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THE FAMILIES OF SOFT L-TOPOLOGIES AND SOFT CLOSURE OPERATORS

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Abstract. In this paper, we study the notions of soft closure operators in complete residuated lattices. We investigate the relations among soft cotopology and soft closure operators. We give their examples.

Keywords: complete residuated lattices; soft closure operators; soft topologies; soft cotopologies.

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1. Introduction

Molodtsov [15] introduced the soft set as a mathematical tool for dealing information as the uncertainty of data in engineering, physics, computer sciences and many other diverse field. Presently, the soft set theory is making progress rapidly [1,4,11-15,22]. Pawlak's rough set [16,17] can be viewed as a special case of soft rough sets [4]. The topological structures of soft sets have been developed by many researchers [3,8,9,19,20,24,25]. Hájek [5] introduced a complete residuated lattice which is an algebraic structure for many valued logic. It is an important mathematical tool for algebraic structure of fuzzy contexts [6,8,21].

Kim [16,17] introduced a fuzzy soft $F : A \to L^U$ as an extension as the soft $F : A \to P(U)$ where L is a complete residuated lattice. Kim [16,17] introduced the soft topological structures in complete residuated lattices.

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In this paper, we study the notions of soft closure operators in complete residuated lattices. We investigate the relations among soft cotopology and soft closure operators. We give their examples.

2. Preliminaries

Definition 2.1. [2,5,6,21] An algebra $(L,\wedge,\vee,\odot,\to,0,1)$ is called a complete residuated lattice if it satisfies the following conditions:

(C1) $L=(L,\leq,\vee,\wedge,1,0)$ is a complete lattice with the greatest element 1 and the least element 0;

(C2) $(L,\odot,1)$ is a commutative monoid;

(C3) $x\odot y\leq z$ iff $x\leq y\to z$ for $x,y,z\in L$.

In this paper, we assume that $(L,\leq,\odot,\to)$ is a complete residuated lattice.

Lemma 2.2. [2,5,6,21] *For each $x,y,z,x_i,y_i,w\in L$, we have the following properties.*

(1) $1\to x=x$, $0\odot x=0$,

(2) If $y\leq z$, *then* $x\odot y\leq x\odot z$, $x\to y\leq x\to z$ *and* $z\to x\leq y\to x$,

(3) $x\odot y\leq x\wedge y\leq x\vee y$,

(4) $x\odot(\bigvee_i y_i)=\bigvee_i(x\odot y_i)$,

(5) $x\to(\bigwedge_i y_i)=\bigwedge_i(x\to y_i)$,

(6) $(\bigvee_i x_i)\to y=\bigwedge_i(x_i\to y)$,

(7) $x\to(\bigvee_i y_i)\geq\bigvee_i(x\to y_i)$,

(8) $(\bigwedge_i x_i)\to y\geq\bigvee_i(x_i\to y)$,

(9) $(x\odot y)\to z=x\to(y\to z)=y\to(x\to z)$,

(10) $x\odot(x\to y)\leq y$ *and* $x\to y\leq(y\to z)\to(x\to z)$,

(11) $(x\to y)\odot(z\to w)\leq(x\odot z)\to(y\odot w)$,

(12) $x\to y\leq(x\odot z)\to(y\odot z)$ *and* $(x\to y)\odot(y\to z)\leq x\to z$.

Definition 2.3. [8,9] Let X be an initial universe of objects and E the set of parameters (attributes) in X. A pair (F,A) is called a *fuzzy soft set* over X, where $A\subset E$ and $F:A\to L^X$ is a mapping. We denote $S(X,A)$ as the family of all fuzzy soft sets under the parameter A.

Definition 2.4. [8,9] Let (F,A) and (G,A) be two fuzzy soft sets over a common universe X.

(1) (F,A) is a fuzzy soft subset of (G,A), denoted by $(F,A) \leq (G,A)$ if $F(a) \leq G(a)$, for each $a \in A$.

(2) $(F,A) \wedge (G,A) = (F \wedge G,A)$ if $(F \wedge G)(a) = F(a) \wedge G(a)$ for each $a \in A$.

(3) $(F,A) \vee (G,A) = (F \vee G,A)$ if $(F \vee G)(a) = F(a) \vee G(a)$ for each $a \in A$.

(4) $(F,A) \odot (G,A) = (F \odot G,A)$ if $(F \odot G)(a) = F(a) \odot G(a)$ for each $a \in A$.

