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TRIPLED COMMON FIXED POINT THEOREM IN M-FUZZY METRIC SPACE

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Abstract: In this paper we introduce the concept of tripled fixed point in the framework of M-fuzzy metric space and establish a tripled fixed point theorem for four self mappings. This result generalizes and extends the existing result of Roldon et al.[10] on tripled fixed point on fuzzy metric space to M-fuzzy metric space using weak compatible mapping. In the support of our result we will provide an example.

Keywords: coupled fixed point; tripled fixed point; M-fuzzy metric space; weakly compatible pairs.
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1. Introduction

Fixed point theorems have been studied in many contexts, one of which is the fuzzy settings. In 1965, Zadeh [17] introduced the concept of fuzzy sets. Using this concept in topology and analysis, many authors have extensively developed the theory of fuzzy sets and its applications. Kramosil and Michalek [8] introduced the concept of fuzzy metric space in 1975.It is well known that a fuzzy metric space is an important generalization of the metric space, In due course of time many researchers like George and Veeramani [4], Grabiec [5], Subrahmanyam [15] and Vasuki [16] used this concept to generalize some metric fixed point results.

In 2006, Bhaskar and Lakshmikantham [2] introduced the concept of coupled fixed point for contractive operators of the form $F: X\times X\rightarrow X$, where X is a partially ordered metric space. Later, Lakshmikantham and Ciric [9] proved coupled coincidence and coupled common fixed point

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theorems satisfying certain contractive conditions. After that many results appeared on coupled fixed point theory for contractions in fuzzy metric spaces [7,9,13].

Berinde and Borcut [1,3] introduced the concept of tripled fixed point in 2011 and proved some important results which extend the results of Bhaskar and Lakshmikantham for nonlinear mappings.

In this paper, we prove a new tripled common fixed point theorem under weaker conditions in the framework of M-fuzzy metric space introduced by Sedghi and Shobe [14], which extends and generalize some recent results [1,3,10].

2. Material and methods

Definition 2.1 ([12]) A binary operation $*: [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norm if it satisfies the following conditions

(1) $*$ is associative and commutative,

(2) $*$ is continuous,

(3) $a * 1 = a$ for all $a \in [0, 1]$,

(4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0,1]$.

Two typical examples of continuous t-norm are $a*b = a\,b$ and $a*b = \min\{a, b\}$.

Definition 2.2 ([14]) A 3-tuple $(X, M, *)$ is called a M-fuzzy metric space if X is an arbitrary (non-empty) set, $*$ is a continuous t-norm, and M is a fuzzy set on $X^3 \times (0,\infty)$, satisfying the following conditions for each x, y, z, a . X and t, s >0,

(1) $M(x, y, z, t) > 0$,

(2) $M(x, y, z, t) = 1$ if and only if $x = y = z$,

(3) $M(x, y, z, t) = M(p\{x, y, z\}, t)$,(symmetry) where p is a permutation function,

(4) $M(x, y, a, t) * M(a, z, z, s) \leq M(x, y, z, t + s)$,

(5) $M(x, y, z, .): (0,\infty) \to [0, 1]$ is continuous.

Remark 2.1 ([14]) Let $(X, M, *)$ be a M-fuzzy metric space. Then for every $t > 0$ and for every $x, y \in X$ we have $M(x, x, y, t) = M(x, y, y, t)$.

Definition 2.3 ([14]) Let $(X, M, *)$ be a M-fuzzy metric space. For $t > 0$, the open ball $B_M(x, r, t)$ with center $x \in X$ and radius $0 < r < 1$ is defined by

$B_M(x, r, t) = \{y \in X: M(x, y, y, t) > 1 - r\}$.

A subset A of X is called open set if for each x . A there exist t > 0 and 0 < r < 1 such that B_M(x, r, t) ⊆ A.

Definition 2.4 ([14]) A sequence {x_n} in X converges to x if and only if M(x, x, x_n, t) → 1 as n →∞, for each t > 0. It is called a Cauchy sequence if for each 0 < ε < 1 and t > 0, there exists n_0 ∈ **N** such that $M(x_n, x_n, x_m, t) > 1-\varepsilon$ for each n, m ≥ n_0. The M-fuzzy metric space (X, M, ∗) is said to be complete if every Cauchy sequence is convergent.

