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OSCILLATION AND NONOSCILLATION PROPERTIES FOR A KIND OF NONLINEAR NEUTRAL IMPULSIVE DELAY DIFFERENTIAL SYSTEMS

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Abstract. This paper is concerned with oscillation and nonoscillation of a kind of nonlinear neutral impulsive differential systems with constant coefficients and constant delays by using the pulsatile constant. The sufficient and necessary conditions for oscillation in the case $\delta \in \mathbb{R}\backslash\{0\}$ are obtained. Two examples are included using the main results.

Keywords: oscillation; nonoscillation; neutral delay differential equations; impulse.

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1. Introduction

Consider a class of first-order nonlinear neutral delay differential equations of the form

$$\big(y(t) - \delta y(t-\tau)\big)' + \beta\left(e^{\gamma y(t-\sigma)} - 1\right) = 0, \tag{1}$$

where $\gamma, \tau > 0$, $\sigma \geq 0$ and β are real constants. Let τ_k, $k \in \mathbb{N}$ with $\tau_1 < \tau_2 < ... < \tau_k < ...$ and $\lim_{k\to\infty} \tau_k = +\infty$ are fixed moments of impulsive effect with the property $\max\{\tau_{k+1} - \tau_k\} < +\infty$,

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and satisfying

$$\Delta\big(y(\tau_k)-\delta y(\tau_k-\tau)\big)+\alpha\big(e^{\gamma y(\tau_k-\sigma)}-1\big)=0,\quad t\neq\tau_k, k\in\mathbb{N} \tag{2}$$

where $\delta\in\mathbb{R}\backslash\{0\}$, $\alpha\in\mathbb{R}$ are constants. For (2), Δ is the difference operator defined by

$$\Delta\big(y(\tau_k)-\delta y(\tau_k-\tau)\big)=y(\tau_k+0)-\delta y(\tau_k-\tau+0)-y(\tau_k-0)+\delta y(\tau_k-\tau-0);$$

$$y(\tau_k-0)=y(\tau_k)\quad \textit{and}\quad y(\tau_k-\tau-0)=y(\tau_k-\tau),\quad k\in\mathbb{N}.$$

The main aim of this work is to study oscillation and nonoscillation properties governing the impulse operators acting on (1) which we denote as the impulsive systems

$$\text{(E)}\qquad \begin{cases}\big(y(t)-\delta y(t-\tau)\big)'+\beta\big(e^{\gamma y(t-\sigma)}-1\big)=0, & t\neq\tau_k, k\in\mathbb{N}\\ \Delta\big(y(\tau_k)-\delta y(\tau_k-\tau)\big)+\alpha\big(e^{\gamma y(\tau_k-\sigma)}-1\big)=0, & k\in\mathbb{N}.\end{cases}$$

Impulsive differential equations are now recognized as an excellent source of models to simulate processes and phenomena observed in theoritical physics, chemical technology, population dynamics, industrial robotic, economics, rhythmical beating, merging of solutions and non-continuity of solutions. Moreover, the theory of impulsive differential equations is emerging as an important area of investigation, since it is much richer than the corresponding theory of differential equations without impulse effect. We mention the monographs ([1], [14], [18]), and ([2]-[4], [19]), where the various properties of their solutions are studied.

In the present years much effort has been devoted to study the oscillatory and asymptotic behaviour of solutions of various classes of functional differential equations of neutral type (see for e.g [16], [17]). However, the impulsive differential equations of neutral type is not well studied. Hence in this work, the author have made an attempt to study the oscillation and nonoscillation properties of solutions of a class of nonlinear neutral first order impulsive differential systems of the form (E).