(6) $\alpha \odot (F,A) = (\alpha \odot F,A)$ for each $\alpha \in L$.

Definition 2.5. [8,9] A map $\tau \subset S(X,A)$ is called a soft topology on X if it satisfies the following conditions.

(ST1) $(0_X,A),(1_X,A) \in \tau$, where $0_X(a)(x) = 0, 1_X(a)(x) = 1$ for all $a \in A, x \in X$,

(ST2) If $(F,A),(G,A) \in \tau$, then $(F,A) \odot (G,A) \in \tau$,

(T) If $(F_i,A) \in \tau$ for each $i \in I$, $\bigvee_{i\in I}(F_i,A) \in \tau$.

A map $\tau \subset S(X,A)$ is called a soft cotopology on X if it satisfies (ST1), (ST2) and

(CT) If $(F_i,A) \in \tau$ for each $i \in I$, $\bigwedge_{i\in I}(F_i,A) \in \tau$.

The triple (X,A,τ) is called a soft topological (resp. cotopological) space.

Let (X,A,τ_1) and (X,A,τ_2) be soft topological spaces. Then τ_1 is finer than τ_2 if $(F,A) \in \tau_1$, for all $(F,A) \in \tau_2$.

Definition 2.6. [8,9] Let $S(X,A)$ and $S(Y,B)$ be the families of all fuzzy soft sets over X and Y, respectively. The mapping $f_\phi : S(X,A) \to S(Y,B)$ is a soft mapping where $f : X \to Y$ and $\phi : A \to B$ are mappings.

(1) The image of $(F,A) \in S(X,A)$ under the mapping f_ϕ is denoted by $f_\phi((F,A)) = (f_\phi(F),B)$ where

$$f_\phi(F)(b)(y) = \begin{cases} \bigvee_{a\in\phi^{-1}(\{b\})} f^{\rightarrow}(F(a))(y), & \text{if } \phi^{-1}(\{b\}) \neq \emptyset, \\ 0, & \text{otherwise.} \end{cases}$$

(2) The inverse image of $(G,B) \in S(Y,B)$ under the mapping f_ϕ is denoted by $f_\phi^{-1}((G,B)) = (f_\phi^{-1}(G),A)$ where

$$f_\phi^{-1}(G)(a)(x) = f^{\leftarrow}(G(\phi(a)))(x),\ \forall a \in A, x \in X.$$

(3) The soft mapping $f_\phi : S(X,A) \to S(Y,B)$ is called injective (resp. surjective, bijective) if f and ϕ are both injective (resp. surjective, bijective).

Lemma 2.7. [8,9] *Let $f_\phi : S(X,A) \to S(Y,B)$ be a soft mapping. Then we have the following properties. For $(F,A),(F_i,A) \in S(X,A)$ and $(G,B),(G_i,B) \in S(Y,B)$,*

(1) $(G,B) \geq f_\phi(f_\phi^{-1}((G,B)))$ *with equality if f is surjective,*

(2) $(F,A) \leq f_\phi^{-1}(f_\phi((F,A)))$ *with equality if f is injective,*

(3) $f_\phi^{-1}(\bigvee_{i\in I}(G_i,B)) = \bigvee_{i\in I} f_\phi^{-1}((G_i,B))$,

(4) $f_\phi^{-1}(\bigwedge_{i\in I}(G_i,B)) = \bigwedge_{i\in I} f_\phi^{-1}((G_i,B))$,

(5) $f_\phi(\bigvee_{i\in I}(F_i,A)) = \bigvee_{i\in I} f_\phi((F_i,A))$,

(6) $f_\phi(\bigwedge_{i\in I}(F_i,A)) \leq \bigwedge_{i\in I} f_\phi((F_i,A))$ *with equality if f is injective,*

(7) $f_\phi^{-1}((G_1,B)\odot(G_2,B)) = f_\phi^{-1}((G_1,B))\odot f_\phi^{-1}((G_2,B))$,

(8) $f_\phi((F_1,A)\odot(F_2,A)) \leq f_\phi((F_1,A))\odot f_\phi((F_2,A))$ *with equality if f is injective.*

3. The families of soft L-topologies and soft closure operators

Definition 3.1. A mapping $cl : S(X,A) \to S(X,A)$ is called a soft closure operator if it satisfies the following conditions;

(SC1) $cl(0_X,A) = (0_X,A)$,

(SC2) $cl(F,A) \geq (F,A)$,

(SC3) If $(F,A) \leq (G,A)$, then $cl(F,A) \leq cl(G,A)$,

(SC4) $cl(cl(F,A)) = (F,A)$,

(SC5) $cl((F,A)\odot(G,A)) \leq cl(F,A)\odot cl(G,A)$.