Lemma 2.1 ([14]). Let (X, M, ∗) be a M-fuzzy metric space. Then M(x, y, z, t) is non decreasing with respect to t, for all x, y, z in X.

Lemma 2.2 ([14]). Let (X, M, ∗) be a M-fuzzy metric space. Then M is continuous function on $X^3 \times (0,\infty)$.

Definition 2.5([6]): Let sup Δ(t, t) =1 for 0 < t < 1. A t-norm Δ is said to be of H type if the family of functions $\{\Delta^m(t)\}_{m=1}^{\infty}$ is equicontinuous at t=1, $\Delta^1(t) = t\ \Delta\ t$, $\Delta^{m+1}(t) = t\ \Delta\ (\Delta^m(t))$,m=1,2,….,t∈ [0,1].

The t-norm Δ_M =min is an example of t-nom of H-type.

Definition 2.6([15]) : The mapping F: X × X × X→ X and g: X→ X is called weakly compatible mapping if F(x ,y, z) = g(x), F(y ,z, x) = g(y) ,F(z ,x, y) = g(z).

implies that gF(x,y,z)= F(gx, gy,gz) ,gF(y, z, x) = F(gy, gz, gx) ,

gF(z, x, y) = F(gz, gx, gy) for all x,y,z ∈ X .

Definition 2.7 ([7]): Define Φ = {ϕ: R^+ →R} where R^+ = [0, + ∞) and each ϕ ∈Φ satisfies the following conditions:

(ϕ-i) ϕ is non-decreasing, (ϕ-ii)ϕ is upper semi continuous from the right,

(ϕ-iii) $\sum \phi^n(t) < \infty$ for all t >0 ,where $\phi^{n+1}(t) = \phi(\phi^n(t))$,n∈ N.

3. Theory and calculation

Lemma 3.1: Let (X, M, ∗) be a M-fuzzy metric space, ∗ is a continuous t-norm of H-type. If there exists ϕ ∈Φ such that M(x, y, z, t) ≤ M(x, y, z, ϕ(t)) for all t >0,then x = y = z

Definition 3.1: An element (x,y,z) ∈ X ×X ×X is called a Tripled fixed point of the mapping F: X ×X ×X→ X if F(x,y,z) = x, F(y,z,x) = y, F(z,x,y) = z.

Definition 3.2: An element $(x, y, z) \in X \times X \times X$ is called a Tripled coincidence point of the mapping $F: X \times X \times X \rightarrow X$ and $g: X \rightarrow X$ if $F(x, y, z) = g(x)$, $F(y, z, x) = g(y)$.
$F(z, x, y) = g(z)$.

Definition 3.3: An element $(x, y, z) \in X \times X \times X$ is called common Tripled fixed point of the mapping $F: X \times X \times X \rightarrow X$ and $g: X \rightarrow X$ if $F(x, y, z) = g(x) = x$, $F(y, z, x) = g(y) = y$, $F(z, x, y) = g(z) = z$.

Definition 3.4: An element $x \in X$ is called a common fixed point of the mapping
$F: X \times X \times X \rightarrow X$ and $g: X \rightarrow X$ if $x = F(x, x, x) = g(x)$.

4. Results

Theorem 4.1 Let $(X, M, *)$ be M-fuzzy metric space where $*$ is a continuous t-norm of H-type satisfying definition (2.5). Let $A: X \times X \times X \rightarrow X$ and $h: X \rightarrow X$, $B: X \times X \times X \rightarrow X$ and $g: X \rightarrow X$ are weakly compatible pairs and there exists $\phi \in \Phi$ satisfying,

(i) $M(A(x,y,z),A(y,z,x),A(z,x,y),\phi(t)) \geq M(hx,hy,hz,t) * M(hy, hz, hx, t) * M(hz,hx,hy, t)$ and

$M(B(x,y,z),B(y,z,x),B(z,x,y),\phi(t)) \geq M(gx,gy,gz,t) * M(gy, gz, gx, t) * M(gz,gx,gy, t)$ for all $x, y, z \in X$, $t > 0$. (4.1)

(ii) $A(X \times X \times X) \subseteq h(X)$ and $A(X \times X \times X)$ or $h(X)$ is complete and

(iii) $B(X \times X \times X) \subseteq g(X)$, $B(X \times X \times X)$ or $g(X)$ is complete.