The motivation of the present work come from the works [25]-[27]. In [25] and [27], Tripathy, Santra and Pinelas have studied first order neutral impulsive delay differential systems with variable coefficients of the form

$$(\mathrm{E}_1)\qquad \begin{cases} \big(y(t)+\delta(t)y(t-\tau)\big)'+\beta(t)G\big(y(t-\sigma)\big)=0, & t\neq\tau_k,\ k\in\mathbb{N}\\ \Delta\big(y(\tau_k)+\delta(\tau_k)y(\tau_k-\tau)\big)+\alpha(\tau_k)G\big(y(\tau_k-\sigma)\big)=0, & k\in\mathbb{N}\end{cases}$$

and established sufficient and necessary conditions for oscillation of all solutions of (E_1) for different ranges of $\delta(t)$. In this direction, we refer to the reader some of the related works [9]-[13]. The objective of this paper is to study (E) and establish conditions for oscillation and nonoscillation of solutions of (E) subject to its associated characteristic equation under

$$u=\gamma^{-1}\big(e^{\gamma u}-1\big),\quad \gamma>0. \tag{3}$$

We may expect the possible solutions of (E) as

$$y(t)=e^{-\mu t}A^{i(t_0,t)},\quad t_0\geq\rho=\max\{\tau,\sigma\}, \tag{4}$$

where $i(t_0,t)=k=$ number of impulses τ_k, $k\in\mathbb{N}$, and $A\neq 0$ is a real number which is called as the pulsatile constant. A close observation reveals that $y(t)=C_1e^{-\mu t}$ is a possible solution of (1) when (E) is without impulses and $y(n)=C_2A^n$ is a possible solution of (2) when $i(t_0,t)=n$ and the impulses are the discrete values only ($\because$ in case (2), $\mu=0$). Therefore, (4) seems to be the possible choice of solution of (E).

A function $y:[-\rho,+\infty)\to\mathbb{R}$ is said to be a solution of (E) with initial function $\phi\in C\big([-\rho,0],\mathbb{R}\big)$ if $y(t)=\phi(t)$ for $t\in[-\rho,0]$, $y\in PC\big(\mathbb{R}_+,\mathbb{R}\big)$, $z(t)=y(t)-\delta y(t-\tau)$ is continuously differentiable for $t\in\mathbb{R}_+$, and $y(t)$ satisfies (E) for all sufficiently large $t\geq 0$, where $\rho=\max\{\tau,\sigma\}$ and $PC\big(\mathbb{R}_+,\mathbb{R}\big)$ is the set of all functions $U:\mathbb{R}_+\to\mathbb{R}$ which are continuous for $t\in\mathbb{R}_+$, $t\neq\tau_k$, $k\in\mathbb{N}$, continuous from the left-side for $t\in\mathbb{R}_+$, and have discontinuity of the first kind at the points $\tau_k\in\mathbb{R}_+$, $k\in\mathbb{N}$.

A solution $y(t)$ of (E) is said to be regular, if it is defined on some interval $[T_y,+\infty)\subset[t_0,+\infty)$ and

$$\sup\{|y(t)|:t\geq T_y\}>0$$

for every $T_y\geq T$. A regular solution $y(t)$ of (E) is said to be eventually positive (eventually negative), if there exists $t_1>0$ such that $y(t)>0$ $(y(t)<0)$ for $t\geq t_1$.

A regular nontrivial solution $y(t)$ of (E) is said to be nonoscillatory, if there exists a point $t_0 \geq 0$ such that $y(t)$ has a constant sign for $t \geq t_0$; otherwise, the regular solution $y(t)$ is said to be oscillatory.

2. Oscillation and Nonoscillation Properties

In this section, we study the oscillatory and nonoscillatory behaviour of solutions of (E) through its associated characteristic equation provided (3) and (4) holds.

Throughout the discussion, we assume that $i(t-\sigma,t) = n_1 > 0$ and $i(t-\tau,t) = n_2 > 0$ are the number of impulses between $t-\sigma$ and t, and $t-\tau$ and t respectively.