The pair (X,A,cl) is called a soft closure space. Let (X,A,cl^1) and (X,A,cl^2) be soft closure spaces. Then cl^1 is finer than cl_2 if $cl^1 \leq cl^2$.

Let (X,A,cl_X) and (Y,B,cl_Y) be soft closure spaces and $f_\phi : (X,A) \to (Y,B)$ be a map. Then f_ϕ is called a soft closure map if, for each $(F,A) \in S(X,A)$,

$$cl_Y(f_\phi(F,A)) \geq f_\phi(cl_X(F,A)).$$

Remark 3.2. [21] If $(L,\odot)$ is a continuous t-norm, then $a\odot\bigwedge_{i\in I} b_i = \bigwedge_{i\in I}(a\odot b_i)$.

Theorem 3.3. *Let $a \odot \bigwedge_{i\in I} b_i = \bigwedge_{i\in I}(a \odot b_i)$ for $a, b_i \in L$ and cl^1 and cl^2 be soft closure operators on $S(X,A)$. Then we have the following properties.*

(1) Define a map $cl^1 \oplus cl^2 : S(X,A) \to S(X,A)$ by

$$(cl^1 \oplus cl^2)((G,A)) = \bigwedge\{cl^1((G_1,A)) \odot cl^2((G_2,A)) \mid (G,A) \leq (G_1,A) \odot (G_2,A)\}.$$

Then $cl^1 \oplus cl^2$ is the coarsest soft closure operator on $S(X,A)$ which is finer than cl^1 and cl^2.

(2) Define $\tau_{cl^i} = \{(F,A) \in S(X,A) \mid (F,A) = cl^i(F,A)\}$. Then τ_{cl^i} is a soft cotopology on (X,A),

(3) $\tau_{cl^1} \oplus \tau_{cl^2} = \{(F,A) \in S(X,A) \mid (F,A) = (F_1,A) \odot (F_2,A), (F_i,A) \in \tau_{cl^i}\}$ is a soft cotopology on (X,A).

(4) $\tau_{cl^1 \oplus cl^2} = \tau_{cl^1} \oplus \tau_{cl^2}$.

Proof. (1) (SC1)

$$(cl^1 \oplus cl^2)((0_X,A)) \leq \{cl^1((0_X,A)) \odot cl^2((0_X,A)) \mid (0_X,A) \leq (0_X,A) \odot (0_X,A)\}$$
$$= (0_X,A).$$

(SC2) and (SC3) are clearly true.

(SC4)

$$(cl^1 \oplus cl^2)((F,A)) = \bigwedge\{cl^1((F_1,A)) \odot cl^2((F_2,A)) \mid (F,A) \leq (F_1,A) \odot (F_2,A)\}$$
$$\geq \bigwedge\{cl^1(cl^1((F_1,A))) \odot cl^2(cl^2((F_2,A))) \mid (cl^1 \oplus cl^2)((F,A)) \leq$$
$$(cl^1 \oplus cl^2)((F_1,A) \odot (F_2,A))\}$$
$$\geq \bigwedge\{cl^1(cl^1((F_1,A))) \odot cl^2(cl^2((F_2,A))) \mid (cl^1 \oplus cl^2)((F,A)) \leq$$
$$cl^1((F_1,A)) \odot cl^2((F_2,A))\}$$
$$\geq (cl^1 \oplus cl^2)((cl^1 \oplus cl^2)((F,A))).$$

(SC5)