Then A, B, g and h have a unique common fixed point in X.

Proof.

Let $x_0, y_0, z_0 \in X$ be three arbitrary points. Since $A(X \times X \times X) \subseteq h(X)$, there must exist points $x_1, y_1, z_1 \in X$ such that $A(x_0, y_0, z_0) = hx_1$ and $A(y_0, z_0, x_0) = hy_1$,
$A(z_0, x_0, y_0) = h z_1$.

Also, $B(X \times X \times X) \subseteq g(X)$, then there exist points $x_2, y_2, z_2 \in X$ such that
$B(x_1, y_1, z_1) = gx_2$ and $B(y_1, z_1, x_1) = gy_2$, $B(z_1, x_1, y_1) = gz_2$.

In general we get sequences $\{x_n\}$, $\{y_n\}$, $\{z_n\}$ in X s.t. $x_{2n} = h(x_{2n+1}) = A(x_{2n}, y_{2n}, z_{2n})$,

$y_{2n} = A(y_{2n}, z_{2n}, x_{2n}) = h(y_{2n+1})$, $z_{2n} = A(z_{2n}, x_{2n}, y_{2n}) = h(z_{2n+1})$ and

$x_{2n+1} = g(x_{2n+2}) = B(x_{2n+1}, y_{2n+1}, z_{2n+1})$, $y_{2n+1} = B(y_{2n+1}, z_{2n+1}, x_{2n+1}) = g(y_{2n+2})$,

$z_{2n+1} = B(z_{2n+1}, x_{2n+1}, y_{2n+1}) = g(z_{2n+2})$, $n = 0,1,2,3,4,\ldots\ldots$ (4.2)

The proof is divided into four parts,

Part- I

We shall prove that $\{hx_n\}$, $\{hy_n\}$, $\{hz_n\}$ are Cauchy sequences .

Since $*$ is a t-norm of H-type, for any $\lambda > 0$ there exists $\mu > 0$ s.t.

$(1-\mu) * (1-\mu) * (1-\mu) * \ldots\ldots. * (1-\mu) \geq 1-\lambda$

(k times) $k \in N$

Since $M(x, y, z, .)$ is continuous and $Lt_{t\to+\infty} M(x, y, z, t) = 1$,for all $x,y,z \in X$, there exist $t_0 > 0$ s.t.

$M(hx_0,hx_1,hx_2,t_0) \geq 1-\mu$, $M(hy_0,hy_1,hy_2,t_0) \geq 1-\mu$, $M(hz_0,hz_1,hz_2,t_0) \geq 1-\mu$,

$M(gx_0,gx_1,gx_2,t_0) \geq 1-\mu$, $M(gy_0,gy_1,gy_2,t_0) \geq 1-\mu$, $M(gz_0,gz_1,gz_2,t_0) \geq 1-\mu$, (4.3)

As $\phi \in \Phi$, by condition (ϕ-iii) ,we have $\sum_{n=1}^{\infty} \phi^n(t_0) < \infty$. Then for any $t > 0$, there exists $n_0 \in N$ such that $\sum_{k=n0}^{\infty} \phi^k(t_0) < t$. (4.4)

From (4.1) we have,

$M(hx_1,hx_2,hx_3, \phi(t_0)) = M(A(x_0, y_0, z_0), A(x_1, y_1, z_1) A(x_2, y_2, z_2), \phi(t_0)) \geq$

$M(hx_0,hx_1,hx_2, t_0) * M(hy_0,hy_1,hy_2, t_0) * M(hz_0,hz_1,hz_2, t_0)$

$M(hy_1,hy_2,hy_3, \phi(t_0)) = M(A(y_0, z_0, x_0), A(y_1, z_1, x_1), A(y_2, z_2, x_2), \phi(t_0)) \geq$

$M(hy_0,hy_1,hy_2, t_0) * M(hz_0,hz_1,hz_2, t_0) * M(hx_0,hx_1,hx_2, t_0)$

$M(hz_1,hz_2,hz_3, \phi(t_0)) = M(A(z_0, x_0, y_0), A(z_1, x_1, y_1), A(z_2, x_2, y_2), \phi(t_0)) \geq$