Theorem 2.1. *Let $\tau > \sigma > 0$, $\delta \in \mathbb{R}\backslash\{0\}$ and $\alpha \neq 0 \neq \beta$. Then the system* (E) *admits an*

(i) *oscillatory solution in the exponential impulsive form* (4) *if and only if the algebraic equation*

$$-\mu\left(1-\frac{\alpha}{\beta}\mu\right)^{n_1} + \delta\mu e^{\mu\tau}\left(1-\frac{\alpha}{\beta}\mu\right)^{n_1-n_2} + \beta\gamma e^{\mu\sigma} = 0 \tag{5}$$

has at least one real root μ with

$$\mu > \frac{\beta}{\alpha} \quad for \quad \alpha\beta > 0 \quad or \quad \mu < \frac{\beta}{\alpha} \quad for \quad \alpha\beta < 0;$$

(ii) *eventually positive solution in the form of* (4) *if and only if the algebraic equation* (5) *has at least one real root μ with*

$$\mu < \frac{\beta}{\alpha} \quad for \quad \alpha\beta > 0 \quad or \quad \mu > \frac{\beta}{\alpha} \quad for \quad \alpha\beta < 0.$$

Proof. (i) Let $y(t)$ be a regular nontrivial solution of the system (E) such that $y(t) = e^{-\mu t}A^{i(t_0,t)}$, $t > t_0 > \rho$. Due to (3), (1) becomes

$$-\mu e^{-\mu t}A^{i(t_0,t)} + \delta\mu e^{-\mu t}e^{\mu\tau}A^{i(t_0,t-\tau)} + \gamma\beta e^{-\mu(t-\sigma)}A^{i(t_0,t-\sigma)} = 0,$$

that is,

$$\beta\gamma e^{\mu\sigma} - \mu A^{i(t_0,t)-i(t_0,t-\sigma)} + \delta\mu e^{\mu\tau}A^{i(t_0,t-\tau)-i(t_0,t-\sigma)} = 0. \tag{6}$$

Indeed,

$$i(t_0,t) - i(t_0,t-\sigma) = i(t-\sigma,t) = n_1$$

and

$$i(t_0,t-\tau)-i(t_0,t-\sigma)=-i(t-\tau,t-\sigma)=-\big[i(t-\tau,t)-i(t-\sigma,t)\big]=n_1-n_2$$

implies that

$$-\mu A^{n_1}+\delta\mu e^{\mu\tau}A^{n_1-n_2}+\beta\gamma e^{\mu\sigma}=0 \tag{7}$$

due to (6). Once again we use (4) in (2) to obtain a relation of the form

$$y(\tau_k+0)-\delta y(\tau_k-\tau+0)-y(\tau_k-0)+\delta y(\tau_k-\tau-0)+\alpha\gamma y(\tau_k-\sigma)=0,$$

due to (3), that is,

$$\begin{aligned}&e^{-\mu\tau_k}A^{i(t_0,\tau_k+0)}-\delta e^{-\mu(\tau_k-\tau)}A^{i(t_0,\tau_k-\tau+0)}-e^{-\mu\tau_k}A^{i(t_0,\tau_k-0)}\\&\quad+\delta e^{-\mu(\tau_k-\tau)}A^{i(t_0,\tau_k-\tau-0)}+\alpha\gamma e^{-\mu(\tau_k-\sigma)}A^{i(t_0,\tau_k-\sigma)}=0.\end{aligned}$$

We may note that $i(t_0,\tau_k+0)-i(t_0,\tau_k-0)=1$. Hence, the last inequality becomes

$$\begin{aligned}A^{1+i(t_0,\tau_k-0)}-\delta e^{\mu\tau}A^{1+i(t_0,\tau_k-\tau-0)}-A^{i(t_0,\tau_k-0)}+\delta e^{\mu\tau}A^{i(t_0,\tau_k-\tau-0)}&\\+\alpha\gamma e^{\mu\sigma}A^{i(t_0,\tau_k-\sigma)}&=0,\end{aligned}$$

that is,

$$(A-1)A^{i(t_0,\tau_k)}-\delta(A-1)e^{\mu\tau}A^{i(t_0,\tau_k-\tau)}+\alpha\gamma e^{\mu\sigma}A^{i(t_0,\tau_k-\sigma)}=0.$$