$$\begin{aligned}
&(cl^1 \oplus cl^2)((F,A)) \odot (cl^1 \oplus cl^2)((G,A))\\
&= \bigwedge\{cl^1((F_1,A)) \odot cl^2((F_2,A)) \mid (F,A) \le (F_1,A) \odot (F_2,A)\}\\
&\odot \bigwedge\{cl^1((G_1,A)) \odot cl^2((G_2,A)) \mid (G,A) \le (G_1,A) \odot (G_2,A)\}\\
&\ge \bigwedge\{cl^1((F_1,A)) \odot cl^1((G_1,A)) \odot cl^2((F_2,A)) \odot cl^2((G_2,A))\\
&\mid (F,A) \odot (G,A) \le ((F_1,A) \odot (F_2,A)) \odot ((G_1,A) \odot (G_2,A))\}\\
&\ge \bigwedge\{cl^1((F_1,A) \odot (G_1,A)) \odot cl^2((F_2,A) \odot (G_2,A))\\
&\mid (F,A) \odot (G,A) \le ((F_1,A) \odot (G_1,A)) \odot ((F_2,A) \odot (G_2,A))\}\\
&\ge \bigwedge\{cl^1((H,A)) \odot cl^2((K,A)) \mid (F,A) \odot (G,A) \le (H,A) \odot (K,A)\}\\
&= (cl^1 \oplus cl^2)((F,A) \odot (G,A)).
\end{aligned}$$

Hence $cl^1 \oplus cl^2$ is a soft closure operator on $S(X,A)$.

For $(G,A) = (G,A) \odot (1_X,A)$, $(cl^1 \oplus cl^2)((G,A)) \le cl^1((G,A)) \odot cl^2((1_X,A)) = cl^1((G,A))$ and $(cl^1 \oplus cl^2)((G,A)) \le cl^2((G,A))$. If $cl \le cl^i$ for $i = 1,2$,

$$\begin{aligned}
&(cl^1 \oplus cl^2)((F,A)) = \bigwedge\{cl^1((F_1,A)) \odot cl^2((F_2,A)) \mid (F,A) \le (F_1,A) \odot (F_2,A)\}\\
&\ge \bigwedge\{cl((F_1,A)) \odot cl((F_2,A)) \mid (F,A) \le (F_1,A) \odot (F_2,A)\}\\
&\ge \bigwedge\{cl((F_1,A) \odot (F_2,A)) \mid (F,A) \le (F_1,A) \odot (F_2,A)\}\\
&\ge cl((F,A)).
\end{aligned}$$

So, $cl^1 \oplus cl^2$ is the coarsest soft closure operator on $S(X,A)$ which is finer than cl^1 and cl^2.

(2) (ST1) is easily proved from $cl^i((0_X,A)) = (0_X,A)$ and $cl^i((1_X,A)) = (1_X,A)$.

(ST2) If $(F,A),(G,A) \in \tau_{cl^i}$, i.e. $cl^i((F,A)) = (F,A)$ and $cl^i((G,A)) = (G,A)$, $cl^i((F,A) \odot (G,A)) \le cl^i((F,A)) \odot cl^i((G,A)) \le (F,A) \odot (G,A)$. $(F,A) \odot (G,A) \in \tau_{cl^i}$

(CT) If $(F_j,A) \in \tau_{cl^i}$ for $j \in J$, i.e. $cl^i((F_j,A)) = (F_j,A)$, $cl^i(\bigwedge_{j\in J}(F_j,A)) \le \bigwedge_{j\in J} cl^i((F_j,A)) = \bigwedge_{j\in J}(F_j,A)$. Hence $\bigwedge_{j\in J}(F_j,A) \in \tau_{cl^i}$.

(3) (ST1) Since $(0_X,A) = (0_X,A) \odot (0_X,A)$ and $(1_X,A) = (1_X,A) \odot (1_X,A)$, $(0_X,A)$, $(1_X,A) \in \tau_{cl^1} \oplus \tau_{cl^2}$.

(ST2) is easily proved.

(CT) If $(F_j,A) \in \tau_{cl^1} \oplus \tau_{cl^2}$ for $j \in J$, i.e. $(F_{ji},A) \in \tau_{cl^i}$, then $\bigwedge_{j\in J}(F_j,A)) = \bigwedge_{j\in J}((F_{j1},A) \odot (F_{j2},A)) = (\bigwedge_{j\in J}(F_{j1},A)) \odot (\bigwedge_{j\in J}(F_{j2},A)) \in \tau_{cl^1} \oplus \tau_{cl^2}$.

Hence $\tau_{cl^1} \oplus \tau_{cl^2}$ is a soft cotopology on (X,A).

(4) Let $(F,A) \in \tau_{cl^1} \oplus \tau_{cl^2}$. Then $(F,A) = (F_1,A) \odot (F_2,A)$ for $(F_i,A) \in \tau_{cl^i}, i = 1,2$, that is, $(F_i,A) = cl^i((F_i,A))$. Thus

$$(F_1,A) \odot (F_2,A) \leq (cl^1 \oplus cl^2)((F,A)) \leq (F_1,A) \odot (F_2,A).$$

So, $(F,A) \in \tau_{cl^1 \oplus cl^2}$. Hence $\tau_{cl^1} \oplus \tau_{cl^2} \subset \tau_{cl^1 \oplus cl^2}$.