$M(hz_0,hz_1,hz_2, t_0) * M(hx_0,hx_1,hx_2, t_0) * M(hy_0,hy_1,hy_2, t_0)$

Similarly,

$M(hx_2,hx_3,hx_4, \phi^2(t_0)) = M(A(x_1, y_1, z_1), A(x_2, y_2, z_2), A(x_3, y_3, z_3), \phi^2(t_0)) \geq$

$M(hx_1,hx_2,hx_3, t_0) * M(hy_1,hy_2,hy_3, t_0) * M(hz_1,hz_2,hz_3, t_0)$

$\geq [M(hx_0,hx_1,hx_2, t_0)]^2 * [M(hy_0,hy_1,hy_2, t_0)]^2 * [M(hz_0,hz_1,hz_2, t_0)]^2$

$M(hy_2,hy_3,hy_4, \phi^2 (t_0)) = M(A(y_1, z_1, x_1), A(y_2, z_2, x_2), A(y_3, z_3, x_3), \phi^2(t_0)) \geq$

$M(hy_1,hy_2,hy_3, t_0) * M(hz_1,hz_2,hz_3, t_0) * M(hx_1,hx_2,hx_3, t_0) \geq$

$[M(hy_0,hy_1,hy_2, t_0)]^2 * [M(hz_0,hz_1,hz_2, t_0)]^2 * [M(hx_0,hx_1,hx_2, t_0)]^2$

$M(hz_2,hz_3,hz_4, \phi^2 (t_0)) = M(A(z_1, x_1, y_1), A(z_2, x_2, y_2), A(z_3, x_3, y_3), \phi^2 (t_0)) \geq$

$M(hz_1,hz_2,hz_3, t_0) * M(hx_1,hx_2,hx_3, t_0) * M(hy_1,hy_2,hy_3, t_0) \geq$

$[M(hz_0,hz_1,hz_2, t_0)]^2 * [M(hx_0,hx_1,hx_2, t_0)]^2 * [M(hy_0,hy_1,hy_2, t_0)]^2$

By above inequalities and by induction, we have,

$M(hx_n,hx_{n+1},hx_{n+2}, \phi^n (t_0))$

$$\geq [M(hx_0,hx_1,hx_2,t_0)]^{2^{n-1}} * [M(hy_0,hy_1,hy_2,t_0)]^{2^{n-1}} * [M(hz_0,hz_1,hz_2,t_0)]^{2^{n-1}}$$

$M(hy_n,hy_{n+1},hy_{n+2}, \phi^n(t_0)) \geq$

$$[M(hy_0,hy_1,hy_2,t_0)]^{2^{n-1}} * [M(hz_0,hz_1,hz_2,t_0)]^{2^{n-1}} * [M(hx_0,hx_1,hx_2,t_0)]^{2^{n-1}}$$

$M(hz_n,hz_{n+1},hz_{n+2}, \phi^n (t_0)) \geq$

$$[M(hz_0,hz_1,hz_2,t_0)]^{2^{n-1}} * [M(hx_0,hx_1,hx_2,t_0)]^{2^{n-1}} * [M(hy_0,hy_1,hy_2,t_0)]^{2^{n-1}} .$$

From (4.3) and (4.4) for $p > m > n \geq n_0$

$$M(hx_n,hx_m,hx_p,t) = M\left(hx_n,hx_m,hx_p,\sum_{k=n_0}^{\infty}\phi^k(t_0)\right) \geq M\left(hx_n,hx_m,hx_p,\sum_{k=n}^{m-1}\phi^k(t_0)\right)$$

$$\geq M\left(hx_n,hx_m,hx_p,\sum_{k=m}^{p-1}\phi^k(t_0)\right) * M(hx_m,hx_{m+1},hx_{m+2},\phi^n(t_0)) * M(hx_{m+1},hx_{m+2},hx_{m+3},\phi^{n+1}(t_0)) *$$
$$\ldots\ldots\ldots\ldots * M(hx_{p-1},hx_p,hx_{p+1},\phi^{p-1}(t_0))$$
$$\geq [M(hx_0,hx_1,hx_2,t_0)]^{2^{m-1}} * [M(hy_0,hy_1,hy_2,t_0)]^{2^{m-1}} * [M(hz_0,hz_1,hz_2,t_0)]^{2^{m-1}} * [M(hx_0,hx_1,hx_2,t_0)]^{2^m} *$$
$$[M(hy_0,hy_1,hy_2,t_0)]^{2^m} * [M(hz_0,hz_1,hz_2,t_0)]^{2^m} * \ldots\ldots\ldots\ldots * [M(hx_0,hx_1,hx_2,t_0)]^{2^{p-2}} *$$
$$[M(hy_0,hy_1,hy_2,t_0)]^{2^{p-2}} * [M(hz_0,hz_1,hz_2,t_0)]^{2^{p-2}} =$$

