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A NOTE ON ADAPTIVE FILTER CONSTRUCTION USING BAYESIAN METHOD

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Abstract. This paper is concerned with constructing the adaptive filters using Bayesian posterior probabilities. First, our results are proposed in normal observations with known variance and then unknown variance case is considered. Finally a conclusion section is also given.

Keywords: Adaptive filter; Bayesian method; Recursive relation; Step size factor

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1. Introduction.

Filtering problems arises in many fields such as engineering, finance and time series. This is the problem of prediction of Y_t based on observations proceed t. Moving average, exponential smoothing and kernel regression estimators are some examples. J. P. Morgan company (1996) proposed an adaptive filter as volatility estimate.

The adaptive filter plays a major role in parameter estimation due to its inherent simplicity. Indeed, it is the procedure of on-line estimation of unknown parameters of a dynamic system and then updating the filter. It has been applied in many fields such as change detection, pattern recognition, prediction of time series and system identification, see Haykin (1986) and Csorgo and Horvath (1997).

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For the available data $(\mathbf{x}_n, y_n)$ at n-th time, we have

$$y_n = \mathbf{x}_n^T \beta + \varepsilon_n,$$

where $\mathbf{x}_n$ and β are $p \times 1$ vectors and ε_n is a sequence of independent error variables with $N(0, \sigma_n^2)$ distribution. The notation T stands for the transpose of a vector or matrix. Following Deng (2006), the adaptive filter is defined as recursive relation

$$\beta_n = \beta_{n-1} + \lambda \mathbf{x}_n \widehat{e},$$

where $\widehat{e} = y_n - \mathbf{x}_n^T \beta_{n-1}$ and λ is a step size factor.

There are other types of adaptive filters such as lease mean square (LMS), normalized and penalized LMS, recursive least square and Kalman filters, see Deng (2006). In this paper, we also derive a version of adaptive filter using Bayesian probability (see Kyriazis (2011)). This paper is organized as follows. In the next section, we propose our method for normal errors with known variance σ_n^2. The unknown σ_n^2 is considered in section 3. The results are also generalized to errors with student-t distribution. Finally a conclusion section is also given.

2. Baysian method

It is easy to see that the likelihood function is

$$\frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\{\frac{-1}{2\sigma_n^2}(y_n - \mathbf{x}_n^T \beta)^2\}.$$

Following Kyriazis (2011), we suppose that β has prior normal distribution with mean vector β_{n-1} and variance matrix Σ_n. Using the Bayes theorem, we conclude that the posterior density is proportional to

$$\exp\{\frac{-1}{2}(\beta^T \Sigma_n^{*-1} \beta - 2\mu_n^{*T} \Sigma_n^{*-1} \beta)\},$$

where

$$\begin{aligned} \Sigma_n^* &= \{\Sigma_n^{-1} + \frac{\mathbf{x}_n \mathbf{x}_n^T}{\sigma_n^2}\}^{-1}, \\ \mu_n^* &= \Sigma_n^* \{\Sigma_n^{-1} \beta_{n-1} + \frac{\mathbf{x}_n y_n}{\sigma_n^2}\}. \end{aligned}$$

Therefore, the posterior distribution is normal with mean μ_n^* and covariance matrix Σ_n^*.

It is seen that the maximum a posteriori (MAP) estimate of β is $\beta_n = \mu_n^*$. One can check that

$$\beta_n = \beta_{n-1} + \Lambda_n \mathbf{x}_n \widehat{e},$$

where the decay factor matrix $\Lambda_n = \sigma_n^{-2}\Sigma_n^*$. That is, we have also find an adaptive filter based on Bayesian theorem. Next, consider the scalar case, when $p = 1$. Therefore,

$$\Lambda_n = \lambda_n = \frac{1}{\gamma_n^2 + x_n^2},$$

where $\gamma_n = \sigma_n / \upsilon_n$.

3. Unknown variance case. In this section, we suppose that σ_n^2 is unknown. There are many selected priors for σ_n^2. For example, we can suppose that σ_n^{-2} has the gamma distribution with parameters ς_n and τ_n. Or, we can assume that it follows a GARCH process. For simplicity reasons, we consider the univariate case when $p = 1$. In these cases, note that the conditional on σ_n^2, we have

$$\widetilde{\beta}_n = \widetilde{\beta}_{n-1} + \lambda_n x_n \widehat{e}.$$

This equation stands for a conditional adaptive filter. For finding a marginal type, it is enough to replace λ_n with $E_{\sigma_n^2}(\lambda_n)$. However, this expectation doesn't have a closed form. Therefore, we can approximate it by a Monte Carlo or bootstrap simulation methods.

Remark 1. Following step by step of Deng's (2006) method, we can replace the normal distribution of ε_n by a student-t distribution.

Remark 2. This adaptive filter may be applied for change point detection in β. For example, we reject the null hypothesis of no change point in β if

$$\beta_n = \beta_{n-1} + \lambda_n x_n \widehat{e} > h,$$

for some suitable thresholds $h > 0$, see Csorgo and Horvath (1997).

4. Conclusions. We propose an adaptive filter for a regression model using Bayesian probabilities. Known and unknown variance cases are studied.

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