Let $(G,A) \in \tau_{cl^1 \oplus cl^2}$. Then $(G,A) = \bigwedge\{cl^1((G_1,A)) \odot cl^2((G_2,A)) \mid (G,A) \leq (G_1,A) \odot (G_2,A)\}$. Since $cl^i((G_i,A)) = cl^i(cl^i((G_i,A)))$ for $i = 1,2$, $cl^i((G_i,A)) \in \tau_{cl^i}$. So, $(G,A) \in \tau_{cl^1} \oplus \tau_{cl^2}$. $\tau_{cl^1 \oplus cl^2} \subset \tau_{cl^1} \oplus \tau_{cl^2}$.

Theorem 3.4. *Let $a \odot \bigwedge_{i\in I} b_i = \bigwedge_{i\in I}(a \odot b_i)$ for $a, b_i \in L$ and τ_1 and τ_2 be soft cotopological spaces on $S(X,A)$. Then we have the following properties.*

(1) Define $\tau_1 \oplus \tau_2 \subset S(X,A)$ by

$$\tau_1 \oplus \tau_2 = \{(F,A) \in S(X,A) \mid (F,A) = (F_1,A) \odot (F_2,A), (F_i,A) \in \tau_i\}.$$

Then $\tau_1 \oplus \tau_2$ is the coarsest soft cotopological spaces on $S(X,A)$ which is finer than τ_1 and τ_2.

(2) Define $cl_{\tau_i}(F,A) = \bigwedge\{(G,A) \in \tau_i \mid (F,A) \leq (G,A)\}$. Then cl_{τ_i} is a soft closure operator on (X,A),

(3) $cl_{\tau_1 \oplus \tau_2} = cl_{\tau_1} \oplus cl_{\tau_2}$.

(4) $\tau_i = \tau_{cl_{\tau_i}}$.

(5) $cl = cl_{\tau_{cl}}$.

Proof. (1) From Theorem 3.3(2), $\tau_1 \oplus \tau_2$ is a soft topological spaces on $S(X,A)$. For $(F,A) \in \tau_1$, $(F,A) = (F,A) \odot (1_X,A) \in \tau_1 \oplus \tau_2$. So, $\tau_1 \subset \tau_1 \oplus \tau_2$. Similarly, $\tau_2 \subset \tau_1 \oplus \tau_2$. If $\tau_i \subset \tau$ for $i = 1,2$, $(F,A) = (F_1,A) \odot (F_2,A) \in \tau_1 \oplus \tau_2$ implies $(F,A) = (F_1,A) \odot (F_2,A) \in \tau$. Hence $\tau_1 \oplus \tau_2 \subset \tau$.

(2) (SC1)

$$cl_{\tau_i}((0_X,A)) = \bigwedge\{(G,A) \in \tau_i \mid (0_X,A) \leq (G,A)\} = (0_X,A).$$

(SC2) and (SC3) are clearly true.

(SC4) Since $cl_{\tau_i}(F,A) \in \tau_i$, we have

$$\begin{aligned} & cl_{\tau_i}(cl_{\tau_i}(F,A)) = \bigwedge\{(G,A) \in \tau_i \mid cl_{\tau_i}(F,A) \leq (G,A)\} \\ & \leq \{cl_{\tau_i}(F,A) \in \tau_i \mid cl_{\tau_i}(F,A) \leq cl_{\tau_i}(F,A)\} \\ & = cl_{\tau_i}(F,A). \end{aligned}$$

(SC5)

$$\begin{aligned} & cl_{\tau_i}((F,A)) \odot cl_{\tau_i}((G,A)) \\ & = \bigwedge\{(F_1,A) \in \tau_i \mid (F,A) \leq (F_1,A)\} \\ & \odot \bigwedge\{(G_1,A)) \in \tau_i \mid (G,A) \leq (G_1,A)\} \\ & \geq \bigwedge\{(F_1,A) \odot (G_1,A) \in \tau_i \mid (F,A) \odot (G,A) \leq (F_1,A) \odot (G_1,A)\} \\ & \geq \bigwedge\{(H,A) \in \tau_i \mid (F,A) \odot (G,A) \leq (H,A)\} \\ & = cl_{\tau_i}((F,A) \odot (G,A)). \end{aligned}$$