$$\left[M\left(hx_0,hx_1,hx_2,t_0\right)\right]^{2^{p-1}-2^{m-1}} * \left[M\left(hy_0,hy_1,hy_2,t_0\right)\right]^{2^{p-1}-2^{m-1}} * \left[M\left(hz_0,hz_1,hz_2,t_0\right)\right]^{2^{p-2}-2^{m-1}}$$
$$\geq (1-\mu)*(1-\mu)*..........\,......*(1-\mu)\geq 1-\lambda.$$

$M(hx_n,hx_m,hx_p, t) > 1-\lambda$. (4.5)

For all m,n,p∈ N with $p > m > n \geq n_0$ and t >0, { hx_n } is a Cauchy sequence. Similarly { hy_n }, { hz_n } are a Cauchy sequences.

Part- II

Now we prove that h and A and g and B have a Tripled coincidence point.

Without loss of generality let us assume g(X) is complete then there exist x,y ,z∈ g(X) and a, b ,c ∈X s.t. $Lt_{n\to\infty} g(x_n) = Lt_{n\to\infty} B(x_n ,y_n, z_n) = g(a) = x.$,

$Lt_{n\to\infty} g(y_n) = Lt_{n\to\infty} B(y_n ,z_n, x_n) = g(b) = y.$

$Lt_{n\to\infty} g(z_n) = Lt_{n\to\infty} B(z_n ,x_n, y_n) = g(c) = z.$ Similarly

$Lt_{n\to\infty} h(x_n) = Lt_{n\to\infty} A(x_n ,y_n, z_n) = h(a) = x.$,

$Lt_{n\to\infty} h(y_n) = Lt_{n\to\infty} A(y_n ,z_n, x_n) = h(b) = y.$

$Lt_{n\to\infty} h(z_n) = Lt_{n\to\infty} A(z_n ,x_n, y_n) = h(c) = z.$ (4.6)

From (4.1) we get

$M(B(x_n, y_n, z_n), B(x_n, y_n, z_n) B(a, b, c), \phi(t)) \geq$

$M(gx_n,gx_n,g(a), t) * M(gy_n, gy_n, g(b), t) * M(gz_n, gz_n,g(b)z, t)$, As M is continuous , $n \to \infty$, we have

$M(g(a), g(a), B(a, b, c), \phi(t)) * M(x, x, g(a), t) * M(y, y, g(b), t) * M(z, z, g(c), t)$

$\geq M(x, x, x, t) * M(y, y, y, t) * M(z, z, z, t)$,which implies

$M(g(a), g(a), B(a, b, c), \phi(t)) = 1$,implies $B(a, b, c)=g(a) = x$,

Similarly $B(b, c, a)=g(b) = y$ and $B(c, a, b)=g(c) = z$.

Also, $A(a, b, c)=h(a) = x$,

Similarly $A(b, c, a)=h(b) = y$ and $A(c, a, b)=h(c) = z$.

Sinc A and h and B and g are weakly compatible we have

hA(a,b,c) = A(h(a),h(b),h(c)) and h(A(b,c,a)) =A(h(b),h(a),h(c)), h(A(c,a,b)) =A(h(c),h(a),h(b)), which implies that

h(x) =A (x,y,z) ,h(y) = A(y,z,x) and h(z) = A (z,x,y) .

Also, $g(x) = B(x,y,z)$, $g(y) = B(y,z,x)$ and $g(z) = B(z,x,y)$.