Therefore,

$$(A-1)A^{i(t_0,\tau_k)-i(t_0,\tau_k-\sigma)}-\delta(A-1)e^{\mu\tau}A^{i(t_0,\tau_k-\tau)-i(t_0,\tau_k-\sigma)}+\alpha\gamma e^{\mu\sigma}=0. \tag{8}$$

Using the fact

$$i(t_0,\tau_k)-i(t_0,\tau_k-\sigma)=i(\tau_k-\sigma,\tau_k)=n_1$$

and

$$i(t_0,\tau_k-\tau)-i(t_0,\tau_k-\sigma)=-i(\tau_k-\tau,\tau_k-\sigma)=-\big[i(\tau_k-\tau,t)-i(\tau_k-\sigma,t)\big]=n_1-n_2,$$

we obtain from (8) that

$$(A-1)A^{n_1}-\delta(A-1)e^{\mu\tau}A^{n_1-n_2}+\alpha\gamma e^{\mu\sigma}=0. \tag{9}$$

If we choose $A = 1 - \frac{\alpha}{\beta}\mu$, then it is easy to verify that (9) reduces to (7). Consequently, (7) is same as the algebraic equation (5). Moreover, (5) is the required characteristic equation for (E). Ultimately, if $y(t)$ is an oscillatory solution of (E) with the pulsatile constant $A = 1 - \frac{\alpha}{\beta}\mu < 0$, where $\mu > \frac{\beta}{\alpha}$ for $\alpha\beta > 0$ or $\mu < \frac{\beta}{\alpha}$ for $\alpha\beta < 0$, then μ satisfies the characteristic equation (5). Conversely, consider the characteristic equation (5) and assume that $\mu = \mu^*$ is the real root of (5) with $\mu^* > \frac{\beta}{\alpha}$ for $\alpha\beta > 0$ and $\mu^* < \frac{\beta}{\alpha}$ for $\alpha\beta < 0$. Then (E) admits an oscillatory solution $y(t) = e^{-\mu^* t}A^{i(t_0,t)}$ with the pulsatile constant $A = 1 - \frac{\alpha}{\beta}\mu^* < 0$.

(ii) The proof of (ii) follows from the proof of (i) and hence the details are omitted.

This completes the proof of the theorem. □

Corollary 2.1. *Let* $\alpha, \beta, \delta \in \mathbb{R}\backslash\{0\}$ *and* $\sigma, \tau \in \mathbb{R}_+$ *such that* $\sigma = \tau \neq 0$ *hold. Then the system* (E) *admits an*

(i) *oscillatory solution in the exponential impulsive form* (4) *if and only if the algebraic equation*

$$-\mu\left(1 - \frac{\alpha}{\beta}\mu\right)^{n_1} + (\beta\gamma + \delta\mu)e^{\mu\sigma} = 0 \tag{10}$$

has at least one real root μ *with*

$$\mu > \frac{\beta}{\alpha} \quad \text{for} \quad \alpha\beta > 0 \quad \text{or} \quad \mu < \frac{\beta}{\alpha} \quad \text{for} \quad \alpha\beta < 0;$$

(ii) *eventually positive solution in the form of* (4) *if and only if the algebraic equation* (10) *has at least one real root* μ *with*

$$\mu < \frac{\beta}{\alpha} \quad \text{for} \quad \alpha\beta > 0 \quad \text{or} \quad \mu > \frac{\beta}{\alpha} \quad \text{for} \quad \alpha\beta < 0.$$

Corollary 2.2. *Let* $\alpha, \beta, \delta \in \mathbb{R}\backslash\{0\}$ *and* $\sigma, \tau \in \mathbb{R}_+$ *such that* $\sigma = 0 \neq \tau$ *hold. Then the system* (E) *admits an*

(i) *oscillatory solution in the exponential impulsive form* (4) *if and only if the algebraic equation*

$$(-\mu + \beta\gamma)\left(1 - \frac{\alpha}{\beta}\mu\right)^{n_2} + \delta\mu e^{\mu\sigma} = 0 \tag{11}$$

has at least one real root μ *with*

$$\mu > \frac{\beta}{\alpha} \quad \textit{for} \quad \alpha\beta > 0 \quad \textit{or} \quad \mu < \frac{\beta}{\alpha} \quad \textit{for} \quad \alpha\beta < 0;$$