(3) Suppose there exists $(F,A) \in S(X,A)$ such that

$$cl_{\tau_1 \oplus \tau_2}((F,A)) \not\leq (cl_{\tau_1} \oplus cl_{\tau_2})((F,A)).$$

There exists $(F_i,A) \in S(X,A)$ with $(F,A) \leq (F_1,A) \odot (F_2,A)$ such that

$$cl_{\tau_1 \oplus \tau_2}((F,A)) \not\leq cl_{\tau_1}((F_1,A)) \odot cl_{\tau_2}((F_2,A)).$$

On the other hand, for $(F,A) \leq (F_1,A) \odot (F_2,A) \leq cl_{\tau_1}((F_1,A)) \odot cl_{\tau_2}((F_2,A))$, since $cl_{\tau_i}(F_i,A) \in \tau_i$, we have

$$cl_{\tau_1 \oplus \tau_2}((F,A)) \leq cl_{\tau_1}((F_1,A)) \odot cl_{\tau_2}((F_2,A)).$$

It is a contradiction. Hence $cl_{\tau_1 \oplus \tau_2} \leq cl_{\tau_1} \oplus cl_{\tau_2}$.

Suppose there exists $(G,A) \in S(X,A)$ such that

$$cl_{\tau_1 \oplus \tau_2}((G,A)) \not\geq (cl_{\tau_1} \oplus cl_{\tau_2})((G,A)).$$

By the definition of $cl_{\tau_1 \oplus \tau_2}$, there exists $(G_i,A) \in \tau_i$ with $(G,A) \leq (G_1,A) \odot (G_2,A)$ such that

$$(G_1,A) \odot (G_2,A) \not\geq (cl_{\tau_1} \oplus cl_{\tau_2})((G,A)).$$

On the other hand, for $(G,A) \leq (G_1,A) \odot (G_2,A)$, since $cl_{\tau_i}((G_i,A)) = (G_i,A)$, we have

$$(cl_{\tau_1} \oplus cl_{\tau_2})((G,A)) \leq cl_{\tau_1}((G_1,A)) \odot cl_{\tau_2}((G_2,A)) = (G_1,A) \odot (G_2,A).$$

It is a contradiction. Hence $cl_{\tau_1\oplus\tau_2} \geq cl_{\tau_1} \oplus cl_{\tau_2}$.

(4) It follows from:

$$\begin{aligned}(F,A) \in \tau_i \quad &\Leftrightarrow cl_{\tau_i}(F,A) = (F,A)\\ &\Leftrightarrow (F,A) \in \tau_{cl_{\tau_i}}.\end{aligned}$$

(5) Since $cl(cl((F,A))) = cl((F,A)) \in \tau_{cl}$, we have $cl_{\tau_{cl}} \leq cl$.

Since $cl_{\tau_{cl}}((F,A)) = \bigwedge\{(G_i,A) \mid (F,A) \leq (G_i,A) \in \tau_{cl}\}$, we have $cl(cl_{\tau_{cl}}((F,A))) = cl_{\tau_{cl}}((F,A))$. Hence $(F,A) \leq cl_{\tau_{cl}}((F,A))$ implies $cl((F,A)) \leq cl(cl_{\tau_{cl}}((F,A))) = cl_{\tau_{cl}}((F,A))$. Thus, $cl_{\tau_{cl}} \leq cl$.

Theorem 3.5. *A map $f_\phi : (X,A,\tau_X) \to (Y,B,\tau_Y)$ be a continuous soft map iff $f_\phi : (X,A,cl_{\tau_X}) \to (Y,B,cl_{\tau_Y})$ is a soft closed map.*

Proof. Since $(F,A) \leq f_\phi^{-1}(f_\phi(F,A))$ and $f_\phi(f_\phi^{-1}(G,B)) \leq (G,B)$ from Lemma 2.7, we have

$$\begin{aligned}&cl_{\tau_Y}(f_\phi(F,A)) = \bigwedge\{(G,B) \in S(Y,B) \mid f_\phi(F,A) \leq (G,B), (G,B) \in \tau_Y\}\\ &\geq \bigwedge\{f_\phi(f_\phi^{-1}(G,B)) \mid (F,A) \leq f_\phi^{-1}(f_\phi(F,A)) \leq f_\phi^{-1}((G,B)), f_\phi^{-1}((G,B)) \in \tau_X\}\\ &\geq f_\phi(\bigwedge\{(f_\phi^{-1}(G,B)) \mid (F,A) \leq f_\phi^{-1}((G,B)), f_\phi^{-1}((G,B)) \in \tau_X\})\\ &\geq f_\phi(cl_{\tau_X}((F,A))).\end{aligned}$$