Part- III

Now we shall prove that $h(x) = y$, $h(y) = z$, $h(z) = x$ and $g(x) = y$, $g(y) = z$, $g(z) = x$. As $*$ is a t-norm of H-type for any $\lambda > 0$, there exist $\mu > 0$ s.t.

$$(1-\mu) * (1-\mu) * \ldots\ldots * (1-\mu) \geq 1-\lambda \text{ for all } k \in N.$$

Since $M(x,y,z,.)$ is continuous and $Lt_{t\to+\infty} M(x, y, z, t) = 1$, for all $x,y,z \in X$ there exist $t_0 > 0$ s.t.

$M(hx, y, z, t_0) \geq 1-\mu$, $M(x, hy, z, t_0) \geq 1-\mu$, $M(x, y, hz, t_0) \geq 1-\mu$.

As $\phi \in \Phi$, $\sum_{n=1}^{\infty} \phi^n(t_0) < \infty$, thus for any $t > 0$ there exist $n_0 \in N$ s.t.

$$t > \sum_{k=n0}^{\infty} \phi^k(t_0).$$

Since

$M(hx,hy_{n+1},hy_{n+1}, \phi(t_0)) = M(A(x, y, z), A(y_n, z_n, x_n), A(y_n, z_n, y_n), \phi(t_0)) \geq$

$M(hx,hy_n,hy_n, t_0) * M(hy,hz_n,hz_n, t_0) * M(hz,hx_n,hx_n, t_0)$

As $n \to \infty$

$$M(hx,y,y, \phi(t_0)) \geq M(hx,y,y, t_0) * M(hy,z,z,t_0) * M(hz,x,x, t_0) \quad (4.7)$$

$$M(hy,z,z, \phi(t_0)) \geq M(hy,z,z, t_0) * M(hz,x,x,t_0) * M(hx,y,y, t_0) \quad (4.8)$$

$$M(hz,x,x, \phi(t_0)) \geq M(hz,x,x, t_0) * M(hx,y,y,t_0) * M(hy,z,z, t_0), \quad (4.9)$$

Similarly

$$M(gx,y,y, \phi(t_0)) \geq M(gx,y,y, t_0) * M(gy,z,z,t_0) * M(gz,x,x, t_0) \quad (4.10)$$

$$M(gy,z,z, \phi(t_0)) \geq M(gy,z,z, t_0) * M(gz,x,x,t_0) * M(gx,y,y, t_0) \quad (4.11)$$

$$M(gz,x,x, \phi(t_0)) \geq M(gz,x,x, t_0) * M(gx,y,y,t_0) * M(gy,z,z, t_0) \quad (4.12)$$

Using (4.7),(4.8),(4.9) and (4.10),(4.11) and (4.12)

$M(hx,y,y, \phi(t_0)) * M(hy,z,z, \phi(t_0)) * M(hz,x,x, \phi(t_0)) \geq [M(hx,y,y, t_0)]^2 *$

$[M(hy,z,z,t_0)]^2 * [M(hz,x,x, t_0)]^2$ and

$M(gx,y,y, \phi(t_0)) * M(gy,z,z, \phi(t_0)) * M(gz,x,x, \phi(t_0)) \geq [M(gx,y,y, t_0)]^2 *$

$[M(gy,z,z,t_0)]^2 * [M(gz,x,x, t_0)]^2$

Thus, we have

$M(hx,y,y, \phi^n(t_0)) * M(hy,z,z, \phi^n(t_0)) * M(hz,x,x, \phi^n(t_0)) \geq [M(hx,y,y, \phi^{n-1}(t_0))]^2 * [M(hy,z,z, \phi^{n-1}(t_0))]^2 * [M(hz,x,x, \phi^{n-1}(t_0))]^2 \geq [M(hx,y,y, t_0)]^{2^n} * [M(hy,z,z,t_0)]^{2^n} * [M(hz,x,x, t_0)]^{2^n}$

and

$M(gx,y,y, \phi^n(t_0)) * M(gy,z,z, \phi^n(t_0)) * M(gz,x,x, \phi^n(t_0)) \geq [M(gx,y,y, \phi^{n-1}(t_0))]^2 * [M(gy,z,z, \phi^{n-1}(t_0))]^2 * [M(gz,x,x, \phi^{n-1}(t_0))]^2 \geq [M(gx,y,y, t_0)]^{2^n} * [M(gy,z,z,t_0)]^{2^n} * [M(gz,x,x, t_0)]^{2^n}$

for all $n \in N$, $\sum_{k=n0}^{\infty} \phi^k(t_0) > t$. Then we have

$M(hx,y,y, t) * M(hy,z,z, t) * M(hz,x,x, t) \geq (1-\mu) * (1-\mu) * \ldots\ldots * (1-\mu) \geq 1-\lambda$,

$M(gx,y,y, t) * M(gy,z,z, t) * M(gz,x,x, t) \geq (1-\mu) * (1-\mu) * \ldots\ldots * (1-\mu) \geq 1-\lambda$ for all $t > 0$.