(ii) *eventually positive solution in the form of* (4) *if and only if the algebraic equation* (11) *has at least one real root* μ *with*

$$\mu < \frac{\beta}{\alpha} \quad \textit{for} \quad \alpha\beta > 0 \quad \textit{or} \quad \mu > \frac{\beta}{\alpha} \quad \textit{for} \quad \alpha\beta < 0.$$

Corollary 2.3. *In Theorem 2.1, let* $\alpha = \beta \neq 0$*. Then the system* (E) *admits an*

(i) *oscillatory solution in the exponential impulsive form* (4) *if and only if the algebraic equation*

$$-\mu(1-\mu)^{n_1} + \delta\mu e^{\mu\tau}(1-\mu)^{n_1-n_2} + \alpha\gamma e^{\mu\sigma} = 0 \tag{12}$$

has at least one real root μ *with* $\mu > 1$*;*

(ii) *eventually positive solution if and only if the algebraic equation* (12) *has at least one real root* μ *with* $\mu < 1$.

Remark 2.1. *Following to Corollary 2.3, we may note that* $\mu = 1$ *if and only if* $A = 0$*, that is,* (E) *has the trivial solution.*

Theorem 2.2. *Let* $\tau > \sigma > 0$ *and* $\alpha = \beta = 0$*. Then*

(i) *for* $\delta \in (-\infty, 0)$ *and* n_2 *odd or* $\delta \in (0, \infty)$ *and* n_2 *even, the system* (E) *admits an oscillatory solution if and only if* $\mu^* \in (1, \infty)$ *is a root of the algebraic equation*

$$-\mu(1-\mu)^{n_2} + \delta\mu e^{\mu\tau} = 0; \tag{13}$$

(ii) *for* $\delta \in (0, \infty)$, (E) *admits an eventually positive solution if and only if* $\mu^* \in (-\infty, 1)$ *is a root of the algebraic equation* (13).

Proof. Proceeding as in the proof of Theorem 2.1 we have the impulsive system

$$-\mu A^{i(t_0,t)} + \delta\mu e^{\mu\tau} A^{i(t_0,t-\tau)} = 0,$$

$$(A-1)A^{i(t_0,\tau_k)} - \delta(A-1)e^{\mu\tau} A^{i(t_0,\tau_k-\tau)} = 0$$

which in turn implies that

$$-\mu A^k + \delta\mu e^{\mu\tau} A^{k-n_2} = 0,$$

$$(A-1)A^k - \delta(A-1)e^{\mu\tau} A^{k-n_2} = 0.$$

Consequently, the above system becomes

$$-\mu A^{n_2} + \delta\mu e^{\mu\tau} = 0,$$

$$(A-1)A^{n_2} - \delta(A-1)e^{\mu\tau} = 0$$

which is equivalent to say that

$$A = 1-\mu, \quad -\mu A^{n_2} + \delta\mu e^{\mu\tau} = 0$$

and hence (13) is the resulting characteristic equation for (E). Clearly, $\mu \neq 1$ for $\delta \neq 0$ in (13). Hence to solve (13), it happens that either $\mu \in (1,\infty)$ or $\mu \in (-\infty,1)$.

(i) Assume that $\mu \in (1,\infty)$. Then $1-\mu < 0$, that is, $A < 0$ and (13) holds true when $\delta \in (-\infty,0)$ with odd n_2 or $\delta \in (0,\infty)$ with even n_2. Therefore, (E) admits an oscillatory solution in the form (4) if and only if $\mu^* \in (1,\infty)$ is a root of (13).

(ii) If $\mu \in (-\infty,1)$, then $1-\mu > 0$, that is, $A > 0$ and (13) holds true when $\delta \in (0,\infty)$. Therefore, (E) admits an eventually positive solution in the form (4) if and only if $\mu^* \in (-\infty,1)$ is a root of (13).