Conversely, let $(G,B) \in \tau_Y$. Since

$$f_\phi(cl_{\tau_X}((f_\phi^{-1}(G,B)))) \leq cl_{\tau_Y}(f_\phi((f_\phi^{-1}(G,B)))) \leq cl_{\tau_Y}((G,B)) = (G,B),$$

we have $cl_{\tau_X}((f_\phi^{-1}(G,B))) \leq (f_\phi^{-1}(G,B))$. So, $f_\phi^{-1}(G,B) \in \tau_{cl_{\tau_X}} = \tau_X$.

Example 3.6. Let $X = \{h_i \mid i = \{1,...,4\}\}$ with h_i=house and $E_Y = \{e,b,w,c,i\}$ with e=expensive,b= beautiful, w=wooden, c= creative, i=in the green surrounding.

Let $(L = [0,1], \odot, \to)$ be a continuous t-norm defined by

$$x \odot y = (x+y-1) \vee 0, \ \ x \to y = (1-x+y) \wedge 1.$$

Let $A = \{e,i\}$ and $(F,A), (F \odot F,A), (G,A) \in S(X,A)$ such that

(F,A)	h^1	h^2	h^3	h^4
e	0.5	0.6	0.2	0.6
i	0.1	0.5	0.5	0.6

(G,A)	h^1	h^2	h^3	h^4
e	0.4	0.5	0.3	0.5
i	0.4	0.5	0.5	0.5

$(F,A)\odot(F,A)$	h^1	h^2	h^3	h^4
e	0.0	0.2	0.0	0.2
i	0.0	0.0	0.0	0.2

$(F,A)\odot(G,A)$	h^1	h^2	h^3	h^4
e	0.0	0.1	0.0	0.1
i	0.0	0.0	0.0	0.1

Define $\tau_1=\{(0_X,A),(1_X,A),(F,A),(F,A)\odot(F,A)\}$ and $\tau_2=\{(0_X,A),(1_X,A),(G,A)\}$. Then τ_i is a soft topology and soft cotoplogy on $S(X,A)$, for $i=1,2$. From Theorem 3.4, we obtain a soft cotopology as

$$\tau_1\oplus\tau_2=\{(0_X,A),(1_X,A),(F,A),(G,A),(F,A)\odot(F,A),(F,A)\odot(G,A)\}.$$

From Theorem 3.4(2), we obtain soft fuzzy closure operators $cl_{\tau_i}:S(X,A)\to S(X,A)$, $i=1,2$ as follows:

$$cl_{\tau_1}((H,A))=\begin{cases}(0_X,A), & \text{if } (H,A)=(0_X,A),\\ (F,A)\odot(F,A), & \text{if } (F,A)\odot(F,A)\le(H,A)\not\ge(F,A)\\ (F,A), & \text{if } (F,A)\le(H,A)\\ (1_X,A), & \text{otherwise,}\end{cases}$$

$$cl_{\tau_2}((H,A))=\begin{cases}(0_X,A), & \text{if } (H,A)=(0_X,A),\\ (G,A), & \text{if } (G,A)\le(H,A)\\ (1_X,A), & \text{otherwise.}\end{cases}$$

Moreover, we have $cl_{\tau_1\oplus\tau_2}=cl_{\tau_1}\oplus cl_{\tau_2}$ from

$$cl_{\tau_1\oplus\tau_2}((H,A))=\begin{cases}(0_X,A), & \text{if } (H,A)=(0_X,A),\\ (F,A)\odot(G,A), & \text{if } (F,A)\odot(G,A)\le(H,A)\not\ge(F,A)\odot(F,A)\\ (F,A)\odot(F,A), & \text{if } (F,A)\odot(F,A)\le(H,A)\not\ge(F,A)\\ (F,A), & \text{if } (F,A)\le(H,A)\not\ge(G,A)\\ (G,A), & \text{if } (G,A)\le(H,A)\not\ge(F,A)\\ (1_X,A), & \text{otherwise.}\end{cases}$$

Conflict of Interests

The author declares that there is no conflict of interests.

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