Hence, $hx = y$, $hy = z$, $hz = x$ and $gx = y$, $gy = z$, $gz = x$.

Part- IV

Now we shall prove that $x = y = z$.

As $(1-\mu) * (1-\mu) * \ldots\ldots * (1-\mu)$ {k times} $\geq 1-\lambda$ for all $k \in N$.

$M(hx_{n+1},hy_{n+1},hy_{n+1}, \phi(t_0)) = M(A(x_n, y_n, z_n), A(y_n, z_n, x_n), A(y_n, z_n, y_n), \phi(t_0)) \geq$

$M(hx_n, hy_n, hy_n, t_0) * M(hy_n, hz_n, hz_n, t_0) * M(hz_n, hx_n, hx_n, t_0)$

As $n \to \infty$

$M(x, y, z, \phi(t_0)) \geq M(x, y, y, t_0) * M(y,z,z,t_0) * M(z, x, x, t_0)$ Thus, we have

$M(x, y, z, t) \geq M(x, y, z, \sum_{k=n0}^{\infty} \phi^k(t_0)) \geq M(x, y, z, \phi^{n_0}(t_0)) \geq 1-\lambda$.

Implies $x = y = z$

Similarly,

Same is true for g function. Hence A, h and B, g have a common fixed point in X. Similarly, we can prove that A, h and B, g have unique Common fixed point.

Corollary 4.1 Let $(X, M, *)$ be M-fuzzy metric space where $*$ is a continuous t-norm of H-type satisfying definition (2.5). Let $A: X \times X \times X \to X$ and $B: X \times X \times X \to X$ are weakly compatible mappings and there exists $\phi \in \Phi$ satisfying,

(i) $M(A(x,y,z),A(y,z,x),A(z,x,y),\phi(t)) \geq M(x,y,z,t) * M(y, z, x, t) *$

$M(z,x,y,t)$ for all $x, y, z \in X$, $t > 0$. (4.13)

(ii) $A(X \times X \times X)$ and $B(X \times X \times X)$ are complete and

Then A, B, have a unique common fixed point in X.

Example 4.1 Let [-2,2], $a * b \geq ab$ for all $a,b \in [0,1]$ and $\phi(t) = t/2$ and $\Phi(t)$ defined by

$\Phi(t) = \{ \alpha\sqrt{t}$ if $0 < t \leq 4$ and $1 - 1/\ln t$ if $t > 4$ where $\alpha = (1/2)(1 - 1/\ln 4)$. Then $\Phi(t)$ is continuous in $(0, \infty)$ and $\lim t \to \infty\ \Phi(t) = 1$.

Let X = [-2, 2]

Let $M(x,y,z,t) = [\Phi(t)]^{|x-y|+|y-z|+|z-x|}$

Let $A(x,y,z) = [2x^2 + 3y^2 + 4z^2] / 12$, $A(y,z,x) = [2y^2 + 3z^2 + 4x^2] / 12$

$A(z,x,y) = [2z^2 + 3x^2 + 4y^2] / 12$ $h(x) = x$ and

Let $B(x,y,z) = [2x^2 + 3y^2 + 4z^2] / 12$, $B(y,z,x) = [2y^2 + 3z^2 + 4x^2] / 12$

$B(z,x,y) = [2z^2 + 3x^2 + 4y^2] / 12$, $g(x) = x$. Here all the conditions of theorem are satisfied and $x=4/3$ is a tripled fixed point of mapping A, B, h and g.

Conclusion

In this study a new tripled fixed point theorem has been proved for four self mappings on new space M-fuzzy metric space. One example is given in the support of the theorem. This theorem can be extended to other spaces with new conditions.

Conflict of Interests

The authors declare that there is no conflict of interests.

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