This completes the proof of the theorem. □

Remark 2.2. *Indeed,* (13) *doesn't hold if* $\delta \in (-\infty,0)$ *and* $\mu \in (-\infty,1)$.

Theorem 2.3. *Let* $\alpha,\delta \in \mathbb{R}\backslash\{0\}$ *be such that* $\alpha\gamma+\delta > 0$. *Assume that* $\tau = \sigma \neq 0$, $\beta = 0$ *and* $n_2 = 1$. *Then*

(i) *for* $\delta \in (-1,\infty)-\{1\}$, *the system* (E) *admits an eventually positive solution if and only if* $4\alpha\gamma \leq (\delta-1)^2$*;*

(ii) *for* $\delta \in (-\infty,-1)$, (E) *admits an oscillatory solution if and only if* $4\alpha\gamma \leq (\delta-1)^2$.

Proof. Let $y(t)$ be a regular nontrivial solution of (E) in the form of (4). Then proceeding as in Theorem 2.1, we have the system of equations

$$-\mu A+\delta\mu e^{\mu\tau}=0,$$

$$(A-1)A+\big[\alpha\gamma-\delta(A-1)\big]e^{\mu\tau}=0. \tag{14}$$

In the above system of equations, we have $\mu=0$. Otherwise, $A=\delta e^{\mu\tau}$ and hence $\alpha\gamma=0$ which is absurd. Consequently,

$$2A=(1+\delta)\pm\big[(1+\delta)^2-4(\alpha\gamma+\delta)\big]^{\frac{1}{2}}.$$

Because, we are concerned with the non-zero real roots, then $(1+\delta)^2-4(\alpha\gamma+\delta)\geq 0$, i.e. $4\alpha\gamma\leq(\delta-1)^2$.

- (i) **Case 1.** If $4\alpha\gamma=(\delta-1)^2$, then $2A=\delta+1>0$ for $\delta\in(-1,\infty)-\{1\}$ (if $\delta=1$, then $A=1$ and from (14), it follows that $\alpha\gamma=0$ ($\because\ \mu=0$), which is impossible), that is, (E) admits a nonoscillatory solution. **Case 2.** For $4\alpha\gamma<(\delta-1)^2$, we have two roots $A_1=\frac{(1+\delta)+\left[(1+\delta)^2-4(\alpha\gamma+\delta)\right]^{\frac{1}{2}}}{2}$ and $A_2=\frac{(1+\delta)-\left[(1+\delta)^2-4(\alpha\gamma+\delta)\right]^{\frac{1}{2}}}{2}$ of A. It is easy to verify that A_1 is positive when $\delta\in(-1,\infty)-\{1\}$. On the otherhand $\delta+\alpha\gamma>0$ implies that $1+\delta>[(1+\delta)^2-4(\alpha\gamma+\delta)]^{\frac{1}{2}}$ and hence $A_2>0$ when $\delta\in(-1,\infty)-\{1\}$.
- (ii) Similar observation can be made when $\delta\in(-\infty,-1)$ for $4\alpha\gamma\leq(\delta-1)^2$.

Hence, the theorem is proved. □

Corollary 2.4. *Let $\alpha,\delta\in\mathbb{R}\setminus\{0\}$, $\tau=\sigma\neq 0$, $\beta=0$, $4\alpha\gamma=(\delta-1)^2$ and $n_2=1$. Then*

- (i) *for $\delta\in(-1,\infty)$, $\delta\neq 1$, the system* (E) *admits an eventually positive solution if and only if the algebraic equation $A^2-A(1+\delta)+\alpha\gamma+\delta=0$ has a real root $A^*\in(0,\infty)-\{\frac{1}{2},1\}$;*
- (ii) *for $\delta\in(-\infty,-1)$,* (E) *admits an oscillatory solution if and only if the algebraic equation $A^2-A(1+\delta)+\alpha\gamma+\delta=0$ has a real root $A^*\in(-\infty,0)$.*

Corollary 2.5. *Let $\delta,\alpha\in(0,\infty)$, $\sigma=0=\beta$ and $n_2=1$. Then*

