Available online at http://scik.org
Eng. Math. Lett. 2016, 2016:5
ISSN: 2049-9337

A NOTE ON THE PAPER "COMMON FIXED POINT RESULTS FOR MAPPINGS UNDER NONLINEAR CONTRACTION OF CYCLIC FORM IN ORDERED METRIC SPACES"

JING LIU*, MEIMEI SONG

College of Science, Tianjin University of Technology, Tianjin 300384, P.R.China

Correction: In this note, we modify the gaps in [W. Shatanawi, M. Postolache, Common fixed point results for mappings under nonlinear contraction of cyclic form in ordered metric spaces, Fixed Point Theory Appl. 2013 (2013), Article ID 60].

Keywords: metric space; contraction; fixed point.

2010 AMS Subject Classification: 47H10.

1. Main results

In [1], subcase 1 and subcase 2 in Step 1 turned out to be not comprehensive, which led mistakes to the procedure $\psi(d(x_{2t+1},x_{2t+2})) \leq \delta\psi(d(x_{2t+1},x_{2t+2})) < \psi(d(x_{2t+1},x_{2t+2}))$. Next, we give the modification.

Theorem 1.1. Let $(X,d,\preceq)$ be an ordered complete metric space and A, B be nonempty closed subsets of X. Let $f,T : X \to X$ be two mappings such that the pair (f,T) is (A,B)-weakly increasing. Assume the following:

(1) The pair (f,T) is a cyclic (ψ,A,B)-contraction;

*Corresponding author

Received September 18, 2014

(2) f or T is continuous.

Then f and T have a common fixed point.

Proof. Choose $x_0 \in A$. Let $x_1 = f(x_0)$. Since $fA \subseteq B$,we have $x_1 \in B$.Also,let $x_2 = Tx_1$. Since $TB \subseteq A$, we have $x_2 \in A$. Continuing this process, we can construct a sequence $\{x_n\}$ in X such $x_{2n+1} = fx_{2n}, x_{2n+2} = Tx_{2n+1}$, $x_{2n} \in A$ and $x_{2n+1} \in B$.
Since f and T are $(A,B)-$weakly increasing, we have

$$x_1 = fx_0 \preceq Tfx_0 = Tx_1 = x_2 \preceq fTx_1 = fx_2 = x_3 \preceq \ldots.$$

We divide our proof into the following steps.
Step 1: We will show that $\{x_n\}$ is a Cauchy sequence in (X,d).
Subcase 1: Suppose that $x_{2n} = x_{2n+1}$ for some $n \in N$. Since x_{2n} and x_{2n+1} are comparable elements in X with $x_{2n} \in A$ and $x_{2n+1} \in B$, we have

$$\begin{aligned}
\psi(d(x_{2n+1},x_{2n+2})) &= \psi(d(fx_{2n},Tx_{2n+1})) \\
&\leq \delta\psi(max\{d(x_{2n},x_{2n+1}),d(x_{2n},fx_{2n}),d(x_{2n+1},Tx_{2n+1}), \\
&\quad \frac{1}{2}(d(x_{2n},Tx_{2n+1})+d(fx_{2n},x_{2n+1})))\}) \\
&= \delta\psi(max\{d(x_{2n},x_{2n+1}),d(x_{2n},x_{2n+1}),d(x_{2n+1},x_{2n+2}), \\
&\quad \frac{1}{2}(d(x_{2n},x_{2n+2})+d(x_{2n+1},x_{2n+1}))\}) \\
&= \delta\psi(d(x_{2n+1},x_{2n+2})).
\end{aligned}$$

Since $0 < \delta < 1$, we have $\psi(d(x_{2n+1},x_{2n+2})) = 0$ and hence $x_{2n+2} = x_{2n+1}$. Similarly, we may show that $x_{2n+3} = x_{2n+2}$. Hence x_n is a constant sequence in X, so it is a Cauchy sequence in (X,d).

Subcase 2: $x_{2n-1} = x_{2n}$ for some $n \in N-\{0\}$. Since x_{2n-1} and x_{2n} are comparable elements in

X with $x_{2n} \in A$ and $x_{2n-1} \in B$, we have

$$\begin{aligned}\psi(d(x_{2n+1},x_{2n})) &= \psi(d(fx_{2n},Tx_{2n-1}))\\ &\leq \delta\psi(max\{d(x_{2n},x_{2n-1}),d(x_{2n},fx_{2n}),d(x_{2n-1},Tx_{2n-1}),\\ &\frac{1}{2}(d(x_{2n},Tx_{2n-1}+d(fx_{2n},x_{2n-1}))))\})\\ &= \delta\psi(max\{d(x_{2n},x_{2n-1}),d(x_{2n},x_{2n+1}),d(x_{2n-1},x_{2n}),\\ &\frac{1}{2}(d(x_{2n},x_{2n})+d(x_{2n+1},x_{2n-1}))\})\\ &= \delta\psi(d(x_{2n+1},x_{2n})).\end{aligned}$$

Since $0 < \delta < 1$, we have $\psi(d(x_{2n+1},x_{2n})) = 0$ and hence $x_{2n+1} = x_{2n}$.Similarly, we may show that $x_{2n+1} = x_{2n+2}$. Hence x_n is a constant sequence in X, so it is a Cauchy sequence in (X,d).

Subcase 3: $x_n \neq x_{n+1}$ for all $n \in N$. Given $n \in N$. If n is even, then $n = 2t$ for some $t \in N$. Since $x_{2t} \in A$, $x_{2t+1} \in B$ and x_{2t}, x_{2t+1} are comparable, we have

$$\begin{aligned}&\psi(d(x_{n+1},x_{n+2}))\\ =\ &\psi(d(x_{2t+1},x_{2t+2}))\\ =\ &\psi(d(fx_{2t},Tx_{2t+1}))\\ \leq\ &\delta\psi(max\{d(x_{2t},x_{2t+1}),d(x_{2t},fx_{2t}),d(x_{2t+1},Tx_{2t+1}),\\ &\frac{1}{2}(d(x_{2t},Tx_{2t+1}))+d(fx_{2t},x_{2t+1})\})\\ =\ &\delta\psi(max\{d(x_{2t},x_{2t+1}),d(x_{2t+1},x_{2t+2}),\frac{1}{2}(d(x_{2t},x_{2t+2})+d(x_{2t+1},x_{2t+1}))\})\\ =\ &\delta\psi(max\{d(x_{2t},x_{2t+1}),d(x_{2t+1},x_{2t+2})\}).\end{aligned}$$

If

$$max\{d(x_{2t},x_{2t+1}),d(x_{2t+1},x_{2t+2})\} = d(x_{2t+1},x_{2t+2}),$$

then

$$\psi d(x_{2t+1},x_{2t+2})) \leq \delta\psi(d(x_{2t+1},x_{2t+2})) < \psi(d(x_{2t+1},x_{2t+2})) \tag{1}$$

which is a contradiction. Thus

$$max\{d(x_{2t},x_{2t+1}),d(x_{2t+1},x_{2t+2})\} = d(x_{2t},x_{2t+1}),$$

therefore

$$\psi(d(x_{2t+1},x_{2t+2})) \leq \delta\psi(d(x_{2t},x_{2t+1}))$$

The rest of proof process is the same with which was given in [1]. Therefore, we omit the proof.

Conflict of Interests

The authors declare that there is no conflict of interests.

References

[1] W.Shatanawi and M.Postolache, Common fixed point results for mappings under nonlinear contraction of cyclic form in ordered metric spaces, Fixed Point Theory Appl. 2013 (2013), Article ID 60.