = \pi_0\tau_0 = \{[q_1, q_2], [q_3], [q_4, q_5], [q_6, q_7, q_8, q_9]\}$

$\eta_1^F(aa) = A_1(a)\eta_1^F(a) = (0.9\ 0.9\ 0.9\ 0.8\ 0.8\ 1\ 1\ 1\ 1)^T$

$\eta_1^F(ab) = A_1(a)\eta_1^F(b) = (0.9\ 0.9\ 0.9\ 0.8\ 0.8\ 1\ 1\ 1\ 1)^T$

$\eta_1^F(ba) = A_1(b)\eta_1^F(a) = (0.9\ 0.9\ 0.9\ 0.6\ 0.6\ 1\ 1\ 1\ 1)^T$

$\eta_1^F(bb) = A_1(b)\eta_1^F(b) = (0.8\ 0.8\ 0.8\ 0.6\ 0.6\ 1\ 1\ 1\ 1)^T$

$\eta_2^F(aa) = A_2(a)\eta_2^F(a) = (0.1\ 0.1\ 0.07\ 0.2\ 0.2\ 0\ 0\ 0\ 0)^T$

$\eta_2^F(ab) = A_2(a)\eta_2^F(b) = (0.09\ 0.09\ 0.07\ 0.2\ 0.2\ 0\ 0\ 0\ 0)^T$

$\eta_2^F(ba) = A_2(b)\eta_2^F(a) = (0.09\ 0.09\ 0.07\ 0.2\ 0.2\ 0\ 0\ 0\ 0)^T$

$\eta_2^F(bb) = A_2(b)\eta_2^F(b) = (0.09\ 0.09\ 0.1\ 0.2\ 0.2\ 0\ 0\ 0\ 0)^T$

$\tau_1 = \{[q_1,\ q_2],[q_3],[q_4,\ q_5],[q_6,\ q_7,\ q_8,\ q_9]\}$,

$\tau_2 = \pi_1\tau_1 = \pi_1 = \pi_F$.

Minimal Machine

$M^M = M/\pi_F = (Q, X, \{A'(a) = (A'_1(a), A'_2(a)) : a \in X\}, \alpha' = (\alpha'_1, \alpha'_2), F'$,

$\eta^{F'} = (\eta_1^{F'}, \eta_2^{F'}))$, where $Q' = \{B_1, B_2, B_3, B_4\}$, $\alpha'_1 = (0.7\ 0.8\ 1\ 0.9)$, $\alpha'_2 = (0.2\ 0.2\ 0\ 0.1)$ and $F' = \{B_4\}$, $\eta_1^{F'} = (0\ 0\ 0\ 1)$, $\eta_2^{F'} = (1\ 1\ 1\ 0)$ and

$$A'_1(a) = \begin{pmatrix} 1 & 0.5 & 0.8 & 0.9 \\ 0.5 & 1 & 0.8 & 0.9 \\ 0.8 & 0.8 & 1 & 0.5 \\ 0.9 & 0.7 & 0.5 & 1 \end{pmatrix} A'_2(a) = \begin{pmatrix} 0 & 0.4 & 0.1 & 0.1 \\ 0.4 & 0 & 0.2 & 0.07 \\ 0.2 & 0.2 & 0 & 0.4 \\ 0.1 & 0.3 & 0.4 & 0 \end{pmatrix}$$

$$A'_1(b) = \begin{pmatrix} 1 & 0.7 & 0.5 & 0.8 \\ 0.7 & 1 & 0.5 & 0.8 \\ 0.5 & 0.3 & 1 & 0.6 \\ 0.8 & 0.8 & 0.5 & 1 \end{pmatrix} A'_2(b) = \begin{pmatrix} 0 & 0.2 & 0.3 & 0.09 \\ 0.3 & 0 & 0.2 & 0.1 \\ 0.4 & 0.6 & 0 & 0.2 \\ 0.1 & 0.2 & 0.2 & 0 \end{pmatrix}$$

$\mu_{\beta_M}(ab) = \alpha_1\eta_1^F(ab) = 0.9,\ \mu_{\beta'_M}(ab) = \alpha'_1\eta_1^{F'}(ab) = 0.9$, and $\gamma_{\beta_M}(ab) = \alpha_2\eta_2^{F'}(ab) = 0.1,\ \gamma_{\beta'_M}(ab) = \alpha'_2\eta_2^{F'}(ab) = 0.1$.

7. Conclusion

By Example 6.1 we conclude that the minimization of number of states in intuitionistic fuzzy automata is always greater than or equal to minimization of number of states in fuzzy automata.

Conflict of Interests

The authors declare that there is no conflict of interests.

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