- (i) *the system* (E) *admit eventually positive solutions if and only if $\alpha\gamma\leq 1+\delta-\sqrt{4\delta}$;*
- (ii) (E) *admit oscillatory solutions if and only if $\alpha\gamma\geq 1+\delta+\sqrt{4\delta}$.*

Remark 2.3. *If we denote*

$$F(\mu) = -\mu\left(1-\frac{\alpha}{\beta}\mu\right)^{n_1} + \delta\mu e^{\mu\tau}\left(1-\frac{\alpha}{\beta}\mu\right)^{n_1-n_2} + \beta\gamma e^{\mu\sigma},$$

then it is easy to verify that $F(0) = \beta > 0$,

$$F\left(\frac{\beta}{\alpha}\right) \to +\infty \ for\ \delta > 0,\ \alpha > 0,\ \beta > 0$$

and

$$F\left(-\frac{\beta}{\alpha}\right) = \frac{\beta}{\alpha}2^{n_1}\left[1-\delta 2^{-n_2}e^{-\frac{\beta}{\alpha}\tau}\right] + \beta\gamma e^{-\frac{\beta}{\alpha}\sigma} > 0$$

for $\delta \in (0,1)$, $\alpha > 0$ *and* $\beta > 0$. *Keeping in view of Theorem 2.1, hence we have proved the following result:*

Theorem 2.4. *Let* $\alpha, \beta > 0$, $\delta \in (0,1)$ *and* $\tau > \sigma > 0$. *Then every solution of* (E) *which is of the form* (4) *oscillates if and only if* (5) *has no real roots* $\mu^* \in \left[-\frac{\beta}{\alpha}, \frac{\beta}{\alpha}\right]$.

3. Examples

Example 3.1. *Consider the impulsive system*

$$\begin{cases} \big(y(t)-\delta y(t-2)\big)' + \beta\left(e^{\gamma y(t-1)}-1\right) = 0, & t \neq \tau_k, t > 2, k \in \mathbb{N} \\ \Delta\big(y(\tau_k)-\delta y(\tau_k-2)\big) + \alpha\left(e^{\gamma y(\tau_k-1)}-1\right) = 0, & k \in \mathbb{N}. \end{cases} \tag{15}$$

where $\delta = 0.03696674041$, $\beta = 4$, $\alpha = 2$, $\gamma = 2$ *and* $\tau_k = k$, $k \in \mathbb{N}$. *We choose* $n_1 = 1$ *and* $n_2 = 2$. *Upon using* (3), *from the characteristic equation of* (15), *it follows that* $A = -1$, $\mu = 4 > \frac{\beta}{\alpha}$ *and*

$$y(t) = e^{-4t}(-1)^{i(2,t)}$$

is an oscillatory solution of (15). *Hence, by Theorem 2.1*(i), (15) *admits an oscillatory solution.*

Example 3.2. *Consider the impulsive system*

$$\begin{cases} \big(y(t)-\delta y(t-3)\big)' + \beta\left(e^{\gamma y(t-2)}-1\right) = 0, & t \neq \tau_k, t > 3, k \in \mathbb{N} \\ \Delta\big(y(\tau_k)-\delta y(\tau_k-3)\big) + \alpha\left(e^{\gamma y(\tau_k-2)}-1\right) = 0, & k \in \mathbb{N}, \end{cases} \tag{16}$$

where $\delta = -1.09741494$, $\beta = 2$, $\alpha = 1$, $\gamma = 3$ *and* $\tau_k = 2k$, $k \in \mathbb{N}$. *We set* $n_1 = 2$ *and* $n_2 = 3$. *Upon using* (3) *from the characteristic equation of* (16)*, it follows that* $A = 0.5$, $\mu = 1 < \frac{\beta}{\alpha}$ *and*

$$y(t) = e^{-t}\big(0.5\big)^{i(3,t)}$$

is an eventually positive solution of (16)*. Hence, by Theorem 2.1*(ii), (16) *admits an eventually positive solution.*

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Conflict of Interests

The authors declare that there is no conflict of interests.

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